

# Laser Cavities and Polarization Optics for Soft X-Rays and the Extreme Ultraviolet

John Paul Braud

Research Laboratory of Electronics, Massachusetts Institute of Technology,  
77 Massachusetts Avenue, Room 38-280, Cambridge, MA 02139, USA

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**Abstract.** Addressed here are polarization optics for extreme-ultraviolet and soft X-ray wavelengths, especially as relevant to laser cavities. It is pointed out that the whisper-gallery mirrors studied by Vinogradov can serve as weak polarizers and, more importantly, as birefringent elements. The application of multilayer technology to polarizing mirrors and beamsplitters is also considered. It is shown that multilayer beamsplitters can function both as reflective and transmissive polarizers. Their behavior is surprising in some cases, with the same polarization being preferred in both reflection and transmission. Three polarizing cavity schemes are proposed, each incorporating a polarizing beamsplitter as its output coupler. Cavity optimization issues are discussed.

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While the vector aspects of electromagnetic radiation typically receive rather less attention than do its wave properties, polarization can nonetheless be an interesting and often important feature at any wavelength – from radio to the ultraviolet and beyond. Polarimetry and ellipsometry can serve, for example, as highly sensitive diagnostic tools. Furthermore, even when polarization issues are not central to the function of a particular system, they may require attention in its actual implementation.

As of yet however, polarization has not often been exploited in the extreme ultraviolet (EUV) and soft X-ray regimes. This is due in part to the fact that direct sources of polarized radiation are not easy to come by at these wavelengths. One example of such a source is the synchrotron, but its size and expense limit its use. Additionally, polarizers and other devices for altering and analyzing the state of polarization have developed only slowly [1, 2]. As with other types of short-wavelength optics, the technology is severely constrained by the highly absorptive and nearly non-refractive behavior of most materials. Sources and optics for EUV and soft X-ray radiation are however becoming more widely available, and as they do so, it

seems likely that polarization issues will assume more significance.

The present paper deals with a promising source of short-wavelength radiation: the X-ray laser. In particular, it addresses the effects of cavity optics upon polarization. The usual cavity schemes, which involve only normal-incidence optics, clearly will not yield polarized radiation. X-ray laser cavities can be designed to produce a polarized output, but the task is trickier than at longer wavelengths. A primary difficulty is that the short lifetime of a typical X-ray laser amplifier allows for at most only a few trips around the cavity, so that optics of strong polarizing effect are required. The present paper treats several varieties of polarizing optics, including both the so-called whisper-gallery mirrors studied by Vinogradov and colleagues [3–7], as well as the more conventional mirrors based on multilayer technology.

In all of the examples discussed here, the operating wavelength is taken to be 194 Å, which corresponds to the  $4d \rightarrow 4p$  transition in nickel-like molybdenum. Hagelstein [8] proposed to use electron-collisional excitation of this line as the basis of a small-scale X-ray laser; the intended pump is a 10-J Nd:glass slab.

The organization of the paper is as follows. Section 1 points out that whisper-gallery mirrors have an effect on polarization, acting both as polarizers (albeit weak ones) and as birefringent elements. The former effect can actually be found in one of Vinogradov's own papers [5], though it is not emphasized there. Section 2 takes up the polarization properties of multilayer mirrors. As discussed by Lee [9, 10] and by Khandar and Dhez [11–13], multilayer mirrors designed for reflection at a 45° angle of incidence can make efficient polarizers. We show here that a similarly-designed multilayer beamsplitter can function simultaneously as a reflective and as a transmissive polarizer. Its properties are surprising – the same polarization is preferred both in reflection and transmission. Such a beamsplitter would serve well as the output coupler of a polarizing laser cavity. Section 3 describes three specific cavity schemes based on this idea; this same section also addresses some issues of cavity optimization. Section 4 is a brief summary and conclusion.

## 1. Whisper-Gallery Mirrors

### 1.1. Description

A whisper-gallery mirror (WGM) is an optical structure which, by means of a series of glancing angle reflections from a concave surface, can deflect light through a large total angle; see Fig. 1. The WGM has been proposed for various uses in the extreme ultraviolet and soft X-ray regimes, most notably as a laser cavity [15, 3]. As an alternative to multilayer technology [16, 17], WGM's would seem to yield comparable reflectivities. They would also offer some potential advantages, one being far greater bandwidth.

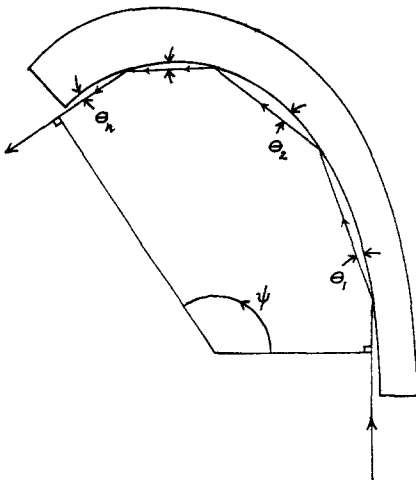


Fig. 1. Whisper-gallery mirror. The sizes of the grazing angles  $\theta_i$  are exaggerated

A WGM would ideally be constructed from a lossless dielectric material having a refractive index of less than unity. Then, if the many glancing reflections all occurred at angles below the so-called critical angle, the beam would be totally reflected at each bounce, resulting in a WGM of perfect reflectivity [18]. While most materials do in fact have a refractive index of less than unity at the wavelengths of interest, they are invariably photoabsorptive and, as a result, the beam loses some fraction of its power at each bounce.

### 1.2. Reflection at Glancing Incidence

The effects of a WGM upon polarization arise simply because the reflection coefficient associated with each bounce depends upon the state of polarization. For a monochromatic plane wave incident from vacuum onto a planar interface with a medium of complex dielectric constant  $\epsilon_1$ , the Fresnel coefficient relating the reflected field to the incident field is given by

$$R_1^{\text{TE}} = (k_{\perp 0} - k_{\perp 1}) / (k_{\perp 0} + k_{\perp 1}) \quad (1a)$$

or

$$R_1^{\text{TM}} = (\epsilon_1 k_{\perp 0} - \epsilon_0 k_{\perp 1}) / (\epsilon_1 k_{\perp 0} + \epsilon_0 k_{\perp 1}) \quad (1b)$$

for the case of transverse-electric (“s”) or transverse-magnetic (“p”) polarization, respectively. In (1), the notation  $k_{\perp i}$  refers to the component of the wavevector  $\mathbf{k}_i$  normal to the interface, with the subscripts 0 and 1 indicating the vacuum and material sides respectively. The projection of  $\mathbf{k}_i$  along the interface is denoted by  $\mathbf{k}_{\parallel i}$ , and naturally  $k_i^2 = k_{\perp i}^2 + k_{\parallel i}^2$ . In the limit of small glancing angles  $\theta \ll 1$  for the incident wave, one has  $k_{\perp 0} \approx k_0 \theta$  and  $k_{\parallel 0} \approx k_0$ . Phase matching along the interface requires  $\mathbf{k}_{\parallel 1} = \mathbf{k}_{\parallel 0}$ , and so we find  $k_{\perp 1}^2 = k_1^2 - k_0^2 = k_0^2 [(\epsilon_1/\epsilon_0) - 1]$ . Finally, defining the dimensionless  $\epsilon = \epsilon_1/\epsilon_0$ , the Fresnel coefficients can be approximated as

$$R_1^{\text{TE}} = - \left( 1 - 2\theta \frac{1}{\sqrt{\epsilon - 1}} \right) \quad (2a)$$

or

$$R_1^{\text{TM}} = - \left( 1 - 2\theta \frac{\epsilon}{\sqrt{\epsilon - 1}} \right), \quad (2b)$$

provided  $\theta \ll |\sqrt{\epsilon - 1}|$  in the TE case or  $\theta \ll |\sqrt{\epsilon - 1}/\epsilon|$  in the TM case.

With conventional glancing-incidence optics, in which typically only a few reflections are involved, it is not usually important to distinguish between  $R_1^{\text{TE}}$  and  $R_1^{\text{TM}}$  since each becomes essentially perfect in the limit of small glancing angles [19]. We shall see however that with whisper-gallery optics the distinction becomes important: the difference in the magnitudes of

the reflection coefficients results in polarization, and that in the phases gives rise to birefringence.

### 1.3. The WGM as a Polarizer

The polarizing effect of a WGM comes about because the TM mode loses slightly more power at each bounce than does the TE mode. The single-bounce reflectivity is given by  $r_1 = |R_1|^2$ , which becomes, using the same approximations involved in (2)

$$r_1 = 1 - 2B\theta, \quad (3)$$

where

$$B^{\text{TE}} = 2 \operatorname{Re} \{1/\sqrt{\varepsilon-1}\} \quad (4a)$$

and

$$B^{\text{TM}} = 2 \operatorname{Re} \{\varepsilon/\sqrt{\varepsilon-1}\}. \quad (4b)$$

The total reflectivity for a path through the whisper-gallery mirror is given by the product of the individual reflectivities for each bounce. Suppose the beam follows a planar path, along which it is deflected through a sequence of small deflections at grazing angles  $\theta_1, \theta_2, \dots, \theta_n$  to yield a total deflection of

$$\psi = 2\theta_1 + 2\theta_2 + \dots + 2\theta_n. \quad (5)$$

The net reflectivity is then given by

$$r = (1 - 2B\theta_1)(1 - 2B\theta_2) \dots (1 - 2B\theta_n). \quad (6)$$

Assuming that the loss at each bounce is small, it follows that

$$\begin{aligned} \ln r &\approx -2B\theta_1 - 2B\theta_2 - \dots - 2B\theta_n \\ &= -B\psi, \end{aligned} \quad (7)$$

or equivalently

$$r^{\text{TE}} = \exp(-2\psi \operatorname{Re} \{1/\sqrt{\varepsilon-1}\}) \quad (8a)$$

and

$$r^{\text{TM}} = \exp(-2\psi \operatorname{Re} \{\varepsilon/\sqrt{\varepsilon-1}\}). \quad (8b)$$

Notice that these net reflectivities depend upon neither the mirror radius nor the number of bounces, but instead upon only the total deflection angle and the mirror dielectric constant. A simple argument to explain this perhaps counter-intuitive result is as follows: the loss experienced at each bounce – as expressed in decibels, for example – is proportional to the glancing angle, and hence the accumulated loss is in turn proportional to the total angle of deflection.

It is clear from (3) and (4) that the TE mode generally experiences a smaller loss at each bounce than does the TM mode, for

$$B^{\text{TM}} - B^{\text{TE}} = 2 \operatorname{Re} \{1/\sqrt{\varepsilon-1}\} \geq 0, \quad (9)$$

with equality holding only in the lossless case. Thus, after a deflection through a total angle  $\psi$ , an initially unpolarized beam will be polarized by an extent

$$\begin{aligned} P &= (r^{\text{TE}} - r^{\text{TM}})/(r^{\text{TE}} + r^{\text{TM}}) \\ &= \tanh(\psi \operatorname{Re} \{1/\sqrt{\varepsilon-1}\}). \end{aligned} \quad (10)$$

As an example, consider reflection of 194 Å radiation from a WGM surface of solid krypton [20], for which  $\varepsilon \approx 0.93 + 0.0077i$  [21]. This value implies  $B^{\text{TE}} = 0.413$  and  $B^{\text{TM}} = 0.442$ , which for deflection through 180° yields reflectivities of  $r^{\text{TE}} = 27.4\%$  and  $r^{\text{TM}} = 25.0\%$ ; the corresponding polarization is  $P = 4.6\%$ . The effect is weak, but if enough cavity passes could be obtained in a laser cavity incorporating a WGM, the resulting beam would be highly polarized. Cavity schemes of greater polarizing effect are discussed in Sect. 3.

### 1.4. The WGM as a Birefringent Element

The relative phase shift between the TE and TM modes at each bounce is found by again referring to the reflection coefficients in (2). Their phases are given in the same small  $\theta$  limit by

$$\arg \{R_1^{\text{TE}}\} \approx 2\theta \operatorname{Im} \{1/\sqrt{\varepsilon-1}\} \quad (11a)$$

and

$$\arg \{R_1^{\text{TM}}\} \approx 2\theta \operatorname{Im} \{\varepsilon/\sqrt{\varepsilon-1}\}. \quad (11b)$$

The phase difference (TM minus TE) accumulated as the WGM deflects the beam through a net angle  $\psi$  is therefore

$$\Delta\phi = \psi \operatorname{Im} \{1/\sqrt{\varepsilon-1}\}. \quad (12)$$

This birefringence can be substantial. Again considering the krypton surface, a deflection of only 340° is required to obtain a quarter-wave phase shift.

## 2. Multilayer Mirrors and Beamsplitters

### 2.1. Reflective Polarizers

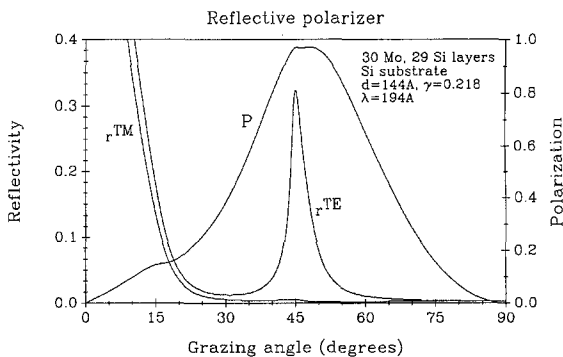
Reflective polarizers based upon multilayer technology have been suggested by Lee [9, 10] and demonstrated by Khandar et al. [11–14]. The principle behind their operation is to exploit reflection at Brewster's angle in order to suppress the TM mode while using an appropriately designed multilayer structure to provide good reflectivity for the TE mode. Recall that the Brewster angle may be characterized as that angle of incidence for which the reflected and transmitted beams are orthogonal; because materials have little refractive effect at the wavelengths of interest here, the Brewster angle is close to 45°.

The polarizers discussed by Khandar and Dhez [11–13] are Hf/Si multilayers optimized for use at a wavelength of 304 Å. However, based upon our intended wavelength of 194 Å, we present here some results of calculations for multilayers of Mo/Si, as used by Ceglio et al. [22–24]. The values assumed for the dielectric constants (relative to vacuum) are  $\epsilon(\text{Mo}) = 0.8115 + 0.1732i$  and  $\epsilon(\text{Si}) = 0.9611 + 0.007112i$ . These were obtained by reference to the tabulations collected by Lynch and Hunter [25] and Edwards [26] respectively; for each element, a cubic polynomial was used to interpolate the value at 194 Å from those at the four surrounding points specified in the tables.

The mirror is taken to be a periodic structure [27] of alternating Mo and Si layers deposited on a Si substrate. A total of 30 Mo and 29 Si layers is assumed; only slightly higher reflectivities result from an infinite structure. The mirror is then optimized by varying the values of both  $d$ , the thickness of a single Mo/Si bilayer, and  $\gamma$ , the fraction of the bilayer thickness taken up by Mo, so as to maximize the TE reflectivity obtained at 45°, the intended angle of incidence.

This process leads to a mirror with  $d = 144$  Å and  $\gamma = 0.218$ . The reflectivities at 45° are  $r^{\text{TE}} = 32.3\%$ , or  $-4.9$  dB, and  $r^{\text{TM}} = 0.525\%$ , or  $-22.8$  dB. (Recall that a ratio like  $r$ , which is a ratio of two powers, is expressed in decibels by taking its logarithm base 10 and then multiplying by 10.) Upon reflection from such a mirror, an initially unpolarized beam would have a TE polarization of  $P = 96.8\%$ . Figure 2 plots the variations in reflectivity and polarization as a function of the angle of incidence.

It should perhaps be mentioned that these calculations, as well as the similar ones which follow, assume the boundaries between the different layers to be perfectly sharp. As discussed in detail by Stearns [28], this is an oversimplification, since the nonideal interfaces actually achieved in fabrication can significantly degrade the mirror's reflectivity. The results here might



**Fig. 2.** Performance of a Mo/Si multilayer reflective polarizer optimized for reflection at 45°-incidence

still be fairly accurate, though, in spite of the idealization: similar calculations for a normal-incidence mirror give reflectivities that agree well with experiment [29]. It could be that the values assumed here for the dielectric constants of Mo and Si are slightly pessimistic and thereby yield realistic reflectivities.

## 2.2. Polarizing Beamsplitters

Hawryluk et al. [30, 31] have constructed normal-incidence beamsplitters for use as laser cavity output couplers. These devices are multilayer mirrors thin enough to allow partial transmission of an incident beam. This same idea could be applied to the 45°-incidence reflector to produce a polarizing beamsplitter.

The behavior of such a polarizing beamsplitter can be surprising. Experience with visible-light optics would lead one to expect this device to preferentially reflect the TE mode while preferentially passing the TM mode. When using Mo/Si at 194 Å, however, the behavior is different: the TE mode is preferred in both reflection and transmission. Presumably this is a general feature of multilayer polarizing beamsplitters made of any sufficiently lossy materials, but the author is currently unable to offer a physical explanation of the effect. Note that with the wavelengths and materials considered by Lee [9, 10], where the refractive indices are closer to unity and the absorptions smaller than those here, the phenomenon does not occur.

As an explicit example, consider an idealized beamsplitter consisting of 6 Mo layers and 5 Si layers, and for which  $d = 144$  Å and  $\gamma = 0.225$ . For 45°-incidence the resulting TE and TM reflectivities are 20.9% and 0.457% respectively, while the corresponding transmissivities are 19.2% and 12.7% respectively. For an unpolarized incident beam, the reflected beam would be TE polarized by 96% and the transmitted beam TE polarized by a more modest 19%. In terms that will be more useful in Sect. 3, the TM mode is suppressed by 16.6 dB on reflection and 1.8 dB on transmission. Figure 3 shows plots of this beamsplitter's reflectivities, transmissivities, and polarization behavior as a function of the angle of incidence. The calculations ignore the effects of overcoat layers, e.g. carbon, and substrate, e.g. silicon nitride [30, 31].

## 2.3. Bounds on Beamsplitter Performance

During the design of a beamsplitter for some particular application, the following question arises: for a given transmissivity, what is the best reflectivity that can be obtained by optimization of the mirror structure? The answer is obvious for perfect dielectrics but not for the lossy materials of interest here.

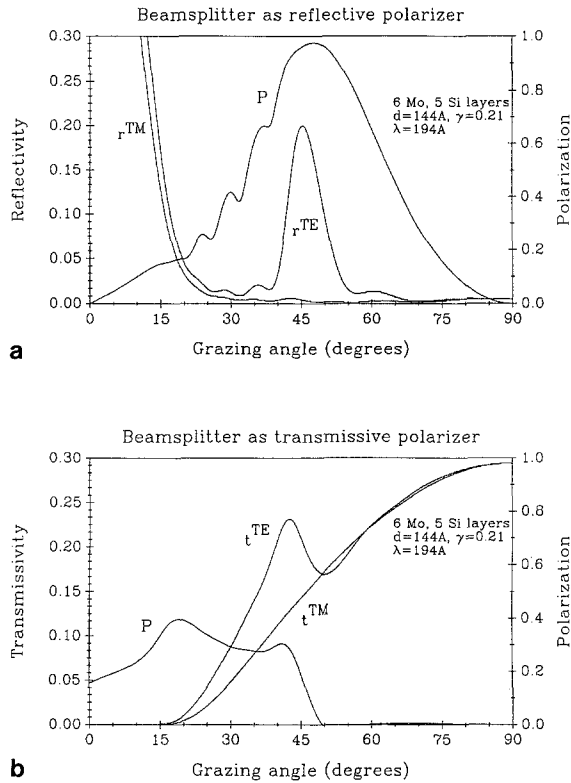


Fig. 3a, b. Performance of a 45°-incidence polarizing beamsplitter: a in reflection, b in transmission

As an extreme case, we know from Sect. 2.1 that it is possible to build a 45°-incidence reflector with  $t=0$  and  $r=32.3\%$ . (When a polarization is not explicitly specified here and below, the TE mode is intended.) Also possible is the degenerate case  $t=1$  and  $r=0$ ; i.e., no beamsplitting effect. Thinking in terms of a reflectivity vs. transmissivity plane, the set of possible 45°-incidence beamsplitters – the non-optimal ones included – occupies a region bounded by three sides; see Fig. 4. One side lies along the  $r$ -axis, ranging from  $r=0$  to  $r=0.323$ . Another lies along the  $t$ -axis, from  $t=0$  to  $t=1$ . The final side is an arc connecting the points  $(t=0, r=0.323)$  and  $(t=1, r=0)$ . This curve represents the best achievable beamsplitters. For a given transmissivity, they are the beamsplitters of maximum reflectivity; the reverse is also true.

This curve showing the best performance obtainable from 45°-incidence beamsplitters (for 194 Å) was calculated by brute force. A program was used to systematically search through a variety of beamsplitters, varying the number of layers as well as the values of  $d$  and  $\gamma$ . The reflectivity and transmissivity were evaluated for each choice of these three parameters, keeping only the best results. Details of the mirror structures giving these optimal values are too lengthy to present here, but all correspond to beamsplitters

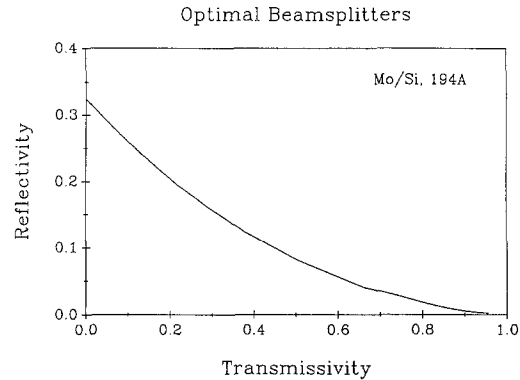


Fig. 4. Curve showing the best performance available from 45°-incidence polarizing beamsplitters

essentially similar to that presented in Sect. 2.2. The number of layers varies from a just a few for the highly transmissive structures to thirty or more for the highly reflective ones, but the values of  $d$  are all around 143 to 145 Å. The values of  $\gamma$  range between roughly 20% and 25%.

### 3. Polarizing Laser Cavities

#### 3.1. Effectiveness of Cavity Feedback

Figure 5 shows three types of polarizing laser cavity. The geometry varies from one cavity design to another, but a 45°-incidence beamsplitter serves as the output coupler for each.

As discussed above, it is possible to make beamsplitters with various values of reflectivity and transmissivity. Which choice is most appropriate for a given laser cavity depends, for example, upon the net gain per pass; i.e., the amplifier gain minus the attenuation of the optics. The number of cavity round-trip times over which the gain is expected to last must also be considered. Higher cavity gains and longer amplifier lifetimes make feedback more attractive; under such conditions a beamsplitter with large  $r$  is preferred, even though  $t$  must be correspondingly small. Inversely, when the cavity gain is low, and particularly when the number of passes available is small, the feedback provided by a cavity is less useful. Indeed, in view of the attenuation inevitably introduced by any optics, it might be better to avoid a cavity entirely when confronted with a situation of low gain.

To quantify these ideas, consider the following scenario of an X-ray laser shot. The amplifying medium is created by the first pulse from a pump laser, producing an X-ray pulse of energy  $E_0$  via amplified stimulated emission (ASE). These X-rays travel through part of the cavity and arrive at the beamsplitter as a pulse of energy  $aE_0$ ; here  $a$  represents the attenuation along the path from the amplifier to the

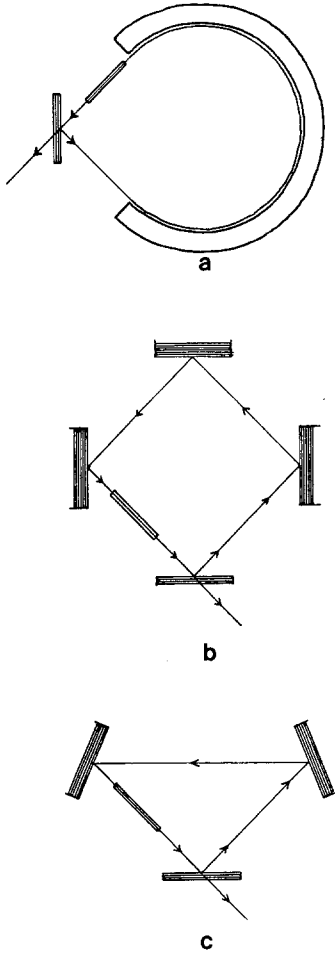


Fig. 5a-c. Geometries for polarizing cavities: a whisper-gallery mirror, b Brewster angle, c triangular. Each has a polarizing beamsplitter as its output coupler

beamsplitter. At the beamsplitter, a fraction  $t$  of the energy is transmitted as output and a fraction  $r$  reflected back into the cavity. The energy returned to the amplifier is then  $braE_0$ , where  $b$  is the attenuation along the path from the beamsplitter back to the amplifier. The amplifier, which has meanwhile been refreshed by a second pump pulse, provides gain  $g$  and the process continues. The total energy output can then be expressed as

$$\begin{aligned} E &= E_1 + E_2 + E_3 + \dots \\ &= taE_0 + tagbraE_0 + tagbragbraE_0 + \dots \\ &= [taE_0] \cdot [1 + (agbr) + (agbr)^2 + \dots]. \end{aligned} \quad (13)$$

Assuming that the amplifier lasts for, say, three passes (including the initial ASE pass) and it provides the same gain on each, the ratio of the total energy output to that of the initial ASE pulse is then given by

$$E/E_0 = ta[1 + (\tilde{g}r) + (\tilde{g}r)^2], \quad (14)$$

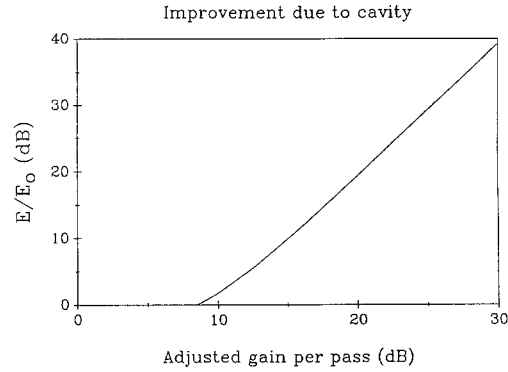


Fig. 6. Curve showing the effectiveness of a cavity as a function of the amplifier gain. For details see Sect. 3.1 of the text

in which  $\tilde{g} = agb$  is an adjusted cavity gain. For a given cavity structure and amplifier gain, the appropriate beamsplitter can be chosen by maximizing (14) with respect to  $r$  and  $t$  subject to the constraint that the pair lie along the curve described in Sect. 2.3.

Because  $a$  is an overall multiplier in (14), it is preferable to avoid putting cavity optics along the path from the amplifier to the beamsplitter; cavity attenuation has less of an effect when it lies after the output coupler. Note, though, that for this to make any sense there must be a preferred direction of travel around the cavity; such a direction might arise from some form of traveling wave excitation of the amplifier or amplifiers, or might simply reflect a choice about which of the two cavity output beams is to be observed.

In any case, if one assumes some value for  $a$ , it then becomes possible to compute the energy enhancement that a cavity can provide. Having taken  $a = 1$ , the ratio  $E/E_0$  is shown in Fig. 6 as a function of the adjusted cavity gain  $\tilde{g}$ . For values of  $\tilde{g}$  less than about 8.4 dB, the cavity is useless: the optimal beamsplitter turns out to be one for which  $t = 1$  and  $r = 0$ . For higher values of gain though, the cavity begins to help. The improvement increases slowly at first, but for large values of  $\tilde{g}$ , the curve of Fig. 6 has a slope of 2. This asymptotic behavior arises simply because when the gain is large, the total energy output is dominated by the twice-amplified final pulse. A less obvious property of the high gain regime is that the best choice of beamsplitter becomes independent of gain: the optimal beamsplitter is that for which  $r^{\text{TE}} = 0.209$  and  $t^{\text{TE}} = 0.192$ ; this is the particular beamsplitter chosen as an example in Sect. 2.2.

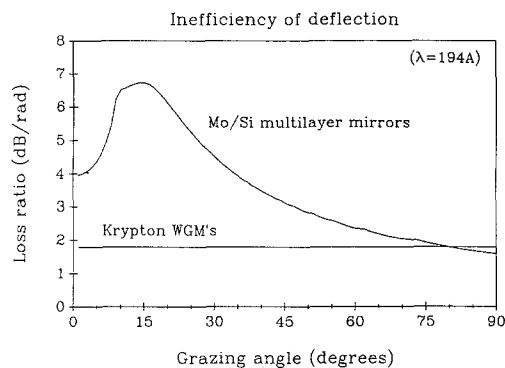
### 3.2. Discussion of Various Cavity Geometries

In the cavity depicted in Fig. 5a, a whisper-gallery mirror deflects the beam through an angle of  $270^\circ$ . The beamsplitter naturally provides the remaining  $90^\circ$  of

deflection required to close the cavity. The losses associated with the WGM are 8.45 dB and 9.04 dB for the cases of TE and TM polarization, respectively. As mentioned in Sect. 1.3, the polarizing effect of the WGM itself is rather weak. For the cavity as a whole, however, the beamsplitter helps enormously to give a polarized output. Using the beamsplitter optimal for high gain, each cavity pass suppresses the TM mode by 17.2 dB relative to the TE mode. The TM mode is knocked down by an additional 1.8 dB upon transmission through the beamsplitter.

If several cavity passes cannot reliably be obtained, it could be useful to replace the WGM with optics of stronger polarizing effect: the geometry shown in Fig. 5b incorporates three Brewster angle reflective polarizers (Sect. 2.1). Each bounce from such a mirror reduces the TM mode by 17.9 dB relative to the TE, giving an enormous net TM suppression of 70.3 dB per cavity pass.

A disadvantage of the Brewster reflector cavity is that the loss for the TE mode is rather large: 4.9 dB for *each* of the three mirrors. A good cavity should, of course, deflect the beam through the required  $360^\circ$  with as little loss as possible. To do so requires cavity elements for which the ratio of loss to angle of deflection is small, and reflection at  $45^\circ$ -incidence is inefficient in this regard. Consider that although the maximum reflection coefficient (32.4%) is higher at  $45^\circ$  than at normal incidence (31.9%), the loss incurred per angle of deflection is 3.20 dB/rad as compared to 1.58 dB/rad. Figure 7 plots this measure of inefficiency as a function of the angle of incidence. Note that this graph does not represent the performance of just a single mirror, but rather of a family of mirrors, each optimized to give maximal reflectivity for the parti-



**Fig. 7.** Curve showing the relative efficiency of reflection from Mo/Si multilayers at different angles. Along the horizontal axis is the grazing angle of the reflection or, equivalently, half the deflection angle. Along the vertical axis is the ratio of the loss experienced to the angle of deflection. The horizontal line at 1.79 dB/rad is the analogous figure for Kr whisper-gallery mirrors

cular angle of incidence. Also, for this curve the multilayer structures are assumed to be infinitely deep. Reflection at normal incidence is readily seen to be of maximal efficiency. The efficiency of krypton WGM's, a figure independent of the net deflection angle [32], is shown for comparison. The WGM's are clearly quite competitive with the multilayers.

The poor efficiency obtained at  $45^\circ$ -incidence suggests that the cavity of Fig. 5b might profitably be deformed so as to change the angles of incidence. This is possible, but the shape with the lowest loss turns out to be one in which the mirror opposite the beamsplitter has a vanishingly small grazing angle. This fact leads to the triangular design of Fig. 5c, which uses just two mirrors in addition to the beamsplitter. These mirrors operate at a grazing angle of  $67.5^\circ$ , and each contributes a TE reflection loss of 4.95 dB. This latter figure is a little higher than that of the Brewster angle reflectors, but here only two mirrors are required rather than three. The net loss is 9.9 dB for the pair, a figure slightly larger than that of the WGM for the same  $270^\circ$  deflection. These multilayers are however superior to the WGM in regard to polarization: the TM reflection loss is 9.68 dB at each mirror, implying a TM mode suppression of 26.1 dB per cavity pass.

To summarize the three designs, the WGM cavity has the lowest TE mode loss per pass but the poorest polarizing ability. Inversely, the Brewster angle cavity has the highest loss but polarizes most strongly. The performance of the triangular cavity makes it something of a compromise between the other two.

#### 4. Conclusion

The whisper-gallery mirror has two effects upon polarization. First, it is a weak polarizer, slightly suppressing the TM linear polarization with respect to the TE. Second and more significantly, it acts as a birefringent element. An appropriately chosen WGM would act like a quarter-wave plate, allowing the conversion of linear polarization to circular polarization and vice-versa. As well as opening the possibility of new sources for circularly polarized radiation, birefringent optics also provide the means for its analysis.

When optimized for reflection near the Brewster angle, multilayer structures can form efficient polarizers. Beamsplitters based on this idea act both as reflective and transmissive polarizers, sometimes in an anomalous manner: for the particular choice of wavelength and materials considered here, the TE mode is found to be preferred in both reflection and transmission. These beamsplitters show promise as output couplers for polarizing laser cavities. It would be

interesting to investigate whether or not a multilayer beamsplitter could be used as a quarter-wave plate.

Polarizing laser cavities appear feasible. Each of three versions discussed here provides a substantial degree of polarization. The losses associated with these cavities are enormous by the standards of visible-light optics, but this is hardly a fair comparison. If the same optimization principles used above are applied to the design of a normal-incidence cavity for 194 Å, the result is a cavity having a loss of 11.9 dB per pass. In light of this figure, the 15.3 dB of the WGM cavity or the 16.7 dB of the triangular cavity does not seem quite so appalling. The price of a polarized output is not all that high: only one additional amplifier gain length is required in order to compensate for the greater cavity loss.

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