Biol. Cybern. 54, 179–187 (1986)

Biological
Cybernetics

A Model of Eye Tracking of Periodic Square Wave Target Motion*

R. Schmid[1] and S. Ron[2]

[1] Dipartimento di Informatica e Sistemistica, Università di Pavia, Pavia, Italy
[2] Occupational Health and Rehabilitation Institute at Loewenstein Hospital, Raanana, Israel

Abstract. When a subject is presented with a visual target moving between two fixed points in a periodic square wave motion, the delay of eye tracking saccades can gradually decrease until the eyes lock on the target or even precede it. For symmetrical target motions (equal duration of the two phases of each cycle), the response time defined as the interval between a target jump in one direction and the beginning of the corresponding saccadic response was almost the same for the two phases of each cycle. This response time was found to depend on the frequency of target motion and to reach a positive value (anticipation) of about 200 ms at about 0.5 Hz. At low and high frequencies eye movement delayed target movement, and the delay was almost that observed for saccades to unpredictable targets. For asymmetrical target motions, the response time was different for the two phases of each cycle. A shorter response delay or a greater anticipation was observed for the response to the shorter phase. The response time to both phases of target motion depended on cycle duration while the response time to the longer phase also depended on the degree of asymmetry of target motion.

After a review of the experimental results, a mathematical model that can help their interpretation is presented. The model also provides a description of the interaction that might occur between the two hemispheres when eye tracking is made by saccades alternately to the right and to the left.

1 Introduction

When an object of interest (target) is made to appear in the visual field of a subject, the subject's eyes make a fast movement (saccade) towards it. The eye movement starts with a latency of about 250 ms with respect to the time of target appearance (Bartz 1962; Saslow 1967) and then develops with an almost stereotyped pattern and a fixed relationship between its amplitude, duration and peak velocity (Bahill et al. 1975).

If the target is made to move between two fixed points following a periodic square wave pattern and the subject is asked to track it with his eyes, after several cycles (2 to 5), saccade latency can be partially or fully recovered (Stark et al. 1962; Dallos and Jones 1963; Fuchs 1967). The results are rather different depending on whether the two phases of the square wave have the same duration (Ron 1982).

The aim of this paper is to review some experimental results obtained by one of the authors (Ron 1982) with both symmetrical and asymmetrical square wave patterns of target motion, and to present a mathematical model that can help the interpretation of these results. The model also provides a description of the interaction that might occur between the two hemispheres when eye tracking is made by saccades alternately to the right and to the left.

2 Methods

Subjects were seated at the center of a circular screen with their head restrained and their chin resting in a chin holder. A light spot (visual target) was projected onto the screen by the reflection of a Laser beam on a mirror moved by a small DC motor with negligible inertia. By feeding the motor with a square wave signal the target was made to jump in the horizontal plane between two fixed points placed symmetrically with respect to the center of the screen. Subjects were asked to follow target movement with their eyes as accurately as possible.

Eye movements were recorded by DC electro-oculography. The bandwidth of the recording system exceeded 100 Hz. Data processing consisted essentially

* Work supported by CNR (Italy) and NCRD (Israel)

of measuring the "*response time*" of each saccade, i.e. the interval between each target jump and the beginning of the corresponding saccadic response. Negative values of the response time were used to indicate delay and positive values to indicate anticipation.

Both symmetrical and asymmetrical square wave target motions were considered, where asymmetry refers to the duration of the two phases of each cycle. When symmetrical patterns were used, the response time of each phase was examined in relation to cycle duration which could vary from 800 ms to 3–4 s. When asymmetrical patterns were used, the response time of each phase was examined either by changing cycle duration and keeping a constant asymmetry or vice versa. The amplitude of target motion was of 15 deg. Different amplitudes were also tested in control experiments to see whether this parameter influences prediction.

Each experimental condition was examined on at least 3 subjects and during at least 25 cycles. Steady state values of the response time were computed after discarding the first 5 cycles which corresponded to the build-up of prediction. A total of 12 subjects participated in the experiments.

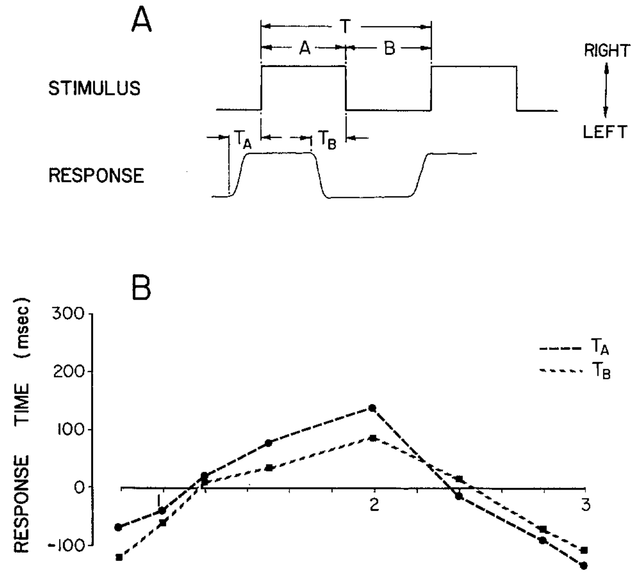


Fig. 2. A Symbols related to target and eye movements. T: cycle duration; A and B: duration of each phase of target motion; T_A and T_B: response times; R_A and R_B: duration of the two phases of eye movement corresponding to the shorter and longer phase of target motion. **B** Response times vs. stimulus duration. Data are mean values from three subjects

3 Experimental Results

The experimental results will be reported separately for symmetrical and asymmetrical patterns of target motion.

3.1 Symmetrical Patterns

When the subjects were presented with a symmetrical square wave target motion, their eyes started following the target with a delay of about 250 ms. This delay could be recovered in a few cycles. Thereafter, the eyes could move locked on the target or even precede it (Fig. 1). The steady state response time depended on cycle duration and was only slightly different for saccades in either direction. The maximal prediction was observed for cycle duration of 1.5–2 s and corresponded to a positive response time of 150–250 ms (Fig. 2). For shorter and longer cycle duration the response time decreased to become negative (delay) and to approach the value of −250 ms observed for the latency of saccades to unpredictable targets. Similar results have been obtained by Zambarbieri et al. (1985) by using both visual and auditory targets.

The influence of target motion amplitude was tested with a 2 s symmetrical square wave. A small increase of anticipation with amplitude was observed (from about 140 ms at 10 deg to about 190 ms at 40 deg).

3.2 Asymmetrical Patterns

When subjects were presented with an asymmetrical square wave target motion, a significant difference was found between the response time T_A to the shorter phase and that T_B to the longer phase (Fig. 3).

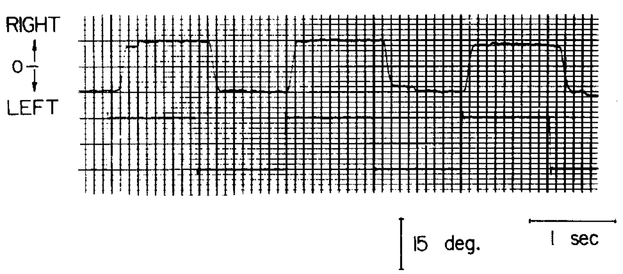


Fig. 1. Eye movement recorded from one subject during the tracking of a symmetrical square wave pattern of target motion

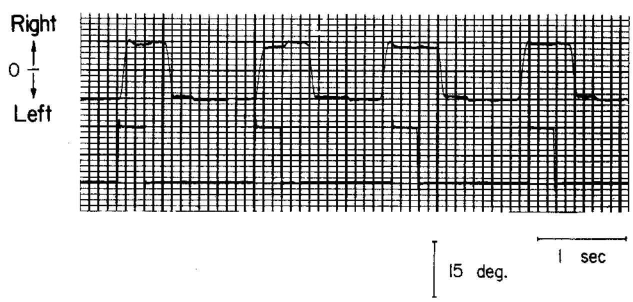


Fig. 3. Eye movement recorded from one subject during the tracking of an asymmetrical wave pattern of target motion

For a given cycle duration, only the response time to the longer phase was strongly dependent on target motion asymmetry (ratio A/B between the duration A of the shorter phase and the duration B of the longer phase). The results obtained with a cycle duration of 1.5 s are shown in Fig. 4. As the duration of the shorter phase was increased in the same session from 250 ms to 750 ms, the response to this phase anticipated target movement by an almost constant amount. The response to the longer phase delayed target movement when the pattern was highly asymmetrical, and became anticipatory as asymmetry decreased. When the duration of the two phases was made to be the same (750 ms), both responses anticipated the target by the same amount as observed in the experiments with a symmetrical pattern of 1.5 s of cycle duration. Since the response times T_A and T_B were significantly different, the durations R_A and R_B of the shorter and longer phases of the eye movement did not coincide with the duration A and B of the corresponding phases of target movement. In general, R_A was longer than A, and R_B shorter than B. The diagram in Fig. 4C gives the values of R_A vs. A measured from the same responses that provided the values of T_A and T_B given in Fig. 4B. The relation between R_A and A was almost linear. R_A was significantly longer than A for high asymmetries and became equal to A as the stimulus pattern became symmetrical.

For a given asymmetry, the response times T_A and T_B depended on cycle duration as shown in Fig. 5 (asymmetry of 1/4). The response time to the shorter phase was always greater (less delay or more anticipation) than that to the longer phase. The change of both response times was maximum for a cycle duration of 1.2–1.5 s. As stimulus cycle duration increased, the response time of both phases approached the latency of saccades to unpredictable targets. A significantly high correlation between T_A and T_B was found at all cycle durations.

The influence of target motion amplitude was tested with asymmetrical square waves of the same period (1.5 s) and different asymmetry. When the amplitude was changed from 15 to 30 deg, the response time of both phases did not change statistically ($p<0.05$) at any value of the asymmetry.

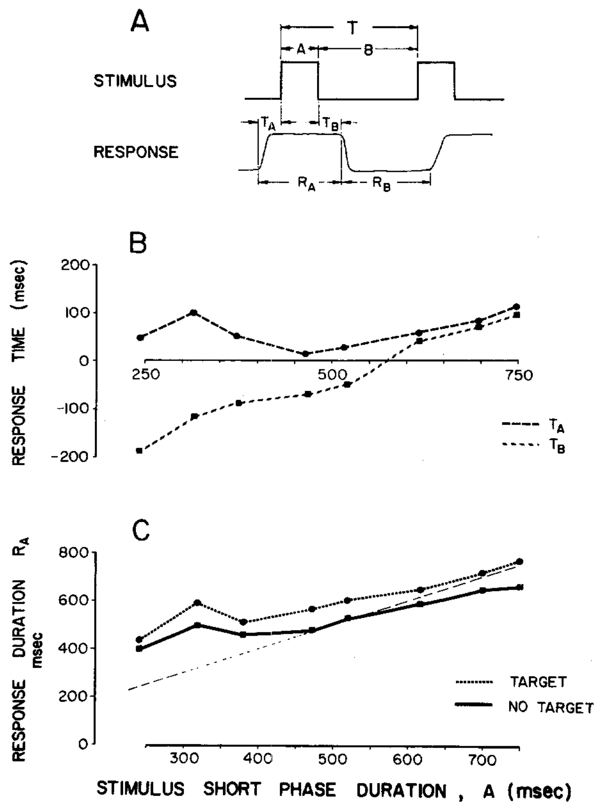


Fig. 4. A Symbols related to target and eye movements. **B** Response times plotted against the duration A of the shorter phase of target motion for a cycle duration of 1.5 s. **C** Duration R_A of the shorter phase of eye movement plotted against the duration A of the corresponding phase of target motion. Data are mean values from three subjects

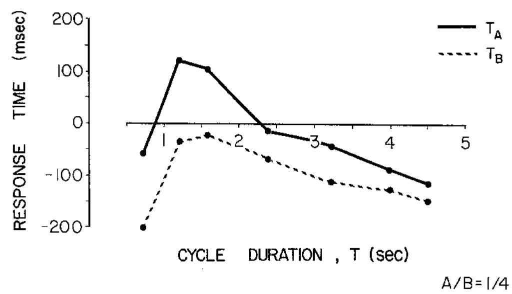


Fig. 5. Eye movement response times vs. stimulus cycle duration for a given asymmetry of target motion ($A/B=1/4$). Data are mean values from three subjects

4 Discussion and Model

The experimental results described in the previous section indicate a high level of synchronization of eye movement when a periodic square wave pattern, either symmetrical or asymmetrical, is presented to the subject. As discussed in a previous paper by one of the authors (Ron 1982), these results suggest the existence in both hemispheres of two distinct mechanisms. The first mechanism would generate an internal reference signal by using prediction to reduce the delay that characterizes the saccadic response to unpredictable targets. The second mechanism would synchronize the eye movement to the internal reference signal.

The presence of a predictor in each hemisphere instead of one predictor serving both hemispheres is suggested by the fact that patients with monolateral traumatic brain injuries may lose the ability to predict a periodic motion of a visual target when the target movement is contralateral to the injured hemisphere (Ron and Glass 1986). The pacing mechanism should also be distinct for the two hemispheres since they have to deal with saccades in opposite directions. Nevertheless, they should interact with each other since a high correlation was found between the response times of the two phases of the responses to an asymmetrical square wave pattern. An attempt to describe the interaction between the two hemispheres and to justify the experimental results reported in this paper has been made by using the model in Fig. 6.

In each hemisphere a predictor provides an internal reference signal which represents an anticipated trace of the relevant phase of target movement. The rate of anticipation τ_a is assumed to depend on both cycle duration T and the degree of asymmetry A/B of the input signal to the predictor. In the case of symmetrical patterns of target motion, τ_a should obviously be the same for the two hemispheres, at least in normal subjects. In the case of asymmetrical patterns, the input signals to either hemisphere have still the same characteristics of periodicity (see Fig. 6). Then, also in this case, it can reasonably be assumed that in normal subjects the rate of anticipation is the same for the two hemispheres. The proposed relationship between τ_a and T for a given asymmetry is shown in the inset of Fig. 6. Up to a cycle duration T^*, τ_a is assumed to be positive (anticipation) and almost constant (300 ms). For longer cycle durations τ_a decreases exponentially to a negative value (delay) of -250 ms which represents the latency of saccades to unpredictable targets. As a first approximation, the dependence of τ_a on the asymmetry of target motion can be described by assuming T^* to decrease with A/B, from a value of 1.8–1.9 s for $A/B=1$ (symmetrical pattern) down to 1.2–1.3 for $A/B=1/4$.

The pacing of saccades in either direction is obtained through a combination of ipsilateral excitation and contralateral inhibition. More precisely, when a saccade in one direction has to be generated to follow the internal reference signal, a build-up of excitation is produced in the relevant hemisphere. As soon as excitation exceeds the residual inhibition from the contralateral hemisphere by a threshold value, a trigger signal is sent to a motor command generator and an appropriate signal is conveyed to the saccadic system. At the same time, a reset signal is sent to the

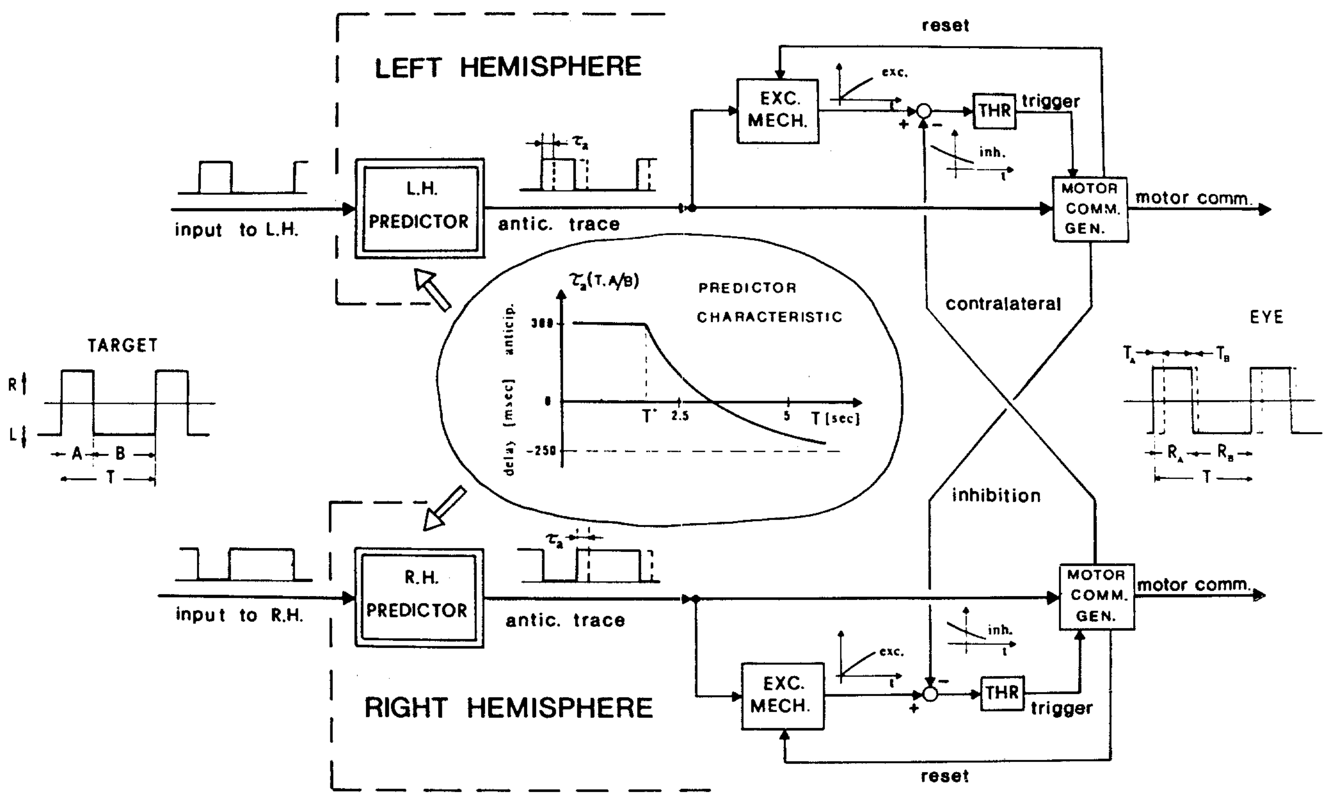


Fig. 6. Model of the interaction between the two hemispheres during the tracking of a square wave pattern of target motion

ipsilateral excitatory mechanism and an inhibitory signal to the contralateral hemisphere. Contralateral inhibition is assumed to decay exponentially with time.

To discuss the model analytically, let us first consider the case of a symmetrical square wave target motion (Fig. 7) where:

$U_e(t)$: excitatory signal in one hemisphere;
$U_i(t)$: inhibitory signal from the contralateral hemisphere;
τ_a: time shift between the internal reference signals and target movement ($\tau_a>0$: anticipation; $\tau_a<0$: delay);
τ^*: time shift between eye movement and internal reference signals;
T_E: time shift between eye and target movement (response time).

Let us neglect the threshold for triggering the motor command generators and let the build-up of an excitatory process started at time t_e be given by

$$U_e(t)=K_e[1-e^{-(t-t_e)/\tau_e}], \quad t\geqq t_e, \tag{1}$$

and the decay of an inhibitory process started at time t_i by

$$U_i(t)=K_i e^{-(t-t_i)/\tau_i}, \quad t\geqq t_i, \tag{2}$$

where τ_e and τ_i are the time constants of excitation and inhibition, respectively. K_e and K_i are gain factors. For the sake of simplicity, we can assume $K_e=K_i=1$ and $\tau_e=\tau_i=\tau$.

At steady state, the contralateral inhibition to one hemisphere is started $T/2-\tau^*$ s before the beginning of excitation in the same hemisphere. Then,

$$t_i=t_e-T/2+\tau^*. \tag{3}$$

By definition of τ^*, it should be

$$u_e(t_e+\tau^*)=u_i(t_e+\tau^*).$$

Then, (1), (2), and (3) give

$$1-e^{-\tau^*/\tau}=e^{-T/2\tau}$$

and

$$\tau^*=\tau\ln(1-e^{-T/2\tau}). \tag{4}$$

The response time T_E is given by

$$T_E=\tau_a-\tau^*. \tag{5}$$

By assuming for τ_a the characteristic shown in the inset of Fig. 6 with $T^*=1.9$ s and a decay factor of 0.7, and by giving τ the value of 0.6, the results reported in Fig. 8 were obtained for the relationship between the response time T_E and the period T of target motion. Model prediction is compared with the experimental data from three subjects (mean values between T_A and T_B).

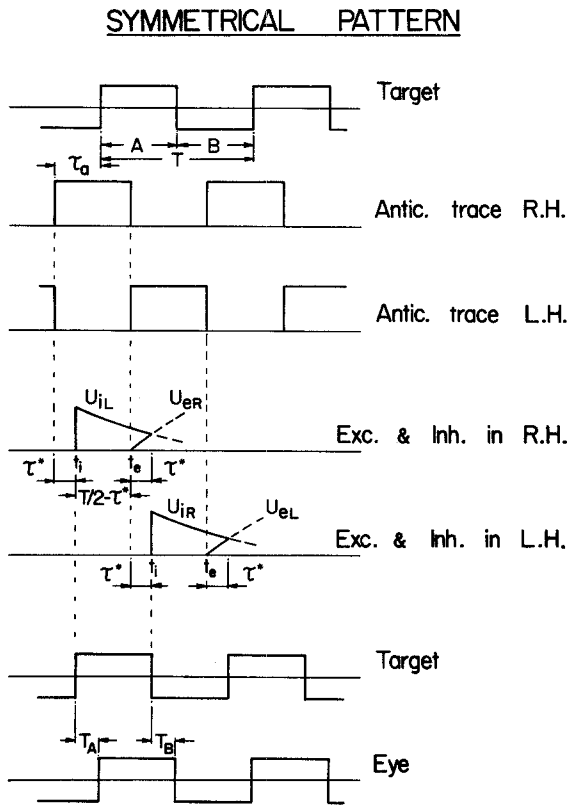


Fig. 7. Excitation and inhibition in the two hemispheres determining the pacing of saccades to the right and to the left during the tracking of a symmetrical square wave pattern of target motion

The case of an asymmetrical square wave pattern can be treated in a similar way (Fig. 9).

For the right hemisphere (RH) it can be written

$$\begin{aligned} &U_{eR}(t)=1-e^{-(t-t_e)/\tau}, \\ &U_{iL}(t)=e^{-(t-t_i)/\tau}=e^{-(t-t_e+A-\tau_A^*)/\tau} \end{aligned} \tag{6}$$

and for the left hemisphere (LH)

$$\begin{aligned} &U_{eL}(t)=1-e^{-(t-t_i)/\tau}, \\ &U_{iR}(t)=e^{-(t-t_i)/\tau}=e^{-(t-t_e+B-\tau_B^*)/\tau}. \end{aligned} \tag{7}$$

By imposing the condition

$$U_{eR}(t_e+\tau_B^*)=U_{iL}(t_e+\tau_B^*)$$

in (6) and the condition

$$U_{eL}(t_e+\tau_A^*)=U_{iR}(t_e+\tau_A^*)$$

in (7), it results

$$\tau_A^*=\tau\ln\frac{1+e^{-B/\tau}}{1-e^{-T/\tau}}, \tag{8}$$

$$\tau_B^*=\tau\ln\frac{1+e^{-A/\tau}}{1-e^{-T/\tau}}, \tag{9}$$

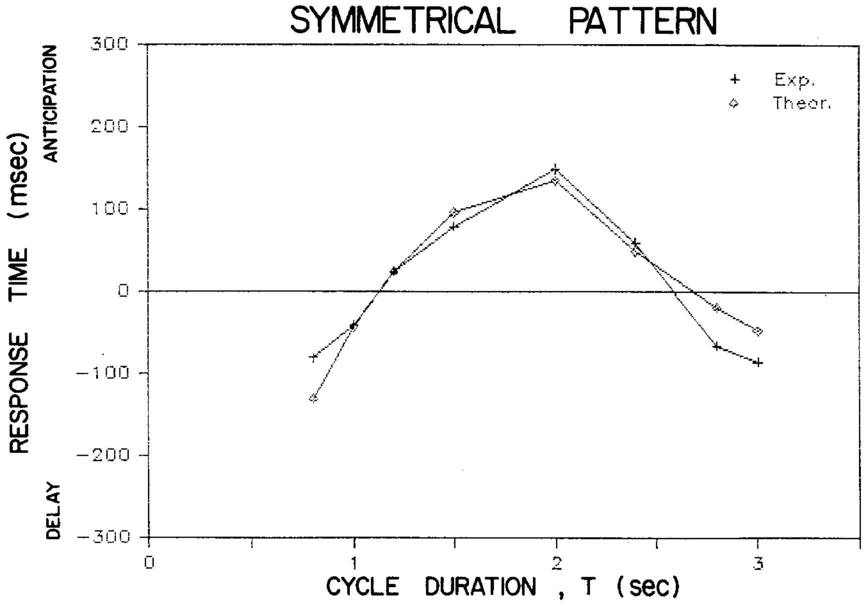


Fig. 8. Predicted relationship between eye movement response time and target cycle duration for a symmetrical square wave pattern of target motion. The experimental data reported for comparison are the average values computed from T_A and T_B recorded from three subjects

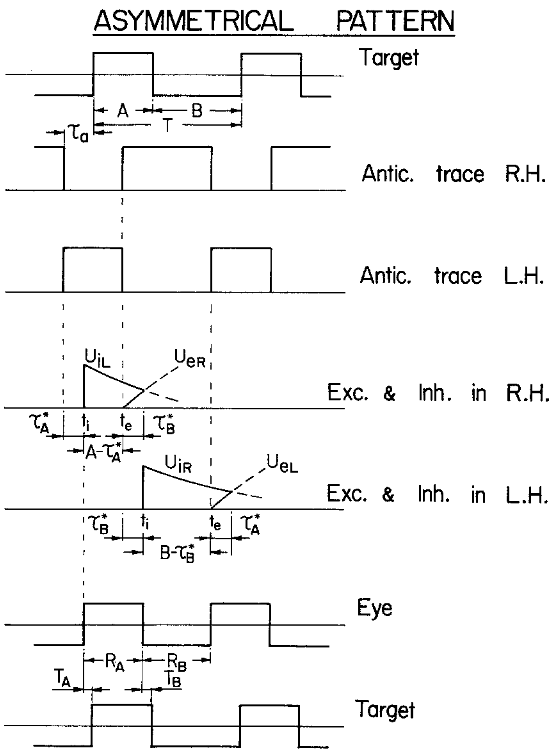


Fig. 9. Excitation and inhibition in the two hemispheres determining the pacing of saccades to the right and to the left during the tracking of an asymmetrical square wave pattern of target motion

and

$$T_A = \tau_a - \tau_A^*, \tag{10}$$

$$T_B = \tau_a - \tau_B^*. \tag{11}$$

Obviously, for $A = B$, (8) and (9) give $\tau_A^* = \tau_B^* = \tau^*$ with τ^* given by (4).

For $A < B$, from (8)–(11) it follows that the response time T_A to the shorter phase of target motion is always smaller than the response time T_B to the longer phase as it was actually observed experimentally. The model in Fig. 6 was used to simulate the two experimental paradigms considered in this study for asymmetrical patterns of target motion, i.e. constant asymmetry with variable cycle duration, and constant cycle duration with variable asymmetry.

The results in Fig. 10 show a comparison between model prediction and experimental data for two values of A/B (1/3 and 1/4) and T varying from 0.75 s to 3 s. The experimental data are the calculated means from three subjects. The values assumed for T^* were 1.2 and 1.4, respectively, whereas the same value of $\tau = 0.6$ s was assumed for the excitatory and inhibitory processes in both simulations.

The results in Fig. 11A show the response time predicted by the model for a cycle duration of 1.5 s and a duration A of the short phase varying from 200 ms to 750 ms (symmetry). T^* was assumed to increase linearly with A/B from 1.2 s to 1.8 s. The experimental data reported for comparison are the calculated mean responses from three subjects. According to the experimental findings, the response time to the shorter phase is always positive and does not vary significantly with the asymmetry whereas the response time to the longer phase increases almost linearly as asymmetry decreases.

The experimental and the predicted relationship between the duration R_A of the short phase of eye movement and the duration A of the corresponding short phase of target motion is shown in Fig. 12B.

One point that emerged from the simulation tests was the high sensitivity of the response times to

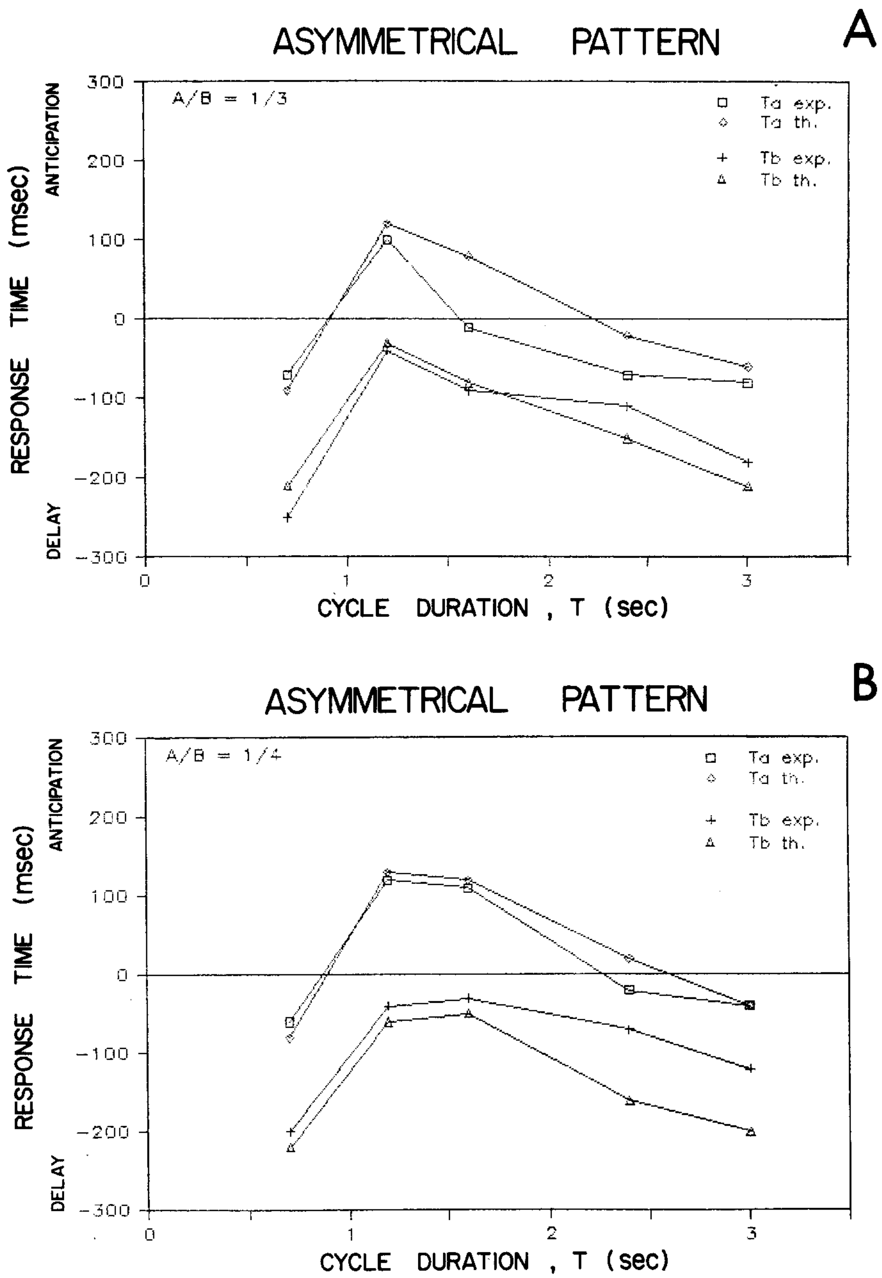
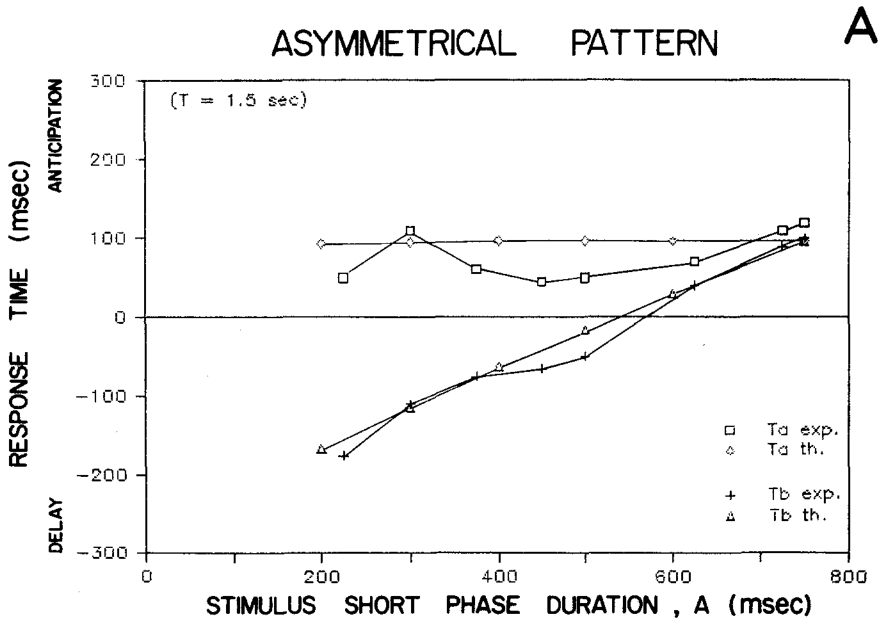


Fig. 10A and B. Comparison between model prediction and experimental data for the eye movement response times during the tracking of asymmetrical patterns with fixed asymmetry. **A** Asymmetry *A/B* of 1/3; **B** Asymmetry *A/B* of 1/4. The experimental data are mean values from three subjects

variations of both the time constant τ of the excitatory and inhibitory processes and the parameter T^* of the predictor characteristic. This finding can justify the high inter-individual variations observed in the experimental data.

The model presented in this paper is based on several assumptions, basically on the existence of two distinct predictors, one in each hemisphere and both generating an internal reference signal that anticipates the relevant phase of target movement by the same amount of time. The rate of anticipation was assumed to depend on cycle duration and on the asymmetry of target motion. Alternatively, it might have been assumed that the rate of anticipation depends, for both hemispheres, on the duration of one phase of target motion, namely the phase the subject is paying attention to. A sudden change in the values of the response times that could sometimes be observed during a session might then be explained by a switch of subject attention from one phase to the other (Ron 1982).

Assumptions can also be made that lead to a different anticipation by the predictors of the two hemispheres. For example, it might be assumed that each predictor introduces an anticipation which de-



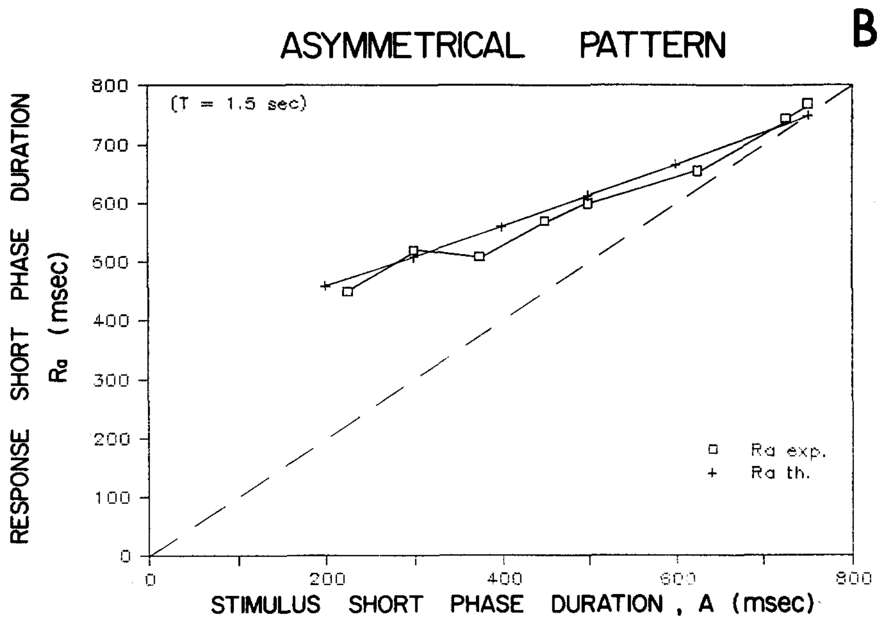


Fig. 11A and B. Comparison between model prediction and experimental data for the tracking of asymmetrical patterns with fixed period (1.5 s) and varying asymmetry. **A** Response times vs. the duration A of the shorter phase of the target motion; **B** Duration R_A of the shorter phase of the eye movement vs. the duration A of the corresponding phase of the target motion. The dashed line represents the condition $R_A = A$. The experimental data are the average values computed from three subjects

pends on the duration of the relevant phase, i.e. on the time the target remains in the receptive field of each hemisphere.

By giving the predictors appropriate characteristics all these alternative hypotheses can be introduced in the general frame represented by the model in Fig. 6. Nevertheless, the experimental results so far made available are not enough to support any of these hypotheses better than that used to develop the model discussed in this paper. This model can therefore be proposed as the simplest way to interpret the general behaviour of a subject tracking with his eyes a symmetrical or asymmetrical square wave pattern of target motion.

References

Bahill AT, Clark MR, Stark L (1975) The main sequence, a tool for studying human eye movements. Math Biosci 24:191–204

Bartz A (1962) Eye movement latency, duration and response time as a function of angular displacement. J Exp Psychol 64:318–324

Dallos PJ, Jones RW (1963) Learning behavior of eye fixation control system. IEEE Trans Autom Control 17C-8:218–227
Fuchs AF (1967) Periodic eye tracking in the monkey. J Physiol 193:161–171
Ron S (1982) Dependence of saccadic prediction on asymmetrical periodic stimulus. In: Roucoux A, Crommelinck M (eds) Physiological and pathological aspects of eye movements. Dr. W. Junk Pub., The Hague Boston London, pp 77–82
Ron S, Glass I (1986) Dynamic changes of saccades timing to periodic stimuli in brain injured patients. In: Ron S, Schmid R, Jeannerod M (eds) Sensorimotor plasticity: theoretical, experimental, and clinical aspects. INSERM, Paris (in press)
Saslow M (1967) Latency for saccadic eye movement. J Opt Soc Am 57:1030–1033
Stark L, Vossius G, Young LR (1962) Predictive control of eye tracking movements. IEEE Trans HHE-3:52–57
Zambarbieri D, Schmid R, Ventre J (1985) Saccadic eye movements to predictable visual and auditory targets. 3rd European Conference on Eye Movements, Dourdan (France), Sept. 1985

Received: January 13, 1986

Prof. R. Schmid
Università di Pavia
Dipartimento di Informatica e Sistemistica
Via Abbiategrasso, 209
I-27100 Pavia
Italy