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TOWARDS A CONSTRUCTIVIST PERSPECTIVE: AN INTERVENTION STUDY OF MATHEMATICS TEACHER DEVELOPMENT

ABSTRACT. A constructivist perspective provided the basis for a four stage intervention with teachers. The intervention which combined intensive summer courses with ongoing support in the classroom was designed to stimulate teachers' development of a constructivist view of learning to serve as a basis for their instructional decision-making. Impact of the intervention was studied through analysis of teachers' writings and the use of an interview-based instrument developed by the researchers. The results indicated that this intervention had an important effect on teachers' beliefs about learning which in turn affected the decisions that they made in the classroom.

Positive reform of mathematics instruction will necessarily require significant new initiatives in teacher development. Cooney (1988) asserts that,

> One of the central issues facing our profession is how the NCTM [1989] *Curriculum and Evaluation Standards for School Mathematics* can influence mathematics teacher education programs .... the task that lies ahead is both significant and awesome (p. 352).

The Educational Leaders in Mathematics (ELM) Project[1], conducted by the SummerMath for Teachers Program at Mount Holyoke College, was designed to begin to address this issue. The project addressed the following three broad goals:

(1) to create an innovative inservice program for precollege teachers of mathematics based on recent research and theoretical work,
(2) to study the effects of this program on teachers' thinking and practice, and
(3) to study the effects of this program on the students of the participating teachers.

This paper addresses the first two of these goals. (A report of the effects on precollege students will appear in a separate article.) First we discuss the theoretical principles that form the basis of the ELM Project and describe the inservice education intervention. Next we analyze the writings of and interviews with participating teachers in order to examine the program's effects on their thinking and practices. Finally, we suggest implications of this work for widespread change in mathematics instruction.

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THEORETICAL FRAMEWORK

Theoretical Considerations with Respect to Learning

A constructivist view of learning was one of the major theoretical components of the ELM Project. Constructivist theory is based on early empirical and theoretical work by Piaget (Pulaski, 1980) and more recent work by researchers and theorists within mathematics education (Cobb et al., 1991; Confrey, 1985; Sinclair, 1987; Steffe, Cobb, and von Glasersfeld, 1988; von Glasersfeld, 1983). The core idea is that learners actively construct their own understandings rather than passively absorb or copy the understandings of others. The construction of new understandings is stimulated by a problem situation that is a situation which disturbs the individual's current organization of knowledge. This disturbance, or "disequilibrium" occurs when his or her current cognitive structures do not adequately solve, explain, predict, or allow for navigation within the situation encountered. Disequilibrium leads to mental activity and a modification of previously held ideas to account for the new experience.

Concurrent with the construction of knowledge by the individual, is the social construction of knowledge by the group or groups to which the individual belongs. Groups of any size and function work to develop "taken-to-be-shared meanings[2]" (Cobb et al., 1991), in order to communicate and progress towards the goals of the group. These social and individual processes are not only concurrent but interactive. As group meanings are negotiated, group members engage in making sense of and resolving disequilibrium caused by differences between their ideas and those of others. Thus, cognitive reorganization is promoted by these attempts at communication and cooperation.

Theoretical Considerations with Respect to Mathematics Instruction

While constructivism does not prescribe explicit instructional strategies, this sense of learning and understanding does imply a new set of goals for the classroom. Teaching mathematics is to be understood as providing students with the opportunity and the stimulation to construct powerful mathematical ideas for themselves and to come to know their own power as mathematics thinkers and learners.

ELM Project staff did not take a prescriptive stance to what teachers should do in their classrooms. Yet they did operate from an instructional model in their own teaching of inservice education classes and in demonstration lessons with school classes.

According to this model, students should be consistently actively engaged in exploring mathematical problem situations, in looking for patterns, in generating ideas and hypotheses, in verifying these hypotheses, generalizing, and justifying the ideas that they come up with. To do so, students manipulate a variety of representations, including physical models, diagrams, computer representations, and mathematical symbols, and develop connections between representations. Such activity goes on at times individually, often in pairs or small groups, and in whole class discussions. Students have the experience of *creating* mathematics, not just *imitating* the mathematics of others. Students regularly communicate their mathematical ideas to the teacher and to their peers orally, in writing, and through the systems of representation that they are using.

Understandings are solidified, not through repetitive drill, but through further refinement of the ideas themselves. Non-routine problems, which encourage the use of new ideas in a variety of contexts, push the understandings to more complex levels.

In contrast to the traditional classroom, the teacher is no longer the teller, but the creator of problem solving situations (Simon, 1986). The teacher must select tasks which are grounded in real world experiences or mathematical objects well known to the students, enabling them to build on already present cognitive structures. The teacher must balance the interests and questions generated by the students with the goals of her curriculum.

In classroom discussions, the teacher is the facilitator, asking probing questions, requesting paraphrases of ideas, managing and focusing the discussion as needed, but avoiding comment on the correctness or the value of particular ideas. The teacher, rather than looking for a simple, short, straightforward path to student success, encourages the exploration of potential pitfalls and misconceptions with the aim of developing broader, more resilient concepts.

No longer is the teacher the sole judge of mathematical validity. The classroom becomes a mathematical community that decides on the truth of mathematical ideas by critically examining the justifications provided by students. The teacher has the added responsibility of helping to develop bridges, where necessary, between the mathematics of the classroom community and the mathematics of larger communities in which the classroom is embedded.

Theoretical Considerations with Respect to Mathematics Teacher Education

Three key ideas guided the development of the ELM inservice intervention.

1. **Teachers must be encouraged to examine the nature of mathematics and the process of learning mathematics as a basis for deciding how to teach mathematics.** Many inservice programs simply introduce instructional strategies or demonstrate materials teachers might use. Such programs seem to have a very localized impact, if any. The ELM Project was based on the assumption that fundamental change in teaching requires growth in teachers' conceptions of mathematics and learning. With such increased understanding, teachers will be better able to make instructional decisions that promote conceptual development and will be able to "debug" decisions that do not yield desired results.

2. **Teachers' learning can be viewed in much the same way as mathematics students' learning.** The preceding discussion of constructivism applies to adult students of mathematics education as much as to young students of mathematics. New insights into pedagogical theory and practice cannot be directly transferred from one person to another by lecture. Inservice programs must provide teachers with activities that help them construct more powerful ideas and develop rich webs of interconnected understandings with respect to mathematics and mathematics learning and teaching.

To this end, it is particularly important that the teachers become mathematics learners in a setting which fosters individual and social construction of mathematical ideas. Teachers must be challenged at *their* levels of mathematical understanding and problem solving ability, allowing them to not only increase their mathematical knowledge, but also to experience a depth of mathematical learning that, for most of them, is unprecedented. Concurrently, teachers are asked to reflect on these learning experiences. The process of making sense of new and discrepant experiences precipitates the modification of previously held ideas about learning and teaching.

3. **Provide follow-up supervision and support.** Integrating new learnings from an inservice course into classroom practice can be extremely difficult. Much as in mathematics learning, the problems that arise when applying new ideas in different contexts (in this case one's own classroom) require considerable refinement of concepts and additional learning. Pressure to cover an existing curriculum, lack of institutional support, resistance from students, and demands on teachers' time all may reduce the effectiveness of application efforts, and implementation may be put off indefinitely. Initial efforts that do not meet with instant success are often abandoned. A

greater impact may be realized when programs integrate clinical supervision of classroom practice with courses on learning and teaching (Joyce and Showers, 1988).

THE INTERVENTION

The ELM Project was a four-stage program designed to combine summer institutes with intensive classroom supervision and to provide a core group of participants with training to conduct inservice workshops in their schools.

Stage One – Initial Level Institute

During their first summer in the program, each participant attended a two-week residential institute for either elementary (K-6) or secondary (7–12) teachers.

The institute schedule included three classes each weekday: Conceptual Understanding and Problem Solving, Logo and Computers, and a physical education class. Each participant kept a journal as a means to reflect upon the experiences of the day.

Conceptual Understanding and Problem Solving provided the core of the institute experience. During the first days, ELM staff taught mathematics to participating teachers (e.g. fractions and area/volume for the elementary teachers, weighted averages (Simpson's paradox) and direct and inverse variation for secondary). This gave the teachers an opportunity to learn mathematics in a classroom where construction of meaning was valued, encouraged, and directly planned for. The mathematical concepts upon which the lessons were based related to the content covered in either the elementary or the secondary curriculum, but were chosen to challenge the teachers. This was not a simulation, but an actual opportunity for mathematics learning.

Mathematics lessons were followed by group discussion, facilitated by a staff member, which focused on the experience of learning, on the structure of the lesson, and on the roles of students and teacher.

Toward the end of the first week, participants were asked to synthesize their reflections in a paper and to describe to the group the understandings of learning and teaching that they developed from the week's work.

The next phase of the program focused on students' learning. Through facilitated discussions of videotaped interviews and live one-on-one interviews with students, teachers studied the understandings and the misconceptions of individual learners.

The final phase of the institute involved a series of tasks designed to develop teachers' abilities to plan lesson sequences which encourage students' powerful constructions. Teachers designed problems to assess understanding; identified key concepts, subconcepts, and prerequisites; designed initial tasks to build on the mathematical or world knowledge of the student; and designed activities which link concrete activities to abstract concepts and symbolism. Lesson sequences were developed in small groups of teachers from the same grade level. They were shared with a larger group of colleagues and staff for the purpose of critique and feedback.

At the end of the second week, teachers were again asked to synthesize their new learnings in a paper.

Logo and Computers. The Logo and Computers class served two purposes: (1) to provide an additional, ongoing opportunity for teachers to learn new concepts in an environment that encouraged the construction of ideas and reflection on the learning processes, and (2) to give teachers hands-on experience with computer environments which can stimulate construction of mathematical concepts and development of problem solving abilities.

Through a series of problem sets, Logo, the educational computer language, was taught for two weeks to elementary teachers and one week to secondary teachers. Secondary teachers spent the second week of the computer class working with mathematical software, particularly the Geometric Supposers and graphing packages. Lessons encouraged the generation and verification of hypotheses.

Physical education classes. Each teacher who was not prevented by physical disability participated in one week of jazz dance instruction and one week of tennis instruction. The physical education class served two main purposes. First, it provided a break from the mental demands of the academic classes and the intense emotional demands on participants who were taking a hard look at the teaching that they had done for an average of sixteen years. Second, it gave the teachers yet another opportunity to reflect on their learning. Many who were comfortable in the domain of mathematics were novices in an area in which they did not feel as comfortable. Teachers were often able to relate their experience of dance or tennis anxiety to the math anxiety experienced by students in their classrooms. It was not the learning of tennis or jazz dance per se that was the key to this component of the program, but rather that the physical education instruction was embedded in an atmosphere of reflection on learning.

Stage Two: Classroom Follow-up

The Classroom Follow-up component of the ELM Project provided extensive support and supervision for participating teachers who had completed the two-week summer institute. One ELM staff member was assigned to each teacher. (Staff members' case loads varied from two to eleven teachers.) The staff person visited the teachers' classroom one class period a week and met for a half-hour with the teacher after the class session. These weekly visits took place from September through May of the year following the institute.

At the beginning of the year, teachers chose from what they found valuable in the summer institute as a starting place for implementation. Staff members provided feedback, demonstration teaching, opportunities for reflection, and suggestions with the teachers' own goals in mind.

Teachers also attended four ELM workshops (two day-long and two half-day) during the academic year. These workshops included collegial sharing about implementation efforts, hands-on lessons related to common concerns, analysis of student learning and misconceptions, and small group planning sessions.

School administrators were invited to join the teachers for one of the afternoon workshop sessions to become more familiar with the program. An additional meeting was set up for the administrators to deal specifically with administrator involvement.

Stage Three: Advanced Institute

Following Stage Two, interested ELM teachers applied for the Advanced Level of the program, which consisted of an Advanced Institute (Stage Three) and the Inservice Leader Apprenticeship (Stage Four). Application to the Advanced Level signified teachers' commitments to continue professional development begun in the first year and to begin to share ELM learnings by conducting inservice workshops.

A major component of the Advanced Institute was once again mathematics lessons for participants (e.g. combinatorics) with reflection on the learning processes. Teachers also engaged in analyzing student learning through videotaped interviews. The greatest emphasis in this institute however was on the development of lesson sequences. Teachers worked in various groupings to plan instruction, receive feedback, and revise instruction. They also analyzed commercial curriculum materials and adapted them according to their ideas about learning and teaching.

In addition to this work, teachers prepared for their roles as inservice leaders. They practiced questioning skills, facilitated groups, and conducted sample lessons. An attempt was made to anticipate difficulties and to discuss feelings that teachers had as they contemplated this new role.

Stage Four: Inservice Leader Apprenticeship

Although Stage Four was designed as a vehicle for wider dissemination rather than participants' further development, it is described here briefly for two reasons. First, ELM participants reported that planning and leading inservice workshops for other teachers helped them to refine their own ideas about learning and teaching. Second, since interviews conducted at the end of Stage Four are discussed below in the analysis of teacher development, knowledge of the participants' intervening experience is relevant to interpreting the data.

During Stage Four, teachers, in teams of two to four, worked with an ELM staff member to plan and lead four afternoon workshops for colleagues. Typically, the amount of responsibility taken by the teachers increased over the course of the four workshops. Planning sessions also served as a support system for the Advanced ELM teachers who received no classroom follow-up during this second academic year.

ELM Staff

The authors, Martin Simon, director, and Deborah Schifter, assistant director (full-time on the project) and three part-time staff members made up the instructional/research team. Staff members had advanced degrees (doctorate or masters) in mathematics, education, and/or psychology. They were selected for their constructivist perspectives on mathematics education and mathematics teacher education and their experience teaching elementary and/or secondary mathematics. The summer institutes and the preparation weeks that preceded the institutes were used for orienting staff members to specific aspects of the program.

ELM Participants

Fourteen ELM teachers began the program during the pilot year, 1985, and thirty teachers each of the next three years. (One teacher dropped out because she did not feel that her participation was worthwhile and one teacher dropped out because of illness in the family.) Approximately one-third of the ELM participants were secondary teachers and two-thirds elementary. They came from public and private schools within a forty-five minute radius of Mount Holyoke College and from one school district in Southern Connecticut. The geographic regions served were determined by availability of staff to conduct follow-up visits, and represented inner city, suburban, university, and rural communities. Teachers who applied in groups of two or more from the same school or school district were given priority in admission to the program. No personal or professional qualities of the individual teachers were considered for selection.

Advanced Institutes were held during the summers of 1986–88 with eight, seventeen, and fourteen ELM participants respectively. All participated in Stage Four. Each year, more teachers were interested in the Advanced Level of ELM than the number supported by the grant. Funds for additional teachers were contributed by participating school districts.

DATA COLLECTION AND RESULTS

In order to study the impact of the ELM program on teachers, two sources of data were analyzed: teachers' writings and interviews with teachers. Below are descriptions of the data collection methods and results of the study.

Teachers' Writings

Teachers' writings were examined as an indication of their learnings, understandings, and implementation experiences. For Stages One and Three, synthesis papers, written after each week of the institutes, were analyzed. For Stage Two, responses to open-ended questions on anonymous questionnaires were used to collect self-report data.

Writings were analyzed following the phenomenographic methods outlined by Marton (1988). Specifically, data relevant to the program's impact on teachers were identified. These data were categorized into themes by each of the researchers independently. Marton points out,

> . . . we are not merely sorting data; we are looking for the most distinctive characteristics that appear in those data; that is, we are looking for structurally significant differences that clarify how people define some specific portion of the world. (p. 146)

Finally, the researchers negotiated differences in their initial schemes to arrive at a consensual set of themes.

Stage One: Initial Level Institutes. For synthesis papers in the Initial Level Institutes, teachers were asked to consider their experiences each week and reflect upon how these experiences affected their views of learning and teaching. A tremendous diversity of content and method of expression were in evidence. This seems to support the notion that the papers conveyed teachers' personal constructions and were not merely a repetition of ideas passively received.

As several themes emerged in the analysis of the papers, data sorting was complicated by the fact that many of the excerpts illustrated more than one theme. Yet such interconnection is perhaps additional evidence of the significance of viewing the data in this way. Each theme is identified below, followed by representative excerpts.

1. Teachers tended to develop a more critical perspective on their own past practices.

> Having been a third grade teacher for the past seventeen years, it is somewhat disconcerting to come to the realization that I have not been providing my classes with the optimum learning experiences in math. My methods over the years have been fairly traditional. I presented new math concepts in various ways, such as using the overhead, drawing symbols on the board, etc. I provided much practice in computation. I programmed them with many rules or tricks for computation and for solving word problems, so that many of them even became quite proficient. But only a few very capable children could tell me why they were doing what they were doing. This is clearly not the way math should be taught.

> I have had to look at . . . how much of my desire to help people by showing them is probably not helping them but robbing them of their own experimentation and discovery.

2. Being a learner during the mathematics lessons stimulated important changes in many teachers' relationships to mathematics. Particularly elementary teachers who had come with a history of mathematics avoidance and mathematics anxiety addressed this change.

> I realized as I worked through the math problems that I felt more empowered than I think I ever have felt before in mathematics. That is not to say that I feel totally confident about math. But I realize that I have ways and means to approach solving a problem and because of that I have more willingness to do so. I'm finding mathematics somewhat less intimidating and more exciting and fascinating.

3. Many teachers reflected on the effect of the learning environment.

> I am willing to take risks because of the positive environment. This serves to show how crucial it is to maintain that special atmosphere in the classroom.

> [My] anxiety was quickly replaced with excitement because the focus has been on thinking about how to get where you want to go, rather than on evaluating whether you've already gotten there . . . .

> The teaching that's being modeled puts the teacher in the role of a fellow student who's also struggling to understand . . . . Granted the teacher already understands the math, but they do not understand what's inside our head when we approach the math.

4. Some teachers focused on teaching strategies that they were experiencing and learning about.

. . . manipulatives are very important as a thinking aid . . . . A diagram or the actual physical situation can offer so much more toward my understanding of a problem.

Group work, a method of teaching I rarely use, seems now to be more important than I had previously believed. So much can be gained from having to explain your reasoning to another person and also from listening to others' conflicting arguments. Having someone question my thinking and motives helps me better understand my own thoughts.

5. Most teachers arrived at the institute without having deeply considered the question, "How do people learn mathematics?" Through their institute experiences, they began to develop new personal theories about learning.

I'm afraid I had never given a lot of thought as to how children learned math, or even, actually, what was really meant by learning math . . . . Since I have had the opportunity to be a learner for the past week . . . it has become obvious to me that in my math classes many children were not actually learning, but were being programmed with information. Processes were being memorized but not understood.

I never realized what a passive learner I've become, and how much more I get out of a situation or experience when I am involved in discovering the solutions.

6. A few teachers were able to articulate not only new ideas about learning mathematics, but also about their own processes of constructing new perspectives on learning and teaching. One elementary teacher described her evolving insights into the institute's design and its relation to her learning.

You are doing much more than modeling questioning and teaching techniques . . . . You are creating a "disequilibrium" for us . . . . Our SM learning doesn't fit our past experiences and our present concepts of what teaching/learning is. Answers aren't given. We're not being lectured to. We're not passive. We are learning, (We are active. We are reflecting. We are thinking.) but the way we are learning contradicts our previous experience, our scheme. Yet it works – we know that. Therefore, we must "accommodate." We must change our concepts of teaching/learning so that our SM experience fits in. Then equilibrium will be found again.

7. Other teachers showed evidence of having one foot in each world. They had come to believe that there is value in students developing their own mathematical ideas, yet they were unable to think of their own learning in these terms. They wanted to be *told* how to do this kind of teaching, to learn how to ask the *right* question. This view may be characteristic of a teacher who is viewing student discovery as a teaching strategy that is not grounded in a view of learning.

Whether it be the problem solving portion, tennis, or Logo, I believe that the main thrust of this institute was discovery . . . However it has not been made clear to me, in a lot of instances, how I am to go about this.

8. In contrast, one teacher made explicit the difference between her previous use of particular teaching strategies and her newly developed view of mathematics learning.

Prior to my experiences here, I felt children could be successful learning math as a result of a hands-on approach .... I felt that since I was fostering active learning the children really could understand what they were doing .... My students appeared successful, at least by conventional standards .... I understand now that although my classroom instruction provided for cooperative learning and "appropriate" learning activities, there was little emphasis on thought. The best I was able to do was to have kids understand my understanding of what was going on. This realization really pulled the rug out from under me. It is no longer a question of whether or not I can express things at a concrete level, but rather that I allow kids to construct their own understandings of what they are doing.

9. Teachers began to anticipate changing their role in the classroom.

I wonder how we can say our job is to teach people to learn. Each of us has known how to learn since birth. Our job is to help the learning to take place – to structure the activities so that learning is not random.

... [I need to] remove myself from what feels like the active role of instructor to the more subtle role of facilitator. I ... need to focus ... on forming questions so that I'm still mentally engaged while the kids work through the math which I guess is how to keep myself from butting into the kids work. That role is far from passive, actually; though it often appears disengaged, it is anything but.

Stage Two: Academic Year Follow-Up. After a full year of applying in the classroom ideas which were developed during the institute, teachers were able to look back on actual changes in their work with students. Because Stage Two data was collected from feedback forms, as opposed to papers written during the institutes, the ideas expressed by teachers were not elaborated to the same extent. Their comments reflected several themes:

1. Many reported that they had integrated teaching strategies learned in ELM.

The biggest effect is my use of writing in learning math. I haven't ever [before ELM] had the students verbalize their thinking. It's amazing how much they learn by writing or speaking their thinking process.

Wait-time, paraphrasing, problem-solving used as everyday lesson components.

2. The most commonly reported impact on their teaching was that they were listening more to students, focusing on the students' ideas and understandings.

I listen much more carefully to students' explanations.

An appreciation for the way people naturally approach math problems vs a somewhat artificial methodology imposed by formal mathematics ... A listening to what the student is really saying when an error is committed. The effect has been profound.

3. Teachers became more committed to the development of understanding and thinking.

Am more concerned with thinking process rather than answer.

I feel that it has helped me see math as less of a preprogrammed objective subject that is mainly oriented toward skill development. I see it as fitting more into the pattern of experimentation that I use in science . . .

4. Teachers indicated a new view of learning in which the student was more active and responsible.

I've been more willing to take time to let concepts develop . . . letting kids work through things rather than explaining them.

Students, for the first time, are "discovering" mathematical concepts.

5. Teachers enjoyed teaching mathematics more and were more comfortable with mathematics.

It has made this year the most enjoyable year I have had in 19 years.

I like math much better and that attitude is felt by my students.

6. Teachers felt that this year of development was only a beginning and that the learning will continue.

One year with a consultant is just not enough time. I feel like I have just begun.

. . . I will have to continue to plan, keep notes, and work hard over the next several years to feel really comfortable and confident, but at least I feel I have a beginning.

Some teachers indicated aspects of their prior teaching which have changed. They no longer direct all classroom processes, explain ideas to students, provide closure at the end of each class, "cover" quantities of material, focus on correct answers, or assign large amounts of drill. Some of the elementary teachers reported that the program had affected the way they teach subject areas other than mathematics.

A detailed analysis of the follow-up program is found in Simon (1989).

Stage Three: Advanced Institute. In their papers for the Advanced Institute, teachers were asked to describe their current models of learning and teaching. Many of them also discussed the processes by which these ideas had evolved. In contrast to the Initial Level Institute in which teachers were confronted with many new ideas and experiences, the Advanced Institute allowed them to integrate those ideas with their classroom experiences during the intervening year and more finely articulate a coherent perspective.

Teachers continued to provide evidence of considerable variation in what they had learned in the program. We found that data from Stage Three could be characterized by its distribution over a particular continuum. At what we would call the less sophisticated end of the spectrum, teachers' writings focused on what works. To the extent that they considered why a particular approach to instruction worked, their reflections tended to be specific to that approach. Broader conceptions of mathematics and about how mathematics is learned were not in evidence. Some examples follow:

From combinatorics, I appreciated the value of carefully laid out sequential questions [problems]. Our group moved forward rapidly, without teachers leading us, by the way they were ordered. I also saw the value of backing up a little [starting at a basic level] as a warm-up for any group's understanding, both to establish and build group dynamics.

I've been keeping a list of questions that [staff members] ask in discussions after a solving session. I've got a list of about 10 so far – they're really neat because they are not only open-ended and inviting, but they give a lot of power to the students to determine the thrust of a discussion.

At the other end of the spectrum were teachers who articulated an integrated understanding of the nature of mathematics, of the learning of mathematics, and of methods of instruction. They characterized mathematics as the study of patterns and relationships, as a way of thinking, and a way to describe the world. For them, mathematical activity involves collaboration and examination by the mathematical community. It involves prediction, analysis, experimentation, observation, estimation, use of multiple representations, justification, generalization, and evaluation. (These ideas are synthesized from the writings of several teachers.)

Mathematics learning was characterized as a process of student construction, building on previous knowledge, moving from concrete and real-life experiences to abstractions, developing mental images, and resulting in a modification of beliefs.

The teacher's role, according to these teachers, is to organize appropriate experiences; pose problems that students see as real and important; determine the sequence of activities and amount of individual, group, and whole class work; probe thinking; facilitate discussions; assess understanding; provide materials; and create a safe environment for mathematical activity.

Even though the ELM staff attempted to avoid articulating explicit ideas about mathematics, learning, and teaching during the institutes, there was always the possibility that teachers had learned to repeat the "party line". However, the interconnectedness of ideas and the sometimes idiosyncratic nature of their writings were evidence that these teachers were articulating personally meaningful ideas. For example, one teacher expressed her ideas as follows:

A. Students construct ideas about their mathematical world/mathematical ideas about their world.

B. As a result of interacting with and actively thinking about their environment
C. Observing, predicting, experimenting, exploring
What happens when they operate on it, manipulate it, "mess about"?

A second teacher expressed her reflections on the nature of mathematics and the appropriate relationship of the student to the study of mathematics.

Learning takes place when someone connects new information with old and rearranges what they believe .... Students will only come to understand mathematics as a style of thinking, of analyzing the world if that is in fact what they do in math class. The questions of how much, how many, how big, how long are the direct objects of math experience, yet what they come to study in math class is how to. Teachers must start by asking the questions of how much, how long, etc. Students must create ways to find how to ....

A third teacher expressed the meaning he had made of his experiences by characterizing learning in terms of "mental models" and mathematics as a jungle to be explored.

All five senses are involved in the process of mathematics as the student manipulates various objects attempting to discover a pattern or method ... the learner is constantly developing theories and discovering patterns in math based on the real world. I believe that using objects such as blocks, cubes, rods, etc. allows the learner to develop mental models of the math discoveries that he makes. Mental models ... are the property of the learner and therefore have greater meaning to the learner than ... rules of mathematics that the learner may be forced to memorize. Because these mental models are based on very concrete representations of the real world they provide a strong foundation on which to build more abstract models of mathematics.

... mathematics is like a huge jungle and the teacher is a person who has already explored many, but not all parts of that jungle .... The teacher is there to aid the learner as he attempts to explore and map his way through the uncharted jungles of mathematics. There are many discoveries for the student to make and to add to his internal map of the world of math ...

Interviews with Teachers

Methodology. ELM researchers evaluated participating teachers' implementation of instructional strategies learned in ELM *and* their use of a constructivist view of learning as a basis for their instructional decisions. The former was designated "strategies" and the latter "epistemology." While the implementation of strategies modeled in ELM was viewed as a significant step, the development and use of a constructivist view of learning was the principal objective of ELM.

To assess implementation of strategies, ELM adapted the Levels of Use (LoU) measure, developed by Hall et al. (1975), which consists of a structured interview and a classification scheme for rating teachers' responses (see Table I).

TABLE I
LoU scale

Level 0:	NONUSE does not use the strategy.
Level III:[3]	MECHANICAL USE uses the strategy; struggles with problems of classroom management with respect to the strategy.
Level IVA:	ROUTINE has incorporated the strategy and worked out any mechanical problems.
Level IVB:	REFINEMENT fine tunes strategy to meet the specific needs of students.
Level V:	INTEGRATION assists colleagues with implementation of the strategy or collaborates with them in implementing the strategy.

LoU ratings were based on the following strategies which were modeled during ELM instruction:

1. Using non-routine problems
2. Exploring alternative solutions
3. Asking non-leading questions
4. Using manipulatives, diagrams, and alternative representations
5. Having students work in groups and pairs
6. Pursuing thought processes on both "right" and "wrong" answers
7. Working with Logo
8. Employing wait time
9. Encouraging student paraphrase of ideas expressed in class

During the LoU interviews this list of strategies served as a guide for the interviewers, but was not seen by the teachers. An audio tape of the interview was given a LoU rating based on the strategy of highest[4] level of use.

To assess the use of a constructivist epistemology, ELM developed a new instrument, the Assessment of Constructivism in Mathematics Instruction (ACMI) which has a parallel format to the LoU. ACMI data were obtained during the same interview session and were rated according to the classification scheme shown in Table II.

These levels represent a rudimentary taxonomy of observations of and discussions with project teachers.

The ACMI required an explicit, working definition of constructivism which would allow raters to determine whether teachers' decision-making was based on a constructivist view. The following two-part definition

TABLE II
ACMI scale

Level 0:	does not have/use a constructivist epistemology.
Level III:[5]	attempts to modify instruction based on a general view that instruction should involve students in active construction; struggles with how to integrate this view with teaching style and curriculum.
Level IVA:	has modified teaching style to include regular activities to foster construction by students; focuses primarily on teaching behaviors.
Level IVB:	focuses on student learning rather than teaching behaviors to shape instruction from a constructivist perspective.
Level V:	assists or collaborates with colleagues to implement instruction based on a constructivist view.

was adopted:

1. Constructivism is a belief that conceptual understanding in mathematics must be constructed by the learner. Teachers' conceptualizations cannot be given directly to students.
2. Teachers strive to maximize opportunities for students to construct concepts. Teachers give fewer explanations and expect less memorization and imitation. This suggests not only a perspective on how concepts are learned, but also a valuing of conceptual understanding.

Interviews for LoU and ACMI assessment were conducted and scored by a staff member who had not worked with the teacher as a classroom consultant. A second scorer (staff member) listened to the audiotape of the interview and scored it independently to check for reliability. Inter-rater reliability was .99 (Spearman rho).

The ACMI instrument and its development are discussed more completely in Simon and Schifter (1990).

LoU and ACMI data. LoU and ACMI data were collected at the end of Stages Two and Four. (These data are from the two full years of the project, spring 1987 and 1988. Data from the pilot year is not included because of the smaller number of teachers and because the pilot year was a developmental year for the assessment instrument).

LoU results (see Table III below) indicate that 98% of the teachers who completed the classroom follow-up implemented strategies modeled in ELM (those listed above). Level IVB, which indicates not only stable use but internalization of the innovations, was achieved by 52% of the teachers with respect to these strategies.

According to the ACMI results (see Table III), 64% showed evidence of at least a rudimentary constructivist view of learning as the basis for their

TABLE III

Summary of LoU and ACMI data (end of Stage Two)

Level	LoU (strategies) #	%	ACMI (epistemology) #	%
0	1		20	
III	10	(98)	6	(64)
IVA	16	(80)	7	(54)
IVB	21	(52)	21	(41)
V	8	(14)	2	(4)
		$n = 56$		

Note. # refers to the number of teachers at that level. (%) refers to the percent of teachers at that level or higher.

Based on interviews in the Spring of 1987 and 1988.

teaching (Level III or above) while 41% were facilitating the constructions of their students by focusing directly on student learning (Level IVB or higher).

LoU and ACMI data at the end of Stage Four, in contrast to those at the end of Stage Two, reflect selection of teachers (usually self-selection) to continue in the program as well as further development in implementation. To examine the effect of the Advanced Level of ELM (Stages Three and Four), we compared the LoU and ACMI scores of the Stage Four teachers from 1988 with 1987 scores from the *same* teachers. The results appear in Table IV.

Participation in the Advanced Level seemed to support teachers' development as measured by ACMI (e.g., an increase from 40% to 87% at Level IVB or higher). Since this group self-selected, no appropriate controls were available. Therefore, it is not possible to compare the effects of the Advanced Level on these teachers with an equivalent group of ELM teachers who had an additional year of experience without participating in Stages Three and Four.

DISCUSSION OF RESULTS

Through their writings, teachers have indicated their belief that the ELM Project experiences had a substantial impact on their teaching. The changes

TABLE IV
Comparison of Stage Two and Stage Four LoU and ACMI data

Level	Stage Two[a]				Stage Four[b]			
	LoU (strategies)		ACMI (epistemology)		LoU (strategies)		ACMI (epistemology)	
	#	%	#	%	#	%	#	%
0	0		2	(100)	0		1	(100)
III	2	(100)	3	(87)	0		1	(93)
IVA	4	(87)	4	(67)	1	(100)	0	(87)
IVB	7	(60)	6	(40)	2	(93)	2	(87)
V	2	(13)	0	(0)	12	(80)	11	(73)
				$n = 15$				

[a] Data collected at the end of Stage Two, May, 1987.
[b] Data collected at the end of Stage Four, May, 1988.
Note. # refers to the number of teachers at that level. (%) refers to the percent of teachers at that level or higher. Only data from teachers who continued on through Stage Four are included for Stage Two.

that were reported began in the Initial Level Institute. Some teachers reported that they learned new teaching techniques while others reported that changes occurred in their views of how children learn, their understanding of the nature of mathematics, their feelings about doing mathematics, and their ideas about what constitutes good teaching. The diversity of expression seemed consistent with a population of teachers who had entered the program with varied conceptions of mathematics learning and teaching, and who consequently constructed different meanings from their ELM experiences.

The LoU and AMCI data provide further evidence of a dichotomy between those whose learning was restricted to the acquisition of new teaching strategies and those who made fundamental shifts in their views of mathematics learning and teaching. As expected, the changes in teaching strategies were more easily and more rapidly made than changes in their views of learning with its concomitant effect on instruction. However, while the latter is seldom the focus of inservice education, we believe that it signals the potential for significant change.

When implementation reaches a level of IVB or above, the likelihood that the innovation will result in lasting changes is greatly increased. After only one year in the program, 52% of the teachers had already reached this level with respect to strategies and 41% with respect to epistemology, an indication that modifications in instruction should be sustainable.

Teachers reported consistently that Stage Three, the Advanced Institute, was an important step in their development. At the second institute, many of the ideas with which they had been wrestling during the year came together and became more fully operational.

ACMI data after Stage Four indicated that teachers had continued their development during their second year in the program. The number of teachers at level IVB or above with respect to use of a constructivist epistemology had doubled. This continued growth is probably the result of both the changes set in motion during the first year and the impact of the Advanced Level of ELM. The data does not allow us to distinguish the effect of each.

CONCLUSIONS

The ELM Project provided evidence that through an inservice program in mathematics education, teachers can develop a vision of mathematics learning and teaching consistent with recent reform movements (as expressed in NCTM, 1989; NRC, 1989). ELM demonstrated the effectiveness of a model which combines summer institutes based on constructivism with intensive, ongoing follow-up support.

Almost all participants in the project adopted new strategies in their mathematics teaching. More importantly, a significant number of these teachers came to base their instructional decisions on a view of learning as construction. We suggest that the latter represents a fundamental change and, for teachers who reached ACMI Level IVB, is likely to be a lasting one.

The ELM results can be viewed as an existence proof that change of this nature can be brought about through inservice education. The question of whether such a change will be brought about on a large scale, however, is grounded in an appreciation of the difficulty of such change. It is important to note that not all ELM teachers made significant changes. It is difficult to draw conclusions from the negative data. Were these teachers in the early stages of a more gradual change? Could the program have been conducted in a way that would have had wider impact? The answer to both of these questions is probably yes with respect to some of the teachers. It is also likely that interventions of this nature will be ineffective with a non-negligible percentage of teachers.

The model provided by ELM, while successful in engendering change in many of the participants, is one which is labor, cost, and time intensive. Although the ELM results do not demonstrate that this level of commitment of time and resources is required, such a conclusion is not inconsistent with the project's findings. The educational opportunities and support available to ELM participants were far greater than what is usually provided for inservice teachers. If visions such as those presented in the *Standards* (NCTM, 1989) are to become reality, we must rethink not only the nature of teacher education efforts, but also the scale of such efforts.

The success of the ELM Project should be interpreted in light of the fact that teachers volunteered for the program. Although the majority of them initially understood little more than that ELM was a chance to learn some new ideas for teaching mathematics, this population was select. These teachers were interested in improving as teachers of mathematics, were willing to devote two weeks of their summer to such improvement, were willing to work with an ELM staff member on a weekly basis, and were willing to have that staff member come into their mathematics classes. ELM does not offer information about how results would have been different if participation had been mandated. What is more, there would have been potential for conflict between the empowerment that ELM teachers experienced in the program and the notion of mandatory participation.

Many factors undoubtedly contributed to the ELM Project's impact on teachers' thinking and practice. However, we single out one feature which we see as central: that teachers were encouraged to develop their own theories of learning as the basis for their curriculum and instructional decisions. How teachers think about mathematics learning is a key determinant of how they teach. In addition, teachers' development of personally meaningful views of learning alters their relationships to their profession and strengthens their self-concepts as teachers of mathematics. Whereas previously teachers may have looked to be told what to teach and how to teach it, the development of their own epistemological view enables them to base decisions on their own, informed, professional judgment. We have observed excitement in teachers as they realized that they could evaluate students' understandings; design appropriate instruction; and justify their instructional decisions to colleagues, administrators, and parents. Empowerment of this type may contribute to teachers' development as educational leaders.

NOTES

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[2] "Taken-to-be-shared" reflects the different perspectives of the participant and the observer. The participants use certain representations (e.g. vocabulary, symbols, behaviors) as if their meanings are shared by all members of the group. (We are doing so with all of the language in this article that we have not defined.) From the perspective of the constructivist observer, individuals' understandings are idiosyncratic and not directly observable, thus, "taken-to-be-shared."
[3] LoU Levels I and II refer to the stages of becoming familiar with an innovation and preparing to use it. Since LoU data were collected after a full year of classroom follow-up, these levels were not relevant to our situation. A teacher who had not implemented any of the strategies after a year was rated at Level 0.
[4] Faced with the problem of compiling data from teachers who had implemented different numbers of the strategies at different levels, we decided that the highest levels would be the most useful in assessing what extent these strategies were integrated into classroom practice.
[5] Since the ACMI scale was generated to be parallel to the LoU scale, levels were labeled accordingly (see note # 3).

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