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A SOCIOLOGICAL ANALYSIS OF SCHOOL MATHEMATICS TEXTS

ABSTRACT. In this paper I introduce some of the central features of a theoretical frame-work – a language of description – which has been developed for the sociological analysis of school texts, in particular, school mathematics texts. My approach is to introduce the language through the practical description of extracts from the texts which were analysed in its development. This also allows me to make some empiricial claims about the texts themselves and, insofar as they are representative of the wider practice, of school mathematics, at least in England and Wales. At the end of the paper, I also give some indication of how the language of description is articulated with broader sociological issues.

1. INTRODUCTION

The project from which this paper derives was concerned with the construction of a sociological language of description for the analysis of school texts.[1] A language of description may be glossed as a conceptual structure which enables the reading of empirical information in terms of a coherent theoretical position.[2] In this case, the empirical information is given by a set of school mathematics texts, *SMP 11-16*.[3] My intention, here, is to provide an introduction both to the language and to some of the results of its application to these texts. The language and the analysis were developed together in a process which might be described as a dialogue between theoretical concerns and the empirical texts. Here, I shall adopt an inductive approach in which I shall try to build some of the main features of the language through a practical analysis of extracts from the texts.[4] I hope that this will reveal the motivating value of the language in giving a particular bias and coherence to the description.

Although the empirical focus here and in the original development of the language was on a specific school mathematics text, work has been and is currently being done in which its descriptive power is extended to other mathematics texts (see, for example, Dowling, 1995b), to non-mathematical texts (Dowling, 1994b, 1995c) to the mathematics classroom (Coombe and Davis, 1995; Johnston, 1993), and to the recontextualising of pedagogic practices in initial teacher education and initial teaching appointments (Ensor, 1995). The work presented here should therefore be understood as illustrative rather than definitive.

The *SMP 11–16* scheme is very popular in the UK and elsewhere. The texts for the first two years of secondary school (years 7 and 8) are in the form of booklets. These are divided into four main levels. Students proceed through the booklets, moving from the lower to the higher levels, but not in any specified order or at any prescribed pace. In the third year, the scheme is differentiated by student 'ability' into three series of more conventional textbooks. These series account for the bulk of the mathematics curriculum in the last three years of compulsory schooling (years 9–11).[5] The main focus of my analysis was on the upper and lower series, the 'Y' and 'G' series, respectively. I shall begin with a discussion of the front covers of the first book in each series.

2. INTRODUCING THE Y AND G BOOKS

The two covers in Figures 1 and 2 bear obvious similarities and obvious differences. The images on the covers are very different, but neither are unambiguous indicators of the contents. The G1 cover is fairly straightforward, presenting three, ordinary-looking timepieces. A rather mundane[6] image. Y1 is different. It appears to be a contour map of a face. This is an unexpected, perhaps enigmatic image. The fact that G1 has a chapter called 'Time' seems to account for its cover illustration. This particular illustration of 'time' might suggest that the topic is to be treated in a rather mundane way. Section E of Chapter 11 in Y1 is called 'Contours and profiles'. This explains the contour map on the cover, but not why the map is in the form of a human face. A humanising of mathematics, or a mathematising of humanity? In any event, something is left unexplained about the illustration. We are invited to resolve the mystery by reading the book. The G book will tell us about watches and clocks, in case we didn't know already.

It is very easy to announce these obvious differences between the two textbook covers. It is rather more difficult to describe the differences in sociological terms. It might even be difficult to believe that they have any sociological significance: they are, after all, only cover illustrations. However, these two texts may be described as recruiting different 'ideal' readers in terms of their respective relationships to school and everyday practices. Once we know that it is a school textbook, we can see that the G1 cover displays items that will certainly be very familiar to the readers. Presenting the reader with objects that are within their own domain of everyday participation is recruiting a reader who is concerned with the mundane. The Y1 cover, on the other hand, is different, enigmatic and ambiguous. The reader's participation signified by the presence of the
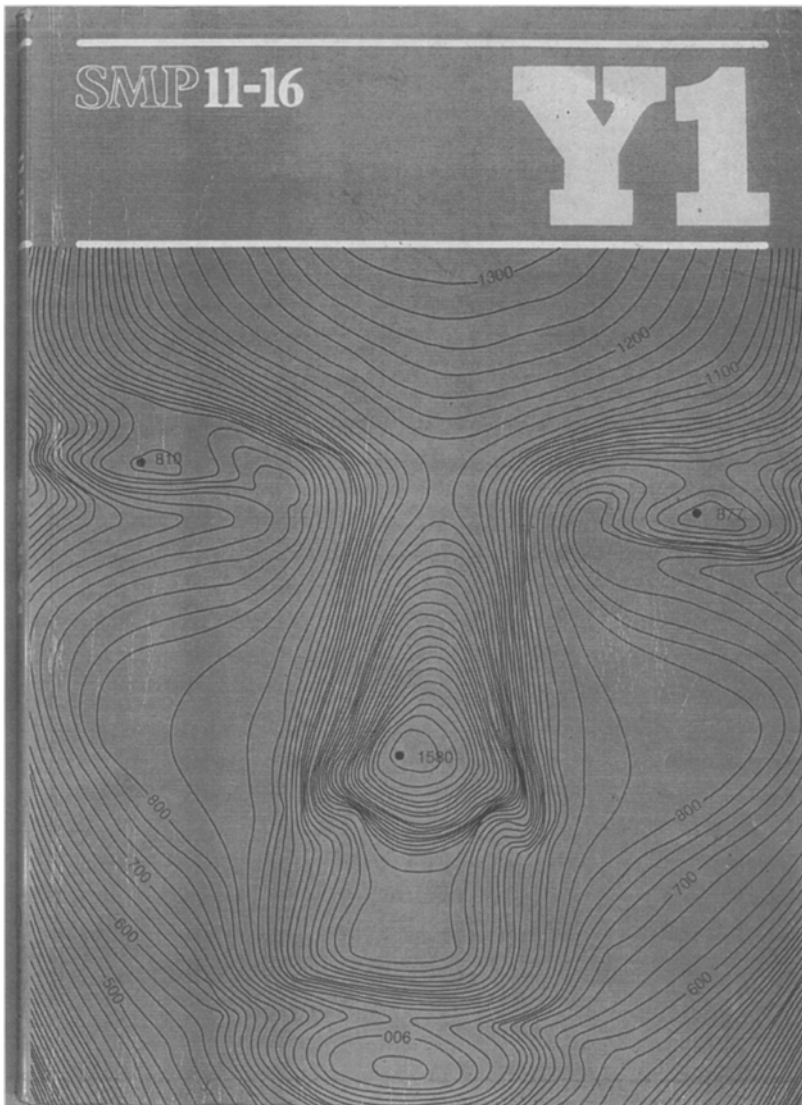
Figure 1. SMP: 1985, *SMP 11–16 Book Y1, CUP*, Cambridge, front cover.

face on the cover is within an arcane practice that is beyond their current experience. Read the other way around: a gaze is being projected from that arcane region onto the reader themself, describing them in its own terms.

The Y1 cover recruits a reader who is concerned with knowledge that they do not already possess, an intellectual reader that is to be apprenticed

Figure 2. SMP: 1985, *SMP 11–16 Book G1*, CUP, Cambridge, front cover.

to an esoteric practice. The G1 cover recruits a reader who might be concerned to optimise their participation in practical, manual matters with which they are already familiar. The arcane/mundane opposition recruits intellectually-oriented and manually-oriented readers to the Y and G series respectively.[7]

Thus, the SMP scheme constructs a hierarchy of readers which connotes the intellectual-manual opposition in the broader division of labour. I want to refer to these as superordinate and subaltern *voices*. These are *positioned* by the text. The scheme has also signalled a division in terms of mathematical practice. Mathematics may be concerned with the everyday, *public domain* which is known and in which potential readers already participate. On the other hand, it may be concerned with an as yet unknown, *esoteric domain*. Mathematical knowledge – which, in its textualised form, I want to refer to as *message* – is *distributed* to the voices such that esoteric domain message and public domain message are associated, respectively, with superordinate and subaltern voices. The text is comprised of *textual strategies* which position voices (positioning strategies) and which distribute message to the respective voices (distributing strategies). But the text is a cultural product which is understood to be producing and reproducing a social structure, which I shall refer to as an *activity*. The activity, in this case school mathematics,[8] regulates who can say or do what. That is, it comprises *subject positions*[9] and *practices*. In the text, subject positions and practices are realised by voices and message, respectively.

The description of the covers may now be summarised in the terms of the language of description. Positioning strategies constitute superordinate and subaltern voices by connoting the intellectual/manual division of labour within the field of material production as a corresponding hierarchy. Distributing strategies constitute superordinate and subaltern voices via the distribution of esoteric and public domain message. I shall now move on to consider two extracts from much later in the Y and G series.

3. RECRUITING TO CAREERS

Figure 3 shows a page from Book G6 – the sixth book in an eight book series. Figure 4 is from Y5, which is the final book in the Y series. Both texts denote the same practice and in this respect, are similar. However, on closer inspection, we find that, again, they are recruiting different reader voices. The use of a photograph, in Figure 3, localises the situation. These are real police officers measuring a real skid. In other words, this is a recording of a particular event. This kind of photograph employs a mode of signification which I shall refer to as a *visual code of presence*. In order to decode the image, the reader must place themselves in the position of the camera. In this case, the result is that the reader is standing at the shoulder of the nearest policeman, squatting down with him. The reader is thus placed in the position of an apprentice to the policeman.

F   Skids

When a car is in an accident,
there are often skid marks on the road.
The police measure the length
of the skid marks.
Then they can work out how fast
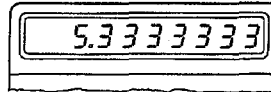the car was going when it started to skid.

This formula tells you roughly how long a skid will be.

$$\text{length of skid} = \frac{\text{speed} \times \text{speed}}{75}$$

The car's speed must be in **miles per hour** (m.p.h.).
The skid length will be in **metres**.

> **F1**   A car is moving at 30 m.p.h. and then skids to a stop.
> Roughly how long will the skid marks be?

> **F2**   (a) Work out how long a skid
> will be from 20 m.p.h.
> Check you get this display
> on your calculator.
>
> | 5.3333333 |
> |---|
>
> (b) **Roughly** how many metres long will a 20 m.p.h. skid be?

> **F3**   (a) Copy and complete this table.
> It shows the length of skid
> from different speeds.
>
> (b) The police measure
> some skid marks.
> They are 27 m long.
> About how fast do you
> think the car was going?

Speed (m.p.h.)	Skid (metres)
20	
30	
40	
50	
60	
70	
80	

> **F4**   Here is a picture of some skid marks.
> 1 cm on the picture is 1 m on the real road.
>
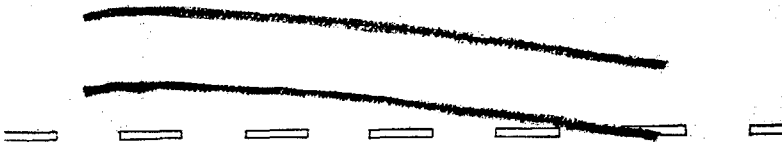> About how fast do you think the car was going?

Figure 3. SMP: 1987, *SMP 11–16 Book G6*, CUP, Cambridge, page 6.

The effect of apprenticeship to a non-intellectual occupation[10] is further
enhanced by the form of the task that the reader is required to perform. The
manual aspect of the task is very much a part of the G text. The table that is

to be completed is presented in manuscript; this might be a page from the policeman's notebook. Finally, a physical act of measurement is required, on the part of the student. This involves a simulation of the policeman's work, albeit recontextualised by the printed page.

The text in Figure 4 is quite different. To begin with, a drawing is used, rather than a photograph. This has the tendency to generalise the situation somewhat. A reduction in modality[11] has been achieved by the interruption of the visual code of presence. The drawing, in other words, does not make the same claim to reality as the photograph. These are faceless, police offers in general, rather than particular individuals – and there is now one of each gender. Furthermore, the viewpoint is now at a distance from and above the action. We are looking down on the police officers at work, objectifying rather than joining them. The Y text, furthermore, backgrounds the manual. Whilst the G text prints a formula in words, the Y text moves immediately into mathematical symbols. The final part of the Y task, although it has not fully left the public domain, is much more like the work of mathematicians than police work.

The G text, then, positions its reader voice within a particular, comparatively low status, career trajectory and distributes public domain message to this voice: mathematics is presented as being for participation in the public domain. The Y text positions its reader as the subject of a gaze which objectifies the public domain setting. The distributing strategy moves the reader away from this setting towards the esoteric domain. Mathematics is presented as describing the public domain, but not as a requirement for participation in it.

The structure of these texts, thus described, is similar to that of the G1 and Y1 covers. Two new features of the language of description have been introduced. The first involves the use of visual code of presence and its possible interruption in weakening the modality of the text. The second aspect concerns the opposition, generalising/localising. This will be of particular interest in considering the next set of texts.

4. GENERAL AND LOCAL ALGEBRA

Figure 5 shows the first page of Chapter 3 in Book Y1. Unlike the other texts discussed so far, there is no iconic signification on this page; all of the text is symbolic.[12] The section is headed 'a review of some shorthand', but the esoteric domain 'shorthand' has been embedded in what are likely to be very unfamiliar settings. In recognising these unfamiliar texts as nevertheless mathematical, the reader is introduced to the generalisable nature of mathematics. The section carries an internationalising of mathematics

itself, specifically, esoteric domain mathematics. Algebra has the status of a language which is, furthermore, international and here it is, appearing in an unidentified European language, in Japanese and in Arabic. 'Language', in the chapter heading, might signify simply a code, like a computer language. However, the play on the word language in the first two lines of the section and the use of Japanese and Arabic text emphasise 'language' and 'international' as foregrounded significations.

To the extent that the foreign language texts are expressed in non-mathematical terms (the foreign language words and symbols are not unambiguously mathematical since they cannot be recognised as such within an English language context) and signify a non-mathematical content (foreignness) they are located within the public domain.[13] However, these texts also implicate specifically and recognisably mathematical signifiers (the letters 'x' and 'y' are conventionally italicised) which are examples of algebra, to which attention is also drawn in the main text. In this respect, the foreign language texts are within the esoteric domain. The foreign language texts thus carry contradictory significations.

Within the textual context, there is no elaboration of the public domain element other than the note that 'Arabic is written from right to left' and the associated irony in a 'translation' of the Arabic algebra as mirror writing. This ironic 'translation' effects a projection out of the public domain context (the foreign language) thus diminishing its priority. The irony also emphasises the foreign connotation of the extract and so enhances the signification of mathematics as international. The esoteric domain significations, on the other hand, are elaborated within the English text. Furthermore, the final quarter of the page (after the mirror writing) is more of less exclusively esoteric domain and so confirms the priority of the esoteric over the public domain. This is a generalising strategy, which lends to esoteric domain mathematics a universal currency; it doesn't matter what the *words* in the text mean, the foreign texts themselves stand as examples of the language of algebra.

The text also carries an invitation into the esoteric domain practices of an international community of mathematicians[14] through the emphasising (in red print) of its agreed rules of expression: 'It would be confusing if ...'. This exposition is followed by task A1 which requires the reader to rehearse these rules, confirming her/him as an apprentice mathematician. This positioning strategy makes visible the authority of the discourse: this is what mathematicians (the dominant voice of the text) have agreed; now you do it.

The rules employ different variable labels from the preceding exposition. This marks them out in the text as does their emphasis in red print.

M13
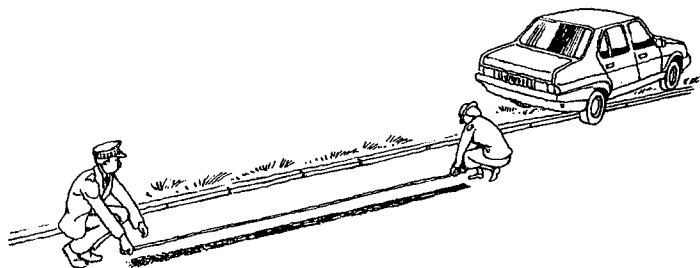
When measuring skid marks, the police can use this formula to estimate the speed of the vehicle.

$$s = \sqrt{(30fd)}$$

s is the speed in miles per hour (m.p.h.).
d is the length of the skid, in feet.
f is a number which depends on the weather and the type of road.

This table shows some values of f.

		Road surface	
		Concrete	Tar
Weather	Wet	0·4	0·5
	Dry	0·8	1·0

(a) A car travelling on a wet concrete road makes a skid mark of length 80 feet. How fast was it travelling?

(b) (i) When the road surface is tar and the weather is dry, the formula may be written

$$s = \sqrt{(30d)}$$

Complete this table to show the values of s for the given values of d, to 1 decimal place.

d	50	100	150	200	250
$30d$	1500				
$s = \sqrt{(30d)}$	38·7				

(ii) Draw axes, with d from 0 to 250 (use 2 cm for 50) and s from 0 to 100 (use 1 cm for 10).
Draw the graph of (d, s).

(iii) Use your graph to find how many feet a car would skid on a dry tar road at 75 m.p.h.

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Figure 4. SMP: 1987, *SMP 11–16* Book Y5, CUP, Cambridge, page 181.

3 The language of algebra

A   A review of some shorthand

The language of mathematics is international.
You may not know what language this problem is written in,
but the algebra is the same as you would find in an English book.

I.82. Să se arate că expresia

$$E = 4n^2 - 2n + 13$$

nu se imparte exact la 289, pentru nici un număr n intreg.

The only recognisable parts of the extract below are the letters x and y,
and the formula $y = x^2$ (unless you can read Japanese!).

y が x の関数であるとき，x の変域をこの関数の**定義域**といい，

定義域の各値に対して定まる y の値の集合をこの関数の**値域**という。

関数 $y = x^2$ で，定義域をすべての数の集合とすれば，値域は負

でない数全体の集合になる。

The next extract is in Arabic. At first sight only the signs $-$ and $=$ are recognisable.

٦. نعود للمرة الثالثة الى منحنى الدالة

$$ص = \dfrac{١}{٣ س ٢ - ١}$$

But س and ص are the equivalents of x and y, ٢ and ٣ are
the numerals 2 and 3. Arabic is written from right to left,
so the formula above is

$$y = \dfrac{1}{3x^2 - 1}$$

It would be confusing if everyone used their own shorthand, so
mathematicians have agreed that

$3a$ means $3 \times a$	a^2 means $a \times a$
ab means $a \times b$	a^3 means $a \times a \times a$
$\dfrac{a}{b}$ means $a \div b$	$2ab$ means $2 \times a \times b$

A1   If $a = 4$, $b = 2$ and $c = 3$, calculate the value of
    (a) $4b$ (b) c^2 (c) b^3 (d) ac (e) $3ab$
    (f) abc (g) $\dfrac{a}{b}$ (h) $\dfrac{b}{a}$ (i) $\dfrac{c}{b}$ (j) $3a^2$

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Figure 5. SMP: 1985, *SMP 11–16 Book Y1*, CUP, Cambridge, page 21.

3 Formulas

A Hiring

Peter works for the
Blagdon Hire Company.
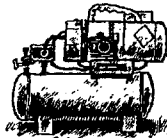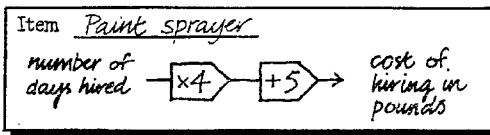They hire out ladders,
paint sprayers, floor cleaners
and so on.

You can hire them for
a number of days.

When you bring them back
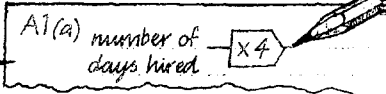Peter has to work out how much you pay.

There is a card for each item the company hires out.
On each card there is a machine chain.
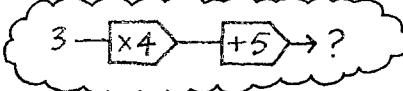The machine chain shows how to work out the cost of hiring the item.

Here is the card for a paint sprayer.

A1 (a) Copy the machine chain
         for the paint sprayer.
         Write it like this.

    (b) How much does it cost to hire a paint sprayer
         for 3 days?

A2 How much does it cost to hire a paint sprayer for 4 days?

A3 Work out how much it costs to hire a paint sprayer
    (a) for 6 days   (b) for 5 days   (c) for 7 days

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Figure 6. SMP: 1985, *SMP 11–16 Book G1*, CUP, Cambridge, page 23.

Both of these contribute to their exemplary status, as does their subse-
quent rehearsal in specific tasks. The rules are thus produced as general
principles, rather than local statements.

Figure 6 shows the opening page of Chapter 3 of Book G1, 'Formulas'.
Although this chapter also concerns algebra, this text is quite different from

the Y text. To begin with, half of the page is taken up by icons – cartoons and drawings. This mode of signification tends to localise the setting within the public domain.[15] The text is concerned with a particular company and with particular individuals. This localising is supported by the content[16] of the first cartoon and by the opening exposition. These specify the setting and its principal character in some detail.

The cartoon of the 'Blagdon Hire Company' is at the head of the chapter, almost on a level with and much more prominent than the chapter title. This is a scene setter. It announces the public domain setting of the chapter far more forcefully than the title announces its esoteric domain topic (six of the seven pages of the chapter concern the Blagdon Hire Company). As is the case with the mirror writing in the Y text, the cartoon incorporates irony, this time through the anachronistic appearance of the proprietor and the inclusion of a broom against the items for hire. It is not at all clear who might want to hire a broom; the manager appears to be looking at the broom with raised eyebrows, perhaps he wonders what it is doing there as well. These ironies effect a projection out of the public domain setting. However, whereas the irony in the Y text is resolved via a prioritising of the esoteric over the public domain, the ironies in the G text can be resolved only within the public domain, because there is no esoteric domain text. The irony signifies 'not this', allowing a connotative signification to come to the foreground. This connotative signification is a mapping of the G text onto comic literature. This is a positioning strategy which transforms the setting into a joke. The setting which motivates almost the entire G chapter is satirised at the very instant of its announcement.

The drawing of a 'card' connotes the esoteric domain through the use of the 'flowchart'[17] symbols referred to as 'machines'. The strong classification[18] (with respect to the non-mathematical) of the flowchart formula is muted by its appearance on a public domain object (the card). Both the formula itself (incorporating non-mathematical expressions) and its signifying mode (as manuscript on a drawing) are markedly less mathematical than the formulae in the Y text. The G text thus draws away from the esoteric domain, again foregrounding the public domain. The text moves on from the card (which is further localised by the drawing of the paint sprayer) to an instruction to the reader to copy it. Accompanying this instruction is a drawing denoting the appearance of the current page of the reader's exercise book when they are part way through copying the algorithm. A pencil point is poised in mid-trace. The second part of this task asks how much it costs to hire the paint sprayer for three days and includes a cartoon showing Peter thinking about the problem. The final

two tasks on the page require the reader to rehearse the same algorithm for different numbers of days.

Mathematically, we might describe the formula as linear in one variable and might generalise it as $C = An+B$. This is the mathematical form of all of the formulae in the chapter, yet there is no exposition concerning either its esoteric domain structure or (more pertinently) its public domain origin. It is not clear, from the text, why this particular formula should be used by the Blagdon Hire Company to price the hire of its paint sprayer (and, indeed for all of its hirings). The text is procedural rather than principled. This, again, localises the message.

Peter appears as a junior in the company and so stands in a possible career relationship with the reader: *you* might be doing this 'manual' job when you leave school. Metonyms that are thereby attached to Peter thus 'ionise'[19] the character with respect to the reader. This identification is sustained in the rest of the chapter, where the other characters are at a greater semantic distance. The manager and supervisor are Peter's superiors in age and rank and all of the hirers have titles (Mr, Mrs, Miss) rather than first names, Mr Evans appears to be the school caretaker.[20] The surrogate relationship between Peter and the reader (and the weaker relationship between Peter and the hirers) projects the subjectivity of the reader into the public domain narrative which is a possible realisation of the reader's own future. The text thereby objectifies the reader. Additionally, the public domain context is satirised, as has been noted, so that the reader's subjectivity is, at least partially, shifted within the public domain to that of reader of comic literature, which is an infantilising of the reader.

Both the Y and the G pages use exemplars. However, whilst the algebraic recognition rules of the Y text have general significance within the esoteric domain, the G exemplar (the algorithm which appears on the 'card', in the 'think bubble' and, partially, on the exercise book fragment) is highly localised). The algorithm is confined not only to the public domain setting, but to a specific item of equipment.

The opening pages of the two chapters exhibit both distributing and positioning strategies. The G text positions its reader by projecting its reader voice onto junior clerical work and onto infantile behaviour (comic reading). These projections are established denotatively and connotatively, respectively. That is, Peter, the junior clerk, is denoted in the text, whilst comics are connoted. These are categories of positioning strategy which will be referred to as *denotative mapping* and *connotative mapping*, respectively. Thus, the Y text positions its reader voice by producing a denotative mapping onto mathematical work through the invitation to join the community of mathematicians.

The G text localises mathematical message via the foregrounding of the iconic, via its detailed and closed narrative[21] relating to the public domain setting, and via its reliance upon a highly local procedure. The Y text generalises mathematical message in a number of ways: it excludes all but the symbolic mode of signification; it internationalises esoteric domain message and despecialises language; and it introduces mathematical principles. Crucially – and as is also the case with the police text in Figure 3 the G reader is to recognise her/himself in the text and is thus objectified by it. The Y reader is to recognise mathematics in the text, so that her/his individuality remains exterior and irrelevant to it. The Y reader is, thereby, an apprentice subject of mathematics.

Generalising and localising are textual strategies that have a particular importance in the language of description. This is because they derive from a modality of practice which can be indexed, but not defined, by the opposition abstract/concrete.[22] The distinction that I want to make is between practice that is highly organised at the level of language, and practice that generally lacks systematic organisation in language. Esoteric domain mathematics is an example of the former. It is able to generate texts which are relatively context-independent. This is not the case with respect to practices relating to more concrete practice, such as the practice of domestic activities. Texts within these practices are, as a rule, relatively context-dependent. Practice which exhibits highly systematic organisation of language is to be described as practices having *high discursive saturation*. Practice that does not organise language in the same way exhibits *low discursive saturation*. The high/low discursive saturation dimension is, it is suggested, aligned with the intellectual/manual opposition in the division of labour. This point will be discussed further later in this paper.

Localising strategies essentially render the text more context-dependent. Icons and closed, public domain narratives are examples of *textual resources* which are recruited by localising strategies within the SMP texts to far discussed. Generalising strategies render the text less context-dependent. The SMP Y texts achieve generalising through the introduction of esoteric domain principles, as is the case in the algebra text in Figure 5. Other Y texts generalise by elaborating connectivity within the esoteric domain. An example of this kind of strategy would be the presentation of multiple descriptions of a circle (as a locus, as the limiting polygon of n sides as n increases, as a conic section, as a plane curve having a particular generic form of equation) and the cross-referencing of the mathematical expressions entailed in such a multiple description.

However, generalising strategies may also incorporate public domain message. This is illustrated in the closing sections of the Y1 chapter on

algebra. Prior to this point, the chapter has approached algebra primarily through the incorporation of esoteric domain generalising strategies. In the final sections, however, the text introduces tasks which involve divergent public domain settings. Unlike the G text description of the Blagdon Hire Company, however, the narratives are all more or less open, giving little detail of the settings. The settings include: engineering problems, such as the supporting of a weight in the middle of an oak beam; professional practices, such as the calculation of the appropriate child's dose of a drug given the child's age and the adult dose; commercial problems, such as the ordering of bread by a café. These tasks all recontextualise their respective settings in ways which will be discussed later. However, the text also includes tasks which are clearly fictitious, such as this one:

There are x children in one room
and y in another.
A trifle weighing t kg is divided
equally between all the children.

Write an expression for

 (a) the total number of children
 (b) the amount in kg which each
      child gets
 (c) Check that your expression for (b) gives
      the correct answer when there are
      7 children in one room and 13 in
      the other, and the trifle weighs 60 kg.

(Book Y1, p. 33)

The setting is ludicrous: why are the children in separate rooms; why do we not, initially, know how many there are or how much the trifle weighs; whoever heard of a trife weighing 60 kg; how are the children going to eat 3 kg of trifle each? The verbal text is accompanied by a cartoon, which signifies two doorways out of each of which children are staring at a huge trifle in the foreground. The task connotes a recontextualised domestic setting, but is clearly ironic, alienating the public domain.

The use of open narratives – giving minimal setting detail, fast switching between settings, and irony ensures that the public domain takes no hold in the text. At the same time, it constitutes a generalising of a mathematical gaze, a triangulation which pinpoints the esoteric domain as the centre of subjectivity of the activity. Again, whereas the G text presents mathematics as being for participation in the public domain, the Y text invites the reader into the esoteric from which location they can describe the non-mathematical world.

The page opposite the final page of the Y chapter is the first of three individual pages entitled 'Mathematics of Yesteryear', which implicate extracts from old school mathematical textbooks, this one apparently from *Rural Arithmetic*, published in 1916. The extract appears on a black background which is bordered by classical Greek architecture. The title, 'Mathematics of yesteryear' appears on a scroll. The extract includes formulae similar to those which appear in the chapter, thus linking this page with the chapter and effecting its punctuation. The chapter thus opens with a synchronic universalisation of mathematics – the language of mathematics is international – and ends with a European diachronic universalising extending from ancient Greece through early twentieth century Britain to contemporary times.

5. RECONTEXTUALISING THE DOMESTIC

I now want to pay some attention to the nature of the public domain. This domain concerns the articulation between mathematical and other practices. I will focus primarily on domestic settings. Pervasive throughout the G series are settings related to shopping, almost invariably produced as narratives 'from the shopper's perspective'. Here is an example from Book G4, the shoppers being represented, in this instance, in the third person:

Soap powder is sold in 'Euro-sizes'.
These are Euro-sizes E5, E10 and E20.
In the McGee family
there are 2 adults and 3 children.
The McGees use six E10
packets of soap powder each year.

C5 What weight of soap powder do the McGees use in a year?

C6 The E10 packet Costs £ 2.59.
     How much does soap powder cost the McGees each year?

C7 The E20 packet contains twice as much
     powder as an E10 packet. It costs £ 3.89.
     (a) How many E20's would the McGees use in a year?
     (b) How much would powder cost them if they used E20's?

C8 (a) How many E5 packets would the McGees
          use in 1 year?
     (b) The E5 packets are on special offer.
          At the moment, an E5 packet costs 95 p.
          How much would the McGees soap powder
          cost in a year if they used E5 packets?

                                        (Book G4, p. 18;[23] drawing omitted)

Now, there is evidence that 'best buy' decisions are efficiently and appropriately made in context (that is, in the supermarket) with very little reference or debt to school mathematics (see Lave, 1988). However, recourse to such research is hardly necessary to reveal some of the violence that has been done to the setting in this case. For example, the long timescale of this narrative (a year) ought, perhaps, to make allowances for inflation,[24] and the answer to C8 (b) given in the Teacher's Guide (£ 11.40) suggests that either the 'special offer' lasted all year (rendering it rather 'ordinary') or the McGees bought a year's supply all at once. This, despite the family's apparent storage problems indicated by their customary use of the middle sized packet rather than the 'better value' E20 size.

Clearly, however, the conditions have to be fixed in order to allow the exercising of the mathematical content of the chapter, which is 'to provide pupils with experience of ratio problems' (*Teacher's Guide to Book G4*, p. 14). But this restructuring is apparently belied by the realism of the task formulation. The reader is provided with some 'factual' information about shopping (Euro-sizes) and is introduced to a family. The task, itself, apparently concerns the optimising of the Mc Gees' domestic routine. The setting, in other words, is highly localised in terms of both task detail and the community space of the family. Thus the narrative is comparatively closed: this might be the reader's family, either now or in the future; or it might connote a 'soap' family with which the reader may identify.

This is a mathematics task, but the text is silent on the matter of how it is to be read and on how the task is to be carried out. There is, deliberately, no explicit pedagogising of method in this chapter, as is common in the G series:

There are many methods of solving problems where ratio is involved. We have deliberately not set out a 'standard method'. After pupils have done a few questions, we hope that discussion with them will bring out these various methods, and thus help them tackle the next batch of problems. (*Teacher's Guide to Book G4*, p. 14)

There is an assumed transparency in the task text with regard to its reading and a repertoire of 'strategies' within the classroom. The repertoire is to be activated by the 'realistic' task and made explicit in 'discussion'. Thus, pedagogic practice, here, is a celebration of essential competencies through official recognition and sharing in discussion. At the end of the chapter there is a set of 'discussion points, the first two of which are, 'Why are things cheaper when you buy them 'in bulk'?' and 'Sometimes it is not sensible to buy in bulk. When is it silly to buy huge packets?' The criteria in addressing this question (as indicated by the Teacher's Guide) are outside of school mathematics. However, the inclusion of such discussion

only serves to accentuate the domestic setting which has apparently been incorporated by school mathematics.

In other words, we might ask why shopping should appear at all in a mathematics textbook. One reason might be to initiate a route into the esoteric domain. Alternatively, the performance of the shopping tasks might itself be constructed as mathematical learning. However, the text does not leave the public domain and, furthermore, there is very little (if any) explicit articulation of these shopping tasks with other tasks and exposition in the G series. Thus the tasks seem to be constituted for the benefit of optimising shopping practices themselves. Mathematics is being constructed as a prior condition for optimum participation in the public domain. I have referred to this as the *myth of participation.*

There are two points to be made, here. Firstly, the ideal reader already shares in a repertoire of strategies, so that their domestic competence is constructed as a prior condition for mathematics. At the same time, mathematics is being constructed as the prior condition for the reader's domestic participation. The myth thereby deconstructs itself.[25] Secondly, since the criteria for succesful completion of the tasks within the mathematics context are not the same as those structuring the domestic activity, there is no basis for the reader's competence. The myth of participation is precisely a shibboleth.[26]

There are very few shopping settings in the Y series, here are two instances taken from the first 'review' section in Book Y1:

Shopkeeper A sells dates for 85 p per kilogram. B sells them at 1.2 kg for £ 1.

(a) Which shop is cheaper?
(b) What is the difference between the prices charged by the two
    shopkeepers for 15 kg of dates?

(Book Y1, p. 55)

Britannia best British flour cost £ 0.71 for 12.5 kg.
Uncle Sam's best American flour cost $ 1.30 for 3.5 lb.
If 1 kg = 2.2 lb and £ 1 = $ 1.85, which brand of flour
was cheaper, and by how much per kilogram?

(Book Y1, p. 56)

Here, there is a clear semantic distance between the reader and the shopping setting and the narratives have become far more open. The first example uses a recontextualised algebraic resource, generalising through the use of letters which depersonalise the shopkeepers. This example also refers to a slightly exotic commodity and to a highly unrealistic quantity. The second case presents a transatlantic comparison which clearly cannot be relevant to most shoppers who would not be expected to board Concorde

in order to get a cheaper pack of flour. In other words, the tasks include some element of strangeness which ensures their dislocation from the domestic setting to which they now only indirectly relate.

Furthermore, as has been illustrated in the previous section, the Y text presents a highly varied public domain, minimising the importance of any given setting. The mathematical gaze is thus presented as having potentially universal descriptive power. This is the *myth of reference*. Thus the two schemes employ distributing strategies which specialise their respective mythologising of the non-mathematical: the localising myth of participation in the G books; the generalising *myth of reference* in the Y series. Crucially, however, in neither case do public domain settings coincide with the non-mathematical setting which is indexed. The mathematical gaze has effected a *recontextualising* of domestic (and other) practices, such that the principles for their elaboration are more or less structured by mathematical, that is, esoteric domain principles.

Domestic activities, then, are not, in themselves, incorporated into the public domain of school mathematics. Rather, the esoteric domain of school mathematics casts a mythologising gaze onto these activities, imposing its own structure on them and thus constructing settings for its public domain. The settings may be constituted as mythical planes of participation. In this schema, subjects are constructed as optimally behaving according to mathematical principles in the elaboration of their own interests. Alternatively, the settings may be constituted as mythical planes for description. Here, subjects' interests are more or less ignored. Rather, the planes are populated by objects which can be organised into mathematical relationships with each other, whilst retaining a trace of their non-mathematical significations. In this latter case, the mythical plane is constituted as referring to, rather than as constituting the domestic activity.

The two strategies constitute distinctive pedagogic forms. As has been noted, the G text constitutes the class of students, rather than the teacher, as embodying a reservoir of strategies for the solution of domestic and other public domain problems. The acquirer voice is constructed as already competent. All that the mathematics lesson has to achieve is to bring the students together in a particular setting on the plane of participation. Acquisition via the sharing of strategies is characteristic of activities exhibiting low discursive saturation. There is no institutionalising of pedagogy within such activities and so acquisition must occur on an informal and so context-dependent basis. It is, furthermore, the context that distinguishes between the esoteric and the public domains of practice. The principles of evaluation, under such conditions, are more with the acquirer than with the

transmitter and are necessarily context-dependent: does it work? Following Bernstein (1995) I want to refer to such activities as *horizontal.*[27]

The Y texts, on the other hand, constitute their acquirer voice as originally lacking in competence. This voice is the acquirer of transmission on the part of the teacher/textbook, for example:

Depending on the level of confidence of the class and the extent of their previous experience of investigations, it may be a good idea to introduce the first problem in Chapter 8 (Cutting a cake) before reaching the chapter, and without any of the assistance given in the chapter. Those who try to solve the problem and 'get lost' are likely to appreciate more the need for a methodical approach. (*Teacher's Guide to Book Y1*, p. 10)

Here, pedagogic action occurs within a more obviously hierarchical relationship between transmitter and acquirer. The principles of evaluation are embodied in the transmitter. This characterises the situation in activities which institutionalise pedagogy as is clearly the case with academic disciplines. The acquirer's interests are constructed as coinciding with those of the transmitter. Insofar as the aquirer may enter the activity via its public domain, their career is within the esoteric. The institutionalising of pedagogy must relieve it of strong context-dependency. This must involve systematic organisation of language, generally, in textbooks, or their analogues. This organisation is precisely the esoteric domain which will comprise principles discourse and canonical texts. Such activities are to be referred to as *vertical* activities.

6. CONCLUSION: THE LANGUAGE OF DESCRIPTION AND THE TEXTS

I am making no claim to have produced a definitive reading of the SMP texts. On the contrary, the reading is explicitly biased, the particular bias being structured by a language of description. It is now possible to assemble the principal features of the language, looking firstly at the structural level. The contextualising basis of social relations and cultural practices is *activity*. Activities are regions of the division of labour, in general, and regulate who can say or do or believe what. Activity, then, constitutes *subject positions* and *practices*. Practices are differentiated in terms of *domain* and *discursive saturation*. The regulating domain of an activity is the *esoteric domain*, which casts a *gaze* upon the practices of other activities. The achievement of this gaze is the recontextualising of practices such that they conform, more or less, to the principles of the esoteric domain. Recontextualised practices constitute the *public domain.*

The activity which has been under consideration in this paper is school mathematics. The school articulates with other institutional locations, in

particular, the family, material production, and the state. These articulations are, undoubtedly, highly complex, nevertheless, several obvious features of the nexus can be mentioned. First, the articulation with the family establishes the teacher in the place of the parent and so in superordinate relationship to the student, but also potentially accountable for their actions to the parent. Second, schooling articulates with material production through the use in the latter of certification for selection purposes. Third, an additional articulation with material production is achieved via the use of textbooks and other pedagogic materials. Fourth, the state employs teachers, inspects schools (or legislates for their inspection), and regulates certification. Fifth, the state also regulates childhood in relation to schooling. Thus the school must incorporate an official pedagogic practice which is more or less visible, which is to say, independent of the specific contexts of its elaboration. Furthermore, this official pedagogic practice must be differentiated, sequenced and paced such that it is consistent with the exigencies of certification and state regulation. School activities are, in other words, *vertical* activities and their practices exhibit high *discursive saturation.*

School knowledge, then, comprises specialised forms which are institutionalised in textbooks and syllabuses. These constitute principles and canons which regulate the official discipline and embody the principles of pedagogic evaluation. In other words, school disciplines are officially regulated by respective esoteric domains. In terms of subject positions, the location of the evaluative principles of pedagogy must be with the teacher, by virtue of their necessarily superordinate relation to the students. Texts which realise this hierarchical relation are referred to as *pedagogic texts.* These texts (re)produce[28] activities. Subject positions and practices are (re)produced as *voices* and *message,* respectively. *Textual strategies position* voices and *distribute* message across the voice structure. This is achieved via the recruitment of *textual resources,* such as settings and signifying modes (iconic, indexical, symbolic).

In my discussion of the school mathematics texts I have revealed a number of features of their mode of working. Positioning strategies map an 'ability' voice hierarchy onto the intellectual/manual hierarchy within the division of labour.[29] Distributing strategies distribute generalised esoteric domain message to the superordinate acquirer voice and localised public domain message to the subaltern acquirer voice, objectifying the latter and constituting the former as a potential subject of mathematical practice. Essentially, the Y book is about mathematics, the G book is about the student.

It has been seen that the G transmitter voice appears, in some instances to act as facilitator only. In this sense, it would appear that the principles of evaluation in the G text are shifted from the transmitter to the acquirer voice. It has been suggested that this characterises acquisition within *horizontal* activities exhibiting low discursive saturation. Horizontal activities, such as domestic activities and manual activities, in general, do not institutionalise pedagogy. Transmission occurs predominantly through the physical copresence of transmitter and acquirer such that the evaluative principles are context dependent and located with the acquirer. There are, by definition, no pedagogic texts generated by these horizontal activities. So, is it the case that the SMP G texts are not pedagogic texts?

It will be recalled that the planes of participation generated by the G texts are products of the recontextualising paze of the esoteric domain of school mathematics. The domestic settings that constitute the public domain are organised according to the principles that predominate within the recontextualising activity – school mathematics – and not those that obtain within the domestic activity itself. This is a crucial feature of the theoretical model being employed here, but it is also empirically supported, here and elsewhere (Dowling, 1995a). This, then is a general proposition: a textualised, which is to say recontextualised practice relating to one activity cannot retain its structure under the gaze of another activity.

Put rather more crudely (and locally), you cannot teach shopping within the context of school mathematics. On the contrary, the mathematical gaze constructs mythical planes of participation which are paraded as substantive, hence the myth; the public domain remains within school mathematics. The text sets up a context for the sharing of strategies, but there is no material basis for the prior acquisition of these strategies by empirical students. This is because they are, of necessity, mathematical and do not derive from the domestic lives of the students. The G text is, in other words, a pedagogic text which is, at least in part, parading as a non-pedagogic text. The subaltern voice is distributed neither shopping nor mathematics; pedagogic action is mythical and, ultimately, alienating.[30]

The Y text, on the other hand, frequently enables the reader to enter through the public domain, but swiftly sublimates them to the esoteric. The mechanism of pedagogic action resonates with Althusser's (1971) metaphor of hailing in the street for the constitution of the subject with respect to ideology in general. Here, it is 'ideology' in particular – school mathematics – and the one-hundred-and-eighty degree turn goes the other way around. The student initially faces the teacher from the floor of the classroom, to be gradually rotated into the teacher's chair, where they take up the subjectivity of the discourse.

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NOTES

[1] In Dowling (1995a) I give a full elaboration of the current level of development of the language of description. I also include an engagement with its theoretical antecedents and a full literature survey of relevant work dealing with textual analysis as well as a more complete analysis (incorporating qualitative as well as quantitative approaches) of the texts considered here. The earlier piece also incorporates an element of quantitative analysis which also cannot be included in the present paper, for reasons of space.

[2] The concept of a language of description was proposed by Basil Bernstein (1995).

[3] The scheme is authored by the School Mathematics Project, Southamptom, and published by Cambridge University Press, Cambridge.

[4] In earlier papers (for example, Dowling, 1992, 1994a) I have adopted a deductive mode of presentation in which the language is elaborated prior to its application to the texts. Because this paper can only illustrate the more complete analysis, readers may have the impressions that there has been a degree of over-reading of some of the extracts. The reader may also justifiably raise questions about the principles of selection of the particular extracts used. In essence, they have been chosen because they are suited to the points that are being made, although in each case alternatives could have been chosen. Nevertheless, they do not, taken together, represent the SMP scheme in total. A different selection might have foregrounded different textual resources. The structure of the analysis (in terms of voices, messages and textual strategies [see below]) and the general description of the scheme, however, would have been the same. In Dowling (1995a) I argue: that the validity of the analysis is constituted by the degree of explicitness of the general methodology and the language of description and that the reliability is constituted by the very substantial number of exemplars (there are over 100 plates in the piece). Clearly, neither of these conditions can be met in the present paper.

[5] An additional series – between the upper and middle tracks – begins in year 10. The books (particularly the lower series) are also augmented by additional material in the form of worksheets and cards and so on. These will not be considered here.

[6] The terms 'mundane' and 'arcane' (used below) may be respectively glossed as 'worldly' and 'secret'.

[7] These are textual constructions rather than statements about empirical readers. The latter would require a different informational base. This is, perhaps, not always recognised in semiotic analysis.

[8] School mathematics is to be distinguished from other mathematical activities, such as academic mathematics. Their distinctive locations within the sociocultural ensures that their respective practices and subject positions are organised differently. Academic mathematics, for example, constitutes a possible career between undergraduate and professor and (to simplify) locates the professor as a producer within a field of peers. The career between school student and mathematics teacher is not contained within the activity of school mathematics and the teacher is, in general, a producer only in the classroom.

[9] This expression derives, essentially, from the work of Michel Foucault. But see Walkerdine et al. (1989), which may be more familiar to readers in the field of mathematics education. See also Dowling (1991a).

[10] Prior to the abandoning of Social Class classifications for UK census purposes, police officers were classified as Social Class III N (OPCS, 1980), that is, skilled, non-manual occupations. Nevertheless, this kind of police work is clearly closer to manual than to intellectual labour.

[11] That is, in its claim to a reality beyond the text. See the discussion on 'modality' in Hodge and Kress (1988).

[12] I make use of Peirce's (1931–58) terms icon, index and symbol, but have defined them differently. In my language, they refer to modes of signification rather than to signs. The *iconic* model incorporates a visual code of presence, as discussed above. The *indexical* mode incorporates a visual code, but no code of presence; graphs and tables fall into this category. The *symbolic* mode is the residual category for which the visual code is reduced to its minimal, linear level (i.e., a verbal text implies a one-dimensional ordering).

[13] Formally, public domain refers to the domain of weak specialisation of mathematical message in terms of expression and content and so does not necessarily imply general familiarity: in most instances, public domain message does have a wider semantic currency than esoteric domain message, but this is clearly not the case in these instances.

[14] Again, it is important to recognise that this refers to 'mathematicians' as constructed in and by the text, not to mathematicians as such (refer also to footnote number 8).

[15] Iconic signification, as defined here, is not possible within the esoteric domain of mathematical practice. This is because mathematical contents are defined formally and cannot be physically realised.

[16] Content refers to that which is signified rather than the mode of signification.

[17] These do not appear to be standard flowchart symbols, but have been pedagogically modified to incorporate directionality.

[18] This term derives from Bernstein (for example, 1977, 1990) and refers, here, to the degree of specialisation of contents.

[19] The metaphor is taken from chemistry and suggests properties (metonyms, electrons) which can be shared by two objects (individuals, atoms).

[20] The reader identification with Peter is somewhat confused by the form of address used in the above extract: *You* can hire them for a number of days ... When *you* bring them back *Peter* has to work out how much *you* pay'. However, the strong career association between Peter and the reader, the fact that the reader is to do Peter's job (i.e., calculate the cost of hiring), the clear semantic distance between all of the hirers and the reader certainly outweigh the apparent addressing of the reader as hirer. In fact, an alternative to 'you' would possibly render the exposition too specific for an exemplar (an individual's name might be used which would also involve an introduction) or, perhaps, too abstract (by using, 'someone'). Thus 'you' is being used as a sort of generalised other which is, nevertheless, not too alien: the reader might, after all, become a hirer in due course; all of the hirings do connote manual labour.

[21] A 'closed' narrative represents an attempt to minimise the scope for alternative interpretations.

[22] This modality of practice has been constituted out of a consideration of the work of a number of authors, in particular: Bernstein (1990), Bourdieu (1990), Foucault (1977), Lévi-Strauss (1972), Luria (1976), Piaget (1972; see also Piaget 1995), and Vygotsky (1986).

[23] The vulgar use of the apostrophe to indicate a plural, as in 'How many E20's', is representative of a common inelegance in language in these texts.

[24] The possibility of prices going down is mentioned in the 'discussion points' in the Teacher's Guide.

[25] See Derrida (1981), Sturrock (1979).

[26] In the Bible, a 'test-word'.

[27] There may appear to be a redundancy in the use of the two expressions, 'horizontal' and 'low discursive saturation'. Horizontal activities are low discursive saturation, but practices which are low discursive saturation are not necessarily within horizontal activities. Practices within the public domain of academic discourses, for example, are likely to exhibit low discursive saturation, but are incorporated in activities that, as I shall indicate below, are *vertical*. Furthermore, 'horizontal' refers to the structure of social relations, whereas 'discursive saturation' refers to the degree of systematic organisation of language within the practices, see Dowling (1995d).

[28] The expression, '(re)produced' is used to distinguish the relation from one of mechanical reproduction.

[29] Elsewhere (Dowling, 1991b, 1995a), I have described the *SMP 11–16* scheme, more broadly, as mapping the high/low 'ability' opposition on to the middle class/working class recontextualising of the division of labour in material production.

[30] This does not, of course, entail that it is impossible for empirical students to acquire these strategies, only that the text comprises insufficient resources to enable them to do so.

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