Wood Sci. Technol. 22: 311–322 (1988)

Wood Science and Technology

A reexamination of some deviations from Darcy's Law in coniferous wood

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Summary. A number of workers have noted a marked decrease of wood permeability with increasing specimen length, and to date a model proposed by Bramhall has been used to explain the phenomenon. This describes the effect of random blockages on the permeability of an anisotropic porous medium. However, the model does not allow for the fact that transverse flow may allow longitudinal flow paths to circumvent blockages to some extent, so some deviation from it can be expected. The issue is an important one if laboratory experiments are to be used as a basis for the prediction of the behaviour of wood during impregnation or drying on a commercial scale. A modified model which allows for transverse flow is therefore developed. New experimental evidence which cannot be explained by the Bramhall model, but which can be explained by the new model, is presented.

Introduction

The flow of gases in the viscous-slip region through a homogeneous porous medium generally obeys a modified form of Darcy's Law:

$$\frac{dQ}{dt} = \frac{K\,A\,\Delta P\,\bar{p}}{L\,p} + \frac{B\,A\,\Delta P}{L\,p} \tag{1}$$

where dQ/dt is the volume rate of flow of a gas measured at pressure p, through a sample of cross sectional area A, and length L, in response to a pressure differential ΔP. K is the superficial viscous permeability constant (given by the specific permeability constant, divided by the gas viscosity), and B the slip constant. The mean pressure in the sample is $\bar{p}$. When such a medium is placed in series with another of viscous and slip constants K_1 and B_1 respectively, the flow through the new heterogeneous me-

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* This paper was written while the author was Heritage Visiting Scientist at the Forest Products Laboratory, U.S. Department of Agriculture, and Visiting Professor at the Department of Forestry, University of Wisconsin, both at Madison, Wisconsin, USA. This support is gratefully acknowledged. The author would also like to thank Messrs. J. Ambler, M. C. Breese and S. H. Turner, who carried out some of the experimental work, and staff at the Commonwealth Forestry Institute, Oxford, who provided the *Juniperus procera* material tested

dium is given by

$$\frac{dQ}{dt} = \left(\frac{K\,K_1\,\bar{p}^2 + (B\,K_1 + B_1\,K)\,\bar{p} + B\,B_1}{(K + K_1)\,\bar{p} + (B + B_1)}\right)\left(\frac{A\,\Delta P}{L\,p}\right) \tag{2}$$

because the resistance of the two homogeneous components in series is additive (Bolton, Petty 1978). The overall superficial viscous permeability constant will then be given by $K\,K_1/(K + K_1)$. A similar approach may be used to derive a form of the Darcy equation, and an overall permeability constant, for gas flow through a heterogeneous medium consisting of any number of homogeneous components in series.

It is now generally accepted that such theory is applicable to wood, and that, in many cases, dQ/dt is indeed proportional to $1/L$ at a given value of $\bar{p}$. However, a number of authors have published data for coniferous timbers suggesting that dQ/dt varies less than proportionately with $1/L$ (Amemiya 1962; Anonymous 1965; Banks 1975; Bramhall 1971; Perng 1980; Petty 1967; Siau 1972). Similar trends may be observed in some published liquid flow data (Booker 1977; Kininmonth 1970). In explaining this phenomenon, Bramhall (1971) used a model of the anatomy of impermeable woods in which "tracheids overlap and communicate with only one tracheid at each end, permitting only axial fluid flow." By then considering the effect of random blockages on the permeability of such a model, he was able to justify the inclusion of an exponential term in modified Darcy equations similar to Eq. (2), thus:

$$\frac{dQ}{dt} = \left(\frac{K\,K_1\,\bar{p}^2 + (B\,K_1 + B_1\,K)\,\bar{p} + B\,B_1}{(K + K_1)\,\bar{p} + (B + B_1)}\right)\left(\frac{A\,\Delta P\,e^{-bL}}{L\,p}\right) \tag{3}$$

where e is the base of natural logarithms, and b is a positive constant. It follows that, if the model holds, the natural logarithm of a conductivity term defined by $(dQ\,p\,L/(dt\,\Delta P))$ should be linear in L at a fixed value of $\bar{p}$. Bramhall provided experimental data showing the variation of the natural logarithm of permeability[1] with L, over the range $L = 5$ mm to $L = 30$ mm, for air dried and solvent dried Coastal and Interior British Columbian Douglas Fir sapwood and heartwood. Bramhall believed that these data were in reasonable agreement with his hypothesis, if high values of permeability observed at $L = 5$ mm were ignored on the grounds that "average tracheid lengths are almost equal to specimen length in this case. Consequently, some tracheids could be expected to be truncated at both ends resulting in an abnormally high flow." This is reasonable. However, reexamination of his data for values of $L > 10$ mm gives the impression that a non-linear relationship might fit many of the data as well as, or better than, the linear one used. Bramhall himself notes that in the case of two of the types of material tested "a significant non-linear component also is found, but its magnitude is very small in comparison with the linear one. This suggests that a minor modification may be required in the proposed model." The question of

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[1] It will be evident from Eq. (3) that at a fixed mean pressure there will be a fixed relationship between the overall viscous permeability coefficient (in this case, $K\,K_1/(K + K_1)$) and this conductivity term. Bramhall investigates the $\log_e$ (specific permeability) vs. specimen length relationship; for analytical simplicity, it was preferred to investigate the $\log_e$ ("conductivity") vs. specimen length relationship at a fixed mean pressure in the present work. This slight difference in approach does not affect the comparability of the present work with that of Bramhall, or in any way influence the apparent deviations from Darcy's Law analysed

the nature and magnitude of this modification, and whether it implies a general flaw in the Bramhall hypothesis, is an important one if laboratory experiments are to be used as the basis for the prediction of the behaviour of wood during impregnation or drying on a commercial scale.

Theory

The anatomical model assumed by Bramhall for impermeable woods is a simplistic one since it implies that such woods have zero lateral permeability, and that blockages are always total. Granted that all woods tested to date have shown a finite lateral permeability, longitudinal flow lines can be expected to diverge around blockages to some extent.

Consider a two dimensional array of cells such as those in wood, N cells high by L columns of cells long (Fig. 1). The array is most permeable in the longitudinal plane. However, as in wood, transverse flow in the N plane is also possible. The precise anatomical nature of any flow paths need not be specified, although it is likely that the transverse plane in question is the tangential plane, since most pits are situated on the radial walls of cells. A finite number, k, of blockages are distributed at random within every column of N cells.

The number of pathways wholly longitudinal (i.e. involving no traverse steps) is given by $N\,e^{-bL}$, where b, as before, is a positive constant which here is a function of k, N. The number of such pathways

at the end of column 1 $= N\,e^{-b}$
at the end of column 2 $= N\,e^{-2b}$
at the end of column 3 $= N\,e^{-3b}$ etc.

Therefore the number of such pathways obstructed

in column 2 but not in column 1 $= N(e^{-b} - e^{-2b})$
in column 3 but not in column 2 or 1 $= N(e^{-2b} - e^{-3b})$ etc.

Therefore the number of pathways obstructed

in column 1 only $= N(1 - e^{-b})\,e^{-b(L-1)}$
in column 2 only $= N(e^{-b} - e^{-2b})\,e^{-b(L-2)}$ etc.

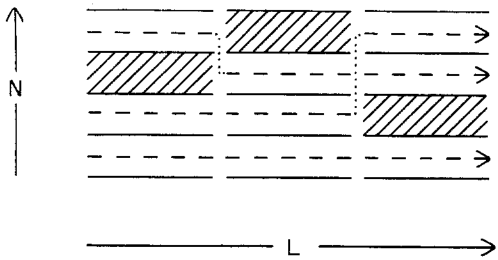

Fig. 1. A small example of a new model, consisting of a two-dimensional array of cells, N cells high, and L cells long in the axial plane. Here N=4, L=3 and k, the number of cells blocked at random in each column of N cells, is eqal to 1. The model is most permeable in the L plane, but flow in the N plane is also possible

Thus, the total number of pathways obstructed once only, C_1, is given by

$$C_1 = N(((1 - e^{-b}) e^{-b(L-1)}) + ((e^{-b} - e^{-2b})^{-b(L-2)}) \ldots + ((e^{-(L-1)b} - e^{bL})) \tag{4}$$

Using a similar approach, the number of longitudinal pathways obstructed twice only, three times only, etc. may be calculated.

If flow paths obstructed one or more times never conduct again, the number left conducting at any length is given by Ne^{-bL}, as stated above, and as defined in the Bramhall model. If, in reality, longitudinal pathways which have been obstructed subsequently conduct to some extent, then the total conductivity of the array will exceed that predicted by the Bramhall model. Two cases are considered below: one where blocked cells are totally impermeable, so that the additive terms arise from divergence around blockages, and one where the additive terms arise from the reduced, but finite, permeability of partially blocked cells.

A. *Divergence around blockages*

With reference to Fig. 1, consider the case where in every column of N cells, all $(N-k)$ unblocked cells conduct and that, as a result, $(N-k)$ longitudinal steps from one column to another involve kX tangential steps, where X is the average number of tangential steps needed to circumvent a blockage.

The reduction in conductivity of a pathway arising from a single blockage will then be given by

$$L\left(\frac{L}{1}+\frac{X}{z}\right)^{-1}$$

where the ratio of tangential to longitudinal permeability of a single cell is z. Thus the number, N_{D1}, of unblocked pathways equivalent to the C_1 blocked pathways will then be given by

$$N_{D1} = zLC_1/(zL+X) \tag{5}$$

Similar equations may be derived for the conductivity of pathways blocked more than once. Using these, the total effective number of pathways, N_e, available at any length in the model can be calculated. This will be directly proportional to the permeability of the model.

B. *Partially blocked cells of reduced permeability*

Reduction in conductivity of a pathway arising from a single blockage will be given by

$$L\left(\frac{L-1}{1}+\frac{1}{z_1}\right)^{-1}$$

where z_1 is the ratio of blocked to unblocked cell permeability. The number, N_{P1}, of unblocked pathways equivalent to the C_1 blocked pathways will then be given by

$$N_{P1} = \frac{z_1 C_1 L}{z_1(L-1)+1} \tag{6}$$

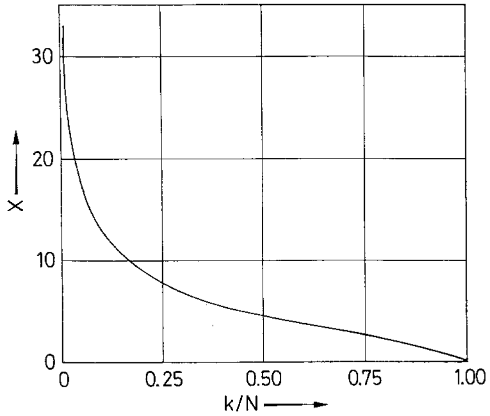

Fig. 2. Variation of X with k/N. From data calculated by Kassab and Bolton (1987)

Similar equations may be derived for the conductivity of pathways blocked more than once. Using these, the total effective number of pathways, N_{P1}, available at any length in the model can again be calculated.

Evaluation of the models

Models based on Eqs. (4)–(6) can very easily be evaluated with a computer, given values for b, N, L, z, z_1, X and k. A value for b may be calculated from N and k. A value for z may be calculated from data showing the permeability anisotropy of timber. From data of Comstock (1970) it seems that z ranges from about 0.1 to 10.0, with the lower values generally being calculated for more impermeable timbers. Values for X, where N is large, have been estimated using a probabilistic approach by Kassab and Bolton (1987). These are plotted as a function of k/N in Fig. 2.

Using model A (divergence around impermeable blockages) with a range of values of k, and values of z of 0.1, 1.0 and 2.5, the variation of the natural logarithm of the total effective number of pathways with length was calculated for up to 20 cells in series (Figs. 3–5). When analysed in this way a material conforming to Darcy's Law should give a horizontal straight line, and the Bramhall model should give a straight line with negative slope (Fig. 3). Deviation from Darcy's Law which does not conform to the Bramhall model is predicted by the present model for many different combinations of k and z. In some cases the predicted curves show curvature at low values of L, and then tend to a horizontal straight line.

Using model B (blockages of reduced permeability) with a range of values of k and z_1, the variation of the natural logarithm of the total effective number of pathways with length was again calculated for a number of cells in series. One example of the data obtained is shown as Fig. 6. Again, deviation from Darcy's Law which does not conform to the Bramhall model is predicted for many different combinations of k and z_1.

To relate these models to reality, reliable estimates of k, z, and z_1 are needed, together with a way of relating the number of cells in series in the model to axial distance in wood. Values of z, the tangential : longitudinal permeability ratio of a

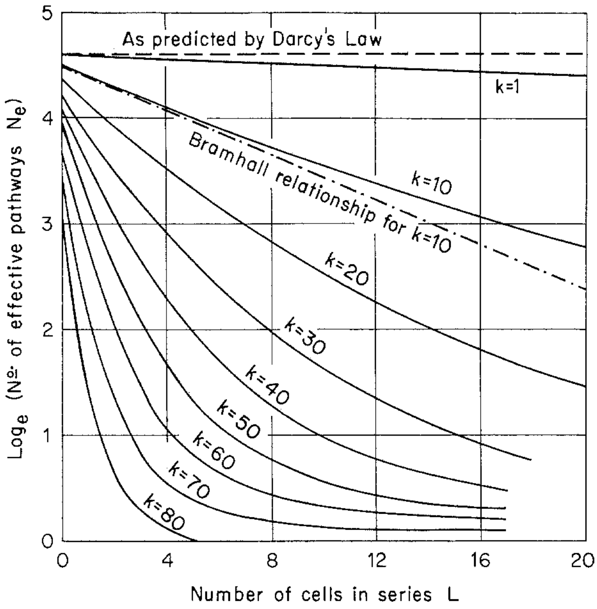

Fig. 3. Model A: variation of predicted number of effective pathways, with the number of cells in series, for z = 0.1 and N = 100. The behaviour expected from Darcy's Law, and from an example of Bramhall's model, is also shown

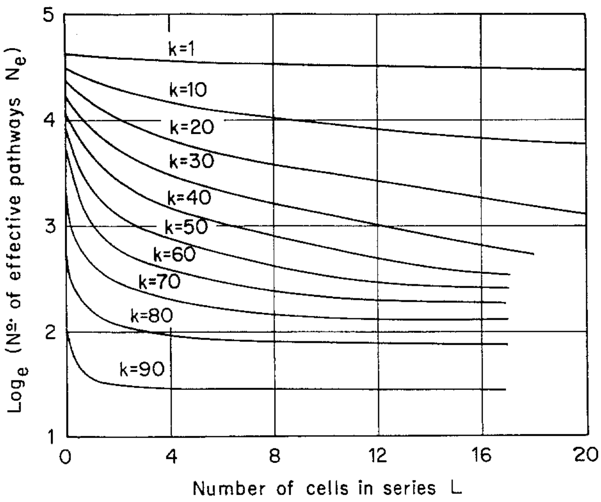

Fig. 4. Model A: variation of predicted number of effective pathways, with the number of cells in series, for z = 1.0 and N = 100

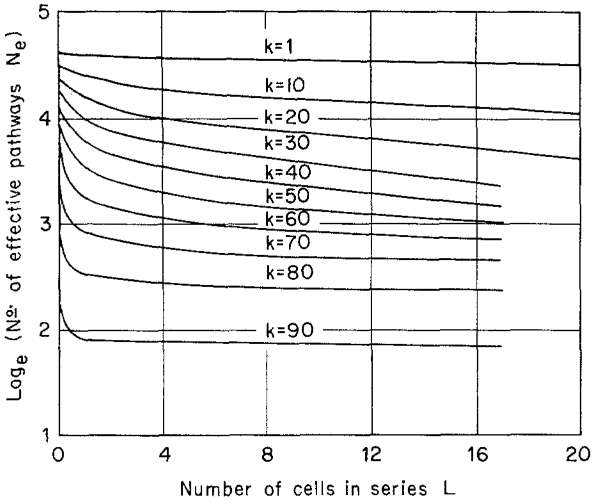

Fig. 5. Model A: Variation of predicted number of effective pathways, with the number of cells in series, for $z = 2.5$ and $N = 100$

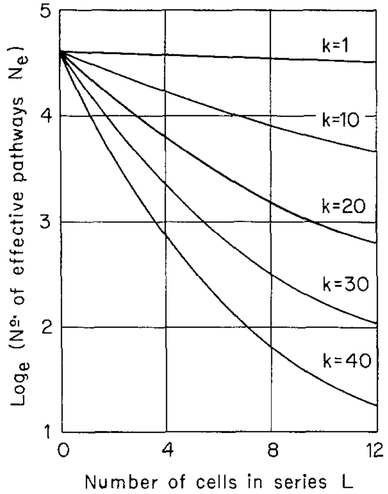

Fig. 6. Model B: Variation of predicted number of effective pathways with number of cells in series, for $z_1 = 0.2$ and $N = 100$

single cell, estimated from experimental data (see above) vary over two orders of magnitude. This may reflect the variability of permeability, and the fact that the longitudinal and tangential permeability are never measured on the same piece of wood. Further experimental work is needed in this area. Realistic values for k and z_1, are very difficult to obtain. To this end attempts were made to impregnate edge sealed impermeable woods axially with plastic monomers, to polymerise these in situ, and then to estimate the number of cells empty, or partially empty, at various depths from the treated surface. However, volatilisation of the resin (forming embolisms in the liquid phase), and the expansion of trapped vapours during the exothermic curing reaction (forcing liquid resin out), made this unreliable.

The relationship between the number of cells in series in the model, L, and axial distance in a coniferous wood will be approximately given by

$$\text{Distance in the wood} = \beta \times \text{mean tracheid length} \times \text{L} \qquad (7)$$

where β is a tracheid overlap factor normally taken as 0.75 (Stamm 1964).

Limitations of the models

1. It is assumed in both models that a given blockage effects a single pathway only. This will never be the case in reality since even the flow conditions in pathways never blocked will be influenced by events in adjacent pathways. However, it can be shown in specific cases that the simplifying assumptions made will always lead to an underestimate of non-linearity, not an overestimate. This can be supported intuitively by considering the fact that events in a pathway adjacent to one completely unblocked can never cause the pathway completely unblocked to conduct less efficiently than it would in isolation.

2. In model A the transverse flow plane is specified above as tangential. In reality, radial flow could also be of some importance. Providing the value of z selected is appropriate to whatever transverse flow occurs, no error will result. The radial : longitudinal permeability ratio for wood is more variable and unpredictable than the tangential : longitudinal ratio (Comstock 1970).

3. In model A, the possibility of random blockages arising in transverse flow paths also is ignored.

4. In both models the wood is assumed to be homogeneous, but prone to random blockages. In reality a number of tissue types (earlywood, latewood, axial resin canals) in parallel may contribute to axial conduction. The different tissue types could have very different permeability vs. length relationships, and this would complicate the predictions of the model considerably.

Experimental evidence

Because the simulations above suggested that behaviour deviating from the Bramhall relationship might reasonably be expected, it was decided to repeat Bramhall's experiments with the heartwood of a number of other species. The first was that of a tropical species, *Juniperus procera*, reportedly one of the most impermeable coniferous woods known (Anonymous 1957). In the material studied, virtually no change in cell shape at the growth ring boundary was discernible under the microscope. The species also lacks resin canals. For these reasons it was thought likely that the material should be relatively homogeneous with respect to permeability. Both the two other species selected, *Chamaecupressus Leylandii* and *Picea abies*, show an indistinct earlywood/latewood boundary, with some narrower latewood cells clearly present; only *Picea abies* contains some axial resin canals (Phillips 1948).

Cylindrical specimens, 40 mm long and about 15 mm in diameter, were cut parallel to the grain. Those with detectable slope of grain were rejected. Acceptable specimens were then carefully microtomed at both ends with a sharp knife, so as to minimise

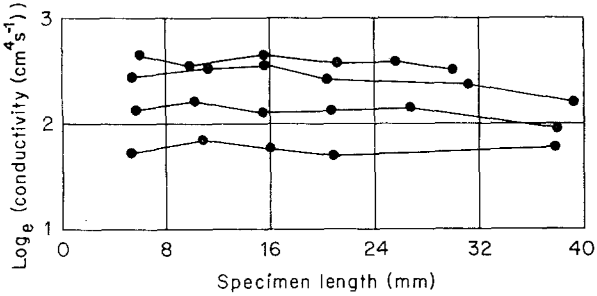

Fig. 7. Observed variation of gaseous conductivity (dQ p L/(dt ΔP)) with specimen length for *Chamaecupressus Leylandii*

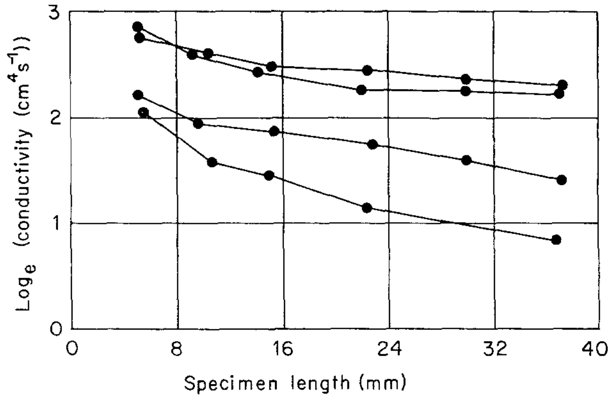

Fig. 8. Observed variation of gaseous conductivity (dQ p L/(dt ΔP)) with specimen length for *Juniperus procera*

surface damage (Choong, McMillan, Tesoro 1975) and the consequent creation of end effects (Bramhall 1971). The specimens were next air dried at 40 °C until they achieved constant weight; after this they were inspected under a microscope, and any containing drying cracks were rejected. The gaseous conductivity of acceptable specimens was then determined using techniques similar to those of Petty and Puritch (1970) at mean pressures ranging from 200 to 600 mm Hg. The conductivity at a standard mean pressure (usually 400 mm Hg) was then read off the resultant conductivity vs. mean pressure curve. After this, each specimen was carefully reduced in length by 5–10 mm, the new end remicrotomed, and the specimen redried, checked for cracks, and retested for conductivity. This procedure was repeated until the specimen was 5 mm, or slightly more, in length. The average tracheid length as measured from macerations was considerably less than this (*Juniperus procera*: 2.99 mm; *Chamaecupressus Leylandii*: 1.76 mm; *Picea abies*: 2.62 mm; coefficients of variation all < 11%), so it is unlikely that any tracheids cut at both ends were present in even the shortest specimens tested. The variation of the natural logarithm of gaseous conductivity (dQ p L/(dt A ΔP)) at the standard mean pressure with specimen length is shown for the three species in Figs. 7–9.

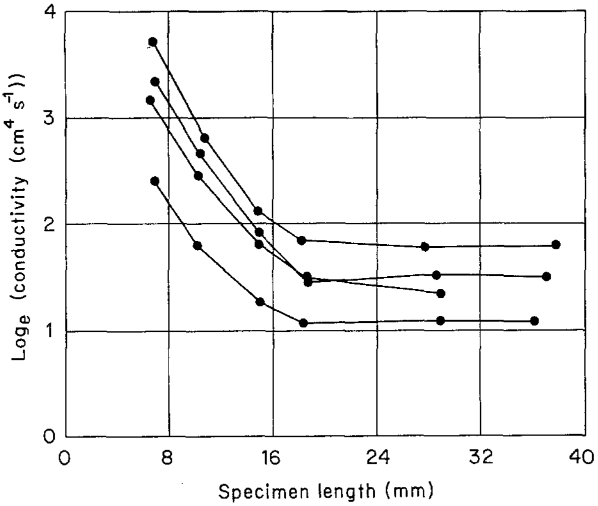

Fig. 9. Observed variation of gaseous conductivity (dQ p L/(dt ΔP)) with specimen length for *Picea abies*

All species proved to be very impermeable, even at short lengths. Data for *Chamaecupressus Leylandii* (Fig. 7) show no clear decrease in $\log_e$ (gaseous conductivity) with specimen length. So this species appears to conform to Darcy's Law. Data for *Juniperus procera* (Fig. 8) show both a decrease in $\log_e$ (gaseous conductivity) with specimen length and some non-linearity. At specimen lengths between 10 and 40 mm there was, however, a tendency towards linearity and conformity to the Bramhall relationship. Data for *Picea abies* (Fig. 9) show distinct non-linearity with a tendency towards a stable value at a specimen length of about 18.5 mm. This raises the question of whether the *Juniperus procera* permeability might not also have tended to a stable value at some specimen length greater than the maximum tested. In any event, the *Picea abies* clearly shows behaviour which cannot be explained by either the Darcy or the Bramhall relationship.

Non-linearity of the sort observed with *Picea abies* was present in some of the model predictions shown in Figs. 3–5. In comparing the experimental data with the theoretical, one of the most important features of the former is the point of stabilisation at 18.5 mm length. Using Eq. (7), this would be equivalent to about 9 cells in series in the models. So the experimental curves should be matched against a model curve which tends towards stability at this number of cells in series. Where $z = 0.1$ this happens when $k = 70$ or more; where $z = 1$, when $k = 60$ or more; where $z = 2.5$, when $k = 50$ or more. On this basis it can be postulated that at least 50–70% of the tracheids in the *Picea abies* tested were not available to flow. It would clearly be very valuable if this could be confirmed experimentally.

It is interesting to compare this postulate of >50–70% tracheid blockage with reported percentages of pits aspirated in impermeable timbers. Phillips (1933) reports that the proportion of air dried *Picea abies* earlywood and latewood pits aspirated were 93% and 69% respectively. By applying probability theory to a model of impermeable Douglas Fir earlywood, Meyer (1971) estimates a 20% chance of a tracheid being inaccessible with 95% pit aspiration, and a 73% chance with 99% aspiration. Such calculations will be in error if pit membranes have a susceptibility to aspiration which varies systematically within the growth ring (Bolton, Petty 1977). In this case some tracheids could become inaccessible at much lower average levels of pit aspiration.

Another way of looking at the *Picea abies* curves in Fig. 9, which are first curvilinear and then linear and horizontal, is that this shape could be the result of the combination of two components in parallel. One component would have to be curvilinear (not necessarily tending to a stable limit); the second would then have to tend to a stable limit, or could even be the horizontal line characteristic of a pathway which obeys Darcy's Law. If the axial resin canals in the species conduct significantly, they might provide such a second component. In this case, it would be the first component only, not the whole wood structure, which would be demonstrating behaviour which cannot be explained by the Bramhall model.

Conclusions

1. Impermeable softwoods should not necessarily be expected to deviate from Darcy's Law in their behaviour.

2. The Bramhall model may explain the deviation from Darcy's Law observed in some impermeable softwoods. However, this model does not allow for transverse flow. If flow in this plane is of any importance in axial flow through an impermeable timber, a deviation from the Bramhall model must be expected. It seems likely that some transverse flow does take place because the same pits are involved in tangential and longitudinal flow paths.

3. The gaseous conductivity of *Picea abies* heartwood clearly cannot be explained by either the Darcy equation, or the Bramhall model as originally conceived. A modification of this model, which allows for tranverse flow, can predict a conductivity vs. length relationship similar to that observed.

4. Further work on the tangential to longitudinal permeability ratio of single pieces of wood (rather than of matched samples), and on the frequency of blockages in impermeable timbers, is needed to refine the model.

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(Received December 15, 1987)

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