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Internal Models and Intermittency: A Theoretical Account of Human Tracking Behavior

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Abstract. This paper concerns the use of tracking studies to test a theoretical account of the information processing performed by the human CNS during control of movement. The theory provides a bridge between studies of reaction time and continuous tracking. It is proposed that the human CNS includes neuronal circuitry to compute inverse internal models of the multiple input, multiple output, dynamic, nonlinear relationships between outgoing motor commands and their resulting perceptual consequences. The inverse internal models are employed during movement execution to transform preplanned trajectories of desired perceptual consequences into appropriate outgoing motor commands to achieve them. A finite interval of time is required by the CNS to preplan the desired perceptual consequences of a movement and it does not commence planning a new movement until planning of the old one has been completed. This behavior introduces intermittency into the planning of movements. In this paper we show that the gain and phase frequency response characteristics of the human operator in a visual pursuit tracking task can be derived theoretically from these assumptions. By incorporating the effects of internal model inaccuracy and of speed-accuracy trade-off in performance, it is shown that various aspects of experimentally measured tracking behavior can be accounted for.

Introduction

In an attempt to understand the neurophysiological processes underlying the movement disorders of cerebral palsy and in the spirit of a paper by Marr and Poggio (1977), entitled "From understanding computation to understanding neural circuitry", we have developed a computational model of information processing performed by the human central nervous system (CNS) during control of movement (Neilson et al. 1985; Neilson et al. 1987). We refer to it as "Adaptive Model Theory". Although the theory was evolved largely de novo, it shares significant philosophical bonds with others in the field (e.g. Kleinman et al. 1970; Pew 1974; Raibert 1978; Saltzman 1979; Greene 1982; Hinton 1984). It is a basic hypothesis of Adaptive Model Theory that the human CNS includes neuronal circuitry (modelling circuitry) to compute, store in memory and adaptively maintain the accuracy of sets of parameters describing the dynamic nonlinear relationships between multiple sensory and motor signals. A detailed description of one possible solution of this problem was presented previously (Neilson et al. 1987). Once computed, the parameters can be recalled from memory and used to control the information transfer characteristics of neural networks during analysis of sensory signals and during movement execution. The neural networks come to behave like internal models of relationships between sensory and/or motor signals and can be employed by the CNS for a variety of purposes both in perception and control of movement. Any change in the relationships between sensory and/or motor signals leads automatically to an adaptive recalibration of the stored internal models. The models are stored in the inverse form. Thus, for example, during movement execution, models stored in the inverse sensory-to-motor form translate preplanned trajectories of desired perceptual consequences into appropriate outgoing motor commands. The inverse internal models compensate for the dynamics and nonlinearities in muscles, biomechanics and any external systems being controlled, so that the overall performance of the human operator should be relatively independent of these factors. This reflects the ability of the human operator to adapt his behavior to suit the responsiveness of controlled systems.

The model is similar to the intermittent, three-stage model developed by experimental psychologists to account for the results of discrete response, double stimulation reaction time experiments (Vince 1948; Hick 1948; Welford 1959, 1980; Karlin and Kestenbaum 1968; Herman and Kantowitz 1970; Kantowitz 1974; McLeod 1977). It has been demonstrated repeatedly from the earliest experiments onwards (see Welford 1980 for review) that during a reaction time to a stimulus, the human CNS can execute a response to a previous stimulus and receive and register a subsequent stimulus which appears and disappears during the reaction time. This implies the existence of three processing stages – sensory analysis (SA), response planning (RP) and response execution (RE) – operating in parallel. Intermittency is introduced because the RP-stage operates discretely and requires a finite interval of time (typically 100–200 ms) to preplan a movement before passing the information on to the RE-stage. Although the RP-stage operates intermittently, the SA- and RE-stages operate continuously in real-time, like systems in cascade (McClelland 1979; Miller 1982). The only time delay in SA- and RE-stages is due to the transmission time required for the flow of information through neural networks, analogous with the flow of signals through filters. The information-transformation characteristics of the neural networks are controlled by the outputs from "modelling circuitry".

In this paper we show that an intermittent, three-stage model can account not only for the results of discrete response, double stimulation reaction time experiments (as discussed in detail by Welford 1980), but also for the behavior of human operators performing continuous tracking tasks. It has been claimed that it is difficult to distinguish between discrete time models of human tracking behavior and continuous models with comparable time delays (see Pew 1974). Continuous tracking models (for example, Kleinman et al. 1970; McRuer 1980), however, do not account for the "psychological refractory period" measured in double stimulation reaction time experiments. In such models the inter-response interval for a continuously responding system equals the inter-stimulus interval and there is no psychological refractory period. An intermittent reaction time model has the advantage that it can account for continuous tracking behavior as well as the psychological refractory period. Thus, an important feature of Adaptive Model Theory is that it provides a theoretical bridge between psychological studies of reaction time processes on the one hand and engineering investigations of man-machine systems on the other.

Much has been written in recent years concerning the limits of human processing capacity and the distribution of processing resources between parallel processors (Norman and Bobrow 1975; Navon and Gopher 1979; Wickens 1980; Gopher et al. 1982; Gopher and Sanders 1984). An individual occupied with one task often lacks the capacity to perform others, although under other circumstances many tasks can be performed concurrently (Wickens et al. 1983). Central processes are assumed to consume resources that are in limited supply and the degree to which parallel processes interfere with each other depends on the extent to which they compete for a common supply of resources. McLeod (1977) proposed variable allocation of a finite capacity between processing stages in order to account for the lengthening of reaction times to both the first and second stimuli in double stimulation reaction time experiments. Variable allocation of finite capacity between the three parallel processing stages mentioned above (and between substages operating in parallel within each stage) also accounts for the ability of subjects to perform multiple tasks simultaneously. In our view, the flexibility of a variable allocation parallel processing model does not diminish the importance of the single channel hypothesis as recently described by Welford (1980). For tasks in which performance depends on a minimum response time delay (such as continuous tracking), maximum capacity must be allocated for response planning if the response is to be initiated in the shortest possible time. A minimum response time delay is obtained when the RP-stage plans one response at a time and does not commence planning a second response until it has completed planning the first. In other words, to achieve a minimum response time delay, the variable allocation three-stage parallel processing system must operate in a manner identical to the operation of the single channel mechanism.

In this paper we show that an intermittent three-stage mechanism which plans one response at a time is consistent with both the open-loop and closed-loop transfer function characteristics of the human operator performing continuous tracking tasks. It may be that some continuous tracking models such as the optimum control model (Kleinman et al. 1970) can account for continuous tracking behavior as well as does Adaptive Model Theory. This would not be surprising because Adaptive Model Theory is a discrete equivalent of the continuous optimum controller based on Kalman filter theory. It incorporates adaptive inverse internal models to compensate for system dynamics and adaptive stochastic model predictors to compensate for time delays, just as in Kalman filter theory. We show here that the behavior of subjects performing continuous pursuit tracking can be predicted from Adaptive Model Theory. From the assumptions of intermittency and internal models we derive theoretically the gain and phase frequency response characteristics for both the open-loop and closed-loop transfer functions of the human operator in a pursuit tracking task. The effects of model inaccuracy and speed-accuracy trade-off are included in the derivation. Adaptive Model Theory is then tested by comparing the theoretical performance characteristics with those measured experimentally.

Application of Adaptive Model Theory to Pursuit Tracking

In a visual pursuit task the subject moves a control (joystick) to keep a response marker aligned with a continuously moving target. Using the constructs of Adaptive Model Theory, the information processing performed by the human CNS during a pursuit tracking task can be represented schematically as in Fig. 1. For simplicity, the SA-stage has been omitted from the schematic diagram in Fig. 1. The RP-stage is represented by the circle labelled RP and the RE-stage is represented by the boxes (drawn with dotted lines) labelled "adaptive controller" and "controlled system". The substages of the controlled system are muscle control system (MCS), biomechanics (BM) and tracking system or external system (E). MCS transforms outgoing motor commands (M) into a pattern of muscle tensions (T). BM transforms T into body movements or joystick movements (θ). E transforms θ into movements of the response marker (R) on the display screen. In many analyses of tracking behavior, θ is regarded as the output from the human operator and the system is differentiated into two subsystems, the "human operator" and the "tracking system" (illustrated by dashed lines in Fig. 1), rather than into the adaptive controller and the controlled system as described here. The latter differentiation is more appropriate in this context because it emphasizes the importance of MCS and BM as substages in the controlled system.

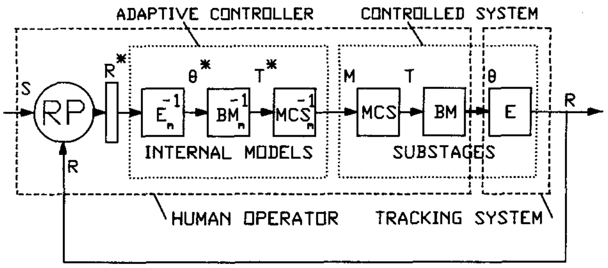

Fig. 1. Schematic diagram of central processes involved in the performance of a visual pursuit tracking task. RP = response planning stage, S = stimulus signal, R = response signal, rectangular block = short-term memory buffer, E_m^{-1} = inverse internal model of E, BM_m^{-1} = inverse model of BM, MCS_m^{-1} = inverse internal model of MCS, R* = preplanned desired response, θ^* = desired movement, T* = desired muscle tensions. Other abbreviations as in text

The basic assumption of Adaptive Model Theory is that the human CNS includes neuronal circuitry (not shown in Fig. 1) which monitors feedback of the multiple input and multiple output signals from each substage in the controlled system and computes, stores in memory and adaptively maintains the accuracy of inverse internal models of the dynamic, nonlinear, varying relationships between them. In a previous paper (Neilson et al. 1987) we have presented a detailed description of computational algorithms and neuronal circuitry capable of performing this operation. The technical feasibility of the algorithms and circuits was confirmed by simulation in a digital computer. Basically, a subcortical circuit involving a cerebellar microzone region was assumed to function like an adaptive filter, as suggested by Fujita (1982). The Wiener kernel weights of the filter are retrieved from memory and used to set the gains of Purkinje cells via the inferior olive and climbing fibers (see Rawson and Tilokskulchai 1982, 1985). The axons of Purkinje cells in a microzone converge on to target cells in a cerebellar nucleus where their signals add, producing the output signal from the filter. If the stored Wiener kernel weights accurately describe the relationship between the input and output signals of a substage, then the subcortical circuit functions as an accurate internal model of that substage. The outputs from the model match the outputs from the substage and the cross correlations between the input signals and the discrepancy signals (that is, discrepancies between the model output signals and the substage output signals) are zero. If the internal model is inaccurate, on the other hand, the cross correlations (and higher order cross correlations) between the (pre-whitened) input and discrepancy signals are nonzero and their values are precisely those that must be added to the stored Wiener kernel weights to correct the model. The circuit functions in an iterative fashion and quickly converges the stored Wiener kernel weights to their correct values. The circuit operates to maintain an accurate internal model of the multiple input, multiple output, dynamic, nonlinear characteristics of the substage and any change in the dynamic responsiveness of the substage leads to an automatic adaptive recalibration of the model. Similar modelling circuitry is employed to compute forward and inverse models.

During response execution, the inverse internal models (E_m^{-1}, BM_m^{-1}, MCS_m^{-1} in Fig. 1) of the substages of the controlled system are employed (as substages in the adaptive controller) to translate the desired response (R*) – preplanned in terms of the desired visual movement of the response marker on the display screen – into appropriate M to activate the controlled system and generate R equal to R* (Fig. 1). The transfer function relationship between R* and R can be expressed as

$$R/R^* = E_m^{-1} \cdot BM_m^{-1} \cdot MCS_m^{-1} \cdot MCS \cdot BM \cdot E\,, \qquad (1)$$

where E is the transfer function of external system, BM the transfer function of biomechanical system, MCS the transfer function of muscle control system, E_m^{-1} the transfer function of inverse internal model of E, BM_m^{-1} the transfer function of inverse internal model of BM, and MCS_m^{-1} the transfer function of inverse internal model of MCS.

If all the inverse internal models are accurate, the transfer functions of the inverse internal models cancel the transfer functions of the substages and (1) simplifies to

$$R/R^* = 1\,. \qquad (2)$$

Thus, the actual response, R, equals the desired response, R*, and the inverse internal models (MCS_m^{-1}, BM_m^{-1}, E_m^{-1}) compensate for the dynamic responsiveness of MCS, BM, and E. But if any of the inverse internal models are inaccurate, for example, due to inexperience with the task or as a consequence of neurological damage, R will not match R* and (1) can then be written as

$$R/R^* = K\,, \qquad (3)$$

where K is the resultant transfer function due to model inaccuracies.

The RE-stage just discussed (and also the SA-stage not described here) employs networks of adaptive internal models which, using terminology employed by McClelland (1979), behave like "cascade" information processing stages. That is, their outputs are influenced continuously by their inputs and the processing time equals only the transmission time required for spread of activity through the networks.

The RP-stage, on the other hand, behaves like a "discrete" information processing stage in that it requires a finite interval of time (T_p) to preplan a desired response, R*, and to store it in short-term memory ready for execution. It plans only one R* at a time and does not commence planning a second R* until it has completed planning the first. (This is not to imply that the RP-stage cannot plan multiple responses in parallel during the performance of simultaneous tasks or for that matter, during performance of a single task. During a single task, however, planning one response at a time leads to a minimal response time delay.) A desired response may initially be represented as an abstract hierarchical structure (Klapp and Wyatt 1976; Sternberg et al. 1978) of goals and subgoals, each subgoal leading to the next as suggested by Kelley (1968). It may involve a heterarchical activation of "frames" and "agencies" as described in theories of artificial intelligence (viz. Minsky 1986). At the level of the most immediate subgoal, however, R* is preplanned as a trajectory of desired perceptual consequences to achieve the subgoal. That is, R* is preplanned in terms of the same sensory feature code in which feedback of R will eventually be represented. In the pursuit tracking task being discussed, R* is preplanned in terms of the desired visual movement of the response marker on the display screen. Thus, central images of intended movements and feedback of actual movements are expressed in the same sensory code and can be compared directly. This overcomes the sensory-motor language problem discussed by Schmidt (1976).

Intermittency

The discrete behavior of the RP-stage introduces intermittency into movement programming. SA-, RP-, and RE-stages operate continuously and in parallel but, because of the discrete behavior of the RP-stage, information is only transferred intermittently, via short-term memory buffers, from the SA-stage to the RP-stage and from the RP-stage to the RE-stage. Although voluntary movement may be executed continuously, the underlying movement programs preplanned by the RP-stage (trajectories of desired perceptual consequences) are only updated intermittently at planning time rates. Thus, according to Adaptive Model Theory, all voluntary movements, from those in simple reaction time experiments, through continuous movements in tracking experiments, to the complex movements of singers, dancers, gymnasts and pilots, are comprised of a concatenated sequence of submovements, each planned in advance by the RP-stage and executed in an open-loop fashion by the RE-stage. This notion of intermittency in movement programming is not new. It has been proposed by many movement control theorists and is well supported experimentally. It was discussed cogently with respect to tracking behavior by Craik in the 1940s (Craik 1947a, b).

Planning a Submovement in Tracking

The following description of the information processing performed by the RP-stage during the planning of a submovement in a pursuit tracking task will provide the background for a subsequent derivation of the error to response and the stimulus to response transfer functions.

The RP-stage reads the most current information available in short-term memory at the output of the SA-stage. This information includes the motions of both the target and response markers extracted from the visual input. Using a previously computed statistical model of the motion of the target (the algorithms involved will be discussed in a subsequent paper), the RP-stage forecasts the future motion of the target for at least a reaction time interval into the future. The RP-stage also forecasts the future motion of the response marker. This is achieved by adding the *change* in desired response planned during the previous planning interval to the current position of the response marker available at the output of the SA-stage (prediction of disturbance perturbations are not considered in the present context). The RP-stage then computes R* to move the response marker from its predicted path into alignment with the forecasted motion of the target. This is analogous to the problem of shooting at a moving target from a moving platform, where the motions of both target and platform must be taken into account. It is equivalent to the control problem of computing an optimum trajectory to move a controlled system between an initial dynamic state and a final dynamic state in such a manner that some performance criterion is maximized.

Speed-Accuracy Trade-Off

Selection of an optimum R* depends on the subject's performance criterion. This criterion can change, depending on the conditions. In order to reduce the tracking error as quickly as possible, the fastest movement would appear to be the best. For human subjects, fast movements to a target are characterized by a relatively constant duration independent of the amplitude of the movement (Freund and Budingen 1978; Freund 1983). By implication, it is the duration of the movement and not the maximum velocity that limits fast movements. Therefore, a fast movement to a target can be defined as one that reaches the target within a single submovement, regardless of the distance to the target.

Fitts' law is a well known empirically derived relationship between the distance to the target, the width of the target and the movement time for arm movements to the target (Fitts 1954; Fitts and Peterson 1964). Crossman and Goodeve (1963) and Keele (1968) showed that Fitts' law can be explained (at least for long duration movements) if it is assumed that a movement to a target consists of a series of submovements, each of the same duration and relative accuracy. This is consistent with the notion of intermittency discussed earlier. Howarth et al. (1971) and Beggs and Howarth (1972) showed that when the total movement time is increased, subjects usually slow down throughout the entire movement and not just during the final precision placement. The entire movement, including the initial segment which covers most of the distance to the target, is also slowed as the termination point becomes more accurate (Langolf et al. 1976). Greater terminal accuracy is normally achieved by slowing each submovement so as to bring the responding limb closer to the target at the time of the final visually based correction.

Speed-Factor

Incorporating this idea of speed-accuracy trade-off into the planning of R* suggests that in attempting to improve accuracy, a subject might choose R* slower than the fastest possible response. Since the duration of each submovement is assumed to be constant (T_p is determined by the time required by the RP-stage to preplan each submovement), it follows that the speed of each submovement will be slowed and that more submovements will be made to the target. If, for example, a subject has an inaccurate internal model of the controlled system, it is likely that the speed of each submovement will be considerably slowed in an attempt to minimize execution errors. This is consistent with the slowing of voluntary movements observed in patients with movement disorders such as Parkinson's disease, cerebellar dysfunction and cerebral palsy and could explain the compulsive freezing and blocking of movements experienced by subjects wearing inverting prism glasses or attempting to perform mirror writing. To allow for the possibility that subjects can change the speed of R*, we will introduce a "speed-factor" (a) into our equations. By definition, the speed-factor will equal the proportion of the distance to the target moved in a single planning time interval, T_p. When the speed-factor $a=1$, for example, the subject moves to the target in a single submovement with a duration equal to the planning time, T_p. It will be shown below that a tracking response signal generated with a speed-factor $a=1$ reproduces the motion of the target, whereas for any other value $a \neq 1$ the response marker does not exactly reproduce the motion of the target and the response signal contains attenuation and phase distortion relative to the stimulus signal.

Sampled Data Signals and the Backward Shift Operator

Although sensory and motor signals within the CNS are probably sampled at a high frequency (say, 20–30 samples/s), performance of a subject in a tracking task involving an irregularly changing stimulus signal is limited by the time, T_p, required by the RP-stage to preplan R*. This is borne out by spectral analyses of coherent tracking response signals which show negligible power at frequencies above 2–4 Hz (Stark 1968; Neilson 1972). Since T_p is short relative to the highest frequencies contained in a tracking response, no information is lost by sampling tracking signals at T_p intervals and performing a digital analysis of the sampled data. For simplicity, this is the approach adopted in the derivation presented below. (We point out, however, that by including high frequency sampling of sensory signals and incorporating prediction of stimulus and response signals for at least a reaction time interval ahead, predictable high frequency changes occurring within a planning time interval could be taken into account. Thus, the derivation below could be extended to account for "precognitive" tracking of predictable high frequency stimulus signals.)

In Fig. 2, the stimulus and response signals are treated as sampled data signals which are sampled synchronously (as indicated by the filled squares) at planning time intervals, T_p. Thus the signals can be presented as time series and the theoretical derivations can be expressed in the form of difference equations or digital transfer functions. The stimulus signal, for example, can be represented by the time series of sampled data points $S_1, S_2, \ldots, S_t$, and the response signal can be represented by the time series $R_1, R_2, \ldots, R_t$. Each sample interval corresponds to a planning time interval, T_p.

In analysis of time series we shall employ some simple mathematical operators used by Box and Jenkins (1976). First is the backward shift operator, B, defined by $BS_t = S_{t-1}$; hence $B^m S_t = S_{t-m}$ (notice that B is equivalent to the Z-transform operator Z^{-1}). Second is the backward difference operator ∇ which can be written in terms of B, since

$$\nabla S_t = S_t - S_{t-1} = (1-B)S_t . \tag{4}$$

Third is the inverse of ∇, the summation operator ∇^{-1} given by

$$\begin{aligned}\nabla^{-1} S_t &= (1-B)^{-1} S_t \\ &= (1+B+B^2+\ldots)S_t \\ &= S_t + S_{t-1} + S_{t-2} + \ldots .\end{aligned} \tag{5}$$

Using the B notation, the transfer function for a digital filter can be derived. Consider, for example, an exponentially weighted moving average filter defined by

$$Y_t = aX_t + (1-a)Y_{t-1} . \tag{6}$$

Using the backward shift operator this can be written as

$$Y_t = aX_t + (1-a)BY_t . \tag{7}$$

This can be rearranged algebraically to give the digital transfer function

$$Y_t/X_t = \frac{a}{1-(1-a)B} . \tag{8}$$

If we substitute $B = e^{-i2\pi f}$, where $i = \sqrt{-1}$, and f = frequency in Hz, into the digital transfer function and expand, we obtain an expression for the gain and phase frequency response function which can be presented graphically (Box and Jenkins 1976, p. 414). We will now employ these mathematical tools to compute the theoretical gain and phase frequency response curves describing the error to response and the stimulus to response digital transfer functions for subjects performing pursuit tracking tasks. In working through the following derivations the reader will need to make frequent reference to the figures, especially Fig. 2.

Error to Response Digital Transfer Function

First, let us simplify the problem by assuming that the target is moving in an irregular fashion and that it is not possible to predict whether it will move up or down from its current position during the next planning interval. The influence of target prediction will be introduced into our equations in a subsequent article by including a stochastic model predictor. For the moment, however, we shall ignore the influence of target motion prediction and proceed as though there was no statistical model of target motion available to the RP-stage.

Let us consider the processes involved in preplanning a desired response during the planning interval labelled RP2–3 in Fig. 2. The RP-stage reads the past motions of the target (S_1, S_2) and the response marker (R_1, R_2) available in short-term memory at the output of the SA-stage. In the absence of a statistical model of target motion, the best forecast that can be made by the RP-stage is simply a projection of the most recent position of the target available at the output of the SA-stage, as indicated by the horizontal dotted line labelled S_2, $S_2(1)$, $S_2(2)$, $S_2(3)$, ... The RP-stage also generates a forecast $R_2(1)$ of the future position of the response marker by adding the *change* in response marker position planned during the previous planning interval RP1–2 to the most recent position of the response marker R_2 available at the output of the SA-stage. A desired response R_4^* is then planned by the RP-stage to move the response marker from its predicted position $R_2(1)$ into alignment with the forecasted trajectory of the target S_2, $S_2(1)$, $S_2(2)$, $S_2(3)$, ... For illustrative purposes, however, the speed-factor of the desired response shown in Fig. 2 has been set to less than unity ($a<1$), so that the distance between $R_2(1)$ and R_4^* is only a proportion (a) of the distance between $R_2(1)$ and the predicted target position $S_2(2) = S_2$. This relationship can be expressed

$$(R_4^* - R_2(1)) = a(S_2 - R_2(1)) . \tag{9}$$

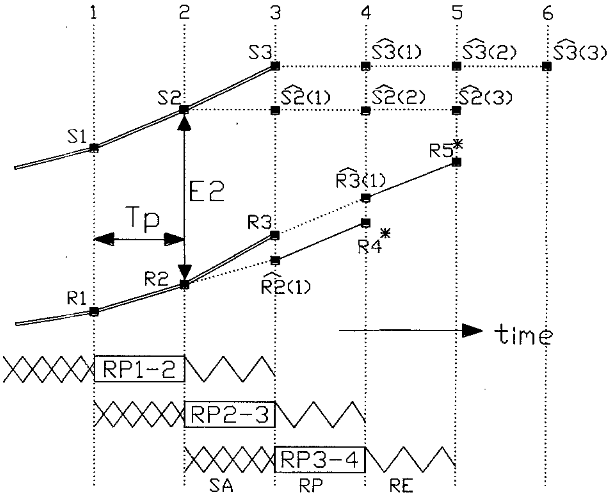

Fig. 2. Schematic diagram illustrating the vertical movement of the target (S) and response marker (R) on the display screen plotted against time. The vertical dotted lines labelled 1, 2, 3, 4, 5, 6 at the top divide the time axis into equal intervals equal to the time (T_p) required by the RP-stage to preplan each R*. Rectangles at the bottom of the diagram labelled RP1–2, RP2–3, and RP3–4 represent the time intervals during which three consecutive submovements are preplanned. The zig zag lines to the right of each rectangle represent the time intervals during which the corresponding submovements are executed. The double zig zag lines to the left of each rectangle represent the time intervals during which sensory information relevant to the planning of each submovement is processed and stored in short-term memory by the sensory analysis (SA) stage. During each interval the RE-, RP-, and SA-stages operate in parallel but are working on different submovements. During interval 2–3, for example, the RE-stage executes the response RP1–2, the RP-stage preplans the response RP2–3 and the SA-stage collects sensory information to be used in planning RP3–4. The filled squares indicate a synchronous sampling of the stimulus (S_1, S_2, S_3) and response signals (R_1, R_2, R_3) at planning time intervals. $S_2(1)$, $S_2(2)$, $S_2(3)$ represent the one-step, two-step and three-step ahead forecasts of future target position computed during RP2–3 based on the current position of the target S_2. $R_2(1)$ represents the one-step ahead forecast of the future position of the response marker computed during RP2–3. R_4^* represents the desired response preplanned during RP2–3

In parallel with the processes occurring during RP2–3 just described, the RE-stage executes the desired response preplanned during the previous planning interval RP1–2. For illustrative purposes, the inverse internal models in the RE-stage are considered to be inaccurate, so that the actual response R will differ from the desired response R*. Consequently, the change in response marker position $(R_3 - R_2)$ will deviate from the predicted change in position $(R_2(1) - R_2)$ because of the inaccuracy of the inverse internal models ($K \neq 1$ in equation 3). Thus

$$\nabla R_3 = R_3 - R_2 = K(R_2(1) - R_2). \tag{10}$$

The above processes repeat during each planning interval. For example, during the planning interval RP3–4, the most recent position of the target S_3 available at the output of the SA-stage is projected to the right to represent a forecast of future positions of the target S_3, $S_3(1)$, $S_3(2)$, $S_3(3)$, ... The change $(R_4^* - R_2(1))$ in response marker position planned during the previous planning interval RP2–3 is added to the most recent position of the response marker R_3 available at the output of the SA-stage, to obtain the predicted position of the response marker $R_3(1)$. Thus, the predicted change in response marker position $(R_3(1) - R_3)$ equals $(R_4^* - R_2(1))$. The desired response R_5^* is preplanned to move the response marker from its predicted position $R_3(1)$ into alignment with the forecasted trajectory of the target S_3, $S_3(1)$, $S_3(2)$, $S_3(3)$, ... Again, since the speed-factor is assumed to be less than unity, R_5^* is only a proportion (a) of the distance to the predicted position of the target $S_3(2) = S_3$. In parallel with these planning processes during the interval RP3–4, the RE-stage executes the desired response preplanned during the previous interval RP2–3. Again, because the inverse internal models in the RE-stage are assumed to be inaccurate, the actual change in response marker position $(R_4 - R_3)$ (not shown in Fig. 2) will deviate from the predicted change in position $(R_3(1) - R_3)$. Thus

$$\nabla R_4 = R_4 - R_3 = K(R_3(1) - R_3) = K(R_4^* - R_2(1)). \tag{11}$$

Examination of Fig. 2 reveals

$$E_2 = S_2 - R_2 = (R_2(1) - R_2) + (S_2 - R_2(1)). \tag{12}$$

Substitution of (9) into (12) gives

$$E_2 = (R_2(1) - R_2) + (R_4^* - R_2(1))/a. \tag{13}$$

Substitution of (10) and (11) into (13) gives

$$E_2 = \nabla R_3/K + \nabla R_4/(Ka). \tag{14}$$

Equation (14) holds for all planning intervals and so, by using the backward shift operator notation described above, it can be expressed in general form as in (15)

$$B^2 E_t = (1 - B) B R_t/K + (1 - B) R_t/(Ka). \tag{15}$$

Equation (15) in turn can be rearranged algebraically to obtain an expression for the error to response digital transfer function

$$R_t/E_t = \frac{KaB^2}{1 - (1 - a)B - aB^2}. \tag{16}$$

Equation (16) shows that the digital transfer function K, representing the relationship between R* and R with inaccuracies in the inverse internal models (3),

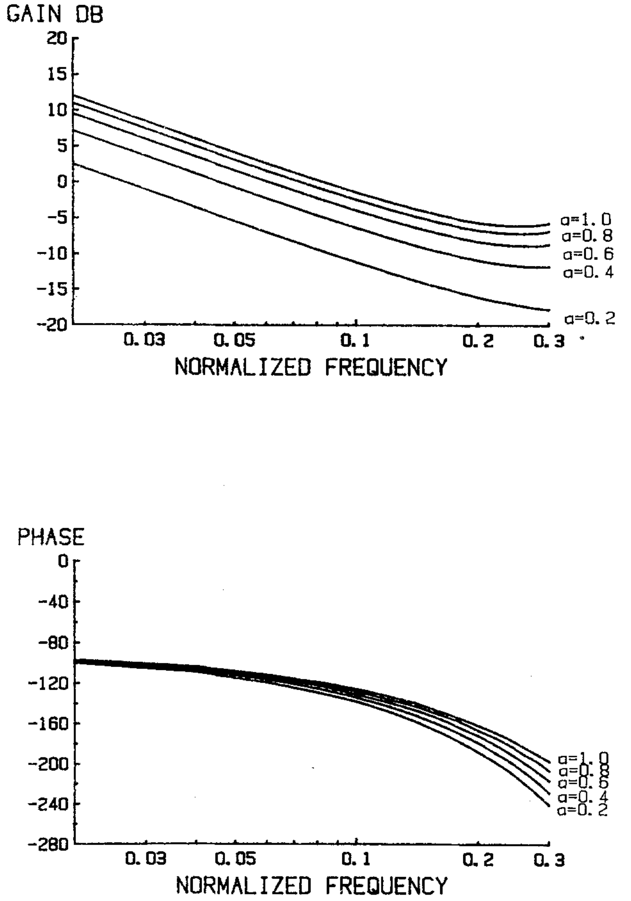

Fig. 3. Gain and phase frequency response curves plotted on semilog scales for the error to response digital transfer function for different values of the speed-factor $a = 0.2, 0.4, 0.6, 0.8, 1.0$ and with accurate inverse internal models. Gain is plotted in decibels, phase in degrees and frequency is normalized by expressing it as a proportion of the sampling frequency ($1/T_p$)

appears in the error to response digital transfer function as a multiplying factor.

The gain and phase frequency response curves for the error to response digital transfer function (16) were computed for a variety of values of the speed-factor a, with $K=1$ (Fig. 3). Examination of these graphs shows that increases in speed-factor produce an increase in the gain and a decrease in the phase lag between the error and the response signal. It shows that, at low frequencies (<0.25 of sampling frequency), the effect of an increase in speed-factor is equivalent to an increase in a proportional gain factor and a decrease in a time delay. It seems reasonable to assume that as the accuracy of tracking improves with practice, subjects will gradually increase the speed of their desired responses. Consequently, one should expect the proportional gain factor to increase and the time delay to decrease as the subject's skill improves with practice.

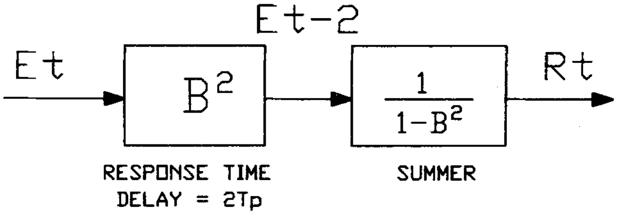

Fig. 4. Information flow block diagram illustrating that the error to response digital transfer function (17) can be represented as a response time delay equal to two planning time intervals in cascade with a digital signal summer (similar to a continuous signal integrator) which sums the delayed error signal at response time ($2T_p$) intervals

Other task, environmental, procedural and subject-centered variables which might influence the subject's choice in a speed-accuracy compromise (for example, anxiety about switching from a simulator to actual conditions) can also be expected to influence the proportional gain factor and time delay.

If it is assumed that the inverse internal models in the RE-stage are accurate and that the subject produces fast responses so that the speed-factor is unity (that is, $K=1$ and $a=1$), (16) simplifies to

$$R_t/E_t = \frac{B^2}{1-B^2}. \tag{17}$$

As illustrated in Fig. 4, the simplified digital transfer function (17) is equivalent to a response time delay equal to two planning time intervals, connected in cascade with a summer which sums the delayed error signal at response time delay intervals.

Stimulus to Response Digital Transfer Function

As can be seen in Fig. 2, the error signal is given by

$$Et = St - Rt. \tag{18}$$

Substituting (18) into (16) and rearranging algebraically gives the stimulus to response digital transfer function

$$R_t/S_t = \frac{KaB^2}{1-(1-a)B-(1-K)aB^2}. \tag{19}$$

With the speed-factor (a) set to unity, the gain and phase frequency response curves for the stimulus to response digital transfer function were computed for a variety of scalar values (k) of the model inaccuracy transfer function K and are presented in Fig. 5. A scalar value of K (other than 1) represents an inaccuracy in the gain of the inverse internal models. Whereas in the error to response transfer function (16) K appears only as a multiplying factor, in the stimulus

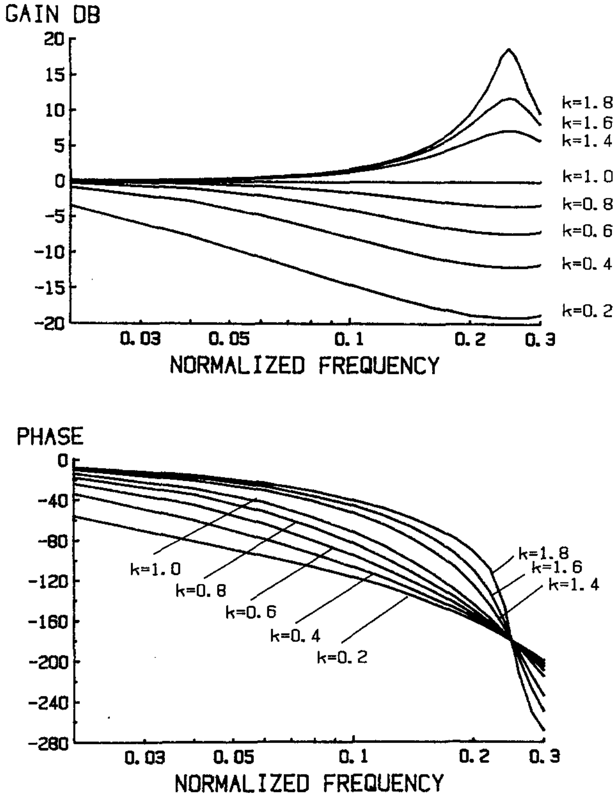

Fig. 5. Gain and phase frequency response curves plotted on semilog axes for the stimulus to response digital transfer function (19) for various scalar values ($k=0.2, 0.4, 0.6, 0.8, 1.0, 1.4, 1.6, 1.8$) of the model inaccuracy transfer function (K) and with the speed-factor $a=1$. Gain is plotted in decibels, phase in degrees and frequency is normalized as a proportion of the sampling frequency ($1/T_p$)

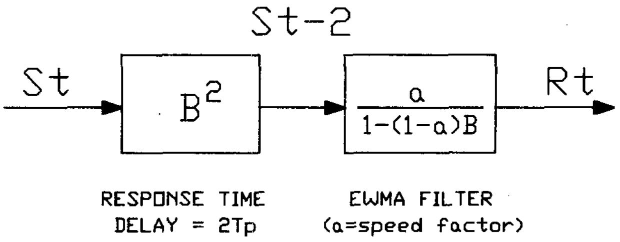

Fig. 6. Information flow block diagram illustrating that the stimulus to response digital transfer function (20) can be represented as a response time delay equal to two planning intervals ($2T_p$) in cascade with an exponentially weighted moving average (EWMA) low-pass digital filter

to response transfer function (19) it also appears in the denominator and so alters the dynamics of the performance characteristics. When the change in R is greater than the change in R* (that is, $k>1$), the stimulus to response transfer function is underdamped and a resonant peak appears in the gain curve at a frequency equal to one quarter of the sampling frequency (Fig. 5). Conversely, when $k<1$, the stimulus to response transfer function is overdamped and the response movements are sluggish. When the model inaccuracy transfer function is either negative or greater than two (that, k is outside the range $0 \leq k \leq 2$), the stimulus to response transfer function is unstable. For example, when $k>2$, the amplitude of the actual response is more than twice the amplitude of the desired response. When the subject attempts to correct the alignment error between the target and response marker, the correction causes an even greater overshoot in the opposite direction. Consequently, a series of corrections with increasing overshoot develops, leading to an unstable oscillation at a frequency equal to one quarter of the sampling frequency. Such oscillations are rarely seen in practice, however, because the subject usually responds by greatly reducing the speed-factor of the preplanned desired response. If the speed-factor reduces to near zero then blocking or freezing of the response occurs. Moreover, as proposed in Adaptive Model Theory, the human CNS rapidly corrects errors in its stored internal models. When $k<0$, any attempt to correct an alignment error will cause the response marker to move in the wrong direction, producing an even larger error. The system is unstable and will be driven into saturation. Such an unstable response can sometimes be seen, for example, when the subject confuses the target marker for the response marker.

When the inverse internal models in the RE-stage are assumed to be accurate (that is, $K=1$), the stimulus to response digital transfer function (19) can be simplified to

$$R_t/S_t=\frac{aB^2}{1-(1-a)B}. \tag{20}$$

Comparison with (8) shows that the stimulus to response digital transfer function (20) is equivalent to a response time delay equal to two planning intervals connected in cascade with an exponentially weighted moving average digital filter, as illustrated in Fig. 6. The gain and phase frequency response curves for this transfer function have been computed for a variety of values of the speed-factor (a). These gain and phase curves (Fig. 7) show that when a subject reduces the speed-factor below unity, in an attempt to improve the accuracy of tracking responses by slowing each submovement and making more submovements to the target, the effect on the stimulus to response performance characteristics is equivalent to introducing a low-pass exponentially weighted moving average filter between the stimulus and response signals. The high frequency response movements are attenuated and the phase lag of the response signal behind the stimulus signal is increased.

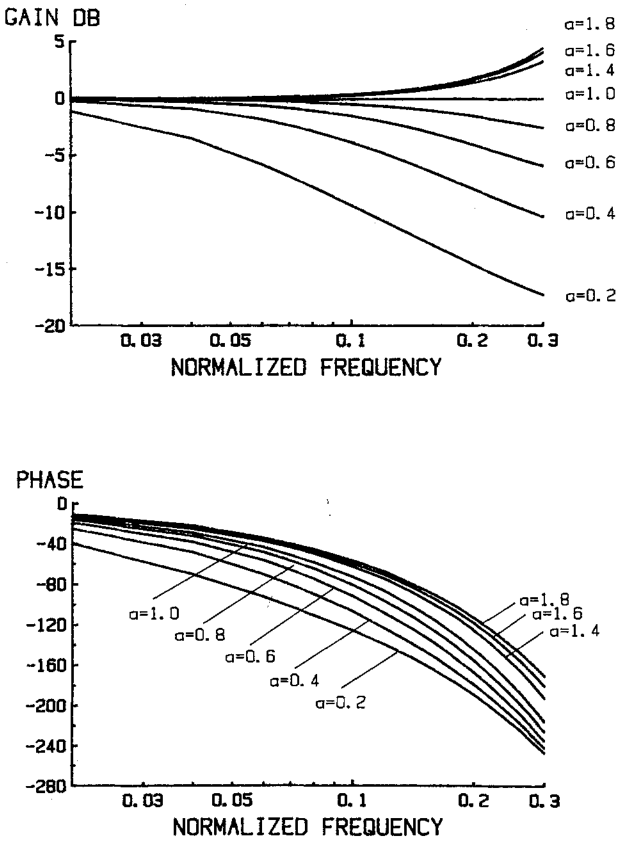

Fig. 7. Gain and phase frequency response curves plotted on semilog axes for the stimulus to response digital transfer function (20) for various values of the speed-factor ($a = 0.2, 0.4, 0.6, 0.8, 1.0, 1.4, 1.6, 1.8$) and with accurate inverse internal models. Gain is plotted in decibels, phase in degrees and frequency is normalized as a proportion of the sampling frequency ($1/T_p$)

If it is assumed that the inverse internal models in the RE-stage are accurate and that the subject makes fast correction movements with a speed-factor equal to unity (that is, $K = 1$ and $a = 1$), the stimulus to response digital transfer function (19) simplifies to

$$R_t/S_t = B^2 . \tag{21}$$

The response signal simply equals the stimulus signal with a response time delay equal to two planning intervals ($2T_p$). From (20) and (21) it can be concluded that when the response signal is generated with a speed-factor equal to unity (that is, the subject corrects the error in a single submovement with a duration equal to the planning interval T_p), the movement of the response marker reproduces the motion of the target with a response time delay. On the other hand, if the subject employs a tracking strategy in which the speed-factor is set to a value other than unity, the movement of the response marker does not exactly reproduce the motion of the target and the response signal contains attenuation and phase distortion relative to the stimulus signal, as well as a response time delay.

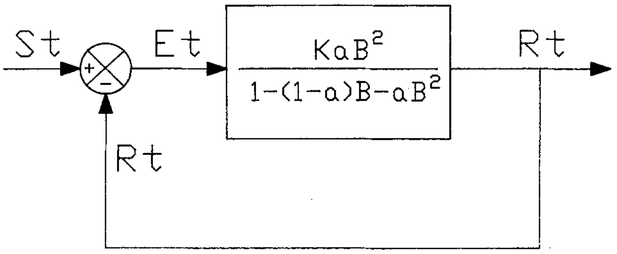

Fig. 8. Information flow block diagram illustrating a sampled data servosystem model of a pursuit tracking task. The open-loop digital transfer function is the error to response digital transfer function (16). The closed-loop digital transfer function, $\mathrm{CLTF} = G/(1 + GH)$, is identical to the stimulus to response digital transfer function (19) derived in the text

Servosystem Model of Tracking Behavior

As discussed earlier, scalar inaccuracies of the inverse internal models in the RE-stage alter the dynamics of the stimulus to response relationship and can cause the system to become unstable. In the region of stability, however, the gain and phase curves of the stimulus to response relationship are relatively insensitive to inaccuracies in the internal models at very low frequencies, as can be seen in Fig. 5. This phenomenon can be attributed to the action of negative feedback (that is, at very low frequencies the subject has sufficient time to detect and correct execution errors) and is consistent with a sampled data servosystem model of tracking behavior illustrated in Fig. 8. It can be shown that the closed-loop transfer function for the sampled data servosystem in Fig. 8 is identical to the stimulus to response relationship (19) derived above. Consequently, a useful extrapolation from Adaptive Model Theory is that the error to response relationship (16) can be regarded as the open-loop digital transfer function and the stimulus to response relationship (19) as the closed-loop digital transfer function in the sampled data servosystem model of tracking behavior shown in Fig. 8.

Validity of Theoretically Derived Performance Characteristics

Many predictions concerning various aspects of tracking behavior can be derived from the theoretical account presented above. To a first approximation at least, the theoretically derived performance characteristics account for the behavior of subjects in tracking tasks. This can be demonstrated by comparison with the empirically determined "crossover" model of tracking behavior described by McRuer and Krendel

(1959a, b, 1974) and McRuer (1980). These authors studied compensatory tracking tasks using a variety of tracking systems with different transfer functions (gain, gain plus integrator, gain plus double integrator, gain plus integrator plus first-order lag filter). They showed that the form of the experimentally measured error to response open-loop transfer function was invariant and that it was insensitive to changes in the response characteristics of the tracking system. Regardless of the characteristics of the tracking system, the response was always proportional to the integral of the error signal with a reponse time delay, as shown by the continuous transfer function (S = Laplace transform operator)

$$Y_p Y_c = \frac{\omega_c e^{-\tau S}}{S}, \tag{22}$$

where Y_p is the transfer function of human operator, Y_c the transfer function of tracking system, ω_c the gain, τ the response time delay, and $1/S$ the integrator.

In other words, the operator's input-output characteristics Y_p are adaptively adjusted to compensate for the characteristics of the tracking system Y_c, so that the overall characteristics $Y_p Y_c$ remain invariant and equivalent to a gain, an integrator and a time delay connected in cascade.

When rearranged as shown in (23), this experimental result implies that the human operator compensates for the dynamic responsiveness Y_c of the tracking system by forming an inverse internal model Y_c^{-1} of the tracking system, just as proposed in Adaptive Model Theory.

$$Y_p = \frac{\omega_c e^{-\tau S}}{S} \cdot Y_c^{-1}. \tag{23}$$

If the gain of the tracking system is doubled, the gain of the operator is halved; if the tracking system includes an integrator, the operator incorporates a differentiator; if the tracking system includes a phase lag filter, the operator behaves like a phase lead filter, etc. The empirical observation that the performance characteristics of the human operator are adapted in this way supports the fundamental hypothesis of Adaptive Model Theory, namely, that the human CNS includes neuronal circuitry to compute, store in memory and adaptively maintain the accuracy of inverse internal models of the dynamic responsiveness of the controlled system (tracking system in this case). Furthermore, the form of the invariant open-loop transfer function (22), measured experimentally, is the continuous signal equivalent of the sampled data digital transfer function (17) derived above. The continuous transfer function (22) states that the response signal is proportional to the integral of the error signal with a response time delay. The digital transfer function (17) states that the response signal is equal to the summation of the sampled error signal with a response time delay.

McRuer and Krendel found that although the experimentally measured form of the open-loop transfer function was invariant, the estimated values of the gain ω_c and the time delay τ in (22) depended on a variety of task, environmental, procedural and subject-centered variables. In measuring the effects of training, for instance, they found that ω_c increased with trials until a stable value was obtained for a particular subject and set of conditions. This is consistent with the theoretical influences of model inaccuracies and speed-factor variations on the error to response digital transfer function (16) discussed earlier. Thus, the empirically determined crossover model, the adaptive behavior of the human operator and the underlying invariant performance characteristics, as well as the effect of training, all are consistent with the performance characteristics derived from Adaptive Model Theory.

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