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A Rheological Model for Mechano-sorptive Deflections of Beams

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Summary

Most of the increase in deflection with time that occurs during the drying of initially green beams under load is known to be due more to the change in moisture content (a mechano-sorptive phenomenon) than to the passage of time. A first approximation rheological model for this deflection is presented. The validity of the model was assessed by tests on small messmate stringybark (*Eucalyptus obliqua*) beams. It was found that during the initial drying about 85% of the total deformation was attributable to the proposed first approximation rheological model.

Introduction

Most measurements of creep in timber have been made with nominally constant stress conditions. In order to use these measurements to predict the long duration effects of varying loads on structures, it is first necessary to identify the sub-components that contribute to the total deformation.

One field of practical application in which the effect of varying stress is of prime importance is the design of timber columns. A structural timber column always contains some initial crookedness or material inhomogeneity which causes it to deflect laterally when subjected to an axial load. This lateral deflection, which increases with time, is of primary structural significance as it leads to bending forces on the column. Reliable measurements of column lateral deflection over periods of several years are not readily available and consequently it is usually necessary to base predictions of column creep on the recorded observations of apparent creep in beams. However, as illustrated by the example shown in Fig. 1, the relationship between a measured beam creep and the corresponding predicted column creep is very dependent on the choice of rheological model for creep.

A few tests to determine the effects of varying stress levels on timber structural members have been reported previously, but these have been concerned primarily with creep at constant moisture content [e.g. Grossman, Kingston 1963]. However, it is now accepted that a far greater portion of the observed apparent creep occurs due to changes of moisture content (see all references cited at the end of this paper) and consequently it is this mechano-sorptive deformation that

* The rheological terminology used in this paper (see Fig. 6) was agreed on by scientists of the Division of Forest Products, CSIRO, working in the area of wood rheology.

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is of major importance in practical applications. In the following, a first approximation rheological model is proposed, and some experiments to assess its validity are described.

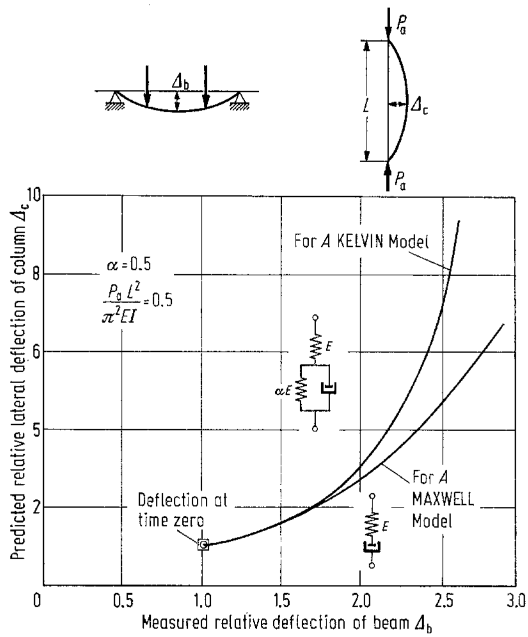

Fig. 1. Example of predicted creep deflections for columns

Nomenclature

f, f_1, f_2, f_3	functions of moisture content defined in Eq. (3) and Fig. 11
EI	lateral bending stiffness of column, Fig. 1
K	constant defined in Eq. (2)
m	average moisture content
P	parameter of force on beam
P'	force on rheological element, Fig. 11
t	time
Δ	parameter of total deflection of a beam
Δ'	deflection of a rheological element, Fig. 11
Δ_e	elastic component of deflection
Δ_{irr}	irrecoverable component of mechano-sorptive deflection
Δ_m	total mechano-sorptive component of deflection
Δ_0	initial elastic deflection
Δ_r	recoverable component of mechano-sorptive deflection
Δ_v	viscoelastic creep component of deflection

Note: The components of deflection are defined by Eqs. (1) to (3) and in Figs. 2, 11 and 12.

A First Approximation Rheological Model

On the basis of preliminary tests and from published works, it was decided to choose the model illustrated diagrammatically in Fig. 2 to describe the deflection of initially green beams during the first drying cycle. It is emphasized that this model is not intended to represent the physical structure of wood. It is purely a diagrammatic representation of the mathematical equations that relate the response of a beam (as described by a deflection parameter Δ) in reply to an input function of the loading paramater P.

The model shown in Fig. 2 contains two elements in series. These will be denoted an elastic and a mechano-sorptive element. The total deflection Δ is taken

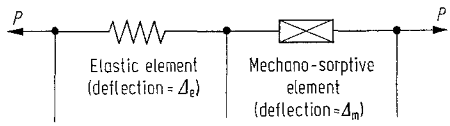

Fig. 2. Schematic representation of a first approximation rheological model

to be made up of an elastic component Δ_e and a mechano-sorptive component Δ_m that are related to the load parameter P as follows.

$$\Delta = \Delta_e + \Delta_m \tag{1}$$

$$\Delta_e = KP \tag{2}$$

$$-\frac{d\Delta_m}{dm} = P \cdot f(m) \tag{3}$$

where m is the average moisture content of the beam, K is a constant and $f(m)$ is a function of moisture content. Eq. (3) describes a type of deformation that will be denoted herein by the term irrecoverable mechano-sorptive deformation. The similarity between the proposed model and a Maxwell rheological model is to be noted. In the two following sections of this paper is a description of tests to assess the validity of the proposed model.

Experimental Procedures

All tests were made with messmate stringybark (*Eucalyptus obliqua*), a hardwood that is susceptible to some collapse on drying. The beams were 25.4 mm wide, 12.7 mm deep and quartersawn to minimize warping due to anisotropic shrinkage. A single log was used as the source for all test specimens.

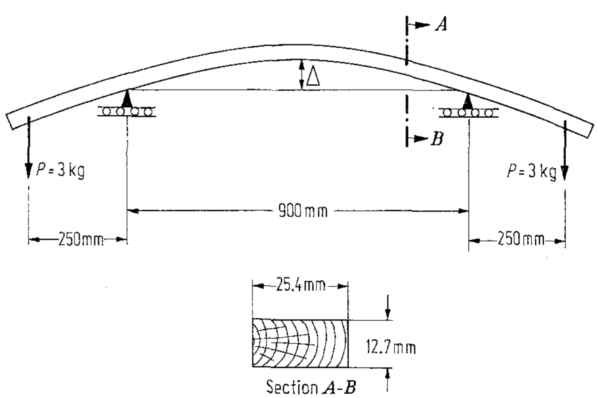

Fig. 3. Method of loading beams

The beams were loaded as shown in Fig. 3, so that the central portions were subjected to bending moments only. The deflections, designated Δ, of the beam midpoints relative to the supports were measured with a cathetometer. Measurements were made for both sides of the beams in order to compensate for warping effects. The load applied to the beams was about 15% of the short term ultimate load capacity of green beams. Preliminary experiments with load levels from 5% — 20% verified that the test beams were being stressed within the known approximate range of constant "relative" creep values [KINGSTON 1962].

The experiments were carried out in a laboratory with a controlled temperature of 21° ± 1° C. There was no other control on the test environment. The relative humidity ranged from 30 to 50% during the overall testing period with a maximum range of 8% during any one test. Each test lasted about 400 hours.

A few tests were made at nominally constant moisture contents by sealing the test beams in polythene wrap. Most tests, however, were made on beams during air-drying in the laboratory. For this case the beams were taken from a saturated environment to the laboratory about 2 to 3 hours before a test commenced. The average moisture contents were in the range 60 to 70% at the start of the tests and 11 to 14% at the end. Moisture contents of continuously loaded specimens were estimated by weighing matched control beams. Preliminary tests indicated that this method gave values accurate to within 1% moisture content.

Experimental Assessment of the First Approximation Model

Method of Assessment

The validity of the first approximation model was assessed by measurement of the magnitude of the rheological effects that are not in accordance with the predictions of the model. These measurements are described in the following sections. All measurements are expressed relative to Δ_0, the initial elastic deflection of green wood at time zero.

Effect of Moisture Content on Elastic Stiffness

An elastic deflection Δ_e is defined herein as the total increment in deflection that occurs up to 6 minutes after the application of an increment in load. For a constant load parameter P, Eq. (2) implies a constant elastic deflection, i.e. $\Delta_e = \Delta_0$. The measured effect of moisture content on elastic deflection was a change of less

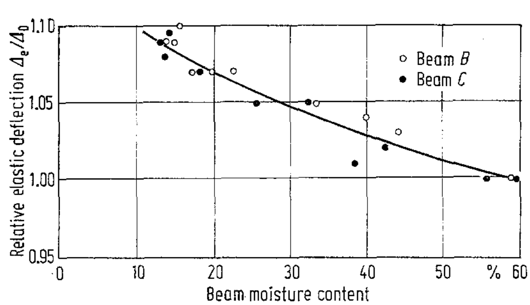

Fig. 4. Effect of drying on elastic deflection

than 10% for the test conditions and is shown in Fig. 4 for two beams. This effect is due to a reduction in cross-section due to shrinkage and the increase in Young's modulus of the wood.

Viscoelastic Creep

The rheological model described by Eqs. (2) and (3) does not predict any viscoelastic creep, defined here as the creep that occurs at constant moisture content and denoted by Δ_v. The measured viscoelastic creep at constant load for two green and two air-dry beams is shown in Fig. 5. The maximum value of relative creep Δ_v/Δ_0 is about 10% for the test duration. On unloading the green beams, it was found that the viscoelastic creep was completely recoverable.

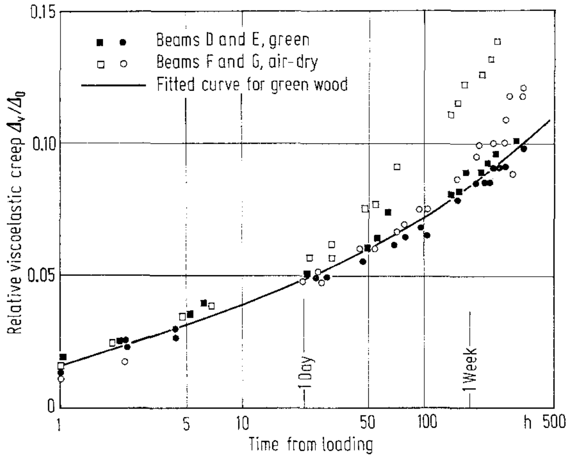

Fig. 5. Viscoelastic creep for green and air-dry beams

The Nature of Mechano-sorptive Deformation

The mechano-sorptive deflection, denoted by Δ_m, was measured on beams allowed to air-dry. This component of deflection will be defined as the residual after subtracting the elastic deflection and viscoelastic creep (predicted from the measurements described in the section on viscoelastic creep) from the measured total deflection. For beams loaded continuously from green (60 to 70% moisture content) to air-dry (11 to 14% moisture content), the measured relative mechano-sorptive deflection Δ_m/Δ_0 was in the range 1.4 to 2.0. In Fig. 6 the significant magnitude of the mechano-sorptive deflection is shown by the comparison with the other deflection components for a typical beam.

Fig. 7 shows the mechano-sorptive deflection for a set of three matched beams that were unloaded sequentially during air-drying. The recoverable component, denoted by Δ_r, that was measured is predicted to be zero by Eqs. (1) to (3); and hence it represents an error in the first approximation model. The recoverable mechano-sorptive deflection measured in this way for two sets of beams is shown in Fig. 8. In a manner analogous to the creep of a Kelvin model this deflection approaches a maximum value asymptotically. The measured maximum relative value is about 0.2.

An alternative estimate of the recoverable component of mechano-sorptive deflection was obtained by sequential loading of a set of matched beams. Typical desorption deflection curves for such a set are shown in Fig. 9. All beams in a matched set commenced drying together and (except for applied loads) were at all times subject to the same environmental conditions. Consequently, within the limitations of matching, all beams in a set had identical moisture contents at all

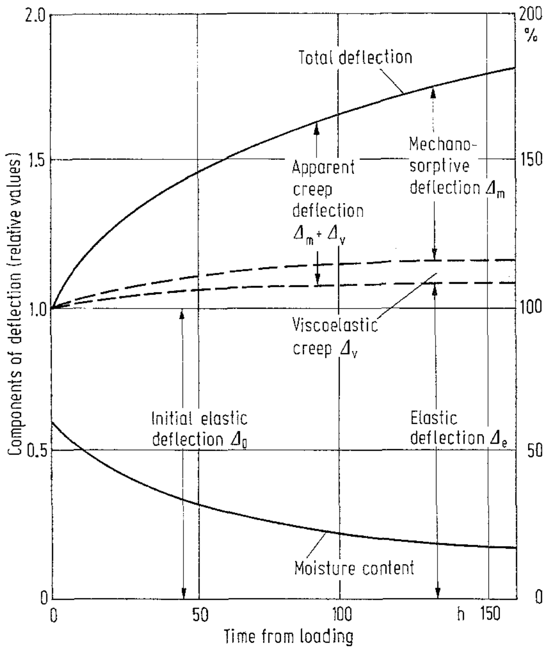

Fig. 6. Components of deflection for beam '*H*'

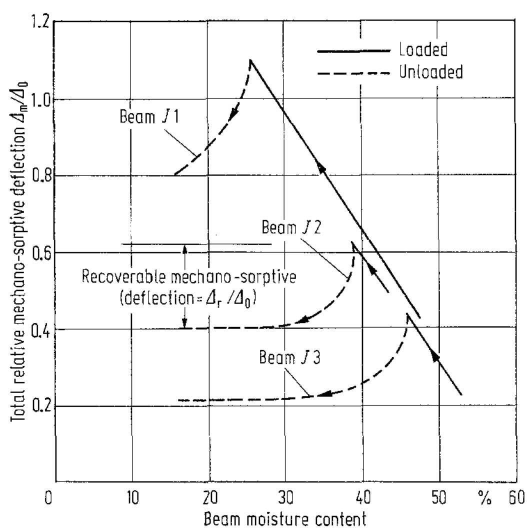

Fig. 7. Mechano-sorptive deflection on unloading beam set '*J*'

times. Eqs. (1) to (3) imply that regardless of previous loading histories, a pair of matched beams with identical moisture contents and loads should exhibit the same increments of mechano-sorptive deflection. Consequently the quantity

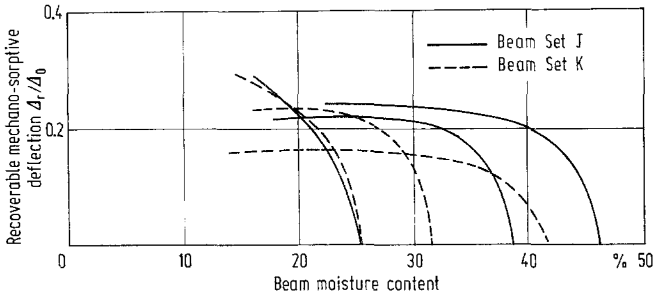

Fig. 8. Recoverable mechano-sorptive deflection measured by unloading

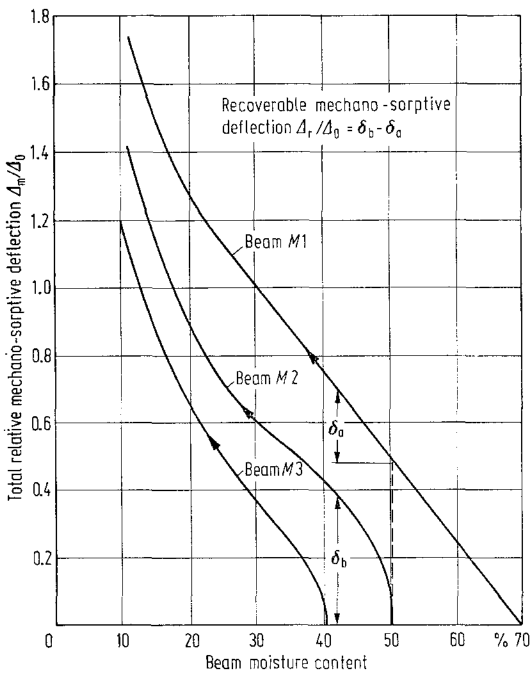

Fig. 9. Total relative mechano-sorptive deflections for sequential loading of beam set '*M*'

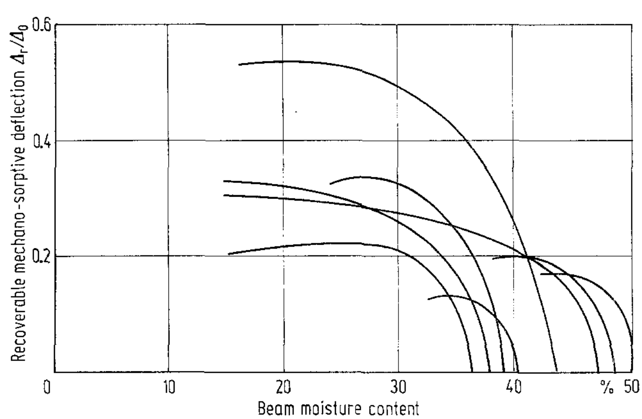

Fig. 10. Recoverable mechano-sorptive deflection measured by sequential loading

$\delta_b - \delta_a$ indicated in Fig. 9 is a measure of the recoverable mechano-sorptive deflection that is not part of the predicted behaviour of the first approximation model. Fig. 10 is a collation of the recoverable deflection measured in this way for four sets of beams. In these tests the average of the measured maximum values of relative recoverable mechano-sorptive deflection was about 0.3.

Contribution of the First Approximation Model

The following are typical measured values of the components of the final total deflection of the continuously loaded, initially green, messmate stringybark beams;

Relative initial elastic deflection, Δ_0/Δ_0	1.00
Relative increment in elastic deflection, $(\Delta_e - \Delta_0)/\Delta_0$	0.10
Relative viscoelastic creep, Δ_v/Δ_0	0.10
Relative recoverable mechano-sorptive deflection, Δ_r/Δ_0	0.25
Relative irrecoverable mechano-sorptive deflection Δ_{irr}/Δ_0	1.25
Relative total final deflection Δ/Δ_0	= 2.70

The contribution of the first approximation rheological model is represented by the initial elastic deflection and the irrecoverable mechano-sorptive deflection. Hence, this model contributes to about 85% of the final total deflection of the beams.

An Improved Rheological Model

Many rheological models may be devised to fit the experimental data reported herein. One example is a model formed of the elements defined in Fig. 11 and connected as shown in Fig. 12. This model consists of a series arrangement of three

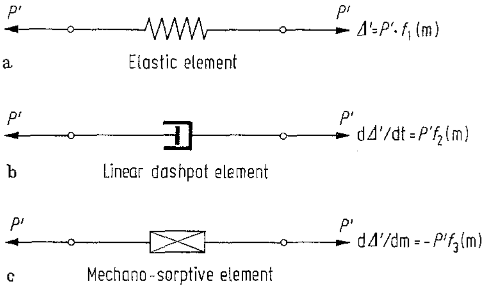

Fig. 11. Basic rheological elements

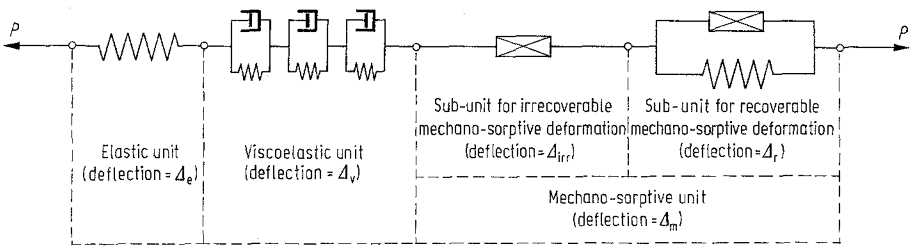

Fig. 12. Improved rheological model for lateral deflection of a beam

units; these are an elastic, a viscoelastic, and a mechano-sorptive unit. The time dependent unit has been completely uncoupled from the mechano-sorptive unit, as the results of rapid desorptions experiments [ARMSTRONG, CHRISTENSEN 1961] have shown that there is little, if any, effect of time on the magnitude of mechano-sorptive deformations. Of course, further tests would be necessary to assess the validity of this or any other proposed improved rheological model.

Conclusions

The experiments have shown that in air-drying over a 2-weeks' period under load, about 85% of the total deformation of green messmate stringybark beams occurs according to the first approximation rheological model defined by Eqs. (1) to (3) and depicted schematically in Fig. 2. A refined model to fit all the experimental data is suggested, but for most practical purposes the first approximation model should be adequate. In the subsequent paper, the author will describe the use of the approximate model to predict the lateral deflections of wood beam-columns during drying. Another interesting application of the model has been made by GROSSMAN [1971].

Comment on the Derivation of a Practical Rheological Model

In the lifetime of practical timber structures, a large proportion of the total creep probably occurs due to small changes in moisture content caused by variations in the relative humidity and temperature of the structural environment after the timber has reached an air-dry condition. This creep may not be similar in nature to that which occurs during the first drying cycle, and indeed there is experimental data [CHRISTENSEN 1962] which shows quite clearly that the first approximation rheological model shown in Fig. 2 cannot adequately describe mechano-sorptive deformations during adsorption. Consequently, it is of considerable practical importance that an investigation be undertaken to determine a rheological model for moisture cycling conditions.

References

ARMSTRONG, L. D., CHRISTENSEN, G. N. 1961. Influence of moisture changes on deformation of wood under stress. Nature **191** (4791): 869—870.

—, KINGSTON, R. S. T. 1962. The effect of moisture content changes on the deformation of wood under stress. Austral. J. of Appl. Sci. **13** (4): 257—276.

CHRISTENSEN, G. N. 1962. The use of small specimens for studying the effect of moisture content changes on the deformation of wood under load. Austral. J. of Appl. Sci. **13** (4): 242—256.

GROSSMAN, P. U. A. 1971. The use of Leicester's rheological model for mechano-sorptive deformations. Wood Sci. Technol. **5** (3): 232—235.

—, KINGSTON, R. S. T. 1963. Some aspects of the rheological behaviour of wood. III. Tests of linearity. Austral. J. of Appl. Sci. **14** (4): 305—317.

HEARMON, R. F. S., PATON, J. M. 1964. Moisture content changes and creep of wood. For. Prod. J. **14** (8): 357—360.

KINGSTON, R. S. T. 1962. Creep, relaxation and failure of wood. Research **15**: 164—170.

LEICESTER, R. H. 1971. Lateral deflections of timber beam-columns during drying. Wood Sci. Technol. **5** (3): 221–231.
SCHNIEWIND, A. P. 1966. On the influence of moisture content changes on the creep of beech wood perpendicular to the grain, including the effects of temperature and temperature change. (CSIRO Translation No. 8361). Holz Roh- Werkstoff **24** (3): 87–98.

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