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Visual Discrimination of Textures with Identical Third-Order Statistics

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Abstract. We found a new class of two-dimensional random textures with identical third-order statistics that can be effortlessly discriminated. Discrimination is based on local "granularity" differences between these iso-trigon texture pairs. This is the more surprising since it is commonly assumed that texture granularity (grain) is determined by the power spectrum which, in turn, can be obtained from the second-order statistics. Because textures with identical third-order statistics must have identical second-order statistics (i.e., identical power spectra), visible texture granularity is not controlled by power spectra, and not even by third-order statistics.

Since 1962 much effort has gone into the generation of texture pairs with identical nth-order statistics (iso-n-gon statistics[1]) but different $(n+1)$th-order statistics (Julesz, 1962). It was assumed that the stochastic constraint of identical nth-order statistics would prevent a texture pair from having different local features that the human visual system could effortlessly discriminate when n was adequately large. In 1962 one-dimensional Markov textures were found with identical n-th order statistics (Rosenblatt and Slepian, 1962). With the help of these Markov processes Julesz observed that textures with identical one-dimensional second-order (iso-dipole) statistics but different statistics of third and higher order usually could not be discriminated in a brief flash (under 160 ms duration to prevent eye movements and shifts of attention) (Julesz, 1962). Since iso-dipole textures have identical power spectra[2], this observation seems to imply that in texture discrimination the phase (spatial position) spectra are ignored, and textures with identical power spectra cannot be discriminated without scrutiny.

Only in 1973 were ways found to generate two-dimensional textures with iso-dipole statistics but different third- and higher-order statistics (Julesz et al., 1973; Julesz, 1975). A typical iso-dipole texture pair is shown in Fig. 1. While the majority of such iso-dipole texture pairs could not be effortlessly discriminated, recently a few iso-dipole textures were found that yielded strong discrimination based on the quasi-collinearity, corner, and closure of local features in one of the textures (Caelli and Julesz, 1978; Caelli et al., 1978), as shown in Figs. 2a, b, and c.

While the iso-dipole constraint in most cases yields texture pairs that cannot be discriminated, the counter-examples given in Fig. 2 clearly show that the iso-dipole requirement is not adequate to suppress a few special local features to which the visual system seems

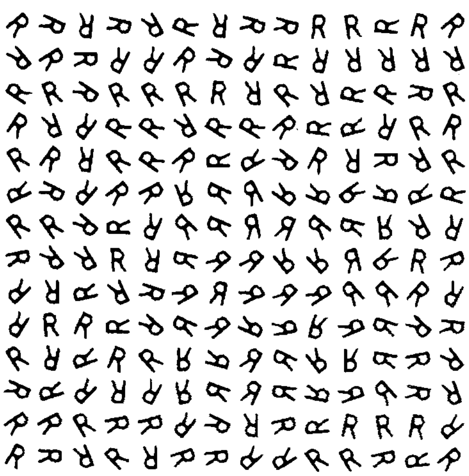

Fig. 1. A typical nondiscriminable iso-dipole texture pair, in which an inserted area is composed of micropatterns that are the mirror images of the micropatterns in the outside area. (From Julesz et al., 1973)

[1] In random geometry n-gons of all shapes are randomly thrown over an image, and the statistics are determined that all n vertices of each n-gon land on a given combination of colors

[2] Dipole statistics determine the autocorrelation function, from which the power spectrum can be obtained by Fourier transformation

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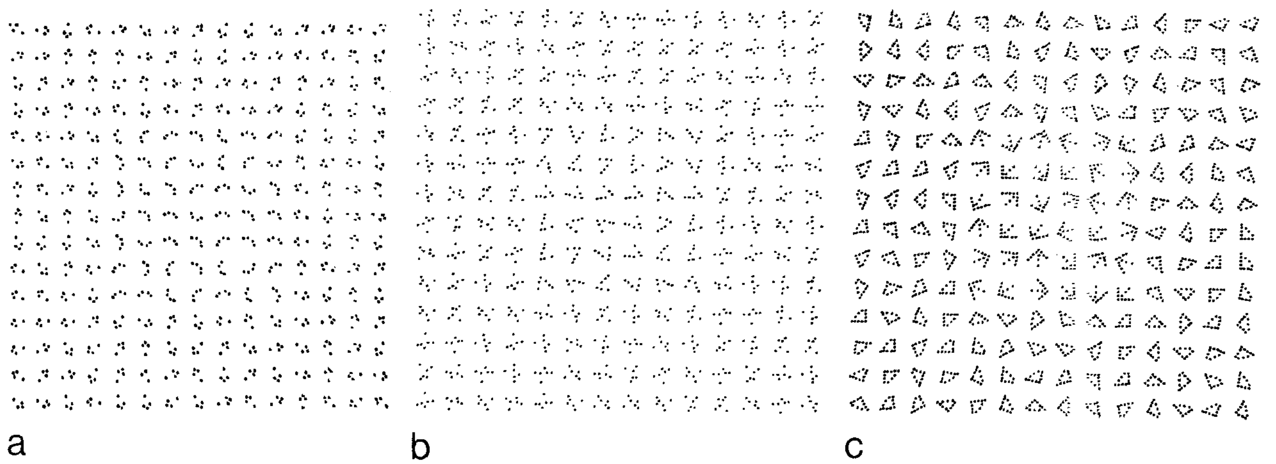

Fig. 2a–c. Discriminable iso-dipole texture pairs, for which discrimination is based on local features of: **a** quasi-collinearity, **b** corner, and **c** closure. (From Caelli et al., 1978)

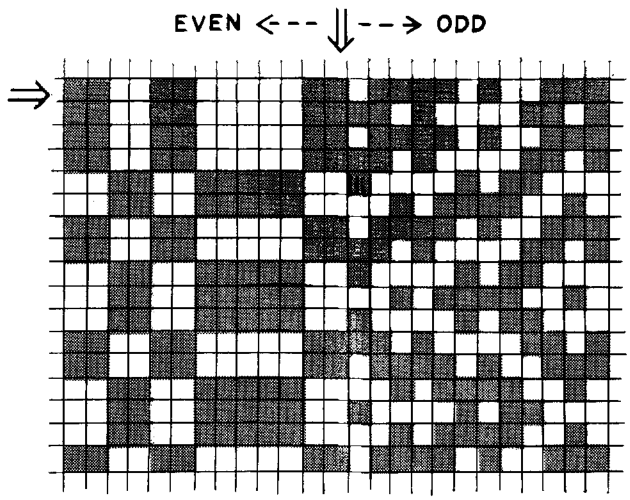

Fig. 3. Discriminable iso-trigon texture pair. The small squares in the first row and middle column are selected black and white at random, while each 2×2 square left of the middle column contains even number of black squares, while odd number of black squares to the right

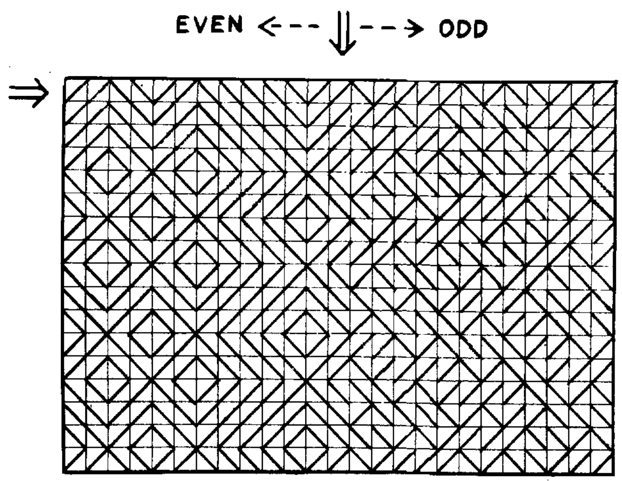

Fig. 4. Same as Fig. 3 except that in place of black and white small squares, squares containing a $+45$ deg or -45 deg diagonal line segment, respectively, are used

to be particularly sensitive. In this article an example is given to show that even a texture pair with iso-trigon statistics can yield strong discrimination. The visual field is cut into an array of small squares (square tessellation), each square to be colored black or white. In one texture, each 2×2 square cell has an *even* number of its 4 small squares colored black. In the other texture each 2×2 square cell has an *odd* number of black squares. Either texture can be determined completely by specifying the colors in one row and column of the array. In Fig. 3 the top row and center column (marked by arrows) were colored by flipping a coin. The squares to the left of center were colored as an even texture and those to the right as an odd texture. It is not hard to show that any three squares have independent equally likely random colors; then both textures have the same trigram statistics. However, the *even* texture is composed of black and white rectangular blocks which make it easy to distinguish from the more erratic *odd* texture. For proof, see Appendix.

Although it is most unlikely that such an artificial texture pair would be encountered in a real-life situation, it serves as the first known example of an effortlessly discriminable texture pair with identical third-order statistics. Furthermore, most observers would consider the odd texture more "granular" than the even texture. It is indeed surprising that not only the iso-dipole requirement but even the iso-trigon stochastic constraint fails to prevent differences in apparent granularity.

One might wonder about the usefulness of dipole statistics in the study of texture perception when even iso-trigon textures can yield texture discrimination. However, the counterexamples found thus far are few, and help to identify those local features that are used by the texture perception system. Until now it was assumed that differences in the size (or aspect ratio) of local clusters were reflected in differences between dipole statistics (or the corresponding spectra). In the light of the present result, however, these local cluster detectors of size have to be added to the already known other local feature detectors tuned to quasi-collinearity, corner, and closure, that seem to operate in spite of the global iso-dipole stochastic constraint.

The method, yielding the iso-trigon texture pairs of Fig. 3, can be used to generate other kinds of iso-trigon textures. For instance, if instead of using black and white small squares in Fig. 3, one draws a $+45°$, or $-45°$, diagonal line segment in the small squares, respectively, an effortlessly discriminable texture pair is again obtained. In the *even* texture closed, diamond-shaped structures are formed, while the *odd* texture appears as a maze-like structure, as shown in Fig. 4. Furthermore, one can generate a new class of iso-dipole textures by applying linear filtering (e.g., blurring) on the texture pair of Fig. 3. While the filtering may change the iso-trigon constraint, the texture pair will remain iso-dipole (since the identical power spectra are multiplied by the same filter characteristics). Thus filtering the texture pair of Fig. 3 gives rise to halftone textures (i.e. many shades of gray) with identical power spectra, and if the filtering is not too drastic the textures will be discriminable. The reader can perform such blurring by defocusing Fig. 3 (e.g. by squinting).

It might still be assumed that as n increases the ratio of discriminable to nondiscriminable iso-*n-gon* textures becomes vanishingly small. Indeed, one can select (in place of the 2×2 square shaped aperture, or "floater", used in our study) some other floaters of more complex shapes, containing 5, 6, or more small squares, in order to generate iso-*4-gon*, iso-*5-gon*, etc. texture pairs. In another study (Julesz, Gilbert, and Schmidt, in preparation) it will be shown that only certain floater shapes can be used. Here it suffices to note, that iso-*4-gon* textures were found that yielded weak discrimination, but in spite of intensive effort, all the iso-*5-gon* textures that were tried could not be discriminated. Nevertheless, the finding that granularity changes of a texture can be controlled only by 4-gon statistics is an unexpected result, and of use to those who want to reduce grain in films or noise in displays.

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Appendix

Dissect the visual plane into an array of squares and color squares black or white to obtain a visual texture. Define $a(x, y) = 1$ if the square at row x and column y is black. Let $a(x, y) = 0$ if this square is white. Define an *even texture* (or an *odd texture*) to be a texture for which

$$a(x+1, y+1) + a(x+1, y) + a(x, y+1) + a(x, y) \tag{1}$$

is even (or odd) for every x, y.

The mathematical condition that a texture be even (or odd) is a binary recurrence equation in which the sum (1) is set equal to 0 (or 1). A solution will express $a(x, y)$ in terms of boundary values, for instance the values $f(x) = a(x, 0)$ and $g(y) = a(0, y)$ along column 0 and row 0. For an even texture

$$a(x, y) = f(x) + g(y) + h \quad (\text{mod}\, 2). \tag{2}$$

where $h = f(0) = g(0) = a(0, 0)$. For an odd texture

$$a(x, y) = f(x) + g(y) + h + xy \quad (\text{mod}\, 2). \tag{3}$$

Let row 0 and column 0 be colored by flipping a coin, so that $f(x)$ and $g(y)$ are 0 or 1 with probability $\frac{1}{2}$.

Pick any three squares (x_1, y_1), (x_2, y_2), (x_3, y_3). The eight possible ways that these squares may be colored are equally likely, both in the even texture and in the odd texture. To prove this, consider any one of the eight colorings, say the one with $a(x_1, y_1) = a_1$, $a(x_2, y_2) = a_2$, $a(x_3, y_3) = a_3$, and count how many choices of h, $f(x_1)$, $f(x_2)$, $\ldots$, $g(y_3)$ produce this coloring. In the odd texture, the choices are solutions h, $f(x_1)$, $f(x_2)$, $\ldots$, $g(y_3)$ of (3), i.e.

$$\begin{aligned} h + f(x_1) + g(y_1) &= a_1 + x_1 y_1 \quad (\text{mod}\, 2) \\ h + f(x_2) + g(y_2) &= a_2 + x_2 y_2 \quad (\text{mod}\, 2) \\ h + f(x_3) + g(y_3) &= a_3 + x_3 y_3 \quad (\text{mod}\, 2). \end{aligned} \tag{4}$$

There are several cases, depending on whether or not some points lie in the same row or column.

Case 1 (no two points in the same row or column). There are seven unknowns in (4): h, $f(x_1)$, $f(x_2)$, $\ldots$, $g(y_3)$. But $f(x_1)$, $f(x_2)$, $f(x_3)$ are determined once h, $g(y_1)$, $g(y_2)$, $g(y_3)$ are given. Then 2^4 out of 2^7 equally likely choices give the desired coloring.

Case 2 (two points in the same row). If $x_1 = x_2$ then $f(x_1) = f(x_2)$ and (4) has only six unknowns. Once h, $g(y_2)$, and $g(y_3)$ are specified, (4) determines $f(x_2)$, $f(x_3)$, and hence $f(x_1)$ and $g(y_1)$. The coloring is obtained with 2^3 out of 2^6 equally likely choices.

Case 3 (three points in one row). If $x_1 = x_2 = x_3$ then $f(x_1) = f(x_2) = f(x_3)$ and (4) has five unknowns. If h and $f(x_1)$ are specified, then (4) determines $g(y_1)$, $g(y_2)$, $g(y_3)$. The coloring is obtained with 2^2 out of 2^5 choices.

Case 4 (one row and one column contain two points each). Suppose $x_1 = x_2$ and $y_1 = y_3$. Then $f(x_1) = f(x_2)$, $g(y_1) = g(y_3)$ so that h, $f(x_2)$, $f(x_3)$, $g(y_2)$, $g(y_3)$ are the unknowns. If h and $f(x_1)$ are specified then (4) determines the other unknowns. The coloring is obtained with 2^2 out of 2^5 choices.

Case 5 (two or three points share a column). The argument is the same as in case 2 or 3.

In cases $1, \ldots, 5$ it was tacitly assumed that the coordinates $x_1, x_2, \ldots, y_3$ are not zero. If $x_1 = 0$ for example, then $f(x_1) = f(0) = h$ and the number of unknowns is reduced. But in every case, there is a corresponding reduction in the number of free parameters on which the solution of (4) depends. The desired coloring a_1, a_2, a_3 is always obtained from 1/8 of the equally likely ways of determining h, $f(x_1)$, $f(x_2), \ldots, g(y_3)$.

The same proof applies to the even texture. Terms x_1y_1, x_2y_2, x_3y_3 are missing from (4) but that does not affect the argument. Thus odd and even textures, although easily discriminated by eye, have the same third order probabilities.

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