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Temporal Pattern Discrimination of Impulse Sequences in the Computer-Simulated Nerve Cells

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Abstract

This paper deals with some properties of temporal pattern discrimination performed by single digital-computer simulated synaptic cells. To clarify these properties, the Shannon's entropy method which is a basic notion in the information theory and a fundamental approach for the design of pattern classification system was applied to input-output relations of the digital computer simulated synaptic cells. We used the average mutual information per symbol as a measure for the temporal pattern sensitivity of the nerve cells, and the average response entropy per symbol as a measure for the frequency transfer characteristics. To use these measures, the probability of a post-synaptic spike as a function of the recent history of pre-synaptic intervals was examined in detail. As the results of such application, it was found that the EPSP size is closely related to the pattern of impulse sequences of the input, and the average mutual information per symbol for EPSP size is given by a bimodal curve with two maximum values. One is a small EPSP size and the other is a large EPSP size. In two maximum points, the structure of the temporal pattern discrimination reverses each other. In addition, the relation between the mean frequency, or the form of impulse sequences of the input, and the average mutual information per symbol has been examined. The EPSP size at one maximum point of average mutual information is in inverse proportion to the magnitude of input mean frequency which relates to the convergence number of input terminal, while that at the other maximum point is proportional to that of the mean frequency. Moreover, the temporal pattern discrimination is affected remarkably by whether successive interspike intervals of the input are independent or not in the statistical sense. Computer experiments were performed by the semi-Markov processes with three typical types of transition matrixes and these shuffling processes. The average mutual informations in the cases of these semi-Markov processes are in contrast to those of the shuffling processes which provide a control case. The temporal structure of successive interspike intervals of the input is thus a significant factor in pattern discrimination at synaptic level.

1. Introduction

In order to examine the processing of information in the nervous system, we need to know how all-or-nothing impulse sequences in nerve fibers are processed by single nerve cells. The occurrence of a post-synaptic spike of single cells is a probability function of successive inter-spike intervals of pre-synaptic spikes. Hence in this type of junction, the input-output probabilistic properties must be examined. Several statistical and probabilistic measures have been used to describe these properties (Gerstein, 1962; Hoopen, 1966; Werner and Mountcastle, 1965; Stein, 1967; Segundo *et al.*, 1963, 1966, 1968). Werner and Mountcastle (1965) applied the information theory experimentally to mechanoreceptor cells activated by mechanical stimuli. They calculated the average mutual information per stimulus, indicating that the neuronal information capacity is limited to a definite value at a long time. Stein (1967) estimated the magnitude of the difference between the information capacities of single cells using frequency coding, and those using binary or interval coding. Segundo *et al.* (1963, 1966) found ganglion cells of the central nervous system of Aplysia to be pattern sensitive and examined the probability of a post-synaptic spike as a function of the recent history of pre-synaptic intervals.

From this aspect the authors were interested in the temporal pattern discrimination of single nerve cells, and the Shannon's entropy method was applied to input-output relations of the computer simulated synaptic cells (Tsukada and Sato, 1972). In this paper, computer experiments were performed by the following procedure:

(i) The statistical structure of the spike train in a pre-synaptic terminal is fixed on the gamma Order 3. The probability of the occurrence of a post-synaptic spike as a function of the recent history of pre-synaptic interval as well as of the EPSP size is computed by the digital-computer-simulated system. We calculated the average conditional entropy per symbol for the pattern classes, the average response entropy and the average mutual information per symbol. Next, we explore it by the entropy method how many pre-synaptic spikes on the average influence effectively on the occurrence of a post-synaptic spike.

(ii) When the statistical structure (especially, mean frequency, distribution form) of the pre-synaptic spike train changes, it is examined how these entropy functions mentioned above change. In this computer experiment we used the gamma distribution form as the input which is able to vary from exponential to normal.

(iii) We examine how to effect on these entropy whether successive inter-spike intervals of the pre-synaptic train are independent or not in the statistical sense. In order to execute this problem, we simulate in digital-computer semi-Markov processes with three types of transition matrixes which are characterized by physiological data.

2. Methods

2.1. Synaptic Model

The synaptic model which was used in this paper was based on the digital-computer-simulated synaptic cells in which the related experiments had been carried out (Segundo *et al.*, 1966, 1968). The simulated preparation consisted of a single pre-synaptic terminal (the input) which made an excitatory connection with a post-synaptic cell (the output). A post-synaptic cell was represented by a nerve cell whose state was defined by membrane potential $P(t)$ and threshold $H(t)$ at any time t. The $P(t)$ was affected by EPSP's each of which corresponded to the certain pre-synaptic spike and of which the temporal summation was linear. The $P(t)$ between spike firing and EPSP arrival was defined by the following equation:

$$P(t) = P_\infty + (P_0 - P_\infty)\, e^{-\lambda_P (t - t_0)}, \tag{1}$$

where t_0: the absolute refractory period,
P_0: the initial resetting membrane potential,
P_∞: the resting membrane potential,
λ_P: the decay constant of $P(t)$.

The PSP summation of n EPSP's (corresponding to arrival clock times $t_1, t_2, \ldots, t_n$) after a spike firing was defined by the following equation:

$$P(t) = P_\infty + (P_0 - P_\infty)\, e^{-\lambda_P (t - t_0)} + (P_A - P_\infty) \sum_{m=1}^{n} e^{-\lambda_P \left(t - \sum_{i=1}^{m} T_i\right)}, \tag{2}$$

where $T_i = t_i - t_{i-1}$ for $i = 2, 3, \ldots, n$, $T_i = t_1$ for $i = 1$ and the potential P_A is the peak potential of EPSP. Each EPSP size normalized by resting threshold was represented by $A_0 = \frac{P_A - P_\infty}{H_\infty - P_\infty}$. The threshold $H(t)$ after spike firing was defined by the following equation:

$$H(t) = H_\infty + (H_0 - H_\infty)\, e^{-\lambda_H (t - t_0)}, \tag{3}$$

where H_0: the initial resetting threshold,
H_∞: the resting threshold potential,
λ_H: the decay constant of $H(t)$.

The synaptic model which was obtained in the way described above is shown in Fig. 1.

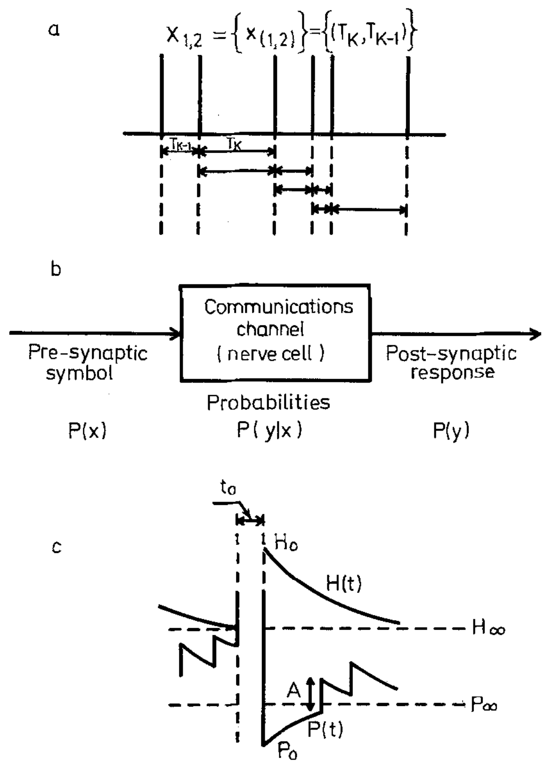


Fig. 1a—c. Digital-computer-simulated system used in this paper. a Random input impulse sequence and joint interval pattern $X_{1,2}$. b Input-output relation of the synaptic channel. c Model of the synaptic cell (see methods)

2.2. Statistical Structures of Pre-Synaptic Impulse Sequences

The statistical structures of pre-synaptic impulse sequences give potent influences on the post-synaptic discharge. Accordingly, the inter-spike interval mean, standard deviation, form, and the statistical dependence of the sequences of inter-spike intervals are important variables of the experiments.

2.2.1. Order-Independent Pre-Synaptic Impulse Sequences. In the experiment (i), the random arrival clock times of a pre-synaptic spike train were determined by drawing the interval from a pseudo-random gamma distribution of Order 3 generated by a digital computer subroutine. In the experiment (ii), the mean intervals and distribution form of the pre-synaptic interval sequences were changed. Let the probability density function (p.d.f.) of gamma distribution be

$$f_{\varrho,\alpha}(x) = \frac{\varrho(\varrho x)^{\alpha-1}}{\Gamma(\alpha)}\, e^{-\varrho x}, \tag{4}$$

where $\alpha > 0$ and $\varrho > 0$ are parameters, $\Gamma(\alpha)$ is a gamma function. The mean μ and variance σ^2 of the distribution are given by

$$\mu = \frac{\alpha}{\varrho}, \quad \sigma^2 = \frac{\alpha}{\varrho^2}.$$

When $\alpha = 1$, the p.d.f. is exponential and when $\alpha > 1$, it is zero at the origin and has a single maximum at $\frac{\alpha - 1}{\varrho}$. If we take the

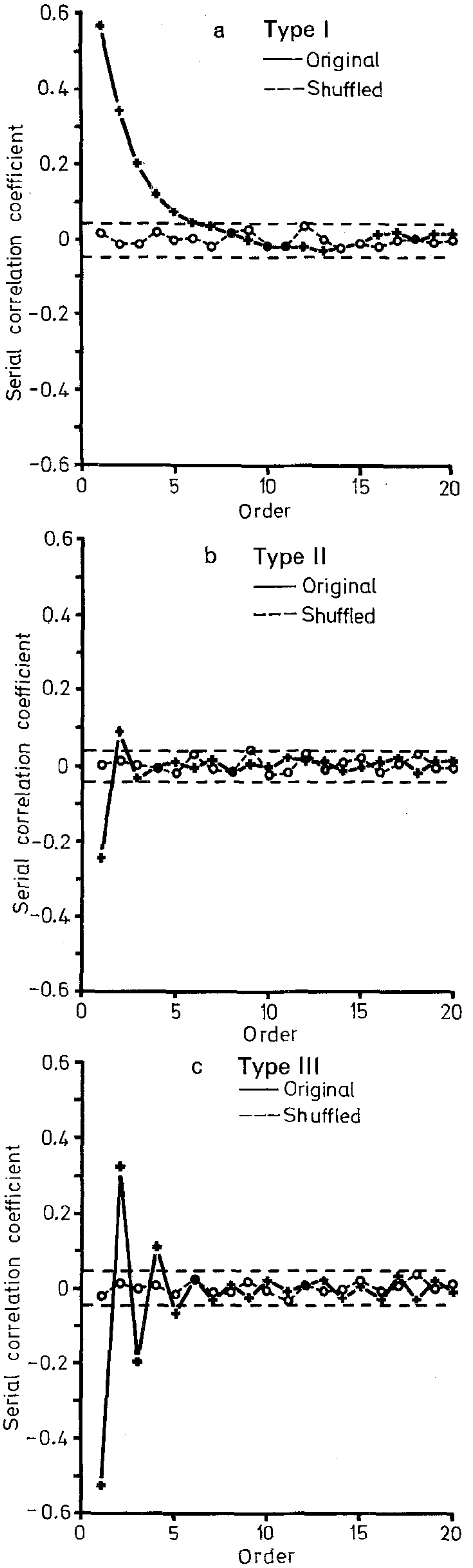


Fig. 2a—c. Typical serial correlograms of the input impulse sequences generated by the semi-Markov processes with three types of transition matrixes. a Type I, b Type II, c Type III

limit of (4) as $\alpha, \varrho \to \infty$ with the mean $\mu = \dfrac{\alpha}{\varrho}$ fixed, the distribution is asymptotically normal around μ with the fractional coefficient of variation $\dfrac{1}{\sqrt{\alpha}}$. In this experiment, α and μ were the variable parameters of determining the form of p.d.f. and its mean interval.

2.2.2. Order-Dependent Statistical Pre-Synaptic Impulse Sequences. In the experiment (iii), the pre-synaptic impulse sequences were generated by the three-stage semi-Markov processes according to the following conditions; we identified State 1 as normal distribution of short interval mean, State 2 as that of middle interval mean and State 3 as that of long interval mean. Transition matrixes of the processes $[p_{ij}]$ were defined by the following three types:

$$\text{Type I} \quad P_{\text{I}} = [p_{ij}] = \begin{pmatrix} 0.70 & 0.20 & 0.10 \\ 0.45 & 0.10 & 0.45 \\ 0.10 & 0.20 & 0.70 \end{pmatrix} \tag{5}$$

$$\text{Type II} \quad P_{\text{II}} = [p_{ij}] = \begin{pmatrix} 0.45 & 0.10 & 0.45 \\ 0.45 & 0.10 & 0.45 \\ 0.70 & 0.20 & 0.10 \end{pmatrix} \tag{6}$$

$$\text{Type III} \quad P_{\text{III}} = [p_{ij}] = \begin{pmatrix} 0.10 & 0.20 & 0.70 \\ 0.45 & 0.10 & 0.45 \\ 0.70 & 0.20 & 0.10 \end{pmatrix}. \tag{7}$$

The Type I is such that a long interval is most likely to be followed by a long interval and a short one by a short one. This type contributes a positive component to each serial correlation coefficient. The Type II is a simulated neuron producing irregular burst (Smith and Smith, 1965) and is characterized by negative serial correlation for the first lag followed by slightly positive and then 0 correlation coefficients. The Type III is a simulated spike train of which the intervals alternate between long and short intervals and gives a strong negative first serial correlation with subsequent alternation in the sign of the coefficients of higher order (Wilson, 1964; Hermann *et al.*, 1967; Perkel *et al.*, 1967). To estimate the order-dependence of inter-spike intervals, each sample was converted into the corresponding renewal process by random shuffling which destroyed serial dependence. The serial correlation coefficients of these examples are illustrated in Fig. 2.

2.3. Input-Output Relations and Definitions

The fundamental approach for the design of pattern classification systems which makes use of the Shannon's entropy method is applied to the input-output relations established at synaptic level (Fig. 1). First problem, therefore, is concerned with the representation of the detailed temporal configuration of input interval patterns $X = \{x_j\}$. In the pre-synaptic spike train, the one-dimensional temporal patterns of interspike intervals X_1 are displayed in an interval histogram, the two-dimensional patterns $X_{1,2}$ in a joint interval histogram, and the n-dimensional patterns $X_{1,2,\ldots,n}$ are represented by pattern points in an n-dimensional space. Here, the conventional expression is introduced as follows: Let the random arrival clock times of a pre-synaptic spike train be $t_1, t_2, \ldots, t_K, \ldots, t_N$ in order, where $T_K = t_K - t_{K-1}$ and N is a large number. We denote the subset of the inter-spike intervals by $X_1 = \{x_1\} = \{T_K\}$, the adjacent inter-spike intervals by $X_{1,2} = \{x_{1,2}\} = \{(T_K, T_{K-1})\}$ (Fig. 1), and similarly the successive inter-spike intervals by $X_{1,2,3} = \{x_{1,2,3}\} = \{(T_K, T_{K-1}, T_{K-2})\}, \ldots, X_{1,2,\ldots,n} = \{x_{1,2,\ldots,n}\} = \{(T_K, T_{K-1}, \ldots, T_{K-n+2}, T_{K-n+1})\}$. The j-th element of input interval patterns X takes on a value x_j with probability $P(x_j)$. The post-synaptic neuron decides to which pattern class (firing a spike or remaining silent) the temporal patterns of pre-synaptic intervals belong.

Secondly, hence, assume that the post-synaptic neuron is to be designed for recognizing two different pattern classes $Y=\{y_i\}=(y_1, y_2)$ (y_1; firing a spike and y_2; remaining silent) and the probability of occurrence of pattern class y_i is $P(y_i)$. Thirdly, the conditional probability of y_i when x_j is observed is $P(y_i|x_j)$ and it specifies the properties of a nerve cell as a communications channel.

2.4. Entropy Methods

A fundamental approach for the design of pattern classification systems which makes use of the Shannon's entropy method is applied to the input-output relation established at synaptic level.

The mutual information per symbol pair (y_i, x_j) is

$$I(y_i, x_j) = \log_2 \frac{P(y_i|x_j)}{P(y_i)} = \log_2 \frac{P(x_j|y_i)}{P(x_j)} \quad \text{(bits)}. \tag{8}$$

The average mutual information is

$$\begin{aligned} I(Y, X) = \langle I(y_i, x_j)\rangle &= \sum_{y_i \in Y} \sum_{x_j \in X} P(y_i, x_j) \log_2 \frac{P(x_j|y_i)}{P(x_j)} \\ &= \sum_{y_i \in Y} P(y_i) \sum_{x_j \in X} P(x_j|y_i) \log_2 \frac{P(x_j|y_i)}{P(x_j)} \quad \text{(bits)}. \end{aligned} \tag{9}$$

Which may be written as

$$I(Y, X) = H(X) - H(X|Y) = H(Y) - H(Y|X) \quad \text{(bits)}. \tag{10}$$

In Eq. (10),

$$H(X) = -\sum_{x_j \in X} P(x_j) \log_2 P(x_j) \quad \text{(bits)}$$

$$H(X|Y) = -\sum_{y_i \in Y} P(y_i) \sum_{x_j \in X} P(x_j|y_i) \log_2 P(x_j|y_i) \quad \text{(bits)}$$

$$H(Y) = -\sum_{y_i \in Y} P(y_i) \log_2 P(y_i) \quad \text{(bits)}$$

$$H(Y|X) = -\sum_{x_j \in X} P(x_j) \sum_{y_i \in Y} P(y_i|x_j) \log_2 P(y_i|x_j) \quad \text{(bits)}.$$

$I(Y, X_{1,2})$ and $I(Y, X_{1,2,3}), \ldots, I(Y, X_{1,2,\ldots,n})$ denote the average mutual information per symbol pair $(y_i, x_{1,2})$ and $(y_i, x_{1,2,3}), \ldots, (y_i, x_{1,2,\ldots,n})$ respectively. The average mutual information is a measure for the discrimination which indicates the difference between the probability distributions $\{P(y_i, x_j)\}$ and $\{P(y_i)\cdot P(x_j)\}$. On the other hand, the measure of $H(Y)$ shows the uncertainty of the pattern classes $Y=\{y_i\}$ and so, from this measure, we can estimate the input-output average frequency transfer characteristic of the nerve cells.

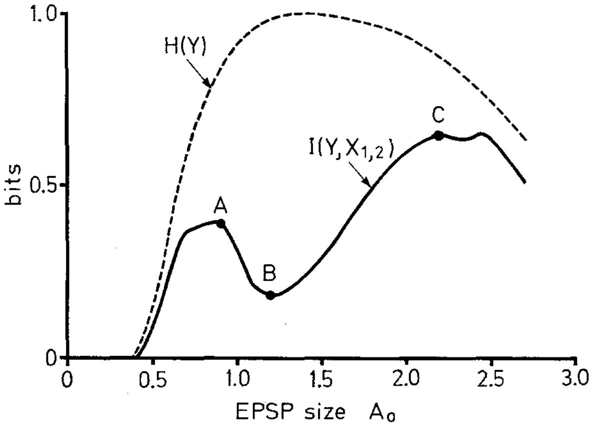


Fig. 3. The average mutual information $I(Y, X_{1,2})$ and average response entropy $H(Y)$ as a function of EPSP size. The input impulse sequence was a pseudo-random gamma distribution of Order 3 with mean interval of 250 msec generated by digital-computer subroutine

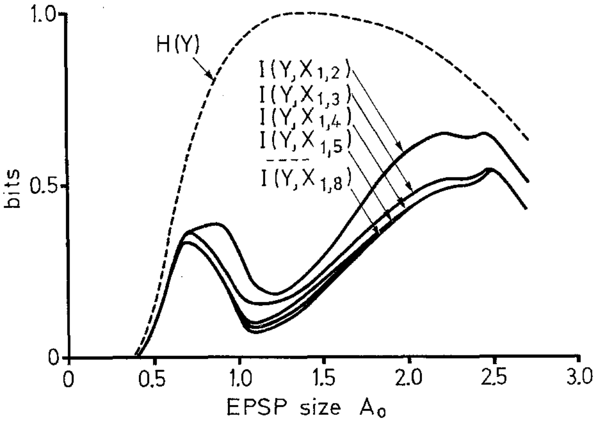


Fig. 4. Estimation of the order of Markov process of the communication channel (synaptic cell). Each of the average mutual information $I(Y, X_{1,n})$ for $n=2, 3, \ldots 8$ was calculated by the same simulated system as that in Fig. 3

3. Results

3.1. Order-Independent Pre-Synaptic Impulse Sequence

3.1.1. Fixed Pre-Synaptic Inputs. The inter-spike interval distribution of pre-synaptic spike train was fixed to gamma distribution of Order 3 with mean 250 msec while the EPSP size A_0 is increased from 0.3–3.0. By the digital-computer-simulation methods described in the previous chapter, the values of the conditional probability $\{P(y_i|x_{1,2})\}$ of y_i when $x_{1,2}$ is observed, the value of the average mutual information $I(Y, X_{1,2})$ and that of the average response entropy $H(Y)$ were calculated as a function of the EPSP size (Fig. 3). Table 1 shows the typical three examples concerning the conditional probability. Each case of A–C in Table 1 corresponds to the Point A–C in Fig. 3 respectively.

As will be seen from Fig. 3, the value of $I(Y, X_{1,2})$ against EPSP size shows a bimodal curve with two maximum values; one in the Point A (small EPSP size), the other in the Point C (large EPSP size), and with a minimum value in the Point B which is a middle EPSP size between A and C. $H(Y)$ gradually increases as the EPSP size, passes through a maximum, and finally falls off. In Case A, the pattern class y_1 concentrates in a partial region of the $T_K - T_{K-1}$ plane, in

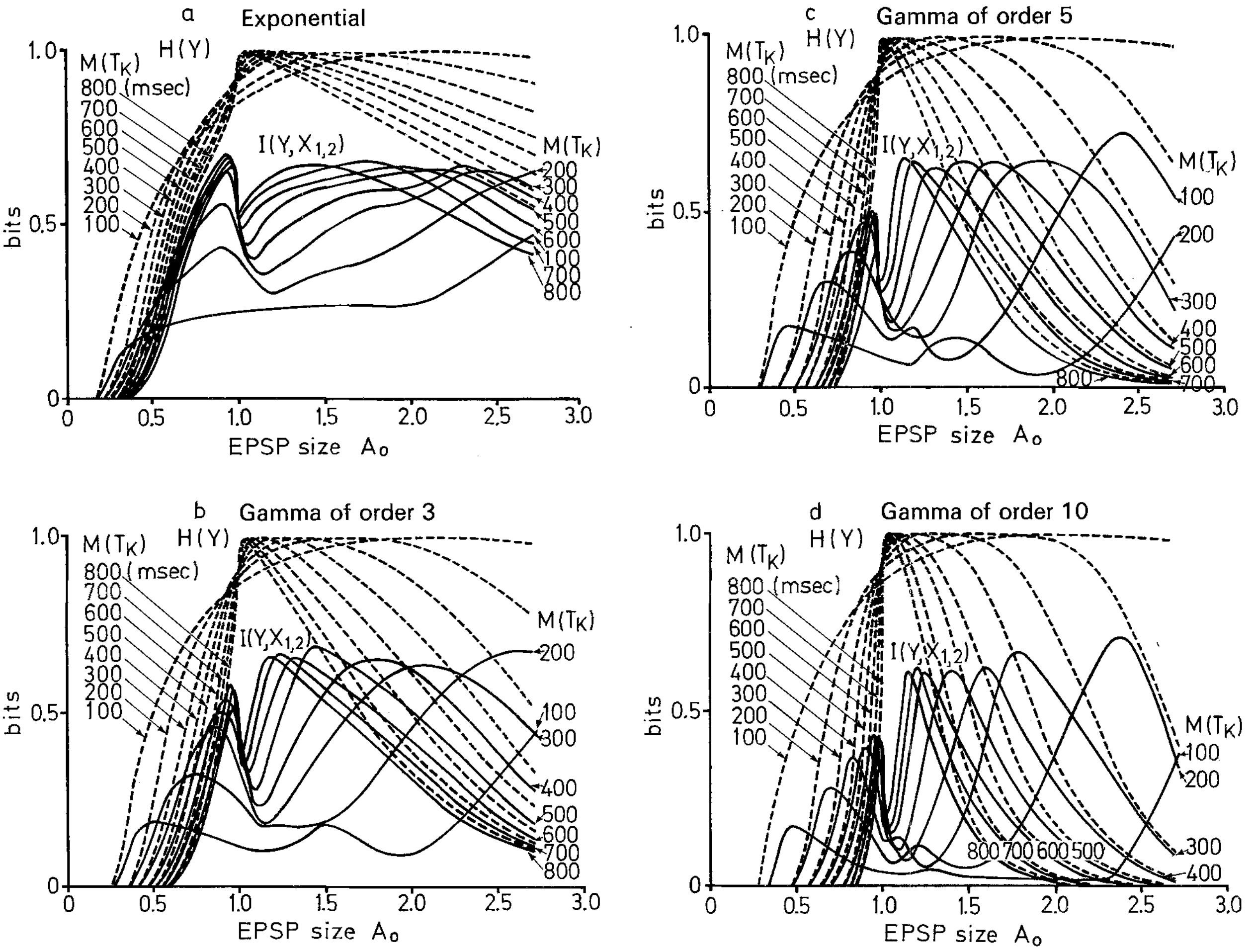


Fig. 5a—d. Relation of the mean frequency and statistical form of input impulse sequence with the entropies $H(Y)$ and $I(Y, X_{1,2})$ as a faction of EPSP size. a Exponential. b Gamma of Order 3. c Gamma of Order 5. d Gamma of Order 10. $M[T_K]$: Mean interval of input impulse sequence

the area below the $T_K = T_{K-1}$ line (heavily outlined in Table 1A). This result agrees with that reported by Segundo *et al.* (1966). In Case B, the pattern classes y_1 and y_2 spread over all area of the $T_K - T_{K-1}$ plane and thus each pattern class cannot be separated from the other. In Case C, they separate from each other; y_1 concentrates in the area above the $T_K = T_{K-1}$ line (full line) and the other class y_2 in the area below the $T_K = T_{K-1}$ line (broken line) in Table 1C. When comparing the structures of pattern classification in the Case A with those in C, the pattern classes y_1 and y_2 in the Case C are inversely related to those in A.

Next experiment was performed to examine how many pre-synaptic spikes on the average influence effectively on the occurrence of a post-synaptic spike. Each $I(Y, X_{1,2})$, $I(Y, X_{1,3}), \ldots, I(Y, X_{1,8})$ per symbol pair $(y_i, x_{1,2})$, $(y_i, x_{1,3}), \ldots, (y_i, x_{1,8})$ respectively was computed by the same simulation models (Fig. 4).

As will be seen from Fig. 4, the average mutual information against each EPSP size is described by following relations:

a) $A \leqq 0.7$ $I(Y, X_{1,2}) = I(Y, X_{1,3}) \ldots,$

b) $0.7 < A \leqq 1.0$ $I(Y, X_{1,2}) > I(Y, X_{1,3}) > I(Y, X_{1,4}) = I(Y, X_{1,5}) = \ldots,$

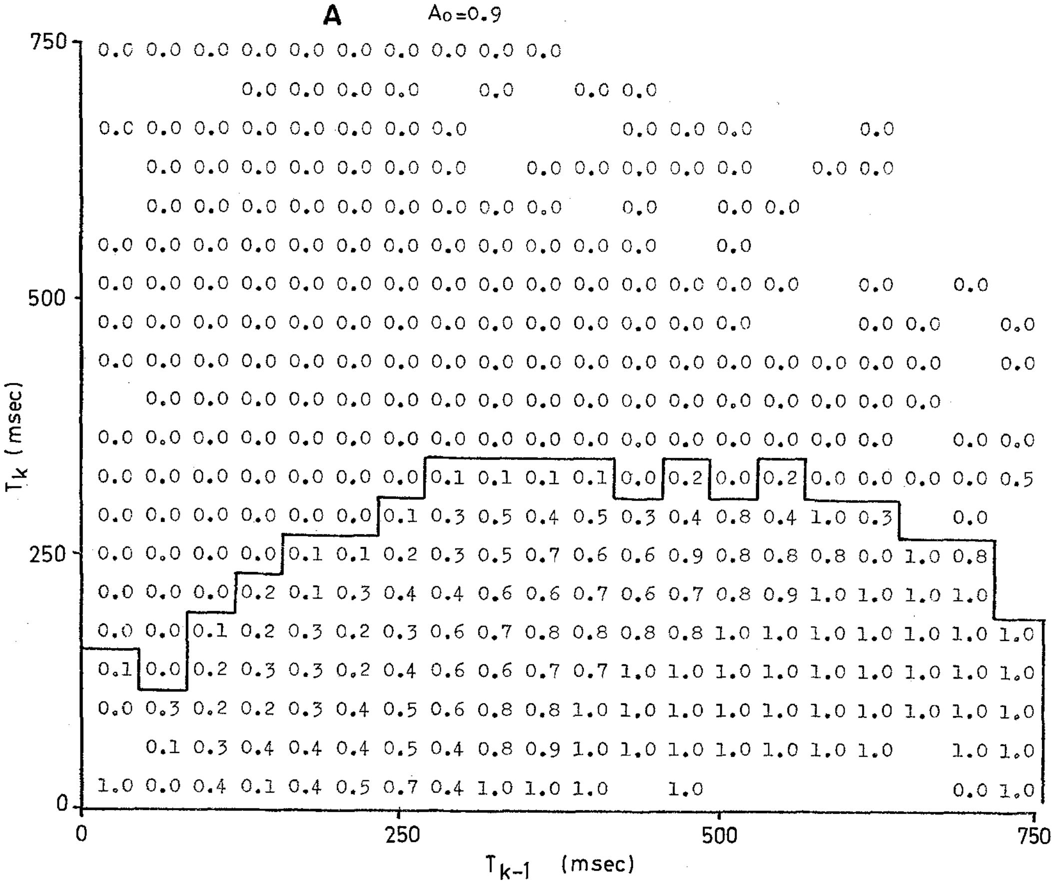
c) $1.0 < A \leqq 1.9$ $I(Y, X_{1,2}) > I(Y, X_{1,3}) > I(Y, X_{1,4}) > I(Y, X_{1,5}) > I(Y, X_{1,6}) = I(Y, X_{1,7}) = \ldots,$

d) $1.9 < A \leqq 2.4$ $I(Y, X_{1,2}) > I(Y, X_{1,3}) > I(Y, X_{1,4}) = I(Y, X_{1,5}) = \ldots,$

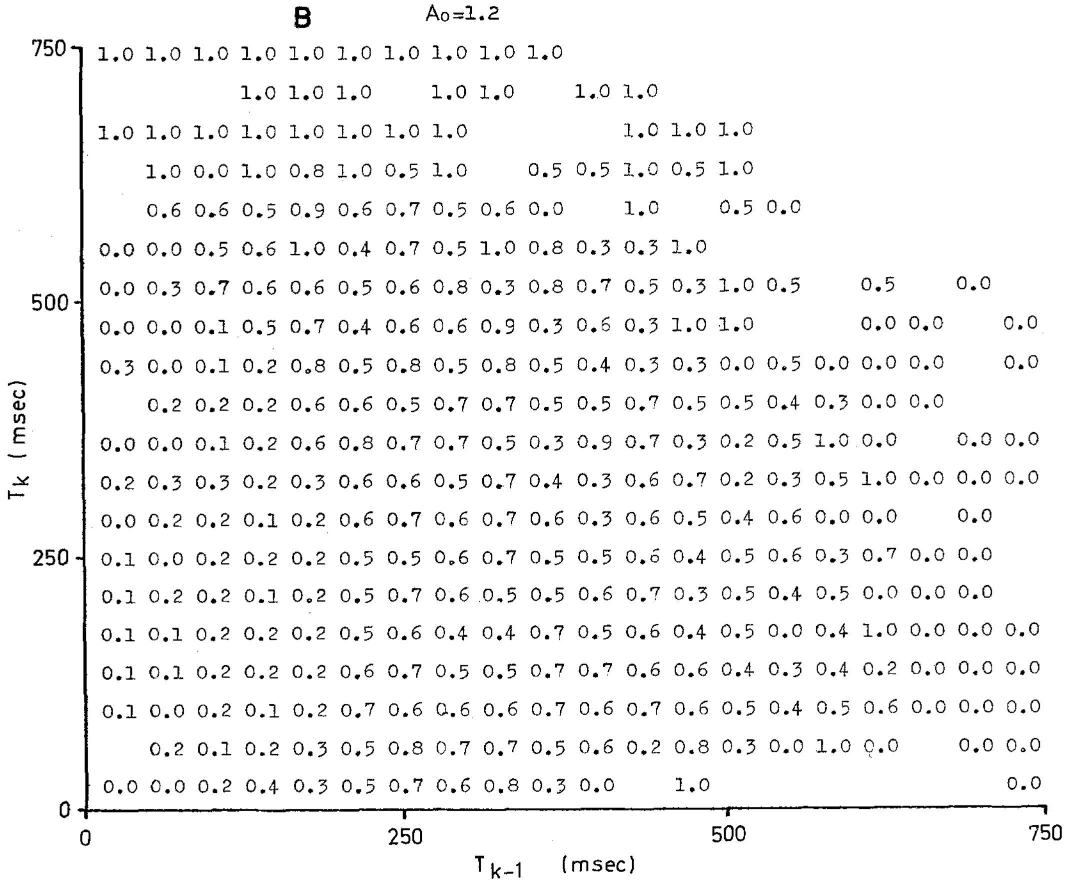
e) $2.4 < A \leqq 2.7$ $I(Y, X_{1,2}) > I(Y, X_{1,3}) = I(Y, X_{1,4}) = \ldots.$

These results suggest that the average effective numbers of the recent history of pre-synaptic intervals which influence on the probability of the occurrence of a post-synaptic spike are one interval in Case a), three intervals in Case b), five intervals in Case c), three intervals in Case d), and two intervals

Table 1A—C. Conditional probabilities $\{P(y_1|x_{1,2})\}$ of y_1 when $x_{1,2}$ is observed (see text)



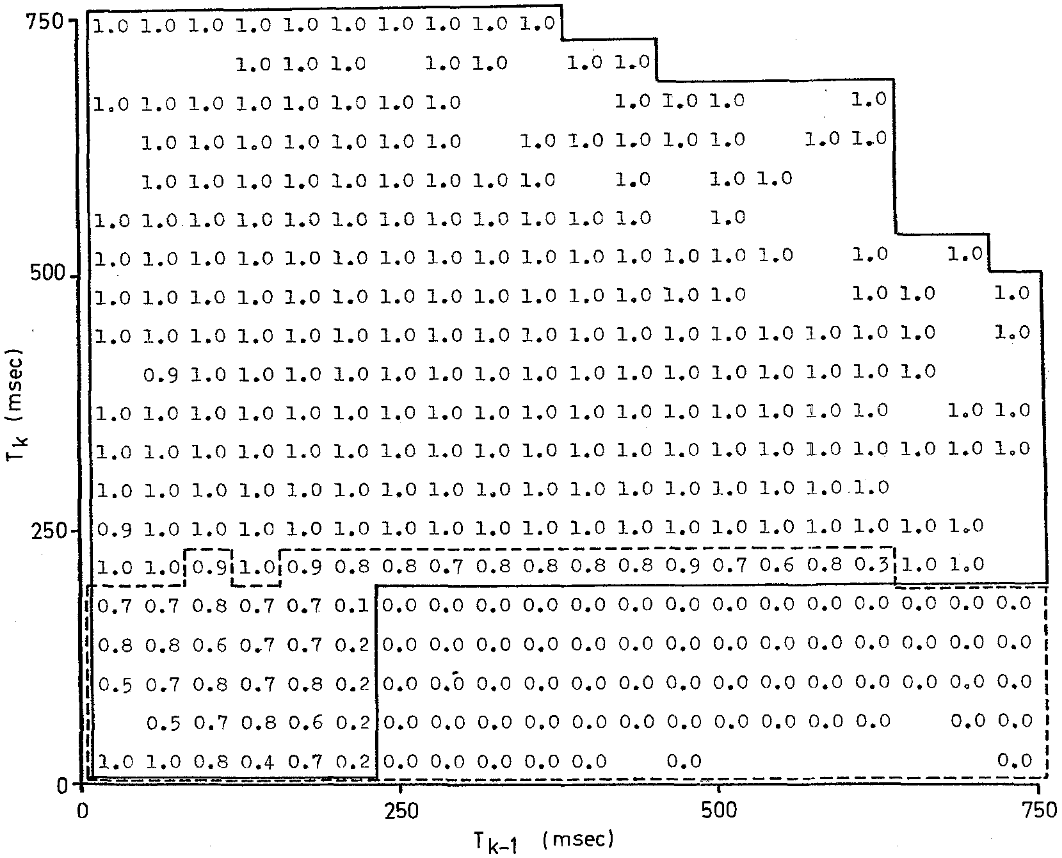
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0										
			0.0	0.0	0.0	0.0		0.0		0.0	0.0								
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				0.0	0.0	0.0			0.0			
	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0	0.0	0.0	0.0	0.0		0.0	0.0			
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0		0.0	0.0					
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0						
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0		0.0	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0			0.0	0.0		0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.2	0.0	0.2	0.0	0.0	0.0	0.0	0.5
0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.5	0.4	0.5	0.3	0.4	0.8	0.4	1.0	0.3		0.0	
0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.5	0.7	0.6	0.6	0.9	0.8	0.8	0.8	0.0	1.0	0.8	
0.0	0.0	0.0	0.2	0.1	0.3	0.4	0.4	0.6	0.6	0.7	0.6	0.7	0.8	0.9	1.0	1.0	1.0	1.0	
0.0	0.0	0.1	0.2	0.3	0.2	0.3	0.6	0.7	0.8	0.8	0.8	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.1	0.0	0.2	0.3	0.3	0.2	0.4	0.6	0.6	0.7	0.7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.0	0.3	0.2	0.2	0.3	0.4	0.5	0.6	0.8	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.1	0.3	0.4	0.4	0.4	0.5	0.4	0.8	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0	1.0
1.0	0.0	0.4	0.1	0.4	0.5	0.7	0.4	1.0	1.0	1.0		1.0						0.0	1.0



1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0										
			1.0	1.0	1.0		1.0	1.0		1.0	1.0								
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0				1.0	1.0	1.0						
	1.0	0.0	1.0	0.8	1.0	0.5	1.0		0.5	0.5	1.0	0.5	1.0						
	0.6	0.6	0.5	0.9	0.6	0.7	0.5	0.6	0.0		1.0		0.5	0.0					
0.0	0.0	0.5	0.6	1.0	0.4	0.7	0.5	1.0	0.8	0.3	0.3	1.0							
0.0	0.3	0.7	0.6	0.6	0.5	0.6	0.8	0.3	0.8	0.7	0.5	0.3	1.0	0.5		0.5		0.0	
0.0	0.0	0.1	0.5	0.7	0.4	0.6	0.6	0.9	0.3	0.6	0.3	1.0	1.0			0.0	0.0		0.0
0.3	0.0	0.1	0.2	0.8	0.5	0.8	0.5	0.8	0.5	0.4	0.3	0.3	0.0	0.5	0.0	0.0	0.0		0.0
	0.2	0.2	0.2	0.6	0.6	0.5	0.7	0.7	0.5	0.5	0.7	0.5	0.5	0.4	0.3	0.0	0.0		
0.0	0.0	0.1	0.2	0.6	0.8	0.7	0.7	0.5	0.3	0.9	0.7	0.3	0.2	0.5	1.0	0.0		0.0	0.0
0.2	0.3	0.3	0.2	0.3	0.6	0.6	0.5	0.7	0.4	0.3	0.6	0.7	0.2	0.3	0.5	1.0	0.0	0.0	0.0
0.0	0.2	0.2	0.1	0.2	0.6	0.7	0.6	0.7	0.6	0.3	0.6	0.5	0.4	0.6	0.0	0.0		0.0	
0.1	0.0	0.2	0.2	0.2	0.5	0.5	0.6	0.7	0.5	0.5	0.6	0.4	0.5	0.6	0.3	0.7	0.0	0.0	
0.1	0.2	0.2	0.1	0.2	0.5	0.7	0.6	0.5	0.5	0.6	0.7	0.3	0.5	0.4	0.5	0.0	0.0	0.0	
0.1	0.1	0.2	0.2	0.2	0.5	0.6	0.4	0.4	0.7	0.5	0.6	0.4	0.5	0.0	0.4	1.0	0.0	0.0	0.0
0.1	0.1	0.2	0.2	0.2	0.6	0.7	0.5	0.5	0.7	0.7	0.6	0.6	0.4	0.3	0.4	0.2	0.0	0.0	0.0
0.1	0.0	0.2	0.1	0.2	0.7	0.6	0.6	0.6	0.7	0.6	0.7	0.6	0.5	0.4	0.5	0.6	0.0	0.0	0.0
	0.2	0.1	0.2	0.3	0.5	0.8	0.7	0.7	0.5	0.6	0.2	0.8	0.3	0.0	1.0	0.0		0.0	0.0
0.0	0.0	0.2	0.4	0.3	0.5	0.7	0.6	0.8	0.3	0.0		1.0							0.0

Table 1 (continued)

C Ao=2.2



1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0										
			1.0	1.0	1.0		1.0	1.0		1.0	1.0								
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0				1.0	1.0	1.0			1.0			
	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0	1.0	1.0	1.0	1.0		1.0	1.0			
	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0		1.0	1.0					
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0						
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0		1.0	
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0			1.0	1.0		1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0
	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0		
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0			
0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
1.0	1.0	0.9	1.0	0.9	0.8	0.8	0.7	0.8	0.8	0.8	0.8	0.9	0.7	0.6	0.8	0.3	1.0	1.0	
0.7	0.7	0.8	0.7	0.7	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	0.8	0.6	0.7	0.7	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.7	0.8	0.7	0.8	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.5	0.7	0.8	0.6	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0	0.0
1.0	1.0	0.8	0.4	0.7	0.2	0.0	0.0	0.0	0.0	0.0		0.0							0.0

in Case e). Thus we can estimate the order of Markov processes of the communication channels (synaptic neurons).

3.1.2. Mean Frequency and Distribution Form. When the statistical structure (mean frequency and distribution form) of the pre-synaptic impulse sequences changed (see Section 2.2), the values of the entropies $I(Y, X_{1,2})$ and $H(Y)$ as a function of EPSP size were computed by the same way of previous experiments (Fig. 5). Four forms of the pre-synaptic impulse sequences were used in this simulation; exponential, gamma of Order 3, gamma of Order 5, and gamma of Order 10. Mean frequency varied from 2 impulses/sec to 20 impulses/sec. In calculating these entropies, two-dimensional pattern $X_{1,2}$ was normalized by the mean intervals as follows: $(T_K, T_{K-1})_{ij}$ shows the pair intervals of i-th and j-th interval lenghs which are counted by steps of τ msec where τ is determined by $3\mu/n$ (μ is mean interval and n is the number of time bins).

As will be seen in Fig. 5, since $I(Y, X_{1,2})$ against EPSP size shows a bimodal curve with two maximum values, one in the small EPSP size is indicated by $I_{MI}(Y, X_{1,2})$ at A_{0I} and the other in the large EPSP size by $I_{M\text{II}}(Y, X_{1,2})$ at $A_{0\text{II}}$. In the mutual relation between the mean frequency of input impulse sequences and the EPSP size A_{0I}, when the mean frequency increases, its EPSP size decreases. On the other hand, in the case of $A_{0\text{II}}$, when the mean frequency increases, its EPSP size increases. Each value of $H(Y)$ in the case of small EPSP size becomes smaller in the order of the magnitude of input mean frequency, while in the case of large EPSP size it becomes larger in opposition.

In the relation between the distribution forms of the input impulse intervals and these entropies as a function of EPSP size, the bimodal curves of $I(Y, X_{1,2})$ against EPSP size become sharper in the order: exponential, gamma of Order 3, gamma of Order 5, gamma of Order 10, and the values of $I(Y, X_{1,2})$ at each fixed EPSP size and input mean frequency satisfy the following relations according to input forms: exponential > gamma of Order 3 > gamma of Order 5 > gamma of Order 10. These order relations coincide with those of the fractional coefficients of variation $\frac{1}{\sqrt{\alpha}}$ in the properties of the input gamma distribution. The values of $H(Y)$ also satisfy the same relations as those of $I(Y, X_{1,2})$.

3.2. *Order-Dependent Pre-Synaptic Impulse Sequences*

Three types of semi-Markov process as the original pre-synaptic impulse sequences and these random shuffled sequences were generated by the computer-simulated subroutines. For each sequence, the average mutual information $I(Y, X_{1,2})$ per symbol pair $(y_i, x_{1,2})$ and the average response information $H(Y)$ were computed by the computer-simulated systems (Fig. 6). For the computation of variability of $I(Y, X_{1,2})_{\text{shuffled}}$ 10 shuffled sequences were used in this paper. Thus, the range of the values of $I(Y, X_{1,2})_{\text{shuffled}}$ was determined by the values of 3σ and -3σ, where σ is the standard deviation of $I(Y, X_{1,2})_{\text{shuffled}}$.

As will be seen in Fig. 6a, each $I(Y, X_{1,2})$ of Type I original impulse sequences against EPSP size is smaller in value than that of shuffled sequences except for the case of very small EPSP size. Considerable discrepancies between $I(Y, X_{1,2})_{\text{original}}$ and $I(Y, X_{1,2})_{\text{shuffled}}$ are noted especially in the following ranges of EPSP size A_0: from 0.95–1.05 and from 1.3–1.6. Those of $H(Y)_{\text{original}}$ and $H(Y)_{\text{shuffled}}$ also show a similar tendency. On the other hand, as will be seen in Fig. 6C, the values of $I(Y, X_{1,2})$ and $H(Y)$ of Type III original impulse sequences are larger in value than those of the shuffled sequences. This difference between two values of each entropy comes out remarkably in the same range of EPSP size as that of Type I. From the facts described above, the values of these entropies in case of Type III present striking contrasts with those in case of Type I and the results obtained here is related to physiological facts that some neuromuscular junctions in crustaceans are highly sensitive to pattern, the contraction being greater many times for a train of alternately long and short intervals than for one of uniform intervals at the same mean rate (Wiesma and Adams, 1950). In case of Type II, the values of these entropies are rather similar to those of Type III but not so remarkable (Fig. 6b).

In addition to these entropy methods for the investigation of the stochastic properties of input-output relations in the single nerve cells, use of the serial correlation coefficients was introduced. The serial correlation coefficients of the pre-synaptic and post-synaptic impulse sequences were calculated

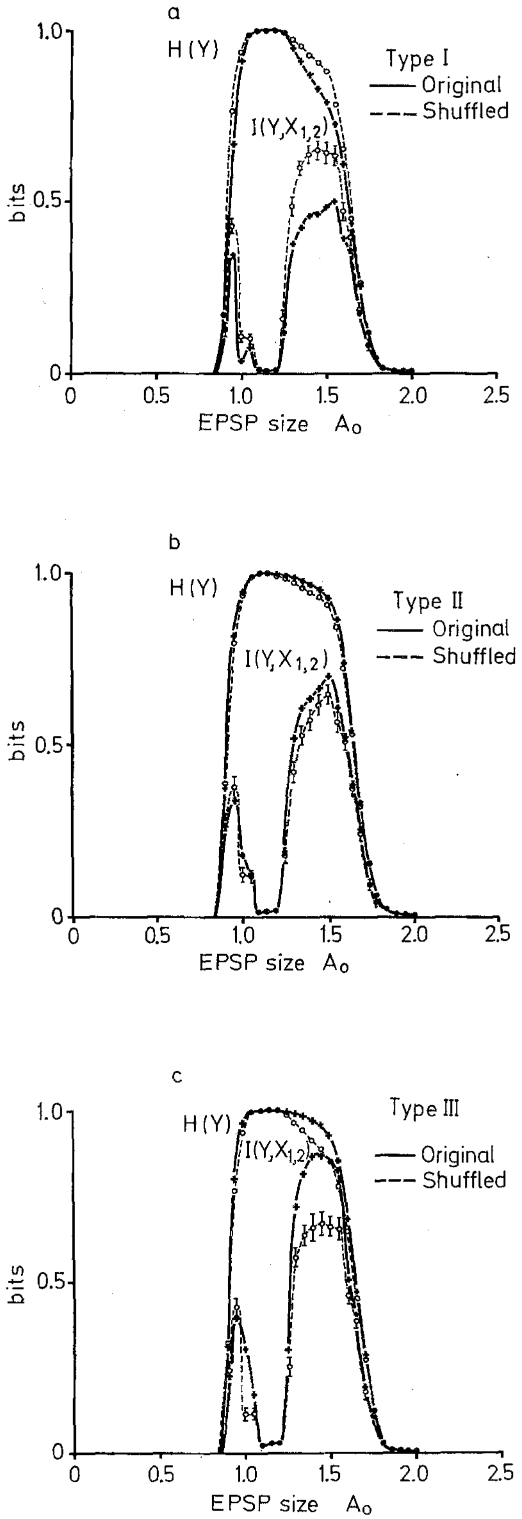


Fig. 6. The entropies $I(Y, X_{1,2})$ and $H(Y)$ for the input impulse sequences generated by the semi-Markov processes with three states with mean intervals as follows: State 1 (short), 0.35 sec; State 2 (medium), 0.40 sec; State 3 (long), 0.45 sec. All intervals are normally distributed with standard deviation of 0.03 sec (see text)

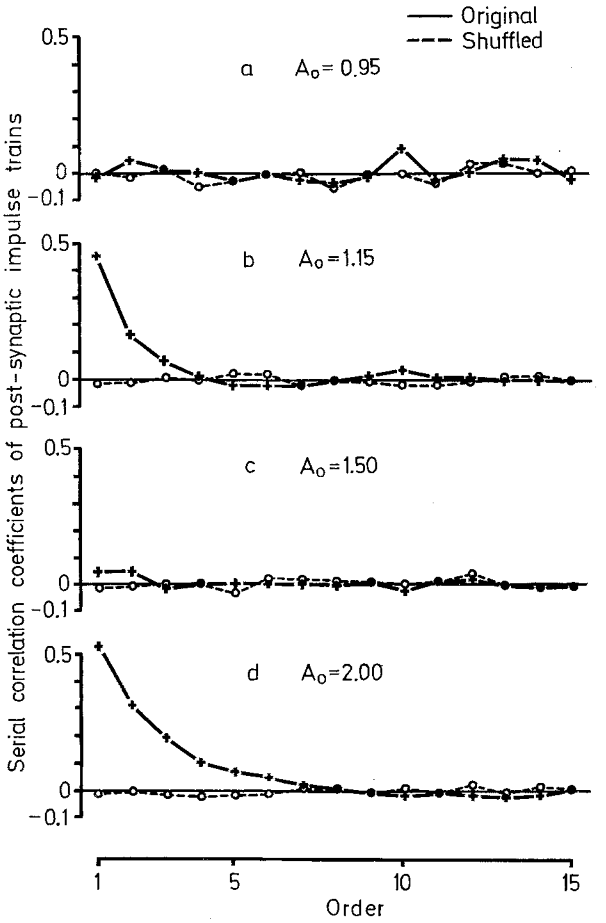


Fig. 7. The serial correlation coefficients of post-synaptic impulse sequences (see text)

against each typical EPSP size which correspond to the maximum and minimum values of $I(Y, X_{1,2})_{\text{original}}$ of Type I (Fig. 7).

As will be seen in Fig. 7, the interval lengths of the post-synaptic sequences exhibit positive serial correlation coefficients for each EPSP size which correspond to b and d in Fig. 7, while for that which correspond to a and c they are approximately zero. This result indicates that the only positive correlation properties of the input interval sequences can be transmitted to the output interval sequences through the synaptic channel in case of the particular EPSP sizes (Points b and d).

4. Discussion

The purpose of the present paper is to investigate some properties of temporal pattern discrimination performed by the single nerve cells. We applied the Shannon's entropy method to input-output relations in digital-computer-simulated networks in the previous paper (Tsukada and Sato, 1972). In this paper, this method was developed to clarify the problem of pattern classification in nervous system. The discussion about this problem will give the following five conclusions:

The first conclusion is that the EPSP size is closely related to the temporal pattern of input impulse sequences, i.e., the results of the average mutual information $I(Y, X_{1,2})$ for EPSP size are shown as a bimodal curve (Fig. 3). This property of temporal pattern classification is characterized by three different types. In the first type, the pattern class y_1 (firing a spike) concentrates on the particular joint patterns of the input impulse sequence which is given by the solid line in Table 1A. This type of pattern classification is obtained from the neuron models for small EPSP size and agrees with that reported by Segundo *et al.* (1966). In the second type, each of pattern classes y_1 and y_2 spreads over all joint patterns of input impulse sequences. This result is obtained from that for middle EPSP size and constitutes no sensitivity to temporal pattern of the input. In the last type, for large EPSP size, pattern classes y_1 and y_2 separate the joint pattern of input impulse sequences into two parts. The structure of this temporal pattern classification contrasts in a striking way with that of the first type.

The second conclusion is that, for each EPSP size, the average effective number of the recent history of pre-synaptic intervals which influence on the probability of the occurrence of a post-synaptic spike was estimated by the Shannon's entropy method.

The third conclusion is that the pre-synaptic statistical structure (mean frequency and distribution form) is an important factor in the temporal pattern discrimination of the single cell. In the relation between the mean frequency of input impulse sequences and the EPSP sizes at the two maximum values of $I(Y, X_{1,2})$, the value of the EPSP size A_{0I} at one maximum is inversely proportional to that of input mean frequency, while that of the EPSP size A_{0II} at the other maximum is proportional to it (Fig. 5). Concerning a number of neurons independently sending impulse to a common nerve cell, it can be treated as a process formed by superposing several renewal processes (Cox, 1962). If the P individual process is a Poisson process of rate ϱ, then the pooled process is a Poisson process of rate $P\varrho$, so that the mean frequency of input impulse relates to the convergence number of input terminals. Concequently, these results suggest that the optimal EPSP sizes of highly sensitive contributions to the temporal pattern discrimination closely relate to the optimal convergence number of excited input terminals. Next, in the relation between the distribution forms of the input inter-spike intervals and the entropies for EPSP size, the values of $H(Y)$ and $I(Y, X_{1,2})$ for each fixed EPSP size and mean frequency satisfy the following relation: exponential > gamma of Order 3 > gamma of Order 5 > gamma of Order 10. Thus, the statistical structures (mean frequency and distribution form) of input impulse sequence and the membrane states (EPSP size and refractory) influence the temporal pattern sensitivity and frequency transfer characteristic.

The fourth conclusion is that the temporal pattern discrimination and the frequency transfer characteristic are affected remarkably by whether or not successive inter-spike intervals of the input impulse sequences are independent in the statistical sense. Each of $H(Y)$ and $I(Y, X_{1,2})$ of the Type I input impulse sequences of positive contribution to the serial correlation coefficients is smaller in value than that of the shuffled sequences, but that of the Type III of a strong negative first serial-correlation with subsequent alternation in sign of the coefficients of higher order is larger in value than that of the shuffled sequences. These descrepancies come out remarkably at certain EPSP size, but go out at some EPSP size, i.e., this simulation model can distinguish some temporal patterns of the input impulse sequences. From these results, we may obtain a new interpretation about the learning system of temporal pattern discrimination.

The fifth conclusion is that the order-dependent statistical property of input impulse sequence was destroyed through the synaptic channel except the case of one-to-one transmission but that of the only Type I input sequence of positive contributions to the serial correlation coefficients can be transmitted to the output impulse sequences in case of the particular EPSP size where the value of $I(Y, X_{1.2})$ becomes minimum.

If one considers information processing of the synaptic cell from the point of view of the temporal pattern sensitivity, the entropy method applied here provides a very useful way to estimate and compare different levels of the nervous system. The properties of the temporal pattern discrimination obtained by this digital-computer-simulated system suggests many physiological possibilities.

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