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HIDDEN DIMENSIONS IN THE SO-CALLED REALITY OF A MATHEMATICS CLASSROOM[1]

"What is a rite?" asked the little prince.
"Those also are actions too often neglected", said the fox. "They are what make one day different from other days, one hour from other hours."

Antoine de Saint Exupéry (1943, p. 84).

Abstract. Teaching and learning of mathematics in classrooms is interpreted as a situation of human interaction in an institutionalized setting. Using theories and categories from different disciplines – e.g., ethnomethodology, linguistics, cross-cultural studies – a documented mathematics classroom episode is re-analyzed. The analysis of the example is used to identify four hidden dimensions in the classroom process and thus deficient areas of research: The constitution of meaning through human interaction, the impact of institutional settings, the development of personality, and the process of reducing classroom complexity. Consequences for teacher training towards more reflected classroom experience are drawn.

1. A NEARLY TRUE STORY

In old Russia two men meet in a train somewhere between Moscow and Warsaw. Since the beaver collars indicate they are both merchants one of them asks: "Where are you going?" "To Moscow," the other replies. "Hey," says the first one, "if you say you go to Moscow you must really want me to believe that you go to Warsaw. But this train is headed for Moscow and this makes certain that you travel to Moscow. So, why are you lying to me?"

These two men are talking not only about directions, but more, they are concerned with their mutual expectations and with their subjective interpretation of what they 'really' do. Though the replying man tells the 'truth' from our contextual view, the asking protagonist understands the utterance as a lie.

Let us try an explanation. (This is not to kill a joke by explaining, but rather to use the explanation to illustrate a more important problem.) Competing merchants will hardly disclose good sources and addresses to each other. The questioner therefore expects a non-destination as answer. Knowing these rules of the interaction and hearing the obviously true destination he

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must construe a lie. Thus we laugh about a man, who seems to be the captive of his expectations. He became accustomed to this game and for him it is reality, his reality. Seen from a more general point of view our 'truth' about the case is no better, nor more valid, than is his 'truth' – although we enjoy a larger majority supporting our interpretation.

This story about 'situations', 'rules', 'expectations and interpretations', and 'subjective realities' brings me directly to my theme: hidden dimensions in the so-called reality of a mathematics classroom. After a short overview of mathematics learning as a social activity and the role of related theory, the constitutive power of human interaction will be concretely demonstrated with a documented classroom situation. Following this, four deficient areas of research in mathematics education will be identified and discussed with a view to changing paradigms of research. Finally, I will come back to my main concern of pre-service and in-service teacher training and make some preliminary conclusions for it.

2. THE CONTRIBUTION OF SOCIAL SCIENCES

To view the learning and teaching of mathematics as a social process – a "jointly produced social settlement" as Lee S. Shulman puts it (see Note 3; also Bauersfeld, 1978) – seems to be a fairly recent issue. Although the ancient Greeks provided us with famous examples of mathematics instruction through dialogue (e.g., Plato's *Menon*) we still do not have much information about the social dimensions of generating mathematical knowledge and of developing individual mathematical power within the classroom. Particularly researchers in mathematics education have not spent much time on these dimensions of human interaction. Other disciplines long since have produced relevant research specific to their disciplines although not to the learning of mathematics. Speaking of hidden or neglected dimensions within mathematics education is only relatively true in the sense that researchers have not made use of relevant developments in the other disciplines.

Examples of such contributions come from symbolic interactionism (Blumer, 1969; Goffman, 1969), linguistics (Gumperz, 1972; Herrlitz, 1977), ethnomethodology (Cicourel, 1974; Mehan, 1975, 1979). The demarcation among these disciplines is difficult, because of their increasing integration through interdisciplinary procedures – procedures which might also benefit mathematics education. Topics such as the generation of meaning and the function of language in social situations, the actual shaping of behavior and cognitive performance through human interaction, the specifity of communication in institutionalized settings, etc., apparently force interdisciplinary approaches and have formed a new type of human science.

There is a final point to make in this initial overview. From the very beginning my concern is both pragmatic, as well as highly theoretical. It is pragmatic since my goal is to improve mathematics teaching and learning through both teacher's and student's actions. It is theoretical because the "improvement" and the "differentiated orientation" require the most sophisticated, reproducible theoretical framework available. Both aspects, the pragmatic and the theoretical, action and reflection, are deeply interwoven. Albert Einstein has put it sharply: "It is always the theory which decides what can be observed" (Mehra, 1973, p. 269). Though from a physicist one might expect to hear the complementary statement: It is always the observation (or the 'reality') which decides the theory. In the human sciences different actions and different concerns often produce different theories, and different theories in turn produce different realities.

This point is often expressed in education by saying that research findings, like a theory on certain classroom events, need special transformation into teaching practice; or, "that there is little direct connection between research and educational practice" (Fred Kerlinger, 1977, p. 5); or, " . . . what is good theory for one purpose is not a good theory for another" (Ernest Hilgard, see Note 2). All these statements are only different expressions in an educational setting of the general point made by Einstein.

3. THE CONSTITUTIVE POWER OF HUMAN INTERACTION

Two dissertations mark cornerstones for the discussion of human interaction in the mathematics classroom: George Bernard Shirk's 'An Examination of Conceptual Framework of Beginning Mathematics Teachers' (1972), and Stanley Erlwanger's 'Case Studies of Children's Conceptions of Mathematics' (1974). Both were directed by Jack A. Easley, University of Illinois, Urbana/ Champaign.

Erlwanger's case studies are related to programs from Individually Prescribed Instruction. His documentation of students' mathematical misconceptions and deficiencies demonstrate how mathematics learning can be damaged by restricted teacher–student communication – a restriction which leads to the nearly total absence of negotiations over meanings. It should be clear that the fading fascination of programmed instruction does not offer a satisfactory explanation for the non-appearance of further research with such case studies.

Shirk's work with beginning teachers gives a striking example of the influence of subjective theories about mathematics teaching, the student's role, and teacher's role. Moreover, his documents give an idea of the fragility of classroom discourse and of the impact of these social situations on mathematics learning. Therefore, I will take a brief example from Shirk's transcripts and use it for comments based on theories from other human sciences.

The episode presents an early part of a beginning teacher's lesson with eighth graders at Urbana Junior High School. The topic is about slides, flips, and turns from *Motion Geometry* written by Russel Zwoyer and Jo McKeeby Phillips (a product of Max Beberman's UICSM). In the preceding lesson Tom, the teacher, has defined parallel lines in terms of slides. The lesson under discussion opens with student's working on positive examples. The episode which I am going to reanalyze starts with line 39 of the transcipt. The teacher presents a counter-example, two intersecting lines (see Figure 1).

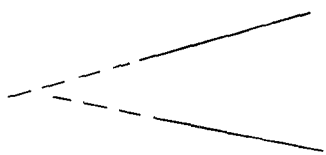

Fig. 1

"Tom, tape of 4/4/72" (from G. B. Shirk, 1972, pp. 173–174).

T – teacher, Tom K – student, Kevin R – student, Reggie

39 T: . . . . . . look at the next figure, right below
40 it. Ya, . . . now, are those two lines parallel?
41 K: Nope.
42 T: Why not?
43 K: They cross each other.
44 T: OK, but, uh, . . . according to what I've said about parallel
45 lines, what can't they do?
46 K: Can't cross them . . . (?)
47 T: What?
48 K: Can't . . . they won't, they won't . . . I'd rather not . . . (ID)
49 T: What Reggie?
50 R: Um.
51 K: . . . They won't come together . . .
52 T: OK, but what did I say, what did I say on the first figure?
53 What could you do to get from one line to the other?
54 K: Slide.
55 T: OK. There's a slide arrow that'll go, . . . that'll take one
56 line into the other.

K: . . . Arrow.
T: Right? Is there a slide arrow, . . . on the second figure? . . . Reggie? On the second figure, can you draw a slide arrow that'll go from one of these lines to the other?
R: (?) Not any more.
T: Like the . . . a slide arrow, . . . will that take the, . . . will that go from one to the other?
R: I don't know.
T: Well, you remember what a slide arrow did?
R: Hm?
T: It, . . . it moved a figure along that slide. Can you draw one that'll do that?
R: Oh.
T: OK, is that a slide arrow?
R: (?)
T: What is that? What, . . . what did you draw?
R: A circle.
T: Well, you remember.
R: (laughter)
T: . . . What that was called?
R: (?)
T: Does anybody remember . . .
R: A rotating . . .
T: OK, we called it a what? . . . A turn?
R: A turn arrow.
T: OK, so what you were starting to draw in was a turn arrow, right? But I'm just talking about slides. Can you draw a slide arrow? Just a straight line, a straight arrow, that'll go from one, that'll take one line to the other? (pause for students to work) . . .

First I shall follow Shirk's own interpretation and then add my comments later:

In this episode, Tom was working for a compound goal which he wanted the students to reach . . . that they recognize the lines illustrated were not parallel and realize there was a reason for it through the definition of slides. The students did accomplish the first part of this goal; but when they invoked a reason other than that which Tom sought, it was rejected by him as being inappropriate. The students weren't connecting the lessons together. Tom could only see this as a completely unexpected deficiency in the student's understanding of a lesson which he believed had been learned earlier, so he turned the lesson toward this deficiency in an effort to correct for it. (Shirk, 1972, p. 46.)

> The critical aspect centers around Tom's expectation that the students would know all of the consequences of the definition of 'parallel'. Not seeing this, Tom interpreted their problem as having to do with slides; and this bothered him for he believed that slides had been adequately covered and, therefore, the students should know them. He was also assuming that the students would appreciate everything that he said and therefore, the problem would have to lie elsewhere, i.e., in their more basic preparations which he thought had been covered earlier. (*ibid*, p. 46.)

For the re-analysis it is useful to note the major shifts in the student–teacher interpretation of the situation. The episode then splits into four parts.

Part I, lines 39–51: The teacher does not succeed in using the counter-example to infer that intersecting lines cannot be parallel (there is no slide arrow which would move the lines together). Unexpectedly he receives a much simpler answer, not invoking motion geometry concepts, "they cross each other" (line 43). Albeit correct, the teacher rejects the answer as inadequate "on the basis only that they should remember what he said in the previous example." (*ibid*, p. 47.)

The students become confused and uncertain as evidenced by K's stammering and brief withdrawal (line 48).

Part II, lines 52–64: "Tom is now directing the student's attention toward the drawing again in an effort to get them to see the connection between it and the slide." (*ibid*, p. 48.) Repeatedly he uses the key word "slide arrow".

The students try to guess the teacher's intentions. Their answers are short and cautious: "Slide" (line 54) and ". . . arrow" (line 57). Their uncertainty increases. Thus under the teacher's pressing Reggie modifies his answers from "Not any more" (line 61) to "I don't know" (line 64). "With their initial efforts rejected, and Tom emphasing slides, the students begin to look around for ways to slide the two lines together for there was nothing in the earlier portions of the lessons about 'no slide' or 'not parallel'." (*ibid*, p. 48.)

Part III, lines 65–86: Still the teacher has not given up his initial aim. His impulse "Well, you remember" (line 74) is an attempt "to get Reggie to put together the formal principles by pointing out to him that what he had drawn were turns rather than slides" (*ibid*, p. 49). Reggie's 'failure' (as seen from the teacher's eyes) justifies the causal ascription that the students have forgotten all about slides. Therefore the teacher begins to 'reteach' the concept towards the end of this section.

The students, however, "in an effort to come up with the answer they thought Tom was looking for, (namely, a slide arrow) invented slide lines between the two intersecting lines" (*ibid*, p. 49), so Reggie at line 69 (see Figure 2). Reggie's misinterpreting the teacher's question (lines 74–76) "What that was called?" is completely in line with this looking for slides. The drawings of the students in this part (see Figure 3) expose the extent to which the

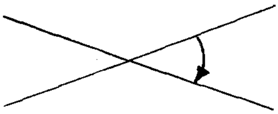

Fig. 2. "Reggie at line 69".

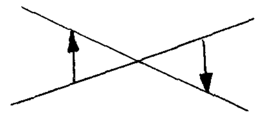

Fig. 3. "Students at line 85"

teacher's pressing has contributed to 'spoiling' the student's concept of 'slide' and of 'slide arrow'. Nevertheless the teacher's interpretation that the students have not learned their slide lesson is not diminished but rather reinforced.

Part IV, lines 87–155: (This concluding portion of the episode is not included in the above quote.) "The remainder of this first episode involves Tom's attempts to reteach the concept of slides to the students." (*ibid*, p. 51.) And he works on that rehearsal until he gets the conviction that "he has reconnected the students with slides and parallel lines." (*ibid*, p. 52.)

On a more general level Shirk explains the episode using the terms 'split personality' and 'guessing ahead' (the latter from John Holt, 1964).

A 'split personality' . . . occurs when the teacher is teaching one lesson and the students, in an effort in 'psych out' the teacher, are actually learning another . . . The 'split personality' in the lesson occurred as a result of the conceptual frameworks which governed Tom's actions, rather than being a part of those frameworks themselves. Tom, acting in accordance with his frameworks, interpreted the student's behavior in a certain way and then acted in a manner consistent with his conceptual frameworks. For their part, the students attempted to guess ahead and therefore acted differently. These actions resulted in one path taken by the students while Tom was trying to lead them along another. (Shirk, 1972, p. 43.)

In the above analysis Shirk uses the classical relation of cause and effect as he matches causes with personal attributes. That is, he traces the outcomes back to properties and actions of single individuals. As a result he is led to somewhat discouraging conclusions and recommendations. From his finding that "there was no change discernible within the conceptual frameworks" he concludes that the future teacher education programs "must be so designed so as to be assimilable to the preexisting conceptual frameworks" of the student teacher (Shirk, 1972, p. 165). I shall try an alternate answer to the teacher training problem later, but first let me give an interpretation of the episode from a different paradigm of social action.

A description of the situation as constituted through the interaction of the participants can challenge the usual causal model, cast doubt on predictive conclusions and possibly shed light on the use of language in the mathematics classroom.

The constitution of the social situation. Principally, and taken as a piece of an ongoing process, the episode cannot be reconstructed sufficiently – neither from personal variables, from characteristics of the single participant, or from the documented speech production. Hence, from additional interviews with the teacher, and from essays and 'comment cards' which the teacher had to write, Shirk has distilled the teacher's conceptualization of mathematics education, of his role as a teacher, and of the student's role. Shirk uses this set of statements only to explain the teacher's moves.

We do not have any comparable information about the students. However, the interactive nature of the process and the mutual relatedness of expectations and interpretations can be partially reconstructed from the transcript. For the purpose of this analysis consider Table I. The first two lines in Table I reconstruct the teacher's interpretation and the student's interpretation of the four parts. (A more detailed analysis can be developed through following the discussion step-by-step.) Comparing the mutual interpretations in columns gives a rough but sufficiently clear idea.

The teacher's immediate objectives change following his changing interpretations of the process. 'Guessing ahead' the student's interpretation of the teacher's intention changes as well. By no means are the actions of the two sides, teacher and students, reactions only to the preceding move of the other side. It is commonly believed that individuals react to the actions of another when in fact they react to their self-constructed interpretation. Yet, 'reaction' is misleading. Far from the simple model of stimulus–response the participant's actions in this social situation are generated through complicated, internal reflective activity. This subjective reflective activity takes into account not only the actual and perceivable moves of the others but also the more general interpretations of the situation, and one's own role in that situation. Furthermore, actual interpretations of related former experiences exercise an influence on the current ongoing interpretation. Each participant's actions contribute to the change of the other's, of their interpretation and their actions. And through this process they contribute to the change of the participant's own interpretation and action. Thus it becomes reasonable to speak of the 'constitution' of the social situation (Mehan, 1975). More precisely: *The social situation is constituted at every moment through the interaction of reflective subjects*. Ethnomethodologists therefore describe "reality as a reflexive activity" (Mehan and Wood, 1975, p. 8).

The episode under discussion is an example of the constitutive power of human interaction, i.e., the interaction constructs the subjects' various realities. Both teacher and students act according to their actual subjective realities. The students draw turn arrows for slides; the teacher diagnoses learning deficiencies; and teacher and students work clearly at cross purposes convinced they understand the situation, clearly.

TABLE I
Interpretations and changes

	Parts of the episode			
Aspects of analysis	I: lines 39–51	II: lines 52–64	III: lines 65–86	IV: lines 87–155
Teacher's interpretation	Use counter-example to strengthen concept. Disappointment about student's failure.	Prompting will help the students catch on. Increasing disappointment	They don't really know what a slide is. Confusion.	Give up and reteach the concept of 'slide'. Resignation.
Student's interpretation	Having not treated counter-examples they think about easier descriptions.	There must be something to say or to do with 'slide arrows'.	He insists on arrows which connect the lines.	He wants us to play the 'recitation game'.
The actual task, teacher's view	Connect the non-existence of a slide to the non-parallelism of the lines.	Draw the student's attention to the role of the slide arrow.	Find out that there is no slide arrow for intersecting lines.	Help the students reconstruct and recorrect the concept of 'slide'.
The actual task, student's view	Tell if intersecting lines are parallel.	Find another description. Later confusion, "Don't know".	Look for new arrows which might match the intersecting lines.	Tell what you have learned previously – "like we said it yesterday . . . ".
Meaning of 'slide arrow' for student	Move in positive examples one line onto the other	(Opening and changing.)	Any move, straight, ↗ or turn ↷ or up-and-down. ⇅	(Like in part I? but scarred and more divergent.)

> Every moment is mysterious, as the understood horizon of the moment is inexhaustible. Every interpretive act indexes this mystery in an unpredictable way. A person's every action is thus creative; it reflexively alters the world. The person begins with certain materials that set limits, and then acts and in acting alters those limits. (Mehan and Wood, 1975, p. 203.)

> Forms of life are always forms of life forming. Realities are always realities becoming. (Melvin Pollner, in Mehan, 1975, p. 32.)

The constitution of meaning. Not only subjective interpretation and assessment change during the process but also the aims, the actual tasks, and even the concepts. A comparison of the teacher's and the student's view during the episode of the task (see Table I, third and fourth line) emphasizes the semantic change of the problem situation. Clearly, each participant's view of the actual task to be done is different and varying during the course of the episode. The task must be understood as a function of the situation.

For the students the concept of 'slide arrow' also varies across the episode (see Table I, fifth line). In the beginning the previous experience with parallel lines (and slide arrows moving them together) is dominant. The intervention of the counter-example and the following discussion with the teacher produce doubts about where to locate the borderlines of the concept. What is and what is not to be included? The student's interpretation of the teacher's insistent questions increasingly spoils the concept and leads to an arbitrary guess as to the true meaning. Any type of arrow, curved or up-and-down, is used by the students (see Figures 2 and 3).

Without further information on the student's thinking the effect of the reteaching is difficult to evaluate. Surely the residual status of the concept 'slide arrow' will differ from its initial status but it may not be improved. Due to the high affective load during Part II, the concept now might be vulnerable to future misunderstanding in similar situations.

Thus, the logical principle of identity is not applicable. The word 'slide arrow' does not mean the same to every participant. Moreover, the meaning changes during the episode repeatedly and remarkably. But, if problems and concepts become functions of the situation instead of being constant and stable, it then becomes necessary to consider the social constitution of meaning, i.e., *the constitution of meaning through human interaction.*

Herbert Blumer makes the same point: "Symbolic interactionism sees meanings as social products, as creations that are formed in and through the defining activities of people as they interact." (1969, p. 5.) However, as a matter of principle, there is small chance of predicting the outcomes of such episodes at their beginning. Nor is there much chance of making predictions about a later stage from the basis of preceding one. Since we cannot ascribe the constitution of meaning to one single participant (e.g., the teacher), we are not in a position to use causal models as adequate descriptors of individual social interaction – particularly not of mathematics teaching and learning. "As every day meanings do not meet the canons of logic, they are transformed by literal description. These transformed meanings are amenable to causal models. Every day life is not." (Mehan, 1975, p. 66.)

At this point the analysis leads into a revolution of fundamental paradigm (following Thomas S. Kuhn, 1962). In the human sciences rules are used differently from in the natural sciences. For example, they are rules about the constituting of situations and meanings rather than rules about the situations and meanings themselves. They are rules about structuring the process rather than about structure of the process. (See Hugh Mehan's title 'Structuring the School Structure', 1978). In the human sciences the interpretation–assessment paradigm will replace the cause–effect paradigm borrowed from the natural sciences.

The role of language. The above episode also prompts a new look at the role of language in mathematics teaching. Years ago linguistic research would have used the utterances in Shirk's episode for a syntactical and a semantic analysis of the material. For example, the analysis might have pointed to a lack of adjectives. Or, it might have noted that the units of speech consist only of short or broken sentences, that paratactical structures dominate, or that many deictic words appear (words which demonstrate or point to something – 'here', 'this', 'there', etc.). Without any additional information the analyst would speak of a 'restricted code' and perhaps a resulting poorly developed meaning.

It is the fundamental idea of Chomsky's theory (1957) that we cannot reconstruct the constitutional process of communication from the surface, that is, from recorded speech. Neither 'discovery-models' (for the discovery of new grammars) nor 'decision-models' (for the decision about the adequacy of a grammar) will work. Chomsky maintains that these models only make use of the linguistic data. Since we have to analyze the rules of structuring communication we must analyze evaluative structures of individual interpretation and assessment and this analysis requires an 'evaluation-model' (for the evaluation among existing grammars). At this point there is a change of paradigm from objective (linguistic) data to interpretative structures as the object of analysis. Since Chomsky's work sociolinguists have increasingly studied the use of language in social interaction.

An interesting and helpful issue is the concept of *indexicality*. If there is no further information, then most people will not understand the discourse in lines 41–51 of the episode:

41 K: Nope.

42 T: Why not?
43 K: They cross each other.
44 T: OK, but, uh, . . . according to what I've said about
45 parallel lines, what can't they do?
46 K: Can't cross them . . . (?)
47 T: What?
48 K: Can't . . . they won't, they won't . . . I'd rather not . . . (ID)
49 T: What Reggie?
50 R: Um.
51 K: . . . They won't come together . . . (Shirk, 1972, p. 173).

Mathematics educators may even become doubtful about the topic and meaning of the discussion if they don't know that the document is related to mathematics instruction. Bar-Hillel has defined such "utterances that require contextual information to be understood" as "indexical expressions". (quoted by Mehan, 1975, p. 93). Thus an 'informed' outsider can even have difficulty understanding what is going on in a discussion. It is difficult to realize what the participants intend to say and to identify the meaning they create in the given situation. As a prerequisite of communication participants have to share common understandings which they take as an implicit basis of reference when speaking to each other. While speaking, each participant anticipates the understanding and the interests of the specific addressee. The speech gets organized through the expectation of what the addressed person already knows. Each speaker uses his interpretation of the given situation and of the addressee as an index from which he forms his utterances and from which he decides his 'choice of grammar'.

In the classroom the "teacher's instructions are indexical expressions which requires teachers and children to employ contextually bound interpretative practices to make sense of these instructions." (Cicourel, 1974, p. 129.) What a participant says not only transports the intended message, but over and above the message the utterance contains information about his understanding of the topic, his interpretation of the situation, his expectations of what the others might know, as well as his present emotional concerns.

Hence, indexicality is another label for the thesis that *the situation influences how language is used*. Not only content and meanings are negotiated and constituted in the social situation, but also the use of language and the performance of the speaker are co-determined. This is true not only for the syntactical structure of the utterances but also for the choice of words. (From this point of view Bernstein's distinction (1965) between 'elaborated' and 'restricted' codes might be more an issue of the indexical and reflexive constitution of the situation rather than of the competence of the speakers.)

Mathematicians, in particular, have invested much effort in producing universal statements, and most school mathematicians would claim any mathematical statement as non-indexical, i.e., as universal and objective. However, this conviction blocks insight into the *irreparable incompleteness* of utterances, and more general, *of any symbolic action.* Each utterance, just as each symbolic form, is necessarily incomplete, because it has to be filled in with meaning via contextual interpretation. Through its genesis and chain of definition a concept inevitably gets infiltrated with contextual information. And "every attempt at repair increases the number of symbols that need to be repaired." (Mehan and Wood, 1975, p. 93.) Therefore, understanding mathematics is not only a case of logic, or of divergent thinking, or of proper definitions. As far as understanding is realized in social interaction (or through communication, which is the very same thing) it inescapably becomes dependent upon the interpretative, indexical, and reflexive constitution of meaning.

4. FOUR DEFICIENT AREAS OF RESEARCH – MATHEMATICS EDUCATION'S HIDDEN DIMENSIONS

The process of teaching and learning mathematics can be viewed most aptly as a highly complex human interaction in an institutionalized setting – an interaction which forms a distinctive part of the participant's life. Four aspects in this issue deserve more detailed discussion since they represent weak areas of research.

1. Teaching and learning mathematics is realized through *human interaction*. It is a kind of mutual influencing, an interdependence of the actions of both teacher and student on many levels. It is not a unilateral sender–receiver relation. Inevitably the student's initial meeting with mathematics is mediated through parents, playmates, teachers. The student's reconstruction of mathematical meaning is a construction via social negotiation about what is meant and about which performance of meaning gets the teacher's (or the peer's) sanction. How can we expect to find adequate information about teaching and learning when we neglect the interactive constitution of individual meanings?

2. Teaching and learning mathematics is realized in *institutions* which the society has set up explicitly to produce shared meanings among their members. Institutions are represented and reproduced through their members and that is why they have characteristic impacts on human interactions inside of the institutional. They constitute norms and roles; they develop rituals in actions and in meanings; they tend to seclusion and self-sufficiency; and they even

produce their own content – in this case, school mathematics. How reliable are studies on the effects of mathematics education if they do not take into account the institutional impact on teacher and student? The question becomes crucial when one thinks about any application of knowledge learned at school to situations outside of school.

3. Mathematics education constitutes a *distinctive part of the student's* life as well as of the teacher's. Anyone who is active in mathematics, will learn something about himself, especially since the activity happens in interactive situations. On the other hand one can learn mathematics only by actively engaging his previous knowledge of related subjects and actions. Therefore mathematics education is deeply related to the man-made world of symbols and meanings, to common sense, and to everyday life. Mathematics education depends on our social and historic conditions. How can we dare to make any prediction about the mathematical abilities of a student and about his chances to develop these abilities if not by carefully relating such statements to his personality and background?

4. Scientists are not the only ones who have difficulties dealing with highly complex issues. The orientation for actions and decisions in the classroom continuously requires the *reduction of complexity*. On the other hand the understanding and the effective reduction of complexities demands their total unfolding and complete reconstruction. To date scientific analysis has been incapable of reducing the complexities of an actual mathematics classroom sufficiently for guiding a teacher's decisions. Yet without such guidance it is impossible to plan effective teacher training program.

5. FUNDAMENTAL PROBLEMS OF RESEARCH AND DEVELOPMENT

Within the last years my view of the structure of the classroom process has been changed as a result of collecting information from several disciplines, participating in mathematics lessons, and analyzing video-taped mathematics lessons. My subjective change includes the aims of my work, the subject to be studied, the methods of research, as well as the underlying paradigms of my thinking. This personal event is worth of mention since discussions with colleagues leads me to believe that my subjective difficulties only mirror much more fundamental difficulties within our profession. Philosophers of science agree that such difficulties within a profession are strong indicators of fundamental change of paradigms.

In the present transition stage three theses seem to be of importance:

1. Mathematics education is deeply in need of *theoretical orientation*. We

have too much research on too small a theoretical basis. Many opening addresses of APA, AERA, and SIG/RME, as well as journal articles from within the last years have complained about this problem, e.g., Lee Cronbach (1975), Lee S. Shulman (1979), Ernest Hilgard (1976). Perhaps there are too many short-termed research contracts. Perhaps there is too much prescription for 'acceptable' research programs. Or, perhaps, there is no support for methodological heretics and thus no encouragement for young researchers to try unusual approaches. For sure, there has yet to appear an adequate forum for theoretical discussion. Whatever the cause, I do not believe that there are not enough new ideas.

2. Research and practical developments follow *different paradigms*. The main stream of research still follows the paradigm of the natural sciences, stating an objective educational reality, using well defined and quantifiable concepts, and analyzing the relationship among them through statistical means. For a long time we have heard and accepted complaints about the complete lack of classroom applicability of research results. (See Kerlinger (1977) for a summary of a researcher's view.)

On the contrary, the majority of a teacher's classroom decisions are made via common sense and intuition rather than through rational analyses by scientific means. If she/he is a good teacher then her/his actions are based on a more differentiated perception of the classroom events that research recognizes. She/he is more open to contextual changes, 'knowing' a student, using "tacit knowledge" (M. Polanyi), and informal reflected experiences. Compared with these 'hard' social facts, current research appears as 'software'. It is necessary that research in mathematics education takes notice of this gap if a claim for practical relevance is to be established.

3. *Interdisciplinary approaches* are promising, if not necessary, for closing mathematics education's 'credibility gap'. Within the broad area of social sciences the discrepancies between rationalistic and hermeneutic descriptions, between naïve and scientific constructs have been realized and investigated much earlier. It is time to integrate these findings into our profession and to transform this knowledge into specific conditions of learning and teaching mathematics.

Unlike the natural sciences the human sciences must deal with an objective social reality on the one hand, yet on the other hand must deal with as many realities as there are reflective subjects. Paradoxically, modern physicists have a highly developed understanding of explanatory models losing their meaning in the light of more comprehensive theories. For example, the question of the 'divisibility' or the 'consistency' of a light quantum (photon) makes no sense in a general theory of elementary particles because the theory describes the relations among elements but is not concerned with the nature of the elements themselves. This is very near to an important issue of constitutive ethnomethodology. The structuring activities of the participants form ('constitute') the social situation among themselves, and the process and rules of these 'structurings' (as Mehan calls them) build a core theory of social action. The theory is related to structurings rather than to structures of the situation in usual social sciences. That is, there is greater generalizability within the process of structurings than within the structures themselves.

6. IMPLICATIONS FOR TEACHER TRAINING

Those who find the discussed theories and interpretations more or less acceptable might find themselves forced to think about consequences. "It seems likely, that innovation in schools will not be of a very radical kind unless the categories teachers use to organize what they know about pupils and to determine what counts as knowledge undergo a fundamental change." (Keddie, 1971, p. 156.) Shirk's findings about the stability of fundamental conceptualizations across teacher training apparently do not leave much chance for that change. How is it possible for a beginning teacher to overcome the sixteen or more years of his own experiences as a student? Particularly, since the contextual force of these experiences often dominates any later verbal information about education and leads the beginner into an almost unconscious reproduction of the school system's characteristics.

But, if we form our cognition and behavior about teaching through social situations, then we can also change this formed cognition and behavior through social situations. We learn to behave in social settings only through the reflected participation and action in social settings. Similarly, a teacher will learn to teach or to change his teaching pattern only through reflected teaching. Yet, this is not the ruling model of present pre-service teacher training.

Usually the teacher student learns about teaching in contexts very different from classroom situations. The organizing interest for picking up knowledge in lectures is more along with passing examinations than related to later classroom application. Through various lectures and seminars the student teacher collects incomplete eclectic knowledge and she/he is left with the unassisted task of integrating this knowledge into an applicable system for the living classroom.

If the constitutive power of social situations on behavior, meaning and language is as strong as assumed here, then the student teacher will have to spend much more time planning, accomplishing and reflecting upon real classroom teaching experience. From the very beginning the teacher to be must encounter an adequate complexity of social classroom exchanges. 'Adequate' means that the complexity of the teaching–learning situation might be reduced in quantity, e.g., via a reduced number of students to teach or a reduced amount of lesson time, but not reduced in quality (as simulation games or video-tape analyses, e.g., would cause). If we claim to educate human beings, then a teacher will have to receive a much more careful, holistic preparation.

This, of course, will require support and development on the side of research as well. And this research, at least a reasonable part of it, will have to follow the interpretative paradigm. "Science at its best is thus like a firm but gentle hand that holds a butterfly without crushing it." (Kenneth S. Bowers, 1973, p. 332.)

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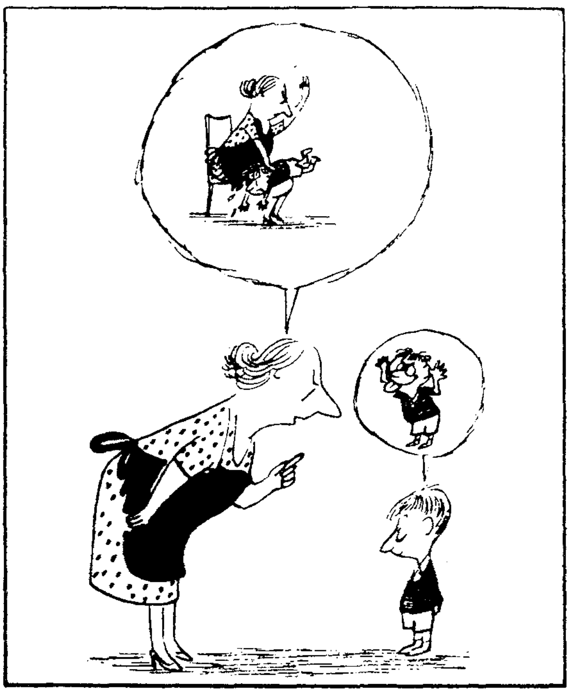

From Sempé, Wie sag ich's meinen Kindern?

NOTES

[1] This paper was developed for an invited address at the pre-session of the Special Interest Group for Research in Mathematics Education (SIG/RME) for the annual meeting of the National Council of Teachers in Mathematics (NCTM), Boston, April 18, 1979.

[2] Hilgard, E. R., Presentation at the Symposium on 75th Yearbook of NSSE, AERA annual meeting, April, 1976.
[3] Shulman, L. S., Presentation at the SIG/RME pre-session of the NCTM annual meeting, Boston, April 18, 1979.

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