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REVIEW ARTICLE

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The relationship between power output and endurance: a brief review

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Abstract It is well established that for work requiring high power output, endurance time is short, and that low power outputs can be maintained for long periods. Parameters describing this relationship are important in characterising work performance and the capacity of humans as a source of mechanical power. The purpose of this paper is to provide a brief review of the available literature investigating this relationship and its parameters. Most experimental data reflect measurements of endurance times over a range of constant power outputs on the cycle ergometer. Early graphical analyses of these data have been superseded by curve fitting, which in turn has led to establishment of the two component hyperbolic model now embodied in the critical power test. This model has been modified and extended in various ways to account for its shortcomings. In addition, a number of different exercise forms have been studied, and the effects of a variety of secondary factors (training status, age, sex, for example) on the parameters have also been investigated.

Key words Anaerobic work capacity · Critical power
Exercise duration · Exhaustion · Fatigue
Hyperbolic model

Introduction

The function of muscles is to convert chemical into mechanical energy, allowing an individual to produce a force and perform work. Four basic interrelated fuel sources are available to muscles for this function. High energy phosphates (phosphagens) contribute alactic anaerobic energy to the muscle system. Very high power outputs can be produced on demand, but the capacity is limited to short durations. Small amounts of blood glucose and large amounts of muscle glycogen can supply energy rapidly also. High power usage can be met almost on demand by lactic anaerobic energy (anaerobic glycolyis accompanied by lactic acid production), while low power usage with an upper limit can be met by an aerobic energy source which relies on oxidation of glycogen. This reliance on oxygen delivery via the lungs and circulation is kinetically slower, taking up to 3 min or so to establish itself. Fuel stored as fat is also available aerobically. Since it is limited to even lower power outputs and may only become available after an hour or more of exercise, it has not been as extensively studied.

Consider the question "Given full fuel reserves, how long does it take to become exhausted at different levels of power output?" It is of course well established that for work requiring high power output, endurance time is short, and that low power outputs can be maintained for long periods. More specifically one should ask, given our knowledge of the chemical and mechanical processes involved, whether the relationship between power output and endurance can be represented by a systems model with meaningful parameters. The relationship is important, not only in characterising work performance, but also in the ability of humans as a source of mechanical power (Wilkie 1960) in activities such as human-powered flight (Nadel and Pierce 1988).

The purpose of this paper is to provide a brief review of the available literature over the last 45 years relevant to the investigation of this relationship and its parameters. Initially we shall consider graphical analyses and empirical curve fitting. System modelling includes the development of the critical power test (Moritani et al. 1981) and a variety of extensions, many of these to account for shortcomings in the assumptions of the critical power test. Since power output need not necessarily be measured on a cycle ergometer, we also discuss the relationship in the context of a number of other forms of exercise. Whatever the model or mode of

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exercise, the parameters of the relationship need to be interpreted, primarily in terms of their correlates with other well-known bioenergetic parameters. We briefly consider also the effects of secondary factors (age, sex, training status, for example) on the parameters as a means of better understanding the relationship. Finally some model methodological questions are addressed.

Empirical modelling

Empirical models are derived by observation and experimentation, rather than by theoretical considerations. Thus this section is brief, and is restricted to purely graphical presentations of data when curves have been drawn by hand, and some analyses where curves have been statistically fitted to the data. Such curves are chosen simply by virtue of goodness of fit and perhaps also of predictive value. No physiological meaning is attached to the form of the curve or to its parameters before the fitting is performed, although a posteriori arguments may be invoked to explain parameter interpretation. Empirical models therefore simply describe the data without explaining them.

Graphical and hand-drawn curves are often used for illustration when no particular analysis is intended. (Kennelly 1906; Ettema 1966; Stegemann 1981; Bellemare and Grassino 1982; Petrofsky and Phillips 1982; Knuttgen 1986; Maughan et al. 1986). More usually since the intention is to investigate the relationship between endurance time to exhaustion, t, at power settings, P (which may also be reflected through the total work performed $W = Pt$), some form of mathematical curve is fitted. At times these curves have become quite complex (Rohmert 1960; Simonson and Lind 1971).

One of the earliest such attempts by Grosse-Lordeman and Müller (1973) has been made by means of two equivalent power curves:

$$\log t = a \log P + b, \quad a < 0 \tag{1a}$$

$$\log W = c \log P + b, \quad c = a + 1 \tag{1b}$$

Parameters a and b were determined empirically from eight experimental points, and considerable interindividual variation was noticed. Others (Kennelly 1906; Pavlat et al. 1993) have also used the power curve. Harmann et al. (1987) have attempted to improve comparability of the power model between individuals by introducing a power scaling factor, F. F is determined using an interactive computer programme to subjectively map each individual's power-endurance curve onto a common standard curve. In effect $F > 1$ stretches an individual's curve to the left, while $F < 1$ compresses it to the right. In this way the curves could be made to overlap, thus reducing interindividual variation. It was observed that individuals with high values of F have good endurance at fractions of their maximal power.

The exponential and/or (natural) logarithmic curves of form:

$$\log t = a - bP \tag{2}$$

$$P = a + be^{-ct} \tag{3}$$

have also been used (Bigland-Ritchie and Woods 1984; Hopkins et al. 1989; Aunola et al. 1990; Mayhew et al. 1992) as an empirical model for the relationship. These curves too have been subject to scaling procedures (Gleser and Vogel 1973; McLellan and Skinner 1985) in an attempt to produce lower relative residual variance. Even double exponential equations have been used (Clarke 1986). In only one case, by Gleser and Vogel (1973), was any attempt made to interpret the fitted curves physiologically.

Figure 1 illustrates fits of Eqs. 1a, 2 and 3 to some data of McLellan and Cheung (1992). Over endurance times ranging from 99 s at 455 W to 804 s at 337 W, all three fit well. Equation 1a has the highest value of R^2 ($= 0.98$) and Eq. 3 the lowest ($R^2 = 0.96$).

Systems Modelling

A systems model differs from an empirical or data model in some important aspects. A systems model is established to theoretically explain the process which produces the data, rather than to describe the data themselves. If the description of the process is sufficiently accurate, then a good fit by data can logically be expected. Although some systems models are derived post facto, the theoretical development usually occurs prior to any data collection or validity analysis. It is

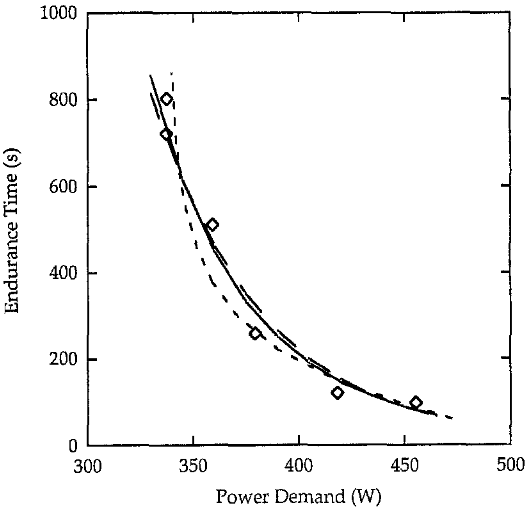

Fig. 1 Empirical power (Eq. 1a, *solid line*) and exponential curves using duration (Eq. 2, *dashed line*) and power (Eq. 3, *dotted line*) as the dependent variable

therefore the data that are fitted to the model rather than the model which is fitted to the data. The parameters of a systems model are biologically meaningful, which in the context of this paper means they represent descriptions of the actual metabolic processes in action during exercise. As a first example these requirements are satisfied over a time range of 1–15 min to an acceptably accurate degree by the simple hyperbolic model, which linearly relates power setting P to the reciprocal of endurance time, $1/t$. More complex hyperbolic type models also satisfy these requirements.

The simple hyperbolic model and the critical power test

This model relating t and P is a two component systems model based on the following two assumptions.

1. There exists an upper limit to the constant power output that could be maintained indefinitely and without exhaustion (at least in theory), depending only on renewable aerobic energy supply. This limit is known as the critical power (Monod and Scherrer 1965), or the threshold of fatigue (Bigland–Ritchie and Woods 1984) and is measured in watts.
2. There is a component of anaerobic energy supply of fixed capacity, known as the anaerobic work capacity, or energy reserve (Monod and Scherrer 1965), measured in joules. This energy is available on demand and can be utilised at as fast or as slow a rate as may be required. Exhaustion occurs when this supply is totally depleted.

Thus for exercise demand in excess of the critical power limit, energy is available through a fixed anaerobic capacity, α, measured in joules, plus a maximal aerobic rate of delivery, β, measured in watts. Mathematically therefore, for fixed $P > \beta$, the total amount of work performed until exhaustion is given by:

$$W = P \cdot t = \alpha + \beta \cdot t \tag{4a}$$

which is the original linear model formulation by Monod and Scherrer (1965), used also by several others (Moritani et al. 1981; DeVries et al. 1982; Whipp et al. 1982; Nagata et al. 1983; Poole et al. 1988; Carnevale and Gaesser 1991; Swanson et al. 1992), and the subject of extensive review by Hill (1993). Dividing by t yields an alternate linear formulation:

$$P = \alpha \cdot t^{-1} + \beta \tag{4b}$$

which is also used by researchers (Poole et al. 1986; Gaesser and Wilson 1988; Housh et al. 1989; Pepper et al. 1992). More correctly, since t is the dependent and P the independent variable (Gaesser et al. 1990) one should write:

$$t = \alpha/(P - \beta) \tag{4c}$$

which is the formulation giving rise to the term "hyperbolic". In all cases, good fits of data to one or other of these model variants have been claimed, such as linear correlations of 0.90 or better (Gaesser and Wilson 1988; Poole et al. 1988), though other goodness of fit measures (such as the relative error of regression) are not given. Nevertheless, it is not clear which of the variants is the best to use (Gaesser et al. 1990; Smith and Hill 1992; Hill 1993).

The critical power test is the exercise testing procedure utilised by many of the above-mentioned researchers to provide data for estimating anaerobic work capacity and critical power. In principle, four or five constant power tests to exhaustion, usually on the cycle ergometer, are performed by the subject(s), in random order and with sufficient spacing only to allow rest and full recovery. Values of P are normally chosen for each subject such that exhaustion is likely to occur in times less than 15 min or so. One or other of the above equations are fitted statistically to the data to obtain the parameter estimates which provide a characterisation of the work performance of each subject.

Extensions of the hyperbolic model

If the hyperbolic model is to be used as a simple illustrative systems model and/or to predict t given moderately high values of P, then it is clearly quite adequate over the stated range. However applied physiologists recognise that the relationship between P and t is not as simple as the hyperbolic model suggests, and a number of other considerations become relevant, particularly if the model is to be used for a more detailed study of the physiological mechanisms of muscular work and fatigue, or to predict t for very high or low values of P. Extensions to take these factors into account may include model segments representing each of the four fuel sources, and/or representing the kinetic delay in availability of one or more of each fuel type, and/or recognising that in prolonged work, critical power (or maximal oxygen consumption, $\dot{V}O_{2\max}$) may not truly represent a continuously attainable maximal rate of aerobic work performance.

The simplest modification to the hyperbolic model is obtained by relaxing the constraint that the time asymptote of the hyperbola defined by Eq. 4c should necessarily be at zero. Thus we propose:

$$t = \alpha/(P - \beta) + k \tag{5}$$

where k, measured in seconds, is the new location of the horizontal time asymptote.

Morton (1996) shows that k is significantly less than zero, which means that the hyperbola crosses the $t = 0$ axis at some value $P = P^*$, which has the natural interpretation of a maximum instantaneous power. Equation 5 can therefore be reparameterised to include P^* in place of k, which enhances its physiological interpretation. Beyond P^* on the cycle ergometer, the subject is unable to turn the pedals, and no work is done. Estimates of the aerobic component (α) from this model are higher and of the aerobic component (β) are lower, than those obtained from the original model (Eq. 4c). Also, it can be shown that at exhaustion, the anaerobic capacity is not all consumed, and that the peak power that can be developed declines from P^* to β as the anaerobic capacity empties progressively during exercise.

Another simple modification of the hyperbolic model by Wilkie (1980, 1981), takes the form:

$$P = \alpha \cdot t^{-1} + \beta[1 - \tau(1 - e^{-t/\tau})/t] \qquad (6)$$

The correction factor applied to the critical power parameter β recognises the fact that aerobic power does not rise instantly to its full rate at the onset of exercise (Ettema 1966), but approaches it with a kinetic time constant τ of around 50 s. Although this term rapidly becomes negligibly small, it is important in exercise of brief duration. Even so, Wilkie noted the fit to be not as good at extremely short durations, or for exercise lasting over 15 min.

A more complex extension has been derived (Péronnet and Thibault 1987; Péronnet et al. 1987; Péronnet and Thibault 1989) involving both the kinetic delay in the aerobic power supply, and the progressive reduction of aerobic power able to be sustained for exercise durations in excess of 7 min (Ettema 1996; Péronnet and Thilbault 1987). Adjustment for the latter incorporated a second exponential term to represent the decrease in the proportion of critical power (or of $\dot{V}O_{2\max}$) which could be maintained over longer and longer durations. In doing so, they produced an index of endurance capability (Péronnet et al. 1987) based on the rate of this decrease with increments in the natural logarithm of t. Their extension takes the form:

$$P = [S/t(1 - e^{-t/20})] + \frac{1}{t}\int_0^t [\mathrm{BMR} + B(1 - e^{-x/30})]\,\mathrm{d}x \qquad (7)$$

with t and x both measured in seconds; where S is the energy from anaerobic metabolism available to the runner, BMR is the basal metabolic rate and B the difference between peak instantaneous aerobic power and BMR.

Although applied to running races rather than to cycle ergometry, the model was extremely accurate in predicting marathon performance and world record running times across a range of distances from 60 m to 42.195 km. The average absolute percentage error was 0.91% for males and 1.32% for females.

Another complex extension producing a hyperbolic type of relationship between P and t has been derived by Morton (1986, 1990). This is derived from a three component system comprising phosphagen utilisation, anaerobic glycolysis and aerobic power, designed originally to represent the bioenergetic processes occurring during exercise. Like the models of Wilkie and Péronnet, it also involves a kinetic delay in the aerobic energy supply. Like Péronnet's model, it also recognises that the maximum sustainable aerobic power declines progressively with longer and longer durations. However the decline is ultimately to an asymptote at around 84% of $\dot{V}O_{2\max}$ which therefore represents a genuine sustainable critical power, rather than declining ultimately to zero as in the Péronnet model. Morton's model is represented, for a "typical" male subject, by the equation:

$$P = \frac{4.5 + 0.17e^{-0.285t} - 3.333e^{-0.00265t}}{0.0216 + 0.00106e^{-0.285t} - 0.0209e^{-0.00265t}} \qquad (8)$$

for $t > 6$ s, displayed as Fig. 2. A critical power ($t \to \infty$) of 208 W is predicted and a maximally achievable power of 972 W for $t \leq 6$ s.

An additional feature of Morton's model is the inclusion of a parameter representing the much debated anaerobic threshold. In the power-endurance relationship, this manifests itself as a time delay in the onset of anaerobic glycolysis. Also unlike any of the above, Morton's model recognises that exhaustion may occur before the anaerobic capacity is totally depleted. As a general rule, anaerobic capacity is much more depleted at exhaustion after long-duration exercise not much above the critical power, while for high-intensity short-duration exercise, anaerobic capacity may still contain significant reserves at the time of exhaustion (Ettema 1966; Saltin and Karlsson 1971).

Also of interest are the asymptotic values of these models. Most have the realistic property that for very long durations as $t \to \infty$, the power asymptote is not zero. At the other time extreme, only the exponential model (Hopkins et al. 1989; Mayhew et al. 1992) and the three component model (Morton 1990) predict

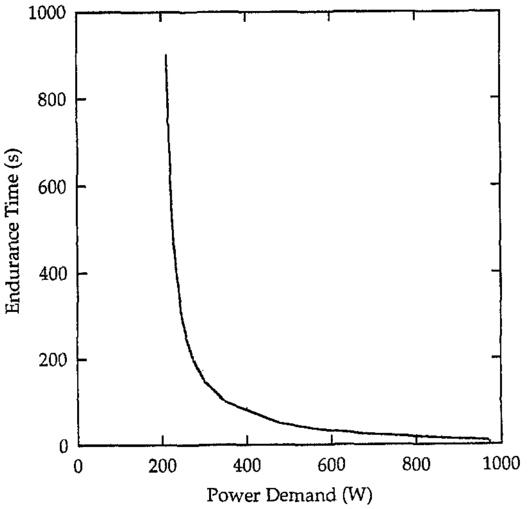

Fig. 2 An illustration of the endurance-time curved derived from Morton's (1990) three component bioenergetic model, Eq. 8

finite P for $t \rightarrow 0$. In the latter case t is approximately 6 s for the maximum attainable anaerobic power.

Other forms of dynamic and static exercise

Dynamic exercise is performed when a muscle or group of muscles contracts resulting in a force which produces usually rhythmic movement, and, as a consequence, work is performed. More specifically, dynamic work is the product of the force and the distance through which the point of application is moved in the direction of the force. More commonly it is defined as the time integral of the power output over the duration of the exercise. It is in this sense that it is used in the hyperbolic model.

Static or isometric exercise on the other hand is performed when usually continuous muscle contraction is required to maintain a fixed position against an opposing force. Squeezing a handgrip spring, or supporting weights are two of the more commonly studied types of static exercise. Strictly speaking, no mechanical work is performed since the force produces no movement, but analogously the time integral of force (rather than power) is regarded as static work.

Eccentric exercise

There have been studies of endurance for both concentric and eccentric muscular contractions using a modified cycle ergometer (Knuttgen et al. 1982; Knuttgen 1986). For a given power output, eccentric exercise power (maintaining resistance at constant pedal frequency against a reverse force) could be maintained for significantly longer than concentric exercise power. However eccentric contractions are known to cause greater muscle filament damage and a rise in serum enzyme activity, and to result in more severe delayed onset muscle soreness.

Ramp exercise on the cycle ergometer

Those doubts about the true nature of the critical power giving rise to the extensions of the hyperbolic model described earlier may be due to the omission of obtaining larger sample values of t for power settings not much above critical power, or perhaps the inaccuracy inherent in such determinations. The assumptions of the hyperbolic model have been adapted for ramp exercise on the cycle ergometer by Morton (1994) where a series of ramp slopes, r in W/s, can be chosen to ensure exhaustion in reasonably short times. Endurance is therefore a function of r rather than P, and results in an equation of form:

$$t = \beta / r + \sqrt{2 \cdot \alpha / r} \tag{9}$$

though this equation has yet to be verified experimentally. Morton's three component model has also been utilised to predict t for a given value of r (Morton 1990), though the derivation is complex.

Running

A straightforward transposition of the hyperbolic model and critical power test has been made for running on the treadmill (Hughson et al. 1984) and studied (Ettema 1966; Housh et al. 1991; Sid-Ali et al. 1991; Housh et al. 1992; McDermott et al. 1993). Velocity takes the place of power as the independent variable, and critical power is replaced by the critical velocity, denoted β^*. The total mechanical work involved in running for various times and at various speeds is awkward to determine, but the product of (constant) velocity and time, measured in distance units, is regarded as the proxy variable for work. Thus anaerobic work capacity is replaced by an anaerobic distance capacity, denoted α^*. Good fits to data (correlations from 0.979 to 0.997) are reported (Hughson et al. 1984) by an equation of the form:

$$V = \alpha^* \cdot t^{-1} + \beta^* \tag{10}$$

A variant of this test procedure has been proposed by Hopkins et al. (1989) where increasing slope on the treadmill at constant velocity takes the place of increasing velocity at constant slope. The approach using incremental velocity at constant slope is a closer analogue to the original critical power model. In addition, the critical velocity parameter is an extremely relevant one for the very popular field of ultra endurance running. On the other hand increasing slope at constant velocity has the advantage that, by trigonometric calculations, the actual power developed can be calculated and this may be preferred from the ergonomic point of view. However practical problems may arise in attempting to get short-duration data at steep slopes.

Swimming

Another straightforward transposition of the hyperbolic model and critical power test has been made by Wakayoshi and co-workers for swimming, both in the pool (Wakayoshi et al. 1992a, 1993a, b) and flume (Wakayoshi et al. 1992b). As with running, critical power is replaced by critical velocity, and anaerobic work capacity by anaerobic distance capacity, and the same equation is utilised. Very good fits are obtained to data from competitive swimmers, and β^* (pool) correlates well with β^* (flume). Others have also examined endurance in the swimming mode (Kennelly 1906; Biggerstaff et al. 1992; Lane et al. 1994; Steward et al. 1994).

Weightlifting

A power-endurance type relationship has been examined for weightlifting (Mayhew et al. 1992). A measure of weight lifted, expressed relative to the one repetition maximum, %1 − RM, takes the place of the independent variable power. The number of repetitions, R, performed in 1 min or until exhaustion if it occurred earlier which it almost always did, takes the place of t. An empirical exponential equation

$$\%1 - \mathrm{RM} = \gamma + \delta e^{-\varepsilon R} \tag{11}$$

was fitted successfully to the data. The parameter γ can be likened to a critical weight, and $\gamma + \delta$ is approximately (a slight underestimate of) the one repetition maximum.

Helmet loading

The application of the strength-endurance relationship in skeletal muscle to helmet design and loading in aviators has been described by Petrofsky and Phillips (1982), though the presentation of the relationship was graphical only. These results were later extended (Phillips and Petrofsky 1986) into a complex three-dimensional model of neck muscle endurance and fatigue as a function of helmet loading; loading being expressed not only as a function of weight, but also of the centre of gravity location.

Other exercise forms

In principle, the hyperbolic model and critical power test or its variants can be transported for any form of exercise in which endurance time (or its analogue) at some measurable power (or one of its analogues) can be determined. One obvious exercise that does not appear to have been recently reported is rowing, though kayaking has been (Ginn and MacKinnon 1989; Clingeleffer et al. 1994). This should not present a problem since rowing ergometers are common, though some aspects of intermittent work mentioned above may impact on the relationship between P and t. Walking, skating and various forms of equine locomotion have also been considered (Kennelly 1906).

Models of static endurance

It appears that the simple hyerpbolic model has not been applied to static exercise. If it were to be, then the concepts of critical power and anaerobic work capacity would need to be reinterpreted say as "critical force", analogously denoted by β^f, and "anaerobic tension capacity", analogously denoted by α^t. However, this apparently simple transposition of the hyperbolic model is not appropriate without some modification to deal with the occlusion of intramuscular blood flow due to local muscle swelling which occurs in static contraction. However some complex empirical hyperbolic type models have been obtained.

The development of an empirical model:

$$t = -1.5 + \frac{2.1}{(F/F_{\max})} - \frac{0.6}{(F/F_{\max})^2} + \frac{0.1}{(F/F_{\max})^3} \tag{12}$$

where t is the endurance time, F the force maintained, and $F_{\max}$ is the maximum force attainable (maximal voluntary contraction) by the subject is attributed to Rohmert (Simonson and Lind 1971). This curve does not tend to a critical force asymptotically, but approaches zero for very long durations. Rohmert however still considered that forces of less than 15% of maximum could be held continuously. When a maximal force is exerted, holding time is of the order of 6 s. For static exercise, a hyperbolic type model of the form:

$$t = k(F/F_{\max} - \beta^f)^{-n} \tag{13}$$

has been developed by Monod and Scherrer (1965), where k is a constant equal to about 2.5, critical force β^f is expressed relative to $F_{\max}$ and n is an exponent equal to about 2.4. Both this and Rohmert's models subsume the question of blood flow occlusion in their empiricism, but it has to be said that fits to their data for endurance times up to about 10 min appeared to be satisfactory. (Graphical presentations only, and no goodness of fit statistics were published in these early papers.)

Intramuscular blood flow occlusion occurs because when a muscle contracts, its fibres which run in a curve attempt to straighten, causing a rise in intramuscular pressure. This may or may not be accompanied by muscle swelling or length changes. Increased pressure restricts the flow of blood to the muscles through the vessels, thus restricting the aerobic energy supply mechanism. If the force of contraction is high enough, the pressure may be sufficient to totally occlude blood flow, the effect on the relationship between F and t being the same as that of artificial occlusion with an inflatable cuff (Royce 1958). Thus the "parameter" β^f is itself a function of the force being maintained. Several studies have investigated the level of force production resulting in total occlusion (Carlson 1969; Heyward 1975). However, occlusion is not an on/off mechanism, for at certain force levels blood flow is only partially occluded.

An extension of the hyperbolic systems model taking occlusion into account has been detailed by Morton (1987). As with the bioenergetic model (Morton 1990), it includes a kinetic delay in the aerobic energy supply via the circulation, and the facility to allow exhaustion without total depletion of α^t at higher values of F. Two critical forces are defined. β_1^f is the upper limit of force which can just be sustained continuously, and below which the notion of exhaustion is vacuous. β_2^f is the force at which blood flow occlusion becomes total. For forces above β_2^f, endurance times can be represented by a simple hyperbola:

$$t = \frac{\alpha^t}{F} - \frac{\alpha^t}{F_{\max}} \tag{14}$$

For values of F between β_1^f and β_2^f, it is assumed that blood flow declines linearly to zero. It is this that gives rise to the complexity of Eq. 2 in Morton (1987). Nevertheless, a fit of data from the literature (Rohmert 1960; Simonson and Lind 1971; Heyward 1975) is extremely good, with $R^2 = 0.98$.

Intermittent static exercise

It is clear that with intermittent static contractions, the muscles would benefit from an interval of relaxation between contractions, and that this would prolong t for a given value of F. This has been reported for handgrip muscles developing a constant force, where a decrease in the contraction time relative to the relaxation time produces an increase in endurance (Park and Rodbard 1962; Rodbard and Bekay-Pragua 1968). The same result has also been reported for contractions of the human diaphragm during inspiration against fixed transdiaphragmatic pressures (Bellemare and Grassino 1982).

Modifications to the equation for endurance to account for intermittent static work (Monod and Scherrer 1965) have been made by introducing a term p. This term represents a duty factor reflecting the proportion of the duty cycle spent when force is developed in the contracted state:

$$t = p \cdot k(F/F_{\max} - \beta^f)^{-n} \tag{15}$$

As above, k is around 2.5, and both β^f and n are acknowledged as functions of p. The relationship between β^f and p was not determined, though it would clearly be inverse, but n was estimated at about $2.4/p$. What is not evident is how this relationship might be affected by the length of the cycle time for fixed values of p, though Monod and Scherrer seem to imply there would be no effect.

Parameter Interpretations

As a general statement, some or all those parameters used for empirical models do not permit any meaningful biological interpretation. Indeed it is difficult to see how any meaning can be put, for example, on the parameters of Rohmert's model Eq. 11 (Simonson and Lind 1971). Apart from an interpretation of the parameters β^f and $F_{\max}$, the same can be said for the other parameters of Monod and Scherrer's model Eq. 14 (Monod and Scherrer 1965). This has not prevented authors attempting a post hoc interpretation, such as for the parameters of the log model Eq. 2 (Gleser and Vogel 1973), who attempted to relate the parameters to anaerobic metabolism and glycogen stores. Some empirical models (Mayhew et al. 1992) do, as described above, permit a reasonable interpretation. Even the parameter c of Eq. 3 which, if regarded as a rate of decline in lifting strength per repetition, can be interpreted as some sort of index of fatiguability. However, these post hoc interpretations are not sufficient to convert an empirical model into a systems model.

Parameters of the simple hyperbolic model

The concepts of both critical power, β, and anaerobic work capacity, α, as described above can be clearly understood, and the critical power test is regarded as valid and reliable as a means of assessing aerobic and anaerobic capacity (Nebelsick-Gullett et al. 1988; Green and Dawson 1993; Green et al. 1994). Nevertheless, a significant body of research is reported as examining their meaning. This is mostly by means of their implications for work performance, and the determination of their correlates amongst other well-known parameters of aerobic and anaerobic function. As a result, both parameters, particularly critical power, have come under fairly close scrutiny.

Some studies (Wilkie 1980, 1981) regard critical power, β, as the maximal aerobic power, or $\dot{V}O_{2\max}$. However it is well known that exercise demanding an oxygen consumption of $\dot{V}O_{2\max}$ cannot be maintained very long; say up to 10 min in normal individuals. Critical power is supposed to represent the maximal rate of fatigueless work, yet several studies demonstrate that exercise at critical power cannot be sustained for very long (Housh et al. 1989; Jenkins and Quigley 1990; McLellan and Cheung 1992; Overend et al. 1992; Pepper et al. 1992), no longer than 33 min on average. Thus the critical power test clearly overestimates the highest power continuously maintainable, in that the model defined by Eqs. 4a–c does not apply very well to durations in excess of 15–20 min. At the other end of the scale, recent unpublished work (D.G. Jenkins and D. Bishop, personal communication) suggests that for very short durations, the amount of work predicted by Eq. 4a is not achieved. It is these inapplicabilities that have led to the extensions to the hyperbolic model described previously. Nevertheless, in general terms, the critical power parameter, β, has an interpretation closely linked to the aerobic fuel supply mechanism. Actual values for critical power derived from a number of studies range between 144.5 ± 20.9 W for a sample of eight females (Moritani et al. 1981) to 31.0 ± 27.9 W for a sample of eight highly trained male cyclists (Jenkins and Quigley 1990). Since critical power varies with body mass, it may be better for comparative purposes to record values on a per kilogram basis. These studies however do not provide this detail of information.

A variety of correlates for critical power have been investigated and found to be significant. It correlates moderately to well with most of the so-called threshold measures: ventilatory anaerobic threshold (Moritani et al. 1981; DeVries et al. 1982; Nagata et al. 1983; Talbert et al. 1991); lactate anaerobic threshold including the onset of blood lactate accumulation, individual anaerobic threshold and maximal lactate steady state (Ginn and MacKinnon 1989; Aunola et al. 1990; Housh et al. 1991; McLellan and Cheung 1992; Wakayoshi et al. 1992a, b, 1993a, b); and the fatigue threshold determine from integrated electromyogram data (DeVries et al. 1982, 1987). In one study, critical power has been found to be no different from $\dot{V}O_{2\,\max}$ (Housh et al. 1991), and the sum of critical power and anaerobic work capacity correlates with $\dot{V}O_{2\,\max}$ (Moritani et al. 1981; Gaesser et al. 1990). Nevertheless, the interpretation of critical power as a "threshold" type measure has been questioned (Housh et al. 1991).

Anaerobic work capacity, α, in the model represents an aggregate work performed by non-renewable fuel supplies regardless of the rate at which these reserves are utilised (Whipp et al. 1982). As such, it is clearly some measure of anaerobic capacity, though specifically mentioned in some (Green et al. 1983) but not other (Vandewalle et al. 1987) reviews of standard anaerobic exercise tests. Nevertheless, it has been found to correlate with several other measures of anaerobic capacity; results from the Wingate test (Nebelsick-Gullett et al. 1988; Vandewalle et al. 1989); the ability to perform repeated intermittent high-intensity exercise (Jenkins and Quigley 1991); and a distance running ability test (Bulbulian et al. 1989; Kolbe et al. 1995). The concept of the maximal accumulated oxygen debt has been introduced (Medbo et al. 1988) to describe anaerobic capacity; this quantity correlates with Wingate test results (Vandewalle et al. 1987). However correlates between it and anaerobic capacity are unclear (Jenkins and Quigley 1991; Hill et al. 1993) and unpublished data (J.I. Medbo, personal communication) suggest that this parameter is only about half of the maximal accumulated oxygen debt. Studies reveal that anaerobic work capacities have been estimated ranging from 8616 (756) J for a sample of eight females (Moritani et al. 1981) to 18 450 (7277) for a sample of eight highly trained male cyclists (Jenkins and Quigley 1990). In any event, the interpretation of anaerobic work capacity as a measure of anaerobic capacity has been questioned also (Johnson et al. 1990; Housh et al. 1992).

Parameters of extended models

Of the extended models described above, only a few have parameters which, in addition to critical power β and anaerobic work capacity α, can be meaningfully interpreted. These include the BMR (Péronnet et al. 1987), an anaerobic threshold (Morton 1986) and a kinetic parameter for the rise in $\dot{V}O_2$ (Wilkie 1980, 1981; Morton 1986). Both these parameters are well known to exercise physiologists. The anaerobic threshold can be considered as describing some form of conceptual boundary between work rates which are performed aerobically and those performed anaerobically. It is the subject of much debate (McLellan 1987; Walsh and Banister 1988). Ventilatory kinetics, involving the rate and manner in which aerobic metabolism rises to meet the imposed work demand, is also a much studied area (Hughson et al. 1988; Hughson and Kowalchuck 1995). A few others have parameters which can be likened to some biological attributes, such as the index of endurance capability (Péronnet and Thibault 1987), or the force at which total blood flow occlusion occurs in static muscular contraction (Morton 1987), or a maximum instantaneous power (Morton 1994, 1996), etc.

Secondary factors affecting the parameters

Many studies examine the influence of a variety of determinants on the parameter values. All but one of the studies quoted have used male subjects. Only one study (Clarke 1986) tested both sexes in static work performance, finding males to have higher maximum strength and absolute endurance. However, females were found to have greater relative endurance; that is, females could maintain a given fraction of their maximum for longer than males. Six to 8 weeks of regular continuous (Gaesser and Wilson 1988; Jenkins and Quigley 1992) or interval (Gaesser and Wilson 1988; Poole et al. 1988, 1990) training, have been found to increase critical power significantly. No significant change to anaerobic capacity was evident in these studies. Strength and high-intensity training have been found to increase anaerobic capacity, but not critical power (Jenkins and Quigley 1993; Stokes et al. 1993). However, the longer term (chronic rather than acute) effects of training on critical power and anaerobic work capacity have not been documented. Overend et al. (1992) found that both critical power and anaerobic capacity were lower in older individuals. However, when critical power is expressed as a percentage of $\dot{V}O_{2\,\max}$, it is higher in the elderly. As with females, elderly were found to have greater relative endurance. Carnevale and Gaesser (1991) found the subjects pedalling at 100 rpm had significantly lower critical power than at 60 rpm, whereas anaerobic capacity was unrelated to the number of revolutions per minute. The most accurate estimates were obtained when subjects were allowed to select and, within each exercise bout, vary their pedal cadence (Hill et al. 1994).

Although stronger individuals can exert a greater maximal handgrip strength, weaker individuals have greater relative handgrip endurance (Tuttle et al. 1955; Carlson 1969; Carlson and McCraw 1971), an effect similar to sex and age differences (Clarke 1986; Overend et al. 1992). These results are supported by a study (Heyward 1975) which found a significant negative correlation between static handgrip strength and endurance times at various percentages of maximal strength. It was suggested that stronger individuals with greater muscle mass produce higher static forces and intramuscular pressures, leading to a higher degree of blood flow occlusion and the resulting lower relative endurance. In dynamic exercise it would be logical to except no such strength effect, but this has not been reported either way.

A study designed to determine critical power and anaerobic capacity for arm cranking exercise, with and without prior exhaustive leg exercise, indicated no significant effect on critical power, but a significant diminution of anaerobic capacity (Swanson et al. 1992). Except for two studies, the parameters critical power and anaerobic capacity have been determined exclusively under normoxic conditions. With lower than normal inspired oxygen concentrations, critical power decreased significantly whereas anaerobic capacity was unaffected (Moritani et al. 1981; Whipp et al. 1982). With higher oxygen concentrations critical power increased significantly, while anaerobic capacity was again unaffected (Moritani et al. 1981; Whipp et al. 1982). Endurance performance at critical power appears subject to learning effects (Scarborough et al. 1991) in that endurance at critical power is significantly longer at the second attempt. This effect also manifests as a significantly higher critical power estimate in a second series of trials on the same subjects (Smith et al. 1991).

Methodological questions

In almost all applications of the critical power test and its variants quoted above, the data analysis have involved linear regression in one form or another. In theory, this usage can be questioned; as detailed below. Whether or not this gives rise to serious errors in practice is another matter.

The appropriate equation

The usual linear regression assumptions include the following; that the independent variable is observed without error, and the dependent variable is observed with additive error (usually assumed to be "white noise"). Nevertheless, statistical theory does show that errors in the independent variable are of minor importance provided they are quantitatively small, say < 10% of the variation or range. In the usual testing procedure on the ergometer, the power setting is set to a known value, that is, it is observed effectively without error; whereas endurance time is observed with a stochastic error and so also is the total work performed. Thus it should be immediately apparent that the most common linear equations used for the estimation of critical power and anaerobic work capacity,

$$W = \alpha + \beta \cdot t \qquad (4a)$$

$$P = \alpha \cdot t^{-1} + \beta \qquad (4b)$$

do not satisfy these usual requirements, and neither would another linear equation,

$$t = -\alpha/\beta + \beta^{-1} \cdot W \qquad (16)$$

which has not been utilised.

However the equation given by

$$t^{-1} = -\beta/\alpha + \alpha^{-1} \cdot P \qquad (17)$$

may be appropriate, though it has not been used, and because of the manner in which the parameters appear, would not likely produce a very precise estimate of critical power. Arguably the most appropriate equation to be fitted is:

$$t = \alpha/(P - \beta) \qquad (4c)$$

which, in recent times, appears to be more commonly used. It is of course non-linear, and requires an appropriate regression fitting routine, a procedure perhaps less familiar than it deserves to be, even though available in most commonly used statistical packages. Currently available information suggests that preferred choice between equations is not clear (Gaesser et al. 1990; Smith and Hill 1992; Hill 1993).

Error structure

The term "white noise" for the additive error of observation in the dependent variable is usually interpreted to mean independent observations, normally distributed, with zero mean and constant variance. The independence requirement is easily satisfied by randomising the order of the tests and allowing sufficient interval for rest and recovery between them. Bishop and Jenkins (1996) suggest the rest periods of 3 h are sufficient, provided subjects are first familiarised with the tests. The normality assumption, though difficult to justify, is probably reasonably well satisfied in most cases; and the zero mean requirement also presents no major difficulty. However, the homoscedastic nature of the error may be a problem. It is an attractive conjecture that the variability amongst repeated observations of time to exhaustion at a given high power, when endurance is short, is likely to be much smaller than at low power when endurance time is long. Indeed, evidence is just now emerging that this seems to be the case (McLellan et al. 1995). That is, the variance of the errors is directly dependent on t. Direct proportionality would make the standard deviation of the errors proportional to $\sqrt{t}$, a not uncommon occurrence.

The violation of any of the above assumptions is usually discovered by residual plotting, a tactic not reported by a single one of the studies quoted. If heteroscedasticity is suspected, weighted least squares or an alternate error structure should be invoked for use in the regression fitting procedure.

The choice of power settings

Most studies, for practical reasons such as time and resource constraints, or to avoid learning and training effects, use four or five values of P over a range such that exhaustion occurs within a reasonable time, say 2–15 min. A recent study (Wakayoshi et al. 1993a, b) has used only two in the estimation of critical velocity in swimming, as has another study (Clingeleffer et al. 1994) with kayakers. While this does provide an estimate, it suffers the distinct disadvantage of giving no indication of its standard error, in which case the choice of location of the two observations becomes crucial. The optimal procedure is to place them as widely apart as is practicable (Housh et al. 1990).

Validity and reliability

In general, the hyperbolic model and its derivatives are claimed to be valid and reliable on the grounds that parameter estimates are found to be stable and within the normal range for humans, are found to correlate well with other similar parameters, and so on as described. One tactic that has not been reported is the training or split-half approach. In such studies the model is estimated using data from one half of the sample, selected randomly, called the training sample, and then cross-validated against the remaining portion of the sample. This approach, assuming success, would seem a potentially useful future contribution to the topic material.

Summary

Earlier empirical modelling yielded a variety of sometimes complex formulations of the relationship between power output and endurance time in both dynamic and static work. In recent years the most widely used model is the two component (aerobic/anaerobic) hyperbolic systems model, characterised by a critical power (or maximal rate of fatigueless work) and an anaerobic work capacity. This model has a simple appeal, its parameters are well understood, and it has always been found to be a good fit to data over the 2- to 15-min range. Extensions to the hyperbolic model incorporate a more realistic representation of the human bioenergetic system, and fit data over a wider range of power and duration, from 5 s to 2 h.

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