International Environment Conventions: The Case of Uniform Reductions of Emissions*

MICHAEL HOEL
University of Oslo, P.O. Box 1095, Blindern, N-0317 Oslo 3, Norway.

Abstract. Several serious environmental problems have a global character. International cooperation to reduce emissions for this type of problems often takes the form of an agreement among the cooperating countries to cut back emissions by a uniform percent rate compared with some base year. This type of agreements has two disadvantages. In the first place, it is well known from environmental economics that equal percentage reductions of emissions from different sources usually gives an inefficient outcome, in the sense that the same environmental goals could be achieved at lower costs through a different distribution of emission reductions. A second problem with agreements of equal percentage reductions is that not all countries will find it in their interest to participate in such agreements. In the paper, it is assumed that the set of countries which participate in an agreement is endogenously determined, with a country participating in an agreement provided that this makes the country better off than it would have been in a situation without any agreement. The agreement among the participating countries is assumed to be a uniform percentage reduction of their emissions. The countries have different opinions about what this uniform percentage should be. In the paper, it is assumed that the outcome is determined by the median country of the participating countries. The assumptions above lead to a particular equilibrium, in which some but not all countries cooperate. The equilibrium reduction of emissions for the cooperating countries is also derived. This equilibrium compared with the first best optimum within the context of simple numerical example.

Key words. International environment conventions, uniform reductions of emissions.

1. Introduction

Several serious environmental problems have a global character. Examples include the effect of emissions of CFC on the ozone layer, and the effect of emissions of carbon dioxide on the global climate. An important feature of these two examples is that it is only global emissions which affect the environment. Another important feature is that consequences of environmental changes may be quite different for different countries.

International cooperation often takes the form of an agreement among the cooperating countries to cut back emissions by a uniform percent rate compared with some base year. This type of agreements has two disadvantages. In the first place, it is well known from environmental economics that

* Presented at the conference "Environmental Cooperation and Policy in the Single European Market", Venice, April 16—20, 1990. The paper is part of the research project "Energy and Society" at the Centre for Research in Economics and Business Administration (SNF), Oslo. I am grateful to Ignazio Musu and Henk Folmer for useful comments on an earlier version of the paper.

Environmental and Resource Economics **2**: 141—159, 1992.

equal percentage reductions of emissions from different sources usually gives an inefficient outcome, in the sense that the same environmental goals could be achieved at lower costs through a different distribution of emission reductions. This source of inefficiency might in fact be quite large: Tietenberg (1985) reports that for 17 types of emissions to air and water, a move from rigid regulations of the type "uniform percentage reductions" to the cost minimizing way of achieving the same environmental goal would reduce costs by 7—95%, with a cost reduction exceeding 50% for 10 of the 17 cases.

A second problem with agreements of equal percentage reductions is that not all countries will find it in their interest to participate in such agreements. A likely minimum requirement for a country to participate in an agreement is that the country is better off under the agreement than it is without any international agreement. The reason for such a requirement is the following. As long as there is no international law to force countries to participate in an agreement, each country can choose to be a free rider outside the agreement instead of participating in the agreement. If the country stands outside the agreement it can enjoy (almost) the same benefits of reduced emissions as if it participates in the agreement, while it doesn't bear any of the costs of reducing emissions. An important motive for a country to participate in an agreement instead of being a free rider is that by being a free rider it increases the risk of the whole agreement breaking down. This motive for participating in the agreement is stronger the more the country has to lose from the agreement breaking down. Obviously, a country which doesn't lose anything from the agreement breaking down has no incentive to participate in the agreement, and it will therefore instead choose to be a free rider.

It is this second issue which is the main topic of the present paper. In order to focus on this issue, it is assumed that all countries have the same costs of reducing emissions. A first best optimum is therefore characterized by all countries reducing their emissions proportionately. However, the countries are assumed to differ with respect to their evaluation of environmental changes. For the case of the greenhouse effect, the reason for this could be that the consequences of climatic changes differ significantly between countries. For instance, some island and coastal countries (the Maldives, Bangladesh) are very exposed to a rising sea level due to global warming. On the other hand, one could argue that parts of, e.g., USSR and Canada would benefit from a warmer climate.

When countries have different environmental cost functions, the first best optimum (maximizing the sum of social welfare) might make some countries worse off than they are in a non-cooperative equilibrium. This is discussed and illustrated with a particular numerical example in Sections 2 and 3. Unless side payments are allowed, this first best optimum is therefore not feasible, cf. the discussion above. In Section 4 we therefore consider the constrained social optimum, in which no country is worse off under the agreement than in the non-cooperative equilibrium. Under this constrained social optimum, emission reductions differ between countries, with larger reductions for the countries most concerned about the environment than under the first best optimum. For the particular numerical example considered, the welfare loss relative to the first best optimum is quite small.

Sections 5—7 consider the case in which all countries participating in the agreement have equal percentage reductions of emissions. In Section 5, it is shown that the stricter the international agreement is (i.e., the smaller are the emissions for each participating country), the fewer countries will participate in the agreement. The uniform emission reduction which maximizes the sum of social welfare for *all* countries is derived in Section 6, and it is shown that emission levels for the participating countries are higher than they are under the first best optimum. Since non-participating countries don't reduce their emissions, total emissions are therefore also higher than they are in the first best optimum. For the numerical example considered, total emissions are 45% higher than they are in the first best optimum.

There is no reason to believe that the participating countries will try to maximize the sum of social welfare over *all* countries. Instead, it is assumed in Section 7 that each participating country is only concerned with its own social welfare. Since the participating countries have different environmetnal cost functions, they also have different opinions about how much emissions should be reduced. In Section 7 it is assumed that the participating countries agree upon the emission reduction which is the most preferred reduction for the median country (among the participating countries). For the numerical example considered, total emissions in this case are only about 3% higher than the level derived in Section 6.

Section 8 gives an analysis of a somewhat stricter requirement for participating in an agreement than the requirement discussed above. Following Barrett (1989), a two stage game is considered, in which the countries decide whether or not to join the agreement in the first stage. The emission levels of the participating countries are decided in the second stage of the game. With this setup, a country will find it better to participate in the agreement than to be a free rider only if the agreement is so much better when it participates that this environmental benefit exceeds the costs of the country participating. An equilibrium with some countries cooperating may exist also in this case. However, for the numerical example considered, only two countries will cooperate, and there will hardly be any difference between this equilibrium and the non-cooperative equilibrium.

Some concluding remarks are given in Section 9.

2. The Model

There are N countries, which have equal revenue functions $R(v_j)$, where v_j measures emissions from country j. It is assumed that $R(0) = 0$, $R'(v) > 0$ and $R''(v) < 0$ for $v < \bar{v}$ and $R'(\bar{v}) = 0$. In other words, $R(v_j)$ measures revenue addition relative to the case of no emissions, and $v_j = \bar{v}$ is the optimal amount of emissions from all countries in the absence of environmental considerations.

Total emissions are

$$V = \sum_i v_i \tag{2.1}$$

and country j's environmental damage function is assumed to be equal to $m_j V$, where m_j is a non-negative constant parameter which measures the marginal environmental cost of country j. It is assumed that countries are numbered so that

$$m_1 \geqslant m_2 \geqslant \ldots \geqslant m_N \geqslant 0. \tag{2.2}$$

In other words, the countries with the lowest numbers are the countries which care most about the environment.

We define the sum of the marginal environmental costs by

$$M = \sum_i m_i > 0. \tag{2.3}$$

In the absence of any international cooperation, emissions are assumed to be determined as the Nash equilibrium of the non-cooperative game in which all countries choose their emissions simultaneously.[1] This Nash equilibrium is characterized by each country j choosing v_j so that $R(v_j) - m_j V$ is maximized, taking all other v_i $(i \neq j)$ as given. This gives

$$R'(v_j) = m_j \quad j = 1, \ldots, N. \tag{2.4}$$

The assumption that each country has constant marginal environmental costs implies that each country's optimal emission level is independent of the emission levels chosen by other countries. When N is large, each m_j will be small relative to M, and we shall assume that m_j is so close to zero that $R'(v_j) = 0$, i.e., $v_j = \bar{v}$ is a good approximation to (2.4).

Throughout the paper, we shall illustrate the general results with a specific example. In this example it is assumed that

$$R(v) = v - \tfrac{1}{2} v^2, \tag{2.5}$$

i.e.,

$$R'(v) = 1 - v \tag{2.6}$$

implying that $\bar{v} = 1$. Moreover, the marginal environmental costs are distributed uniformly between $1/N$ and 0, so that

$$m_j = \frac{N-j}{N-1} \, \frac{1}{N}. \tag{2.7}$$

If country j could choose equal emission levels for all countries including itself, it would choose v_j so that $R(v_j) - m_j N v_j$ was maximized, giving $R'(v_j) = m_j N$. With the present example, this implies (cf. (2.6) and (2.7))

$$v_j = \frac{j-1}{N-1}. \tag{2.8}$$

In other words, the larger j is, the larger is the preferred uniform emission level, with $v_1 = 0$ and $v_N = 1$. For the median country, i.e., $j = (N + 1)/2$, the preferred uniform emission level is equal to 0.5.

Total marginal environmental costs for this example follow from (2.3) and (2.7)

$$M = \sum_i m_i = \frac{1}{(N-1)N}\left(N^2 - \sum_i i\right) = 0.5. \tag{2.9}$$

The non-cooperative equilibrium is given by (2.4), which in the present example gives (cf. (2.6) and (2.7))

$$v_j = 1 - \frac{1}{N}\frac{N-j}{N-1} \geqslant 1 - \frac{1}{N}. \tag{2.10}$$

For, e.g., $N = 100$, all countries thus choose emission levels in the interval [0.99, 1]. The approximation $v_j = 1$ for the non-cooperative case is therefore a good approximation in this example.

3. The First-Best Optimum

The sum of net benefits for all countries is

$$B = \sum_i [R(v_i) - m_i V]. \tag{3.1}$$

where V is given by (2.1). A first best social optimum follows from maximizing (3.1) with respect to all v_j, which gives

$$R'(v_j) = M, \quad j = 1, \ldots, N, \tag{3.2}$$

where M is the sum of marginal environmental costs. In other words, marginal revenues are equal for all countries, and equal to the sum over all countries of the marginal environmental costs of the emissions. Since we have assumed that all countries have equal revenue functions, (3.2) implies that optimal emissions are equal for all countries.

If side payments are allowed, the total net benefit from cooperating may be distributed in any way between the countries.[2] In particular, all countries can be made better off than they are without any cooperation.

Without side payments, some "low m countries" might be worse off under the first best optimum than they are without any cooperation. This is certainly true if $m_N = 0$: In this case country N doesn't care about the environment, and any reduction of v_N below $\bar{v}$ makes this country worse off.

With our numerical example (3.2) may be written

$$1 - v_j = M. \tag{3.2}$$

Since $M = 0.5$ (cf. (2.9)), we thus have $v_j = 0.5$ in this case. In this case the first best optimum thus implies a 50% reduction of emissions for all countries (compared with the non-cooperative case). The optimal emission level is in this case equal to the median countrys most preferred uniform emission level.

It is straightforward to verify that

$$\begin{aligned} v_j &= 0.5 \Rightarrow B = 0.125N, \\ v_j &= 1 \Rightarrow B = 0. \end{aligned} \tag{3.3}$$

The total net gain relative to the non-cooperative case is thus $0.125N$. Denote the corresponding net gain for country j by b_j. We have

$$b_j = [0.5 - 0.5 \cdot (0.5)^2 - m_j N \cdot 0.5] - [1 - 0.5 - m_j N],$$

i.e., using (2.7)

$$b_j = 0.5 \frac{N-j}{N-1} - 0.125. \tag{3.4}$$

From (3.4) it follows that

$$b_j \geqslant 0 \quad \text{iff} \quad j \leqslant 0.75N + 0.25. \tag{3.5}$$

With, e.g., $N = 100$, the 75 countries with highest marginal environmental costs are thus better off under the first best optimum than without any cooperation. The 25 countries with the lowest environmental concern prefer the non-cooperative outcome to the first best optimum.

4. Constrained Social Optimum

Consider the case in which B is maximized subject to

$$b_j = [R(v_j) - m_j V] - [R(\bar{v}) - m_j N] \geqslant 0, \tag{4.1}$$

i.e., that no countries should be worse off under the social optimum than under the non-cooperative equilibrium. The Lagrangian corresponding to this optimization problem is

$$L = \sum_i [R(v_i) - m_i V] - \sum_i \lambda_i b_i, \tag{4.2}$$

where V and b_i are given by (2.1) and (4.1), respectively. The first order conditions are

$$(1+\lambda_j)R'(v_j) = \sum_i (1+\lambda_i)m_i. \tag{4.3}$$

Using $\Sigma_i m_i = M$ it is clear that

$$\sum_i (1+\lambda_j)m_i = M + \sum_i \lambda_i m_i = (1+\lambda_j)M - \lambda_j \sum_i m_i + \sum_i \lambda_i m_i$$
$$= (1+\lambda_j)M + \sum_i (\lambda_i - \lambda_j)m_i.$$

Inserting this into (4.3) gives

$$R'(v_j) = M + \frac{\sum_i (\lambda_i - \lambda_j)m_i}{1+\lambda_j}, \tag{4.3'}$$

where $\lambda_j \geqslant 0$ and $\lambda_j b_j = 0$.

Consider first the "high m countries", i.e., the countries for which the constraint (4.1) in not binding in equilibrium. For there countries $\lambda_j = 0$. As long as $\lambda_i > 0$ for sufficiently large values of i, the second term on the r.h.s. of (4.3′) is positive for countries with $\lambda_j = 0$. These countries therefore have lower emissions in the constrained optimum than they have in the first best optimum.

The countries with the lowest m_j-values have the highest λ_j-values. For these countries the term $\Sigma_i(\lambda_i - \lambda_j)m_i$ is therefore negative, i.e., they have higher emissions than under the first-best optimum. If $m_N = 0$, it follows directly from (4.1) that $v_N = \bar{v}$, i.e., country N has no reduction in its emissions compared with the non-cooperative case.

Figure 1 illustrates the case of a constrained optimum when $m_j - m_{j-1}$ is negative and sufficiently small for all j and $m_N > 0$. In the first best optimum, all countries have emission levels equal to v^0. In the constrained optimum, emissions from country j are given by v_j^c.

In the specific example considered previously the first best optimum gave emissions equal to 0.5 from all countries, and total net benefits of cooperating equaled 12.5 for $N = 100$. For the constrained optimum, numerical calculations show that 71 countries remain unconstrained (for $N = 100$), and that the emissions from these countries are 0.49. The emissions from unconstrained countries are thus slightly lower than they are in the first best optimum. Since emission levels for most of the 29 constrained countries are above 0.5, total emissions in the present case equal 53.8, compared with 50.0 in the first best optimum. Total social gains from cooperating are equal to 11.86, i.e., only 5% lower than the total social gains under the first best optimum.

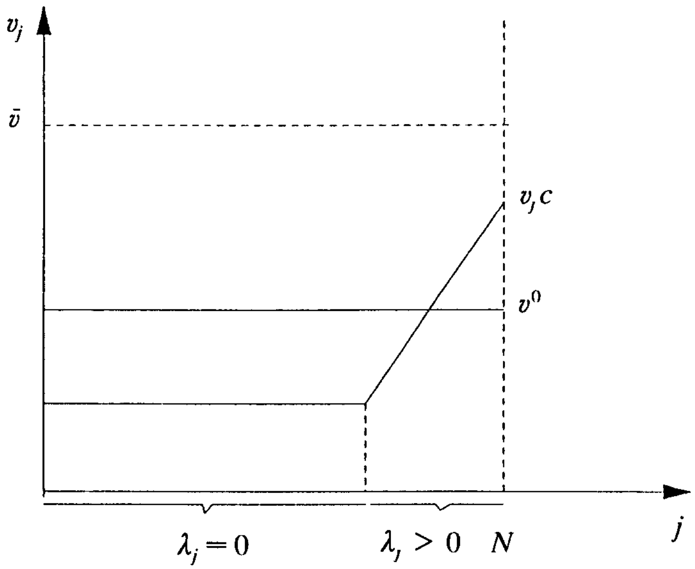

Fig. 1.

5. Participation in an Agreement with Uniform Emissions

The numerical example in the end of the previous section indicates that the lack of side payments does not necessarily imply a large total welfare loss, as long as emissions are allowed to differ between countries. However, if emissions are restricted to be equal for all cooperating countries, the loss might be considerably larger. The reason is that the requirement that no country is worse off under an agreement than without might make several countries prefer not to join the agreement. This section considers the relationship between the emission level agreed upon and the number of participating countries.

Using v to denote the uniform emission level under an agreement, and n as the number of participating countries, we have

$$V = nv + (N - n)\bar{v}. \tag{5.1}$$

The requirement $b_j \geqslant 0$ may be written as

$$R(v) - m_j V \geqslant R(\bar{v}) - m_j N\bar{v}$$

or, using (5.1)

$$nm_j \cdot (\bar{v} - v) \geqslant R(\bar{v}) - R(v) \tag{5.2}$$

or

$$nm_j \geqslant \frac{R(\bar{v}) - R(v)}{\bar{v} - v} \equiv g(v). \tag{5.2'}$$

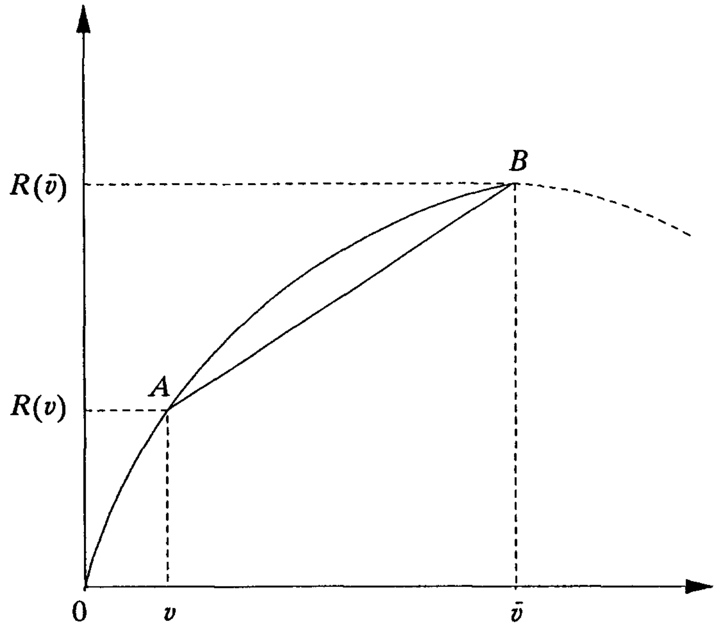

Fig. 2.

From Figure 2 we see that $g(v)$ is given by the slope of the line AB, i.e.,

$$\begin{aligned} &g(\bar{v}) = 0 \\ &g'(v) < 0 \\ &g(v) < R'(v). \end{aligned} \tag{5.3}$$

If (5.2) holds for $j \leqslant n$ for some agreement v', it therefore also holds for an alternative agreement $v'' > v'$.

The marginal country n satisfying (5.2) is given by

$$n(v) = \max\{n \mid nm_n \geqslant g(v)\}. \tag{5.4}$$

Unless the sequence $m_1, m_2, \ldots$ declines very rapidly, nm_n is an increasing function of n for small values of n. We shall assume that nm_n increases monotonically until n reaches a value $\hat{n} \leqslant N$, after which nm_n decreases monotonically if $\hat{n} < N$. Assume that $nm_n < g(0)$ for all n. In other words, for tight enough emission controls, there does not exist any group of cooperating countries which are all better off with the emission agreement than without. The maximal value $\hat{v}$ of v which satisfies (5.2′) for some value of n is thus positive, and given by

$$g(\hat{v}) = \hat{n}m_{\hat{n}} = \max_{n \in [0, N]} \{nm_n\}. \tag{5.5}$$

If $v < \hat{v}$, i.e., $g(v) > g(\hat{v})$, $nm_n < g(v)$ no matter what n is. In other words, for emission levels below $\hat{v}$, there does not exist any group of cooperating countries which are all better off with the emission agreement than without.

If $v = \hat{v}$, $\hat{n}$ countries are at least as well off under an agreement with $v = \hat{v}$ for the $\hat{n}$ participating countries and $v = 1$ for the $N - \hat{n}$ remaining countries as they are with no agreement.

If $\hat{n} = N$, any agreement with $v \geqslant \hat{v}$ makes all countries better off (or equally well off) than they are in the non-cooperative case. No agreement with $v < \hat{v}$ is possible as long as countries which don't gain from the agreement choose emission levels equal to $\bar{v}$.

If $\hat{n} < N$, which seems most plausible, the countries $1, \ldots, \hat{n}$ will want to participate in an agreement giving $v = \hat{v}$ for the participating countries. For higher emission levels, more countries will want to participate, and *all* countries will be better off under the agreement if v is so high that $Nm_N \geqslant g(v)$. Since $g(v) > 0$ for $v < \bar{v}$, there will only exist emission levels below $\bar{v}$ satisfying $Nm_N \geqslant g(v)$ if $m_N > 0$. In other words, all countries can be better off by agreeing to reduce emissions only if all countries care positively about the environment.

The relationship between n and v is illustrated in Figure 3 for the case in which $\hat{n} < N$ and $m_N > 0$. For $v < \hat{v}$, i.e., $g(v) > g(\hat{v})$, no group of countries will find it in their interest to participate in an agreement. For $v \geqslant \hat{v}$, i.e., $g(v) \leqslant g(\hat{v})$, the maximal number of countries participating in the

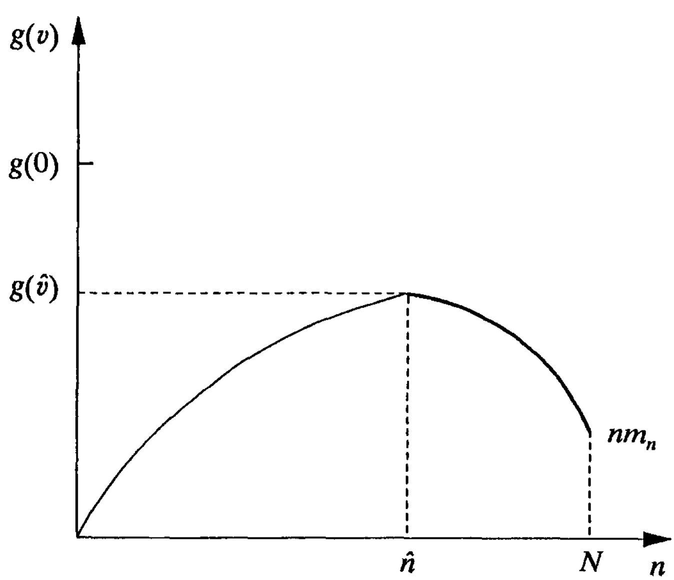

Fig. 3.

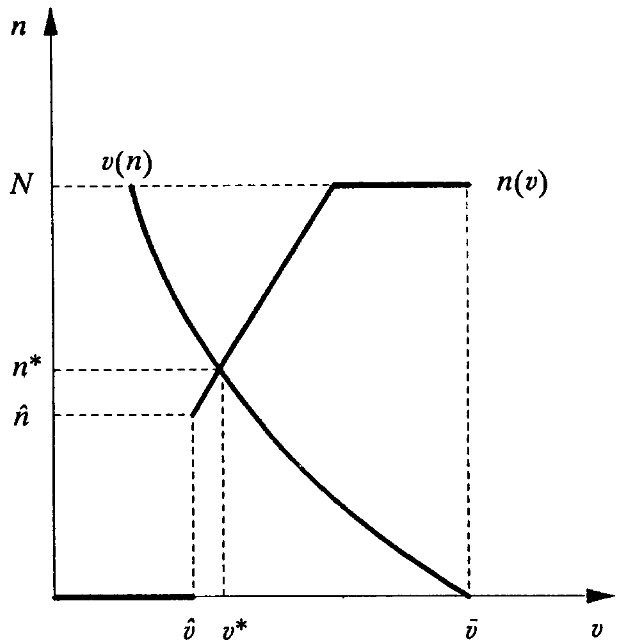

Fig. 4.

agreement is given by the heavily drawn part of the nm_n curve in Figure 3. In the subsequent sections, we shall denote this relationship between v and n by $n(v)$. We thus have $n(v) = 0$ for $v < \hat{v}$, $n(\hat{v}) = \hat{n}$, and $n'(v) > 0$ for $v > \hat{v}$ if $n(v) < N$. This function is given by the heavily drawn discontinuous curve in Figure 4.

For our specific example the revenue functions are given by $R(v) = v - v^2/2$. From (5.2′) it is therefore clear that the function $g(v)$ in this case is

$$g(v) = \frac{0.5 - v + v^2/2}{1 - v} = \frac{1 - 2v + v^2}{2(1 - v)} = \frac{1 - v}{2}. \tag{5.6}$$

Moreover, from (2.7) we find

$$nm_n = \frac{n(N - n)}{N - 1}\,\frac{1}{N}. \tag{5.7}$$

Solving (5.5) for this case thus gives $\hat{n} = N/2$. For N large it follows from (5.5)–(5.7) that

$$\hat{v} = 1 - \frac{N}{2(N - 1)} \approx 0.5. \tag{5.8}$$

In other words, proposals of equal emissions below $v = 0.5$ will not be supported by any countries, while a proposal of $v = 0.5$ will be supported by 50% of the countries.

For $v > \hat{v}$ it follows from (5.5)—(5.7) that the relationship between n and v is given by

$$v = 1 - \frac{2n(N-n)}{N(N-1)} \tag{5.9}$$

or, solving for n

$$n(v) = \frac{N}{2}\left\{1 + \left[1 - 2(1-v)\frac{N-1}{N}\right]^{1/2}\right\} \tag{5.10}$$

or

$$\frac{n(v)}{N} \approx 0.5[1 + (2v-1)^{1/2}]. \tag{5.10'}$$

In other words, $n(0.5) \approx 0.5N$, $n(1) = N$, and $n''(v) < 0$ for $v > \hat{v}$.

Total emissions are equal to

$$V = n(v)v + [N - n(v)]\bar{v}. \tag{5.11}$$

6. Socially Optimal Uniform Reduction

If one restricts oneself to uniform reductions, and participation in the agreement is given by $n(v)$ as explained in the previous section, total welfare is given by

$$B = n(v)R(v) + (N - n(v))R(\bar{v}) - M[n(v)v + (N - n(v))\bar{v}]. \tag{6.1}$$

The level of emissions which maximizes B is given by

$$n(v)R'(v) + [R(v) - R(\bar{v})]n'(v) - M(v - \bar{v})n'(v) - Mn(v) = 0. \tag{6.2}$$

Some manipulation of (6.2) gives, using the definition (5.2) of $g(v)$

$$R'(v) = AM, \tag{6.3}$$

where

$$A = \frac{1 - (\bar{v} - v)\dfrac{n'(v)}{n(v)}}{1 - \dfrac{g(v)}{R'(v)}(\bar{v} - v)\dfrac{n'(v)}{n(v)}} < 1 \tag{6.4}$$

since $n'(v) > 0$ and $g(v) < R'(v)$ (cf. (5.3)). Comparing (6.3) with the first best optimum (3.2), it is thus clear that emissions from the participating countries are higher than under the first best optimum. Since non-participating countries do not reduce their emissions in the present case, total emissions are also higher in the present case than in the first best optimum.

In our specific example, $R'(v) = 1 - v$ and $M = 0.5$, so that (6.3)—(6.4) may be written as

$$1 - v = \frac{1 - (1 - v)\,\dfrac{n'(v)}{n(v)}}{1 - \dfrac{1}{2}\,(1 - v)\,\dfrac{n'(v)}{n(v)}} \cdot 0.5. \tag{6.5}$$

Solving (6.5) for $n'(v)$, we find

$$n'(v) = n \cdot \frac{2v - 1}{v(1 - v)}. \tag{6.6}$$

Inserting v and $1 - v$ from (5.9) gives, after some manupulation

$$n'(v) = \frac{N(N - 1)}{2(N - n)} \cdot \frac{N(N - 1) - 4n(N - n)}{N(N - 1) - 2n(N - n)}. \tag{6.7}$$

From (5.9) we can also derive

$$n'(v) = \left(\frac{dv}{dn}\right)^{-1} = \frac{N(N - 1)}{4n - 2N}. \tag{6.8}$$

Eliminating $n'(v)$ from (6.7) and (6.8) we can solve for n (for any given N). For $N = 100$ we find $n = 77$, which inserted into (5.9) gives $v = 0.64$. Total emissions and total welfare follow from (5.11) and (6.1), giving $V = 72.3$ and $B = 8.85$. Compared with the first best optimum, total emissions have thus increased by 45%, while the net benefits of an agreement have dropped from 12.5 to 8.85, i.e., a reduction equal to 29%.

7. The Most Preferred Uniform Reduction of the Median Country

There is no reason to believe that the n countries participating in an agreement will choose the emission level which maximizes total welfare, as derived in Section 6. In the public choice literature, it is often assumed that it is the preferences of the median voter which determine the outcome in situations of this type (see, e.g., Mueller, 1979). In the present context, this means that the n countries which cooperate agree upon the emission level which is the median value of the most preferred emission levels of the n

countries. For any given value of n, the most preferred emission level of country j ($\leqslant n$) is the value of v which maximizes

$$b_j = R(v) - m_j V$$

or, inserting V from (5.13)

$$b_j = R(v) - m_j[nv + (N-n)\bar{v}]. \tag{7.1}$$

The value of v which maximizes b_j is given by

$$R'(v) = m_j n. \tag{7.2}$$

Since $m_j n < m_j N$, the most preferred emission level of country j is higher than the most preferred emission level for the case in which *all* countries have equal emissions.

Since m_j is declining in j, the most preferred v-value, given by (7.2), is lower the lower is j. Among the participating countries, $j = (n+1)/2$ is the median country. This country's most preferred emission level is therefore equal to

$$R'(v) = m_{(n+1)/2}n \tag{7.3}$$

which gives v as a function of n, denoted by $v(n)$.[3]

Since $(n+1)/2 \approx n/2$, we have $nm_{(n+1)/2} \approx 2(n/2)m_{n/2}$. For $n \leqslant \hat{n}$ the function $(n/2)m_{n/2}$ is increasing in n. From (7.3) and $R''(v) < 0$ it therefore follows that $v'(n) < 0$ for $n \leqslant \hat{n}$, as illustrated in Figure 4. (In Figure 4, it is assumed that $v'(n) < 0$ for *all* n. This will be the case if $\hat{n} \geqslant N/2$, since $\hat{n} \geqslant N/2$ implies that $(n/2)m_{n/2}$ is rising in n for all n up to N.)

The intersection between the curves $n(v)$ and $v(n)$ gives the equilibrium pair (n^*, v^*): When n^* countries cooperate, they will agree upon v^* as their common emission level. With this emission level, the cooperating countries are better off than they are without any agreement, while each of the $(N - n^*)$ remaining countries would be worse off participating in an agreement with emission level v^* than they would be in the non-cooperative equilibrium.

Since the curve $n(v)$ is discontinous, the curves $n(v)$ and $v(n)$ need not have any point of intersection. If $v(\hat{n}) < \hat{v}$, we get the situation described in Figure 5. The Appendix gives an example of such a situation. With this situation, there will be no equilibrium with countries cooperating. Whatever number of countries one tentatively lets cooperate, they agree upon an emission level which either implies that some of these countries would have been better off in the non-cooperative equilibrium or that some of the non-cooperating countries would have been better off participating in the agreement than they would have been in the non-cooperative equilibrium.

When Figure 5 is valid, the non-cooperative equilibrium is the only possible equilibrium: When no countries cooperate, no country can be in a

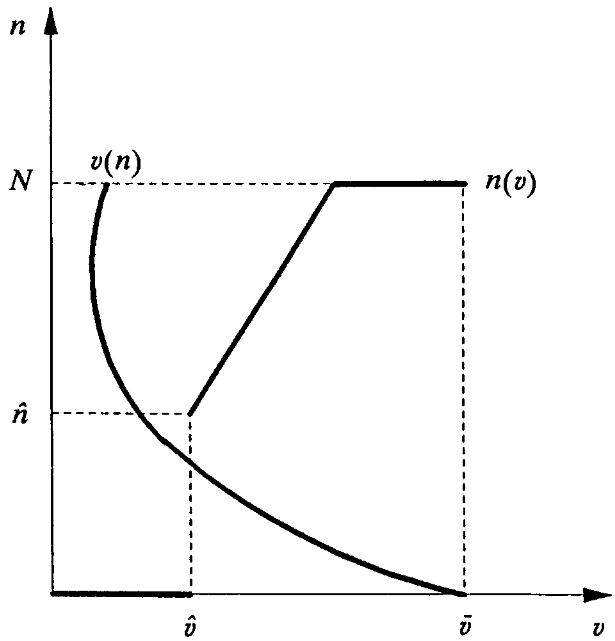

Fig. 5.

position where it would have been better off joining an agreement than it is under the non-cooperative equilibrium.

For our specific example with $R'(v) = 1 - v$, we can insert for $m_{(n+1)/2}$ from (2.7) in (7.3). After same manipulation this yields

$$v(n) = 1 - \frac{(2N - n - 1)n}{2N(N - 1)} \tag{7.4}$$

which is a declining function of n.

The intersection between the $n(v)$- and $v(n)$-curve is found by equating the right hand sides of (5.9) and (7.4), which gives

$$2n^*(N - n^*) = \frac{n^*}{2}(2N - n^* - 1),$$

i.e.,

$$n^* = \frac{2N + 1}{3}. \tag{7.5}$$

Solving for $N = 100$ we thus find $n = 67$, which inserted into (7.4) gives $v = 0.55$. Total emissions and total welfare follow from (5.13) and (6.1), giving

$V = 70.1$ and $B = 8.28$. In this example the total emissions and total welfare are thus quite close to the values which maximize social welfare (derived in Section 6).

8. The Free Rider Problem

Throughout the paper, it has been assumed that a sufficient condition for a country to participate in an international agreement is that the country is better off under the agreement than it is under the non-cooperative outcome. As was pointed out in Section 1, however, this condition is a *minimum* requirement for participation. Even if a country is better off participating in an agreement than it would be without any agreement, it might be even better off if the other countries cooperated while it itself stood outside the agreement. Given the decision of other countries of whether or not to cooperate, a country will find it better to participate in the agreement than to be a free rider only if the agreement is so much better when it participates that this environmental benefit exceeds the costs of the country participating. As Barrett (1989) has shown, one may have an equilibrium with some countries cooperating also in this case. Barrett considers a two stage game, in which the countries simultaneously decide whether or not to join the agreement in the first stage. The emission level of the participating countries is decided in the second stage of the game.[4]

The problem of choosing the appropriate emission level in the second stage of the game was treated in Section 7. One possible choice of emission level at this stage of the game is the most preferred emission level of the median country (among the participating countries). In this case the emission level is given by (7.3), which defines the function $v(n)$ from Section 7.

In the first stage of the game, the countries decide whether or not to cooperate with other countries. If any countries cooperate, it will obviously be the countries with largest concern for the environment, i.e., the countries with the lowest index numbers. Assume that n countries cooperate (i.e., countries $1, 2, \ldots, n$). Country n is better off cooperating than being a free rider if

$$R(\bar{v}) - R(v(n)) \leqslant m_n[(\bar{v} + (n-1)v(n-1)) - nv(n)]. \tag{8.1}$$

The l.h.s. of (8.1) is the revenue increase for country n of being a free rider instead of cooperating with the other $n - 1$ countries. The terms in the square brackets in the r.h.s. of (8.1) give the increase in emissions if country n decides to be a free rider instead of cooperating: If it cooperates, emissions from the n countries are $nv(n)$, while they are $\bar{v} + (n-1)v(n-1)$ if country n chooses to be a free rider. The emissions from the remaining $N - n$ countries are $\bar{v}$ whatever country n chooses to do.

If there are n countries cooperating in equilibrium, country $n + 1$ must be better off being a free rider than it is by cooperating with the n first countries, i.e.,

$$R(\bar{v}) - R(v(n+1)) \geqslant m_{n+1}[(\bar{v} + nv(n)) - (n+1)v(n+1)]. \quad (8.2)$$

The group of countries cooperating in equilibrium is thus $1, 2, \ldots, n$; with n determined by the conditions (8.1) and (8.2). One can of course not rule out the possibility of $n = 1$, i.e., that no countries cooperate.

Using our numerical example, we can insert (2.5) and (2.7) into (7.3) and (8.1)—(8.2). Solving for $N = 100$, we find that $n = 2$.[5] In other words, only two countries cooperate in equilibrium. Moreover, these two countries only reduce their emissions from 0.99 to 0.98. This equilibrium is therefore for all practical purposes equal to the non-cooperative equilibrium in which each country's emission level is given by (2.4). It is also interesting to note that the number of participating countries is almost independent of the total number of countries: Two countries cooperate for $N \geqslant 5$, while there is no cooperation if $N < 5$.[6]

9. Conclusions

Table I below summarizes some of the results for the numerical example considered. It is clear from this table that the welfare loss associated with an agreement of the type "equal percentage reductions" is quite substantial (compared with a first best optimum or the constrained optimum making all countries better off). In considering this welfare loss, one should also bear in

Table I. A First best optimum (Section 3); B Constrained optimum (Section 4); C Maximum social welfare given equal emissions from participating countries (Section 6); D Equal emissions from all participating countries, emission level chosen by median country among participating countries (Section 7)

	A	B	C	D
Percent participation	100	100	77	67
Percent emission reduction by participating country	50	51[a]	36	45
Percent reduction of total emission	50	46	28	30
Welfare relative to no agreement (in % of max)	100	95	71	66

[a] The 71 "unconstrained" countries reduce their emissions by 51%. The remaining countries reduce their emissions so much that they are equally well off under the agreement as they are in the non-cooperative equilibrium. This implies a reduction of emissions from these countries by less than 51%.

mind that the model is constructed so that uniform percentage emission reductions is optimal if side payment can be used so that all countries are willing to participate in the agreement.

Appendix

Consider the revenue function

$$R(v) = \begin{cases} v^{0.2} & \text{for } v \leqslant 1 \\ 1 & \text{for } v > 1, \end{cases} \tag{A.1}$$

i.e., $\bar{v} = 1$ and $R(\bar{v}) = 1$. For this case we have

$$R'(v) = 0.2v^{-0.8} \tag{A.2}$$

and

$$g(v) \equiv \frac{R(\bar{v}) - R(v)}{\bar{v} - v} = \frac{1 - v^{0.2}}{1 - v}. \tag{A.3}$$

For the m-distribution given by (2.7), the value $\hat{v}$ (defined by (5.5)) follows from (A.3), (2.7) and (5.5). Straightforward calculations give

$$\hat{v} = 0.55. \tag{A.4}$$

The function $v(n)$ follows from (7.3), which after inserting from (A.2) and (2.7) gives

$$0.2\, v^{-0.8} = \frac{n}{N(N-1)} \left(N - \frac{n+1}{2} \right). \tag{A.5}$$

This gives v as a declining function of n. For $N = 100$ we find

$$\begin{aligned} &v(1) = 1, \\ &v(50) = 0.45, \\ &v(100) = 0.32. \end{aligned} \tag{A.6}$$

In particular, it follows from (A.4) and (A.6) that $v(\hat{n}) = v(50) < \hat{v}$. In other words, we have the situation which is illustrated in Figure 5.

Notes

[1] See, e.g., Hoel (1991, 1992) and van der Ploeg and de Zeeuw (1991) for discussions of Nash equilibria in similar international environmental games.

[2] The total net benefit from cooperating is equal to the difference between the value of B from (3.1) when all v_j are optimal and the value of B when all $v_j = \bar{v}_j$, which holds in the non-cooperative case.

[3] Instead of assuming that the median country chooses the emission level, one could assume that the emission level is chosen so that the sum of welfare for the participating countries is maximized. This will generally give a different condition than (7.3), and therefore also a different function $v(n)$. However, when the m-values are uniformly distributed as assumed in our specific example (cf. (2.7)), it is easily seen that these two different assumptions lead to the same condition (7.3).
[4] Barrett assumes that all countries are equal, so that an agreement will require all participating countries to have the same emission level.
[5] In the numerical calculation, the true non-cooperative emission levels, given by (2.4), are used instead of the approximation $\bar{v}$.
[6] For very large values of N, we get $v = 3$: $N = 10^8$ gives $v = 3$, while $N = 10^7$ gives $v = 2$.

References

Barrett, S. (1989), 'On the Nature and Significance of International Environmental Agreements', mimeo, London Business School.

Hoel, M. (1991), 'Global Environmental Problems: The Effects of Unilateral Actions Taken by One Country', *Journal of Environmental Economics and Management* **20**, 55—70.

Hoel, M. (1992), 'Emission Taxes in a Dynamic Game of CO_2 Emissions', in R. Pethig (ed.), *Conflict and Cooperation in Managing Environmental Resources*, Springer Publishing House.

Mueller, D. C. (1979), *Public Choice*, Cambridge University Press.

van der Ploeg, F. and de Zeeuw, A. (1991), 'International Aspects of Pollution Control', *Environmental and Resource Economics* **2**, 117—139, this issue.

Tietenberg, T. (1985), *Emission Trading: An Exercise in Reforming Pollution Policy*, Resources for the Future, Washington D.C.