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Combined structures-controls-integrated optimization using distributed parameter models*

A. V. Balakrishnan
Department of Electrical Engineering, UCLA, Los Angeles, CA 90024, USA

Abstract. Distributed parameter methods can offer unique advantages in combined structures–control optimization, particularly in the preliminary design phase where at present complex computer programs based on large-dimension Finite Element Models are currently in vogue, replacing them by closed-form analytic expressions for performance criteria in terms of the structure/controls parameters. In this paper we present an explicit closed-form expression for a lattice-truss clamped at one end with co-located sensors and actuators at the other end, using an equivalent anisotropic Timoshenko beam model. Some generic controls/structures optimization problems are shown to be tractable analytically, exploiting this expression.

1 Introduction

In this paper our aim is to show that distributed parameter models offer unique advantages in combined structures–controls oprimization, particularly in preliminary designs where at the present time complex computer program based on large-dimension Finite Element models are in vogue. We shall develop closed-form analytic expressions for performance criteria in terms of structural parameters with the aid of which it is possible not only to reduce dramatically the computational complexity but also to obtain analytical solutions to the optimization problem thereby gaining valuable insight on the structures–controls interaction.

For prior work on such optimization problems the reader is referred to [1] where the authors offer computer studies considering a deterministic LQG infinite horizon problem using finite-dimensional models. Additional references on work along similar lines can also be found therein.

Here we consider a beam-like truss similar to the EPS (Earth Pointing Satellite) structure. To reduce the complexity, we specialize to the cantilever version with the beam clamped on one end and an offset antenna at the other end which also houses co-located force/moment actuators and rate sensors. The control performance criteria include

(i) mean-square attitude error

and

(ii) sum of the absolute values of the real parts of the closed-loop eigenvalues.

Structural performance criteria include structure mass and control mass (and moment of inertia).

The continuum model we use is an anisotropic Timoshenko beam model. Our major result is the derivation of closed-form formulas for (i) and (ii) in terms of the mass and flexibility coefficients in the Timoshenko model which are then expressed in terms of the truss parameters following [2].

Section 2 begins with the anisotropic one-dimensional Timoshenko model dynamics. Using an infinite-dimensional "state-space" version of the dynamic equation (following [7]) we derive a formula for the minimal attainable mean square attitude error using only rate sensors, as well as for the sum of the absolute values of the real parts of the closed loop eigenvalues.

Section 3 details the features of the EPS truss considered. We derive the equivalent Timoshenko beam elastic constants in terms of the truss geometry and material for the case where the longerons, battens and cross-bars have the same cross-sectional area. The mean-square pointing error

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formulas are then expressed in terms of the truss parameters. This simplicity of formula is compared with the corresponding FEM version where a large dimension matrix version is required. For typical parameters the values obtained by both techniques are presented. They agree within one percent.

Finally an illustrative combined structures-controls optimization problem is examined in Sect. 4, and is shown to have a simple analytical solution.

2 The anisotropic Timoshenko continuum model equations

We begin with the (one-dimensional) anisotropic Timoshenko model following [2], for a beam clamped at one end with an offset mas at the other end, where also the force/moment actuators and sensors are located. Let s denote the space variable along the beam axis, $0 \leqq s \leqq L$. Let t denote time and let

$u(t,s)$ denote elongation along the beam axis—(X-component of the displacement taken as X-axis)
$v(t,s)$ denote the Y-component of the displacement
$w(t,s)$ denote the Z-component of the displacement
$\phi_1(t,s)$ torsion angle about the X-axis
$\phi_2(t,s)$ torsion angle about the Y-axis
$\phi_3(t,s)$ torsion angle about the Z-axis.

Let superdots denote differentiation with respect to time and primes, differentiation with respect to the space variable s. Then the dynamic equations valid for the interior of the beam, $0 < s < L$, are:

$$m_{11}\ddot{u} - c_{11}u'' - c_{14}v'' - c_{15}w'' - c_{15}\phi_2' + c_{14}\phi_3' = 0$$

$$m_{22}\ddot{v} - c_{44}v'' - c_{14}u'' + c_{44}\phi_3' = 0, \qquad m_{33}\ddot{w} - c_{55}w'' - c_{15}u'' - c_{15}\phi_2' = 0,$$

$$m_{44}\ddot{\phi}_1 - c_{66}\phi_1'' - c_{36}\phi_2'' - c_{26}\phi_3'' = 0$$

$$m_{55}\ddot{\phi}_2 + m_{56}\ddot{\phi}_3 + c_{15}u' + c_{55}w' - c_{36}\phi_1'' + c_{55}\phi_2 - c_{33}\phi_2'' - c_{23}\phi_3'' = 0$$

$$m_{66}\ddot{\phi}_3 + m_{56}\ddot{\phi}_2 - c_{14}u' - c_{44}v' - c_{26}\phi_1'' - c_{23}\phi_2'' + c_{44}\phi_3 - c_{22}\phi_3'' = 0.$$

We specialize to the case where the truss is clamped at the end $s = 0$, so that the end contains at $s = 0$ are:

$$u(t,0) = v(t,0) = w(t,0) = \phi_1(t,0) = \phi_2(t,0) = \phi_3(t,0) = 0.$$

The actuator dynamics at $s = L$ are:

$$0 = m_4\ddot{u}(t,L) + c_{11}u'(t,L) + c_{14}v'(t,L) + c_{15}w'(t,L) + c_{15}\phi_2(t,L) - c_{14}\phi_3(t,L) + F_1$$

$$\begin{matrix}0\\0\end{matrix} = m_4\begin{vmatrix}\ddot{v}(t,L) + r_y\ddot{\phi}_1(t,L)\\ \ddot{w}(t,L) + r_z\ddot{\phi}_1(t,L)\end{vmatrix} + \begin{vmatrix}c_{14}u'(t,L) + c_{44}(v'(t,L) - \phi_3(t,L))\\ c_{15}u'(t,L) + c_{55}(w'(t,L) + \phi_2(t,L))\end{vmatrix} + \begin{vmatrix}F_2\\F_3\end{vmatrix}$$

$$\begin{matrix}0\\0\\0\end{matrix} = \hat{I}\begin{vmatrix}\ddot{\phi}_1(t,L)\\ \ddot{\phi}_2(t,L)\\ \ddot{\phi}_3(t,L)\end{vmatrix} + C_3\begin{vmatrix}\phi_1^1(t,L)\\ \phi_2'(t,L)\\ \phi_3'(t,L)\end{vmatrix} + M_c + m_4\begin{vmatrix}r_y\\ r_z\\ 0\end{vmatrix} \otimes \begin{vmatrix}-\ddot{w}(t,L)\\ \ddot{v}(t,L)\\ 0\end{vmatrix}$$

where $\otimes$ denotes vector cross-product,

$$C_3 = \begin{vmatrix}c_{66} & c_{36} & c_{26}\\ c_{36} & c_{33} & c_{23}\\ c_{26} & c_{23} & c_{22}\end{vmatrix},$$

$$\begin{vmatrix}F_1\\F_2\\F_3\end{vmatrix} \sim \text{(Actuator) Control forces,}$$

M_c:(Applied) Control moment,

$$\hat{I} = I + m_4 + m_4 \begin{vmatrix} r_z^2 + r_y^2 & 0 & 0 \\ 0 & r_y^2 & -r_z r_y \\ 0 & -r_z r_y & r_z^2 \end{vmatrix}.$$

The co-ordinates of the antenna c.g. are $(0, r_y, r_z)$. The antenna mass is m_4 and the moment of intertia about the c.g. I.

2.1 *Abstract version*

We proceed immediately to the state space (or "abstract") formulation of these equations. Readers unfamiliar with the necessary background may skip this part and simply accept the abstract version because of the similarity to the finite-dimensional version as in standard texts such as [3]. The novelty in our formulation is the inclusion of the boundary values as part of the state. Thus let **H** denote the Hilbert space

$L_2(0, L)^6 \times R^6$.

Elements in **H** will be denoted $\begin{bmatrix} f \\ b \end{bmatrix}$, $f \in L_2(0, L)^6$, $b \in R^6$. Let A denote the operator with domain in **H** defined by

$$\mathbf{D}(A) = \left[\begin{matrix} f \\ b \end{matrix} \middle| f, f' \in L_2(0, L)^6, f(0) = 0; b = f(L) \right]$$

and

$$\mathbf{A}x = \begin{bmatrix} -A_2 f'' + A_1 f' + A_0 f \\ L_1 f(L) + A_2 f'(L) \end{bmatrix}$$

where

$$\mathbf{A}_2 = \begin{vmatrix} C_1 & 0 \\ 0 & C_3 \end{vmatrix}, \quad \mathbf{C}_1 = \begin{vmatrix} c_{11} & c_{14} & c_{15} \\ c_{14} & c_{44} & 0 \\ c_{15} & 0 & c_{55} \end{vmatrix}, \quad \mathbf{A}_1 = \begin{vmatrix} 0 & C_2 \\ -C_2^* & 0 \end{vmatrix},$$

$$\mathbf{C}_2 = \begin{vmatrix} 0 & -c_{15} & c_{14} \\ 0 & 0 & c_{44} \\ 0 & -c_{55} & 0 \end{vmatrix} \quad \mathbf{A}_0 = \text{Diag.}\,[0, 0, 0, 0, c_{55}, c_{44}] \quad , \mathbf{L}_1 = \begin{vmatrix} 0 & -C_2 \\ 0 & 0 \end{vmatrix}.$$

With this definition, it is not difficult to see that A is self-adjoint, nonnegative definite with dense domain. In particular the potential energy is given by

$$[\mathbf{A}x, x] = \int_0^L \left[\mathbf{H} \begin{vmatrix} f' \\ f \end{vmatrix}, \begin{vmatrix} f' \\ f \end{vmatrix} \right] ds, \qquad x = \begin{vmatrix} f \\ f(L) \end{vmatrix}$$

where

$$\mathbf{H} = \begin{vmatrix} \mathbf{C}_1 & 0 & 0 & -\mathbf{C}_2 \\ 0 & \mathbf{C}_3 & 0 & 0 \\ 0 & 0 & & \\ -\mathbf{C}_2^* & 0 & & \mathbf{A}_0 \end{vmatrix}$$

where the elastic constants c_{ij} are constrained so that the matrix **H** is nonnegative definite. It is necessary in particular that the matrices $\mathbf{C}_1, \mathbf{C}_3$ be nonnegative definite and

$c_{ii} c_{jj} - c_{ij}^2 > 0.$

It is further possible to prove that

$$[\mathbf{A}x, x] = 0 \quad \text{implies} \quad x = 0$$

under our (cantilever) conditions. In other words there are no rigid-body modes. In particular $\mathbf{A}^{-1}$ is well-defined and is compact, linear bounded.

Let

$$\mathbf{M}_0 = \begin{vmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{55} & m_{56} \\ 0 & 0 & 0 & 0 & m_{56} & m_{66} \end{vmatrix}$$

where

$$m_{66}m_{55} - m_{56}^2 > 0$$

and let

$$\mathbf{M}_b = \left|\begin{array}{ccc|ccc} m_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_4 & 0 & m_4 r_y & 0 & 0 \\ 0 & 0 & m_4 & m_4 r_z & 0 & 0 \\ \hline 0 & m_4 r_y & m_4 r_z & & & \\ 0 & 0 & 0 & & \hat{I} & \\ 0 & 0 & 0 & & & \end{array}\right| .$$

Then the abstract equivalent of the controlled Timoshenko beam equations becomes:

$$\mathbf{M}\ddot{x}(t) + \mathbf{A}x(t) + \mathbf{B}u(t) = 0 \tag{2.1}$$

where

$$x = \begin{vmatrix} f(t, \cdot) \\ b(t) \end{vmatrix}$$

where

$$b(t) = \begin{vmatrix} u(t, L) \\ v(t, L) \\ w(t, L) \\ \phi_1(t, L) \\ \phi_2(t, L) \\ \phi_3(t, L) \end{vmatrix} \tag{2.2}$$

$$Bu = \begin{vmatrix} 0 \\ u \end{vmatrix}, \qquad Mx = \begin{vmatrix} M_0 f \\ M_b b \end{vmatrix} \quad \text{and} \quad u(t) = \begin{vmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ M_c(t) \end{vmatrix}$$

the suite of control forces and moments. The tensor model for a co-located rate sensor can then be expressed as:

$$y(t) = \dot{b}(t) + N_0(t)$$

where $N_0(\cdot)$ is white Gaussian noise with spectral density

$$D_o = d_o I_{6 \times 6}.$$

To conclude our model we take actuator noise into account and thus we have finally have the stochastic equation:

$$M\ddot{x}(t) + Ax(t) + Bu(t) + BN_a(t) = 0 \tag{2.3}$$

where $N_a(\cdot)$ is white Gaussian noise with spectral density matrix

$D_a = d_a I_{6\times 6}$.

The mean square attitude error for any control is then defined by:

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T (u(t,L)^2 + v(t,L)^2 + w(t,L)^2 + |r|^2\phi_1(t,L)^2)dt. \tag{2.4}$$

We shall denote this by

σ_a^2.

In terms of the abstract formulation it is shown in [4, 5] that the minimal attainable mean square attitude error

$$= \sqrt{d_a d_o}\,(b_{11} + b_{22} + b_{33} + |r|^2 b_{44})$$

where

$\{b_{ij}\} = \mathbf{B}^*\mathbf{A}^{-1}\mathbf{B}, \quad 1 \leqq i, j \leqq 6.$

Now we can calculate the matrix

$\mathbf{B}^*\mathbf{A}^{-1}\mathbf{B}$

in terms of the elastic constants. We have finally thus:

$$b_{11} = \frac{Lc_{44}c_{55}}{c_{11}c_{44}c_{55} - c_{14}^2c_{55} - c_{15}^2c_{44}}$$

$$b_{22} = \frac{L(c_{11}c_{55} - c_{15}^2)}{c_{11}c_{44}c_{55} - c_{14}^2c_{55} - c_{15}^2c_{44}} + \frac{L^3}{3}\left(\frac{c_{66}c_{33} - c_{36}^2}{d_3}\right)$$

$$b_{33} = \frac{L(c_{11}c_{44} - c_{14}^2)}{c_{11}c_{44}c_{55} - c_{14}^2c_{55} - c_{15}^2c_{44}} + \frac{L^3}{3}\left(\frac{c_{22}c_{66} - c_{26}^2}{d_3}\right)$$

$$b_{44} = L\left(\frac{c_{22}c_{33} - c_{23}^2}{d_3}\right), \quad b_{55} = L\left(\frac{c_{66}c_{22} - c_{26}^2}{d_3}\right), \quad b_{66} = L\left(\frac{c_{66}c_{33} - c_{36}^2}{d_3}\right)$$

where

$$d_3 = c_{66}(c_{33}c_{22} - c_{23}^2) - c_{36}(c_{36}c_{22} - c_{23}c_{26}) + c_{26}(c_{36}c_{23} - c_{33}c_{26}).$$

And correspondingly, the Mean Square:

Longitudinal extension

$$= \sqrt{d_s d_o}\left(\frac{Lc_{44}c_{55}}{c_{11}c_{44}c_{55} - c_{14}^2c_{55} - c_1^2c_{44}}\right)$$

XY-bending

$$= \sqrt{d_s d_o}\left\{\frac{L(c_{11}c_{55} - c_{15}^2)}{c_{11}c_{44}c_{55} - c_{14}^2c_{55} - c_{15}^2c_{44}} + \frac{L^3}{3}\left(\frac{c_{66}c_{33} - c_{36}^2}{d_3}\right)\right\}$$

XZ-being

$$= \sqrt{d_s d_o}\left\{\frac{L(c_{11}c_{44} - c_{14}^2)}{c_{11}c_{44}c_{55} - c_{14}^2c_{55} - c_{15}^2c_{44}} + \frac{L^3}{3}\left(\frac{c_{22}c_{66} - c_{26}^2}{d_3}\right)\right\}$$

YZ-displacement due to torsion

$$= \sqrt{d_s d_o}\left(\frac{(c_{22}c_{33} - c_{23}^2)}{d_3}\right)|r|^2.$$

For a "direct" feedback control of the form

$$u(t) = \gamma y(t)$$

where γ is a scalar, the corresponding mean square pointing error:

$$\sigma_a^2 = \left(\frac{\gamma^2 d_o + d_a}{2\gamma}\right)(b_{11} + b_{22} + b_{33} + |r|^2 b_{44}).$$

The corresponding sum of the magnitudes of the real parts of closed-loop eigenvalues

$$= \gamma \,\mathrm{Tr.}\, M_b^{-1}.$$

If there are no constrains on γ, we note that σ_a^2 is minimized by taking

$$\gamma = \sqrt{d_a/d_o}$$

with the corresponding minimum as already indicated.

3 The EPS truss model

The specific truss we shall consider is patterned after the Earth Pointing Satellite structure, shown in Fig. 1. Each bay is single-laced with the geometry shown in Fig. 2, and the notation used is the Table 1. The elastic constants c_{ij} and the mass constants m_{ij} of the equivalent Timoshenko beam have been derived in [2]. Here to simplify the calculations we shall specialize to the case:

$$A_l = A_d = A_b = A_\delta = A^1$$

$$l = b, \quad d = \sqrt{2}l, \quad \zeta_l = \zeta_d = \zeta_b = \zeta_\delta = \rho$$

number of bays $= n$, $\quad L = nl$, $\quad E =$ Young's modulus.

This yields:

$$c_{11} = \frac{(40 + 24\sqrt{2})EA}{9 + 4\sqrt{2}}, \qquad c_{14} = -c_{15} = c_{44} = c_{55} = \frac{2EA}{1 + 2\sqrt{2}} \quad \text{(Newton)}$$

$$c_{22} = c_{33} = \frac{(2725 + 1476\sqrt{2})EAl^2}{2628 + 1336\sqrt{2}}, \quad c_{23} = -\left(\frac{(97 + 140\sqrt{2})EAl^2}{2628 + 1336\sqrt{2}}\right) \quad \text{(Newton)m}^2$$

$$c_{26} = -c_{36} = \frac{1}{2}c_{66} = \frac{(16 + 33\sqrt{2})EAl^2}{296 + 130\sqrt{2}} \quad \text{(Newton)m}^2$$

$$m_{11} = m_{22} = m_{33} = (8 + 5\sqrt{2})A\rho \quad \text{kg/m}$$

$$m_{44} = 2m_{55} = 2m_{66} = \frac{(20 + 9\sqrt{2})Al^2\rho}{6} \quad \text{kg}\cdot\text{m}$$

$$m_{56} = -\frac{(Al^2\rho)}{6\sqrt{2}} \quad \text{kg}\cdot\text{m}.$$

[1] Not to be confused with the stiffness matrix **A**!

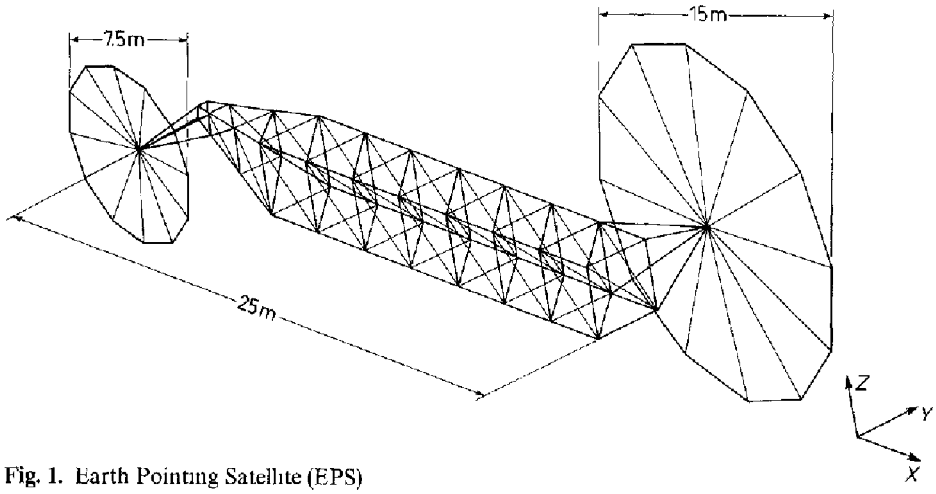

Fig. 1. Earth Pointing Satellite (EPS)

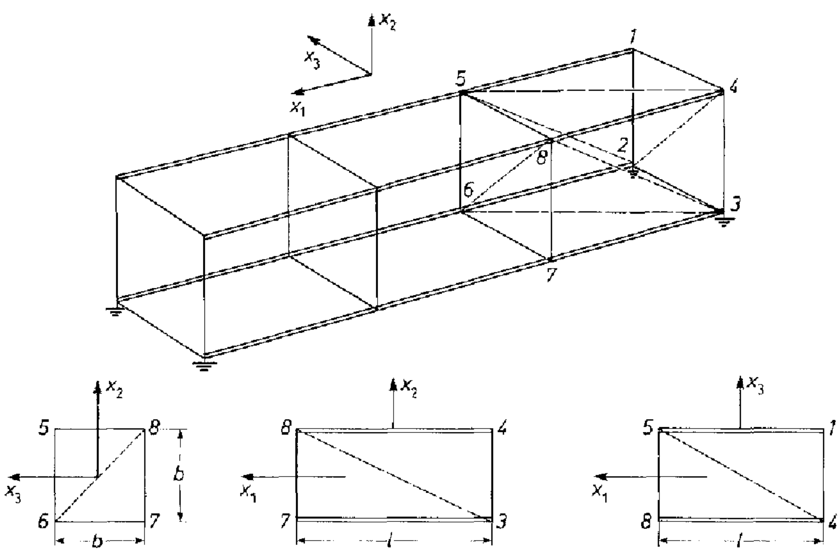

Fig. 2. Beam Geometry

Correspondingly

$$b_{11}+b_{22}+b_{33}+|r|^2 b_{44}=\frac{L}{EA}\left(a_1+a_2\frac{L^2}{l^2}+\frac{a_3|r|^2}{l^2}\right)$$

where a_1, a_2, a_3 and fixed numerical constants. Hence finally the mean square minimal pointing error (Table 1)

$$=\sqrt{d_a d_o}\frac{L}{EA}\left(a_1+a_2\frac{L^2}{l^2}+\frac{a_3|r|^2}{l^2}\right).$$

The mean square pointing error for direct rate feedback with gain γ

$$=\left(\frac{\gamma d_o+d_a}{2\gamma}\right)\frac{L}{EA}\left(a_1+a_2\frac{L^2}{l^2}+\frac{a_3|r|^2}{l^2}\right).$$

The sum of the magnitudes of the real parts of the closed-loop eigenvalues

$$=\gamma\,\mathrm{Tr.}\,M_b^{-1}.$$

Table 1.

	Cross section area	Length	Coefficient of thermal expansion	Material volume density
Longitudinal bar	A_l	l	α_l	ζ_l
Diagonal bar	A_d	d	α_d	ζ_d
Battens	A_b	b	α_b	ζ_b
Cross bracing in battens	A_δ	$\delta = \sqrt{2}b$	α_δ	ζ_δ

Table 2

		FEM $\times 10^{-8}$	Continuum (Timoshenko) $\times 10^{-8}$
b_{11}	(Axial)	9.913	9.913
b_{22}	(Bending)	1131.42	1156.293
b_{33}	(Bending)	1131.42	1156.293
b_{44}	(Torsion)	18.825	19.07
b_{55}	(Bending)	4.376	4.406
b_{66}	(Bending)	4.376	4.406

3.1 Comparison with FEM

We note that for any control "step" input $u(\cdot)$,

$u(t) = u, \quad t \geqq 0; \quad \mathbf{B}^*\mathbf{A}^{-1}\mathbf{B}u$

can be interpreted as the steady-state "output".

$\lim_{t\to\infty} b(t)$

defined in (2.2). For given truss parameters this can be calculated by an FEM (where $\mathbf{A}$ is the "stiffness matrix"), and thus affords a means of comparison between the two techniques. For this purpose we consider the case where

$n = 9, \quad l = 3\,\mathrm{m}, \quad \rho = 3250, \quad E = 2.759 \times 10^{11}, \quad A = 2.468 \times 10^{-4}\,\mathrm{m}^2.$

The values for the diagonal terms b_{ii} computed by both methods are given in Table 2. For the FEM, 40 nodes, 122 three-D truss elements with 6 DOF were used with A being 108×108 which has to be inverted numerically as compared with the trivially simple formulas for the cotinuum model. The values are within 2% accuracy of each other.

4 Combined optimization: an illustration

In this section we shall examine an illustrative example of a combined optimization problem for the EPS truss. Let us begin with a canonical problem of minimizing the mass of the structure subject to a given pointing error requirement.

For our model the mass is

$LA\rho$

and fixing L and ρ, we have the cross sectional area A as the variable. The minimal mean square pointing error formula reduces this to a trivial problem: viz. namely minimize A subject to

$$\sqrt{d_o d_a}\,\frac{L}{EA}\left(a_1 + a_2\frac{L^2}{l^2} + \frac{a_3|r|^2}{l^2}\right) \leqq \delta$$

which then yields an explicit formula for A in terms of δ^2.

A variant on this problem (grossly simplifying [6]) is to minimize

structure mass + control mass

subject to

(i) mean square pointing error $\leqq \delta^2$

and

(ii) the stability requirement that sum of the absolute values of the real parts of the closed-loop eigenvalues $\geqq \sigma_s$.

The optimality of "direct" rate feedback is shown in [4]; we assume this here anyway, γ denoting the gain. The attainable control amplitude is of course dependent on the control mass – the stationary part of it. On the other hand we note that (see [6])

(sum of the absolute values of the real parts of closed-loop eigenvalues) $= \gamma \operatorname{Tr} . M_b^{-1}$

where M_b contains the "moving" or "rotor" mass/inertia of the actuator. These are important details that must be taken into account.

Nevertheless the following simplified problem is still of interest:

minimize

$LA\rho + c_2\gamma$

where c_2 is a given constant, subject to

i) $\left(\frac{\gamma d_o + d_a}{2\gamma}\right)\frac{L}{EA}\left[a_1 + a_2\frac{L^2}{l^2} + \frac{a_3|r|^2}{l^2}\right] \leqq \delta^2$

ii) $\gamma \operatorname{Tr}. M_b^{-1} \geqq \sigma_s$.

This problem has again a clearly analytical solution. Given this enormous simplification, it is fair to conjecture that distributed parameter models can offer significant advantages over Finite Element models in combined Structures-Controls optimization.

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