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Linearity and non-linearity in mechano-sorptive creep of softwood in compression and bending*

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Summary. Non-linearity of softwood in axial mechano-sorptive creep during moisture cycling can be characterised as departure from linear behaviour. Linear behaviour is shown by experimental measurement to be a gradual approach to a creep limit with exponentially decreasing increments of compliance, J_i, per moisture cycle, when tested under a constant stress. On the other hand, when the stress is progressively increased by a small increment $\Delta\sigma$ after each moisture cycle, the compliance increments will progressively increase, having a value of $\Delta\sigma \sum_1^n J_i$, where n is the cycle number. By subtracting one compliance increment from the succeeding one, the value of $\Delta\sigma$ J_i can be obtained. Analysis of experimental results in bending tension and compression showed that the compression test pieces departed from linearity at total strains around 0.14% to 0.15%, the bending test pieces showed slight evidence of non-linearity at about the same strain, whilst the tensile pieces were approximately linear up to 0.18% strain.

Introduction

For various reasons it is important to understand the extent to which the deformation strain of wood is linear with stress.

Linearity in this context means, in effect, that doubling the stress will result in doubling the strain at any point in its history. Various workers have studied linearity in wood. As a general rule, the material's strain response to stress is linear at lower stresses, temperatures and moisture contents (e.g. King 1961; Grossman, Kingston 1963; Bach 1965; Keith 1972). The threshold values of these three parameters, above which non-linearity begins, appears to depend on the mode of stressing. It also varied considerably from one author to another. This variation probably depended mainly on the criterion on which departure from linearity was based, and the accuracy with which it could be measured. Some important results are summarised in Table 1. These values may be compared with a proportional limit around 0.25% strain in a static bending test and failure at around 0.50% strain (Kollmann, Côté 1968).

Kingston and Budgen (1972) noticed non-linearity of creep at about 50% of the ultimate stress in bending and at about 70% of the ultimate stress in compression.

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Table 1. Onset of non-linearity, from previous authors

Authors	Species	Mode of loading	Stress level	Strain level	Moisture content	Remarks
Kingston + Clarke	hoop pine	bending	50% ultimate			
Hearmon + Paton	beech	bending	1/8 to 3/8 ultimate		cycling	
Echenique-Manrique	tropical hardwoods	compression tension		0.25–0.30% near fail	12% 12%	
Kingston + Budgen	hoop pine hoop pine	bending compression	50% ultimate 70% ultimate		9–14% 9–14%	
Keith (1971)	white spruce	compression compression		0.19% 0.30%	18% 9%	visible damage
Keith (1972)	white spruce	compression	17–21 MPa or 57–70% ultimate	0.21–0.33%	18%	
		compression	35 MPa or 81% ultimate	0.30%	9%	
Hunt (1979)	beech	tension	20–30% ultimate		cycling	

Unfortunately the ultimate stress values were not given, but since the ultimate stress in compression is known to be considerably lower than that in bending, the actual stresses at which non-linearity began were probably similar. The authors concluded that the onset of non-linearity in bending was associated with the behaviour of the compression face of the test pieces.

In studying non-linearity, Dinwoodie (1968) looked at the development of dislocations immediately after loading. The first signs appeared at stresses as low as 25% of the ultimate strength in both air-dried (12% moisture) and green norway spruce, representing strains of about 0.15% to 0.20%.

Non-linearity was also studied by Keith (1971, 1972) who made compression-creep tests at two different constant moisture contents and various stress levels. After completion of the tests he sectioned the pieces and studied them for compression damage as recommended by Dinwoodie (1966, 1968). The creep strains at which damage began was about 0.03% strain at 18% moisture content, whilst only borderline damage was observed at 0.05% strain at 9% moisture content. These represent total strains of 0.19% and 0.30% respectively.

Hunt (1986) found close agreement between the mechano-sorptive creep compliances in tension and bending of pine at stresses of 7.5 MPa; and so concluded that the material was linear at this stress level. Some tensile creep measurements on beech indicated a departure from linearity during mechano-sorptive creep between 20 and 30 MPa (Hunt 1979). No other workers appear to have studied linearity in mechano-sorptive creep, although non-linearity was observed by Gril (1988).

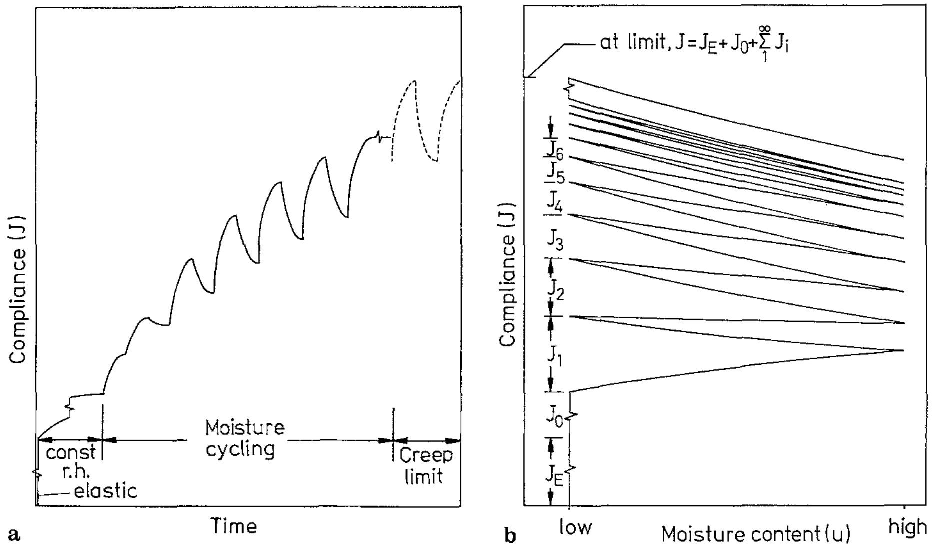


Fig. 1 a and b. Schematic graphs of creep experiment with moisture cycling: **a** compliance versus time; **b** compliance versus moisture content

Linearity in mechano-sorptive creep

In order to study non-linear behaviour, it is first necessary to state what is expected of linear behaviour in mechano-sorptive creep; so that departures from linearity can then be measured. Figure 1 shows typical results of a bending test in the linear region; in which a relatively low stress is applied at a low moisture content, held constant for 24 hours, and then the moisture is cycled between a high and low value. The incremental increases of compliance during each cycle, $J_1, J_2, \ldots J_n$ gradually decrease, and it has been suggested (Hunt, Shelton 1987a) that there is a limiting creep value above which the creep compliance will not go. The slope, $d\varepsilon/du$, of the strain versus moisture content in the right-hand graph, at higher values of compliance approaching the creep limit, has been shown (Hunt, Shelton 1988) to be caused by differences in shrinkage coefficients between the tension and compression faces of the bending test piece. The tension face expands less and the compression face expands more than in the zero-load condition. The sizes of the changes were large enough to explain the apparent creep recovery during subsequent humidifications.

In the papers quoted above, the creep limit was obtained by a load reduction after one moisture cycle, as shown schematically in Fig. 2. However, if a creep limit exists, it should be possible to quantify it by extrapolation from a multi-cycle experiment of the type shown in Fig. 1. There are various ways of approaching this extrapolation. One way is to assume that moisture changes cause rearrangements of molecular bonds under a stress bias (Gibson 1965), and then to base the calculations on probability theory. The gradual exhaustion of the possible internal mechanisms of creep during moisture cycling could give an exponential approach to a creep limit. The equation of the resulting curve could then be

$$J = (J_E + J_c) + J_\infty (1 - e^{-n/N}) \tag{1}$$

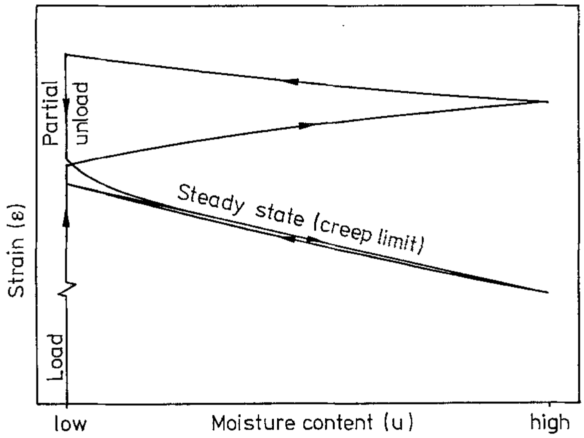


Fig. 2. Schematic graph showing accelerated method of reaching a creep limit

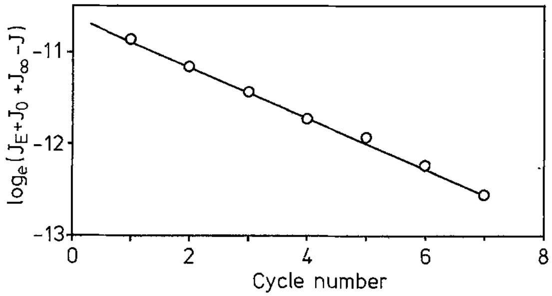


Fig. 3. Results of a multi-moisture-cycle creep test plotted on a logarithmic scale to obtain an estimated creep limit

where n is the cycle number, N is a characteristic cycle number, and the creep limit compliance $(J_E+J_c+J_\infty)$ is the sum of the elastic compliance J_E, a constant J_c and a mechano-sorptive creep-limit compliance J_∞. The constant J_c is obtained from the best fit of Eq. (1) to the experimental data. Since J_c is a compliance, and is evidently neither elastic nor mechano-sorptive, it may be assumed to be a 'normal-creep' compliance. However, Eq. (1) ignores any effect of time on the value of J_c, which is therefore only an approximation. The use of this approximation, for the purpose of data fitting, is partly justified by the knowledge that mechano-sorptive creep is much greater than normal creep (Hearmon, Paton 1964).

Rearranging Eq. (1) gives

$$\log_e\{(J_E+J_c+J_\infty)-J\}=\log_e J_\infty-n/N \qquad (2)$$

Therefore, on the basis of the above theory, it should be possible to analyse the results of a multi-cycle experiment to get a straight-line plot on a graph of $\log_e \{(J_E+J_c+J_\infty)-J\}$ versus the cycle number n. Such a plot is shown in Fig. 3 for a set of *Pinus sylvestris* experimental data. This procedure was found to give a reasonably good fit for 14 separate sets of creep results involving seven or more humidity cycles. However, in all cases of creep, the best-fit line required a large value of the constant J_c; and the large increase in compliance during the first cycle always caused the first point to be above the line as seen in Fig. 3. When some multi-cycle recovery data was analysed in the same way, this effect was even more marked.

The discrepancies quoted above were at first attributed to the simplification of the normal creep behaviour by including it in the simple constant, J_c. It was therefore decided to try to separate the elastic and normal creep compliances from the total compliance at each stage. An analysis had already been made of the normal creep behaviour of the same sample of *Pinus sylvestris* by Shelton (1988). He found by least-square analysis that a basic master curve for constant-load, constant-humidity creep after subtraction of the elastic compliance, took the empirical form at a reference moisture content of 5% of

$$J_N = 8.282(10^{-7}) - 4.266(10^{-6})\,L_T + 2.833(10^{-6})\,L_T^2 - 5.954(10^{-7})\,L_T^3 + 4.632(10^{-8})\,L_T^4, \quad \text{in MPa}^{-1} \tag{3}$$

where L_T is the decimal logarithm of the time in seconds. The effect of material variations on creep behaviour correlated well with the measured elastic compliance J_E, allowing a horizontal shift H_E of the master curve on the logarithmic time (s) axis to the right, of

$$H_E = 3.1033 - 2.004(10^{-4})/J_E \tag{4}$$

A further expansion of the time scale depended on the moisture content of the test and resulted in a logarithmic shift factor H_m, of

$$H_m = -0.06145 + 0.0453\ u \tag{5}$$

where u is the moisture content in %.

The above method of analysis allowed an estimation of the "normal"creep strain after any combination of times and moisture contents, based on the assumption that the normal creep could be separated from the mechano-sorptive creep, and using step-by-step analysis. Two typical sets of results are shown in Table 2 for bending creep of *Pinus sylvestris*.

On examination of these and similar results, the same two problems arose as with the previous analysis, in which the normal creep component was not separated; namely a large constant term J_c and a large amount of mechano-sorptive creep during the first cycle, which caused the first point to lie above the straight line on a graph such as that of Fig. 3. However it was noticed that the constant J_c appeared to be a fixed fraction of the total limiting creep, so that the ratio $J_c/(J_c+J_\infty)$ was constant at about 0.4. The only suitable explanation appeared to be the postulation of at least two

Table 2. Data analysis of two bending tests using a single exponential term and separating the "normal creep" from the mechano-sorptive creep, with Eqs. (3)–(5)

	Test a	Test b
Elastic compliance J_E, 10^{-3} MPa^{-1}	0.0752	0.069
Estimated normal creep compliance after one cycle J_N, 10^{-3} MPa^{-1}	0.0143	0.0122
Mechano-sorptive creep:		
constant term J_c, 10^{-3} MPa^{-1}	0.0203	0.042
cycling creep limit J_∞, 10^{-3} MPa^{-1}	0.0344	0.069
characteristic cycle number N	3.31	3.58

mechano-sorptive creep mechanisms, having different characteristic cycle numbers. This is rather like the well-known curve-fitting procedure for normal viscoelastic time-dependent creep, in which a series of exponential terms is used to obtain a creep distribution function. However, in the present case, in which a function is to be fitted to only a few points in a seven or eight humidity-cycle experiment, it was felt that the use of more than two exponential terms could not be justified. This would give a creep equation of the type

$$J = J_E + J_N + J_1 (1 - e^{-n/N_1}) + J_2 (1 - e^{-n/N_2}) \tag{6}$$

and where the mechano-sorptive creep limit $J_\infty = (J_1 + J_2)$. J_N is the normal creep component obtained from Eqs. (3), (4) and (5) and is a function of time and moisture. An iterative method was built into a computer programme to determine suitable values of J_1, J_2, N_1 and N_2. The number of points on which the analysis could be based was increased by the use of a method suggested by the results of Hunt and Shelton (1988) which showed that the slope of the strain-moisture content graph at a limiting state of creep was approximately proportional to the strain level. The changing slopes were attributed to changing expansion and shrinkage rates with strain, rather than purely creep effects. By estimating the magnitude of the change in slope with strain and adding this value to the compliances at the humid end of the cycle, the effect of these dimensional changes can be almost eliminated, leaving only the creep effects, with reference to the drier condition. For example, Figure 4 shows a plot of an eight-cycle creep test started at a low humidity in which, to all of the compliance values at the humid end of the cycle, a value of 0.088 J was added. The coefficient 0.088 was estimated from the compliance-moisture curves at the eighth cycle. It can be seen that both the dry and humid data points fit reasonably well on a continuous curve. It can also be seen that the fitted curve, using Eq. (6), fits well to the data.

The above analysis is based on the assumption that normal creep is different from mechano-sorptive creep and can be separated from it quantitatively. However, it is often postulated that humidity cycling is merely a means of accelerating normal creep, so that all creep is 'normal'. On this basis, it was found to be possible to fit the measured data equally effectively by modifying Eq. (6) to

$$J = J_E + J_0 + J_1 (1 - e^{-n/N_1}) + J_2 (1 - e^{-n/N_2}) \tag{7}$$

where J_0 is now the creep that took place before moisture cycling started, as shown in Fig. 1. However, in order to fit experimental data, the value of the constant term J_0 needed to be larger than that actually measured. Some typical values and the r.m.s. errors for bending tests at 7.5 MPa surface stress are compared in Table 3 and Fig. 4.

Evidently more work is required to resolve the fundamental question of whether the analysis of Eq. (6) or that of Eq. (7) is more suitable. Both methods of analysis give low r.m.s. errors of around 0.5% or better, and the difference between the extrapolated creep limits is only 2.5% of the overall creep limit.

It should be noted that the above analysis used exponential terms based on the number of moisture cycles, since cycles of a consistent size were used. For more general purposes, the parameter n would need to be replaced by $\sum |\delta u|$, the total of moisture changes including sorption and desorption since the start of the test. The characteristic cycle numbers N_1 and N_2 would then be replaced by corresponding moisture parameters U_1 and U_2.

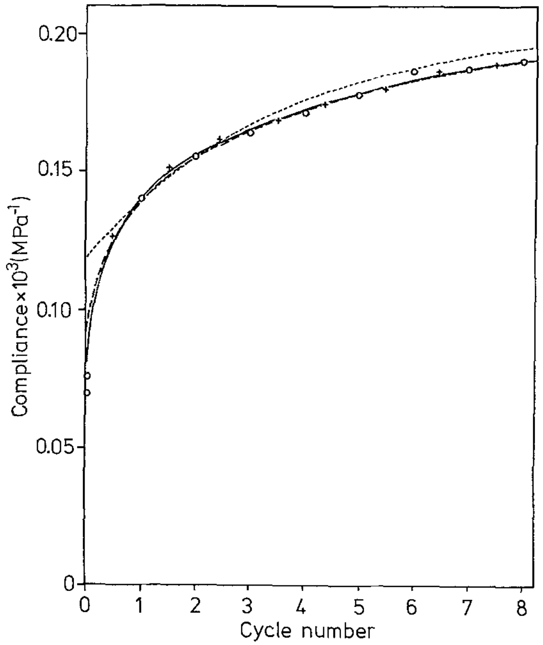


Fig. 4. Results of a creep test at constant load with eight humidity cycles, starting dry. The compliances at the wet end of the cycle have been multiplied by 0.088 to allow for altered dimensional change rates. Fitted curves: with normal creep calculated separately (Eq. 6) ——; with combined normal and mechano-sorptive creep (Eq. 7) – – –; using a single exponential term (Eq. 1) - - - -; Experimental points: dry o; wet +

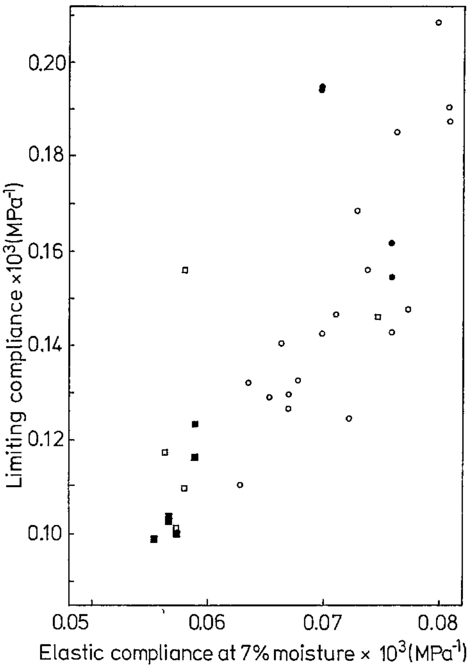
Table 3. Data analysis using two exponential terms

	Test b		Test a	
	Analysis with Eq. (6)	Analysis with Eq. (7)	Analysis with Eq. (6)	Analysis with Eq. (7)
Elastic compliance, J_E, MPa^{-1}	69.77 (10^{-6})	69.77 (10^{-6})	75.94 (10^{-6})	75.94 (10^{-6})
J_N, MPa^{-1} after 8 cycles (Eq. 6)	23.9 (10^{-6})		28.03 (10^{-6})	
J_0, MPa^{-1} (Eq. 7) measured		6.1 (10^{-6})		6.3 (10^{-6})
J_0, MPa^{-1} (Eq. 7) from analysis		24.23 (10^{-6})		26.86 (10^{-6})
J_1, MPa^{-1}	50.5 (10^{-6})	35.5 (10^{-6})	35.4 (10^{-6})	19.59 (10^{-6})
J_2, MPa^{-1}	54.8 (10^{-6})	73.0 (10^{-6})	20.22 (10^{-6})	41.54 (10^{-6})
N_1, cycles	0.34	0.44	0.29	0.44
N_2, cycles	4.2	4.5	4.0	4.7
Total creep limit, MPa^{-1}	199.0 (10^{-6})	202.5 (10^{-6})	159.6 (10^{-6})	163.9 (10^{-6})
rms error, MPa^{-1}	0.94 (10^{-6})	1.13 (10^{-6})	0.56 (10^{-6})	0.66 (10^{-6})

Some creep-limit compliance results for both the accelerated experimental method of Hunt and Shelton (1988) and the above extrapolation method are shown in Fig. 5 plotted against J_E, the elastic compliance. It is interesting to notice that the creep limits predicted by the two methods are similar.

Non-linearity in mechano-sorptive creep

In order to reduce the difficulties of drawing conclusions with such a variable material as wood, the non-linearity was studied by means of incremental increases of stress



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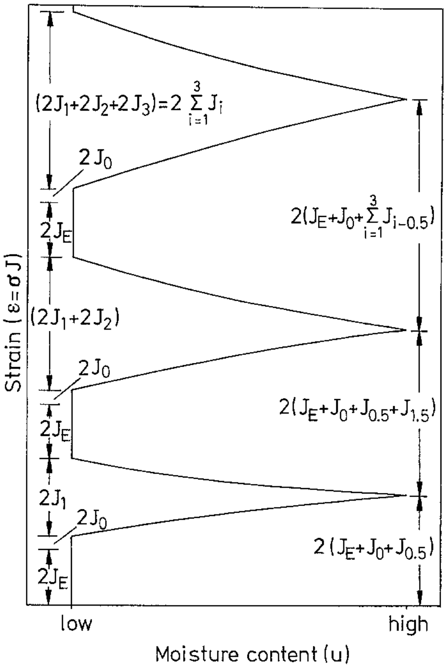

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Fig. 5. Limiting creep compliance plotted against elastic compliance at 7% moisture content: baltic redwood ○ ●, european spruce □ ■; accelerated method ○ □, extrapolation method ● ■

Fig. 6. Expected strain versus moisture graph for linear behaviour during a creep experiment with humidity cycling in which the stress is increased a further 2 MPa after each cycle

after each moisture cycle. By this means, the stress and strain at which departure from the expected linear behaviour, as detailed in the above section, could be observed. This was done in tension, compression and bending on matched samples of baltic redwood (*Pinus sylvestris*), and using stress increases of 2 MPa after each moisture cycle. Using the notation of Fig. 1, the strain versus moisture content graph for linear behaviour should be as shown in Fig. 6. It can be seen that the increase in strain during the first moisture cycle should be $2J_1$, during the second cycle $(2J_1+2J_2)$, and during the n^{th} cycle $2\sum_{i=1}^{n} J_i$. By subtracting the increase in strain during the $(i-1)^{th}$ cycle from that during the i^{th} cycle, the value of $2J_i$ is obtained.

Some typical results are shown in Fig. 7. It must be remembered that these are second differences, and therefore they represent very small compliances. This explains the large amount of scatter in the results and emphasises the need for extreme accuracy in creep measurements. However, by comparison of the experimental results with typical curves for linear behaviour (solid lines) the stress at which non-linearity began can be estimated. Non-linearity in compression appeared to begin at stresses between 8 and 10 MPa, at total strains between 0.14% and 0.15%. Non-linearity in bending was less dramatic but appeared to begin at about the same stress levels, and

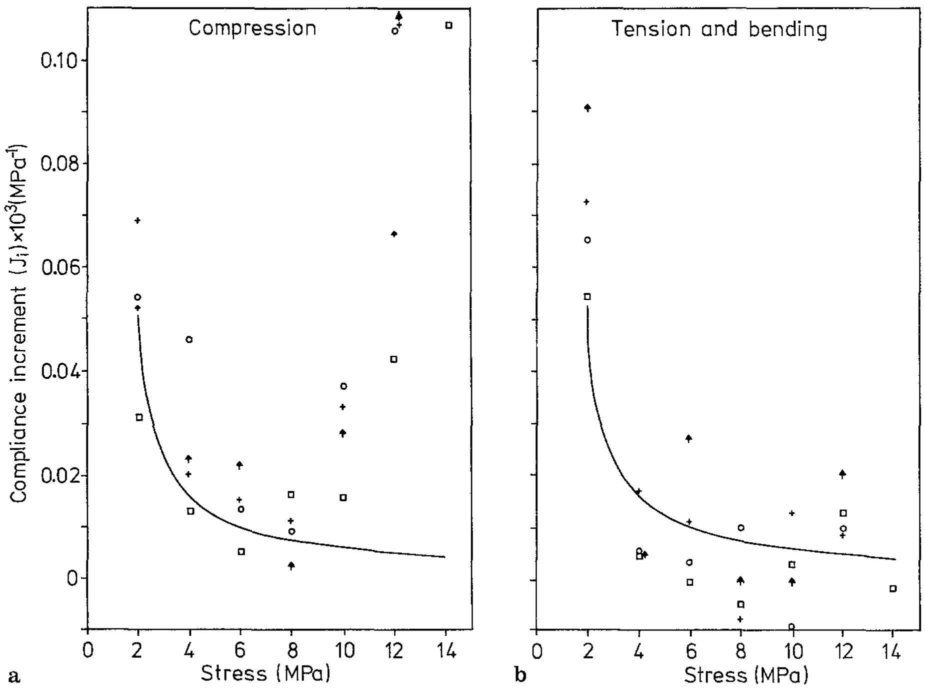

Fig. 7a and b. Mechano-sorptive creep increments during successive moisture cycles of an experiment in which the stress is increased by 2 MPa after each cycle (i.e. the cycle number is half the stress value): **a** compression; **b** tension o □ and bending +/+. The symbols represent different test pieces. Solid curve represents typical exponential approach to a creep limit

strains around 0.16%. These results may be compared with those of previous workers listed in Table 1.

In the present study some compression test pieces that were taken to a total strain of 0.25% to 0.26% at a stress of 14 MPa were unloaded and the humidity was then cycled to observe recovery and to compare with some bending and tension pieces whose creep behaviour had remained linear or nearly so. The purpose was to test whether the non-linearity resulted in irrecoverable creep that could be attributed to compression damage. After four humidity cycles and 670 hours the remaining mean unrecovered strains were approximately as shown in Table 4. Interpretation of these few data items is not easy but they show some consistency in suggesting that the departure from linearity of the compression tests might be associated with creep damage and the resulting irrecoverable strain.

There are some problems associated with the specification of the onset of non-linearity in terms of stress level. Stress level is often expressed as a percentage of ultimate stress; but the latter can vary by a factor of up to three, according to the moisture content and the mode of stressing. For this reason, strain may be a more suitable parameter. On the other hand, for design purposes, stress is a useful parameter, since it can be directly compared with allowable design loadings. In order to obtain both advantages, test stresses and the stresses at the onset of non-linearity could be specified as fractions of the instantaneous elastic modulus. Although the elastic modulus

Table 4. Unrecovered strain for various loading modes

Loading mode	Unrecovered strain	Estimated departure from linearity, strain
Tensile	0.0309%	0.0020%
Bending	0.0328%	0.0040%
Compression	0.0640%	0.0290%

also varies with moisture content, the variation has generally been found to be small (e.g. Hunt, Shelton 1987b and other workers). This means that the onset of non-linearity in the present study, at strains of 0.14% to 0.15%, could also be specified as occurring at 0.75 (10^{-3}) of the elastic modulus. It will be observed that this figure is the ratio of stress to elastic modulus, and so also represents an elastic strain.

Modelling of mechano-sorptive creep

It is hoped that the above analysis of creep during moisture cycling at low stresses, below the onset of non-linearity, makes further progress towards the development of a creep model. It is envisaged that the model could include six components:

(1) The elastic compliance.

(2) A correction for the slope of the elastic compliance vs moisture content graph as a function of material variation.

(3) The normal-creep compliance, which must include the effects of material variation and of moisture content.

(4) The mechano-sorptive creep compliance, possibly based on four parameters, as described in this paper.

(5) Dimensional-change corrections:

(a) For bending only, a correction for the effects of thickness changes with moisture (Hunt, Shelton 1987b).

(b) For direct stresses only, a correction for zero-load dimensional changes.

(6) A limiting value of stress or strain above which non-linearity begins, with possible further non-linear analysis above this limit.

It is evident that there is much work to be done in developing such a model. This model should then help to point the way towards a theoretical explanation based on the structure of the material.

Conclusions

Whilst it has been previously shown that a creep limit can be obtained rapidly by a load reduction after moisture cycling, a creep limit can also be estimated by extrapolation from the results of a multi-cycle experiment.

The extrapolation method uses an exponential approach to a mechano-sorptive creep limit, and therefore suggests a mechanism based on probability. For a good fit to the experimental data, two exponential terms were needed, suggesting that more than one mechanism might operate.

Non-linearity was found to begin in compression at about 0.15% strain or a stress of about 0.75 (10^{-3}) of Young's modulus. All bending tests at stress levels above this are therefore likely to be non-linear and must be interpreted accordingly.

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