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Manipulation of Optical Fields by Atomic Interferometry: Quantum Variations on a Theme by Young

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Abstract. We analyze the principle of a very general and conceptually simple method for manipulating optical fields by coupling them into a matter waves Young double slit apparatus. The field, non resonant with the atoms, acts as a "phase-retarding" medium in one of the arms of the interferometer and shifts the atomic fringe pattern. The method constitutes a simple quantum nondemolition measuring scheme of the photon number. Non classical states such as Schrödinger cats and Fock states of the field are generated in the measurement process. The analysis of the atomic interferometer with optical "retarding" fields provides a very simple and striking illustration of basic concepts of the quantum measurement theory and of the principle of complementarity. This scheme, which would be very difficult to implement in the optical domain, is equivalent to a more feasible and recently proposed Ramsey interference method to measure small microwave fields with beams of Rydberg atoms.

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The manipulation of isolated and simple quantum objects has developed as an important new field in quantum optics. It is now possible to trap single atoms or ions and to study their interaction with light under various circumstances. Conversely, it is also possible to prepare a single field mode containing a few photons in a high Q cavity and to study its coupling with atoms crossing the cavity one at a time. The development of these new "microscopic scale" experiments has shifted the perspective under which quantum phenomena were described in this field of physics. The emphasis was previously placed on the calculation of quantum mechanical mean values, because they were the relevant observed quantities in experiments performed on "macroscopic" ensembles of atoms or photons. The quantum aspects of the phenomena with their remarkable discontinuous jumps were essentially buried in the averaging procedure. It is now possible to focus on the simulation of single realizations of a system "history", revealing its quantum jumps and taking explicitly into account the discontinuity introduced by the measurement process. The "mean value" point of view is of course retrieved by averaging over many such simulations. This change of perspective has been triggered by the amazing improvements of the experimental techniques making experiments on isolated quantum systems possible, and by the parallel and no less amazing progresses in computing facilities which make it nowadays easy to perform lengthy Monte Carlo simulations of a "God playing dice experiment" on a desk top computer!

This new class of quantum optics experiment, together with similar advances in condensed matter physics (single electron tunneling experiments for example) have revived interest in the discussion of fundamental concepts in quantum physics, related to the measurement theory, the complementarity principle, the existence of macroscopic quantum mechanical superpositions etc. The fact that experiments to demonstrate in a direct way some of the less intuitive predictions of quantum theory are no longer of the "gedanken" type, but become feasible, has undoubtedly stimulated a new reflexion on these concepts and clarified their interpretation.

Among the various effects occuring in "microscopic" quantum optics, interference effects are particularly interesting to study. The detection of interferences is indeed a very general way of measuring physical quantities and their analysis, at the microscopic "one particle at a time" level, is very instructive for the deep understanding of quantum concepts. The fact that atomic interferometers can now be practically realized, with the prospect of large correlation lengths in the not too distant future, is a strong incentive to think about new kind of experiments probing the most fundamental aspects of the quantum theory.

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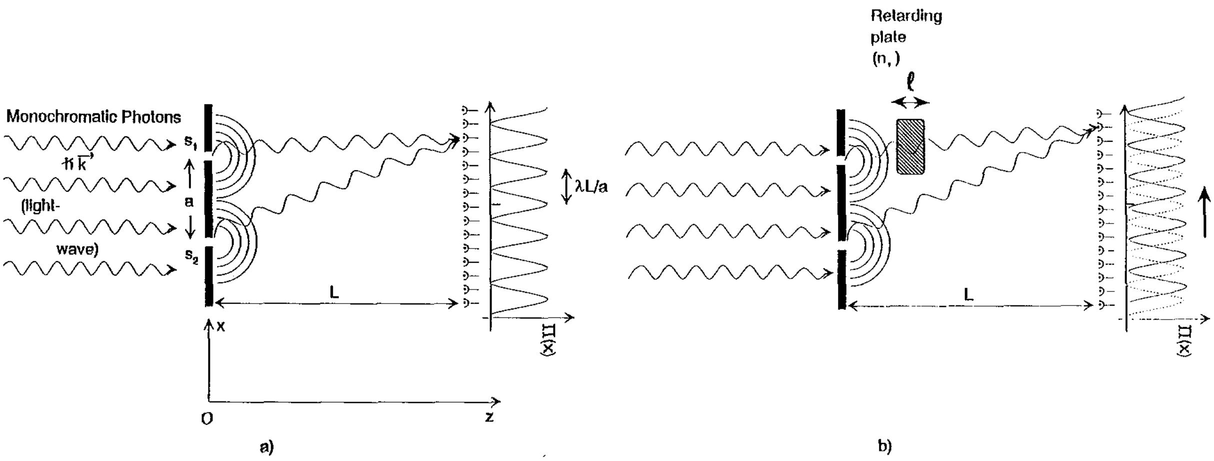

Fig. 1a, b. Scheme of a Young double slit interferometer for optical fields. **a** Symmetrical apparatus with fields propagating in vacuum: the interference pattern has a "bright" fringe at the center of the photodetector array. **b** Same interferometer with a retarding plate in one optical "path": the fringes are translated by an amount proportional to the phase shift introduced by the plate

In this paper, we discuss an experimental situation where an atomic interference effect is employed to manipulate a quantum field made of a few photons and to acquire information on this system. This experiment, which combines interferometry with cavity quantum electrodynamics concepts, helps us elucidate basic quantum ideas in a very pedagogical way. Furthermore, it suggests that atomic interferences blended with microscopic quantum optics can be used to generate various non classical field states which may be of practical interest. The ideas discussed here originate from a recent letter [1] and from a detailed article to be published elsewhere [2]. In this presentation, we will discuss these ideas on a very simple model involving a Young double slit interference experiment. This model is probably too simple to describe a realistic situation with present state of the art atomic beams and cavities. It has however the merit of being familiar to all physicists and is thus particularly well suited for a simple understanding of the physical ideas underlying the more realistic experimental situation discussed in the above mentioned references.

1 Dual Versions of the Young Double Slit Experiment

In an ideal optical Young interferometer (schematized in Fig. 1a), a plane wave of monochromatic radiation (wavelength and wave vector in vacuum respectively λ and $k = 2\pi/\lambda$) propagates along Oz and impinges on an opaque screen presenting two small slits s_1 and s_2 parallel to Oy, having their centers aligned along Ox at distance a from each other. The light scattered by these slits is detected at a distance L from the screen by an array of photon counters aligned along the Ox direction. The probability $\pi(x)$ of detecting a photon at point x exhibits spatial oscillations resulting from the interference between the two amplitudes corresponding to the scattering of the light by s_1 and s_2. The fringe spatial period is $\delta x = \lambda L/a$ and there is a "bright" fringe at the center of the photodetector array. A brief quantitative description of this interference process will be useful for the discussion to come. The Huyghens principle states that the field in the interferometer is the sum of two "cylindrical wave" contributions originating from virtual sources placed at s_1 and s_2. For infinitely narrow slits and for $z \gg a, x$ ("far field" limit) the components emitted by the virtual sources take a simple form and the field in the interferometer is

$$\Psi(\mathbf{r}) = \mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{\mathrm{i}kx^2/2z}[\mathrm{e}^{\mathrm{i}kxa/2z} + \mathrm{e}^{-\mathrm{i}kxa/2z}]. \tag{1}$$

Assume now that a small volume in front of the slits is filled by a transparent medium with a spatially inhomogeneous index of refraction $n_i(\mathbf{r})$. The fringe pattern is then altered in shape and/or position. The new pattern is quite generally obtained by solving, with the boundary conditions imposed by the slits, the Helmholtz equation determining the propagation of the field in the medium

$$\Delta\Psi(\mathbf{r}) + n_i^2(\mathbf{r})k^2\Psi(\mathbf{r}) = 0. \tag{2}$$

A particularly simple case arises when the alteration merely consists in placing a retarding plate with a real index n_i and a width ℓ in front of one of the slits (e.g. s_1, see Fig. 1b). In this case, the amplitude corresponding to the photon scattering through s_1 is dephased by an amount $k(n_i - 1)\ell$. The wave function in the interferometer is now, under the same assumptions as the ones leading to (1),

$$\Psi(\mathbf{r}) = \mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{\mathrm{i}kx^2/2z}[\mathrm{e}^{\mathrm{i}kxa/2z}\mathrm{e}^{\mathrm{i}k(n_i-1)\ell} + \mathrm{e}^{-\mathrm{i}kxa/2z}] \tag{3}$$

and the whole fringe pattern in the $z = L$ plane is simply translated by the amount $(n_i - 1)\ell L/a$. The measurement of such fringe translations is of course the basis for a variety of interferometric measurements of lengths and indices of material media.

Let us now exchange the roles of matter and radiation and consider a situation we might call "dual" to the previous one (see Fig. 2). Monokinetic atoms of mass m and velocity v propagating along Oz constitute a matter-wave (with wave number $k = mv/\hbar$) which replaces the light wave of the first experiment. This wave impinges on a screen which is, as in the optical case, pierced by two small slits at distance a from each other. The atoms are in a quantum state g, supposed for simplicity to be non degenerate, which is either a ground or a metastable level. They are detected by an array of counters placed at distance L behind the screen. Spatial oscillations in the probability of detection along Ox

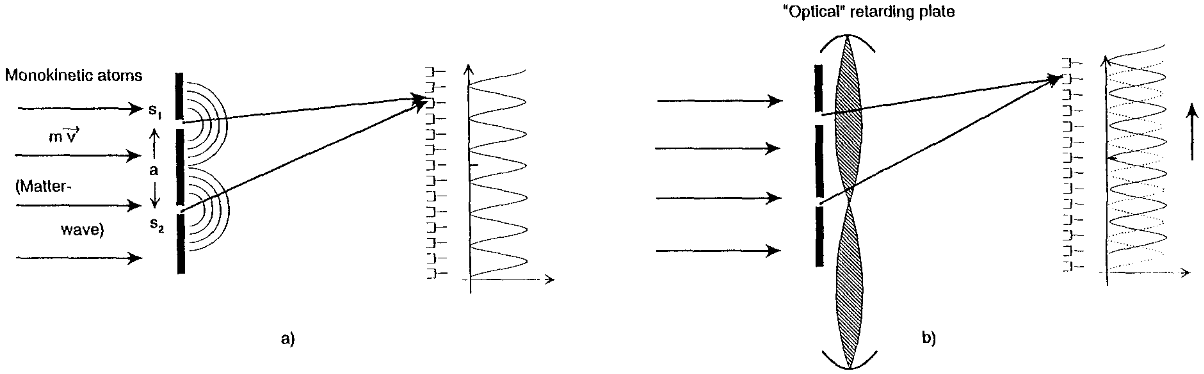

Fig. 2a, b. Scheme of a Young double slit experiment for atomic matter-waves. **a** Symmetrical apparatus with atoms propagating in vacuum: the interference pattern exhibits a "bright" fringe at the center of the atomic counter array. **b** A non resonant field, acting as a "phase-retarding" plate for the de Broglie wave is introduced behind the slits with an antinode facing s_1 and a node facing s_2: the fringe pattern is shifted by an amount proportional to the field intensity

are also obtained in this case, as recently demonstrated in an experiment performed with metastable helium atoms [3].

To carry on the analogy with the optical experiment, a small volume close to the screen can be "filled" with non-resonant light which is not absorbed by the atoms, but which shifts the energy of state g and dephases the atomic wave function. This volume "filled" with light can be described as a medium with an inhomogeneous index for the matter-wave which alters the interference pattern on the screen. This alteration depends of course on the field intensity, i.e., on its photon number. It is possible to conceive a field distribution which alters the phase of the matter-wave component "going" through s_1, without perturbing the one "going" through s_2. Figure 2b represents such a situation, which replicates for matter-waves the retarding plate configuration of Fig. 1b. A small cavity of the Fabry-Perot type placed close to the screen has its axis aligned along Ox. The standing wave mode of this cavity presents an antinodal line aligned along Ox in front of s_1 and a nodal line in front of s_2. If the slits width is small enough compared to the optical wavelength, the perturbation on the atomic wave can be considered as nearly homogeneous in front of s_1, and practically negligible in front of s_2. We thus expect that the component of the matter-wave going through s_1 will be dephased by an amount proportional to the number of photons in the cavity, without alteration of the phase of the component going through s_2. This effect is the basis of the phenomena we intend to analyze in detail in this article. We start this analysis by describing a very simple situation in the next section.

2 Matter-Wave "Retarded" by an Optical Field Having a well Defined Intensity

Suppose that the cavity represented on Fig. 2b stores a non-resonant field with a well defined number of photons n (Fock state). We do not address here the question of the preparation of this field, which will be considered below. To make the analysis as simple as possible, the atoms are supposed in this section to be sent one by one through the apparatus, the interference pattern in the detection plane being reconstructed by a statistical averaging over a large number of single particle events. Moreover, the cavity is assumed to have a very large quality factor Q, so that the photon number does not decrease appreciably during the time required to record an interference pattern on the detector array (the field is of course assumed to be decoupled from its source). These assumptions are obviously very difficult to achieve simultaneously in practice, but we are concerned here only with questions of principle (we will discuss at the end of this article the importance of these restrictions and the possibility of relaxing them in more realistic experiments).

The electric field distribution in the cavity is defined by a dimensionless function $\mathbf{f}(\mathbf{r})$. The cavity mode frequency ω is supposed to be close enough to the Bohr frequency ω_{eg} of the transition linking g to an excited atomic level e so that all other levels can be ignored in the estimation of the light shift produced on level g. The energy shift $\mathcal{U}_n(\mathbf{r})$ for an atom at point $\mathbf{r}$ can be expressed as

$$\mathcal{U}_n(\mathbf{r}) = n\,\frac{\omega}{\omega - \omega_{ge}}\,\frac{|D_{ge}|^2}{2\epsilon_0 \mathcal{V}_{\text{cav}}}\,|\mathbf{f}(\mathbf{r}) \cdot \mathbf{e}_{ge}|^2. \tag{4}$$

In this equation, D_{ge} is the magnitude of the atomic electric dipole matrix element between g and e, $\mathbf{e}_{ge}$ is the unit vector defining the polarization of the transition and $\mathcal{V}_{\text{cav}} = \int |\mathbf{f}(\mathbf{r})\cdot\mathbf{e}_{ge}|^2 d^3\mathbf{r}$ is the "effective volume" of the cavity mode. Equation (1) is a simple perturbative result, valid provided $|\mathcal{U}_n(\mathbf{r})|$ remains always much smaller than $|\omega - \omega_{ge}|$. This "off-resonance" condition means that real absorption of a photon cannot occur in the cavity.

$\mathcal{U}_n(\mathbf{r})$ appears as a spatially inhomogeneous "optical potential" for the atoms whose spatial wave function $\Psi(n;\mathbf{r})$ obeys the Schrödinger equation

$$\Delta\Psi(n;\mathbf{r}) + \left[1 - \frac{2m}{\hbar^2 k^2}\,\mathcal{U}_n(\mathbf{r})\right] k^2 \Psi(n;\mathbf{r}) = 0. \tag{5}$$

The analogy with the light-wave case is striking (compare with (2)). It allows us to define an effective inhomogeneous index of refraction for the matter-wave:

$$n_i(\mathbf{r}) = \sqrt{1 - \frac{2m}{\hbar^2 k^2}\,\mathcal{U}_n(\mathbf{r})} \cong 1 - \frac{m}{\hbar^2 k^2}\mathcal{U}_n(\mathbf{r}). \tag{6}$$

The simplifying expansion made in the above equation is justified if we assume that $|\mathcal{U}_n(\mathbf{r})| \ll \hbar^2 k^2/2m$ for all $\mathbf{r}$ values, which means that the light forces acting on an atom are a small perturbation to its motion. The introduction of this potential results in a fringe pattern alteration quite similar to the one discussed above in the light-wave case. The important point for our analysis is that the $\mathscr{U}_n(\mathbf{r})$ potential – and the resulting index – are proportional to the photon number, so that a determination of the fringe alteration yields a direct information on n.

The atomic wave function in the interferometer is determined by solving (5) with boundary conditions accounting for the presence of the opaque screen and the two slits in the $z = 0$ plane. This might be a very complicated problem to solve which, fortunately, the optical analogy allows us in simple cases to bypass. In the configuration of Fig. 2b, the dephasing $\Delta\varphi(n)$ produced by the cavity field on the component of the atomic wave function "going" through s_1 can be written as

$$\Delta\varphi(n) = \int_{x=a/2} [n_i(\mathbf{r}) - 1]k dz \cong -\frac{m}{\hbar^2 k} \int_{x=a/2} \mathscr{U}_n(\mathbf{r}) dz. \tag{7}$$

In this equation, the integral is carried along the line $x = a/2$ parallel to Oz. The integral of $|\mathbf{f}(\mathbf{r}) \cdot \mathbf{e}_{ge}|^2$ across the mode defines an effective "cavity length" $\ell_{\text{cav}} = \int |\mathbf{f}(\mathbf{r}) \cdot \mathbf{e}_{ge}|^2 dz$ and $\Delta\varphi(n)$ is finally equal to $n\varphi_0$ where φ_0 is the "phase shift per photon" induced on the component of the matter-wave crossing the field antinode:

$$\varphi_0 = -\frac{\ell_{\text{cav}}}{\hbar v} \times \frac{\omega}{\omega - \omega_{ge}} \times \frac{|D_{ge}|^2}{2\varepsilon_0 \mathscr{V}_{\text{cav}}}. \tag{8}$$

We recognize in ℓ_{cav}/v the characteristic atom-cavity crossing time and the dephasing $n\varphi_0$ essentially appears as a phase shift accumulated on one component of the atomic wave function during the "crossing" of the field antinode. Carrying further the optical analogy, we finally find an approximate expression for $\Psi(n; \mathbf{r})$, valid for $z \ll x, y, a$:

$$\Psi(n; \mathbf{r}) = \mathscr{A} \mathrm{e}^{ikz} \mathrm{e}^{ikx^2/2z} [\mathrm{e}^{ikxa/2z} \mathrm{e}^{in\varphi_0} + \mathrm{e}^{-ikxa/2z}]. \tag{9}$$

$\mathscr{A}$ is a normalization constant. This result assumes of course translational invariance along Oy. In a more realistic situation, the length and width of the slits are finite and the above expression must be corrected for diffraction effects, which we will neglect here since we are concerned only with questions of principle. Equation (9) shows very clearly that the probability amplitude of detecting an atom at point x in the $z = L$ plane results from the interference of two contributions, one of which depends upon the photon number in the cavity and the other not. As a result, the statistical interference pattern on the screen is shifted with respect to its position in the absence of photons by an amount proportional to n. Of course, the analogy between the light- and matter-waves experiments is not fortuitous. It is well known that light shifts are the counterpart on atoms of the dispersive propagation effects which are desribed on the field by the index of refraction. The fact that the field has to be non-resonant in the second experiment is equivalent to the statement that the retarding plate is transparent in the first one.

3 Matter-Wave "Retarded" by an Optical Field Presenting Intensity Fluctuations

This "experiment" becomes really interesting if we consider the more complicated- and more realistic-situation where the initial field in the cavity is not in a photon number state but rather exhibits fluctuations of n values. For sake of simplicity, we restrict our analysis to "pure" field states expressed as

$$|\Psi_{\text{field}}\rangle = \sum_n C_n^0 |n\rangle \tag{10}$$

with $p(n) = |C_n^0|^2$ being the probability for the field to contain n photons. As an example, let us recall that a quasi-classical field with complex classical amplitude α is a superposition of n states with quantum mechanical amplitudes C_n^0 given by a Poisson law:

$$C_n^0 = e^{-\alpha^2/2} \frac{\alpha^n}{\sqrt{n!}}. \tag{11}$$

This field has a mean photon number $\bar{n} = |\alpha|^2$ and a characteristic photon number dispersion $\Delta(n) = \bar{n}^{1/2}$. Its phase ψ is defined with a precision $\Delta\psi = 1/\Delta(n)$. Such a field can be prepared by coupling the cavity to a classical current source which is "switched off" before the atomic interference "experiment" begins. Here again, we assume that the relaxation time of this field in the cavity is much longer than the duration of the interference recording.

As a consequence of photon number dispersion, the phase shift experienced by the matter-wave during the cavity crossing becomes now also a fluctuating quantity. The situation looks at first sight analoguous to a light-wave Young double slit experiment with a retarding plate presenting width (or index) inhomogeneities. From this analogy, we would expect to observe an interference pattern with a somewhat reduced fringe contrast if the photon number dispersion is not too important, and a complete "washing out" of the fringes when the atomic phase dispersion $\varphi_0 \Delta n$ is of the order of π. Depending upon the experimental conditions – to be specified below – this prediction may or may not be right. The analogy with an inhomogeneous retarding plate is indeed too simple or too naive since it overlooks an important point. Whereas it is legitimate to assume that an optical retarding plate is a classical object whose fluctuations are uncorrelated to the light wave, this is not the case for the matter-wave-retarding-cavity-field. Its fluctuations, of an intrinsic quantum nature, are intricately correlated to the state of the atom interacting with the apparatus and are strongly affected by the atomic position measurement process.

Using (10) and exploiting the linearity of quantum mechanics, we readily obtain the quantum state of the single atom+field system in the interferometer

$$|\Psi_{\text{atom+field}}\rangle = \sum_n C_n^0 \iiint \Psi(n; \mathbf{r}) |n; \mathbf{r}\rangle d^3\mathbf{r}, \tag{12}$$

where $|n : \mathbf{r}\rangle$ represents the atom at point $\mathbf{r}(x, y, z)$ with n photons in the cavity and $\Psi(n; \mathbf{r})$ is the matter-wave function introduced above, which takes a simple form for an interferometer with narrow slits, in the far field limit [see (9)]. The combined atom + field system described by (12) is ob-

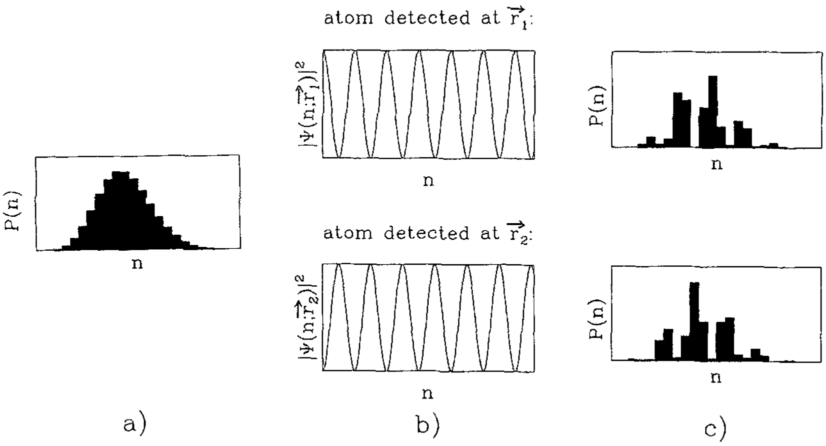

Fig. 3. Change of the photon number distribution produced by the first detected atom in the Young interferometer. The initial distribution, displayed in a), corresponds to a coherent field with $\bar{n} = 10$. It is multiplied by the fringe function $|\Psi(n;\mathbf{r})|^2$, represented versus n in b), for two positions of the detected atom ($\varphi_0 = \pi/2$). We have chosen two values $\mathbf{r}_1$ and $\mathbf{r}_2$ separated by about half the fringe spatial period. In the resulting distributions, shown in c), photon numbers closest to the "dark" fringes have been "decimated". These numbers are "complementary" in these two realizations

viously in an "entangled" state presenting strong correlations between the matter and field parts.

The postulates of quantum mechanics allow us to make the following basic predictions: (i) the probability density $\pi(r)$ of detecting at point $(\mathbf{r})$ an atom interacting with the interferometer containing a field described by (10) is

$$\pi(\mathbf{r}) = \sum_n |C_n^0|^2 \, |\Psi(n;\mathbf{r})|^2; \tag{13}$$

(ii) moreover, once the atom has effectively been detected at a given point, say $\mathbf{r}_1$, the state of the atom+field system "collapses" into the decorrelated state (defined within a normalization constant)

$$|\Psi^{\text{(after detection)}}_{\text{atom+field}}\rangle \sim \sum_n C_n^0 \Psi(n;\mathbf{r}_1)|n;\mathbf{r}_1\rangle. \tag{14}$$

It represents the atom at point $\mathbf{r}_1$ (obviously) and the field in a state described by the new amplitudes

$$C_n^1 = \frac{1}{\mathcal{N}_1} C_n^0 \Psi(n;\mathbf{r}_1) \tag{15}$$

with $\mathcal{N}_1$ being the normalization constant:

$$\mathcal{N}_1 = \sqrt{\sum_n |C_n^0|^2 |\Psi(n;\mathbf{r}_1)|^2}. \tag{16}$$

Coming back to the specific geometry of the Young double slit experiment (amplitudes $\Psi(n;\mathbf{r})$ given by (9)), we see that the initial field amplitudes have been essentially multiplied by the fringe function characteristic of the interferometer, computed at the position where the atom happens to have been detected. Interpreted as a function of n, the multiplying quantity is an oscillating function with zeros close to some n values (dark fringes) and maxima close to others (bright fringes). The n-position of these minima and maxima depends of course on the $\mathbf{r}$-position of the detected atom. Figure 3 shows the changes of the photon probability distribution when the atom is detected at a point $\mathbf{r}_1$ or a point $\mathbf{r}_2$, these positions being separated by about half the fringe spatial period. The initial field is coherent, with $\bar{n} = 10$ and $\varphi_0 = \pi/2$. Different photon numbers are efficiently "decimated" for these two detection outcomes, the state of the field being in any event strongly modified. This decimation process is, as we show below, the basis of a very simple quantum nondemolition measurement method for the field intensity.

4 Photon Decimation and QND Measurement of Field Energy

The main feature of the process we have just outlined is that the probability amplitudes C_n are multiplied after each detection event by a function of n presenting strong minima. It is also important to realize that the position of these minima is random and depends upon a quantum mechanical "dice casting" since the position at which any atom is detected cannot be known for sure in advance. Moreover, the probability of finding atom number $m+1$ at a given point depends upon the state of the field left in the cavity by atom number m, i.e., upon the outcome of the previous atomic detections. In other words, the "dice casting" process has a "memory" More quantitatively, after a sequence of atoms has been detected at points $\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_m$ the new field amplitudes have become

$$C_n^m = \frac{1}{\mathcal{N}_m} \prod_{1 \le i \le m} \Psi(n;\mathbf{r}_i) C_n^0, \tag{17}$$

where $\mathcal{N}_m$ is the normalization constant. According to (13), the conditional probability that the $(m+1)^{th}$ atom will be detected in turn at point $\mathbf{r}_{m+1}$ is

$$\begin{aligned}\pi(\mathbf{r}_{m+1}) &= \sum_n |C_n^m|^2 \times |\Psi(n;\mathbf{r}_{m+1})|^2 \\ &= \frac{1}{\mathcal{N}_m^2} \sum_n \left(\prod_{1 \le i \le m+1} |\Psi(n;\mathbf{r}_i)|^2 \right) |C_n^0|^2. \end{aligned} \tag{18}$$

It is easy to simulate this process numerically. At each stage, a computer determines the probability distribution for the position of the next atom in the detection plane, then "decides" where this atom is actually found by making a random choice according to the corresponding probability law. Once the new $\mathbf{r}$ value has been obtained, the computer determines the new C_n amplitudes, from which the probabiltiy distribution for the next atom is obtained and so on. Figure 4a shows a typical sequence of photon number probability distributions obtained after 1, 10, 30, 50, 100, and 2000 atoms have been detected in the case where the initial field is a coherent state with $\bar{n} = 10$. The value of φ_0/π is chosen to be 0.312. After about 50 atoms have been counted, the field is "reduced" into a pure photon number state (here $n = 8$). Another sequence will converge, after a comparable

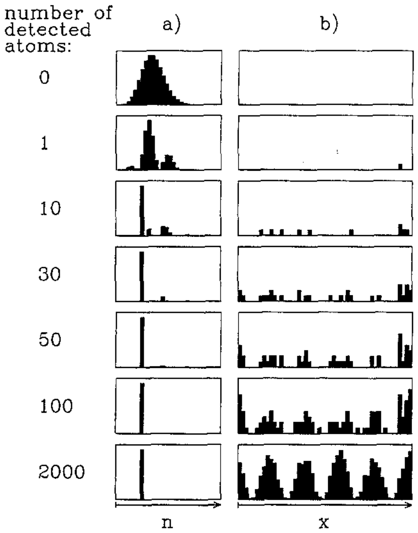

Fig. 4a, b. Evolution of the photon number distribution with the number of detected atoms in one simulation of the Young atomic interferometry experiment ($\varphi_0/\pi = 0.312$). The initial distribution (top frame) corresponds to a coherent field with $\bar{n} = 10$. The field collapses, in this case, into the $\bar{n} = 8$ Fock state after about 50 detected atoms and remains in this state thereafter. **b** Evolution of the distribution of the positions of the detected atoms in the same simulation. The fringe pattern is "revealed" at the same time as the photon number is reduced to a single value. Another simulation would converge to another Fock state, with a different phase spatial for the fringes

number of steps, towards another n value and the distribution of Fock states obtained by repeating a large number of times this procedure reconstructs the Poisson law of the initial field photon number distribution. The mechanism of the field state "reduction" in this process is simple to understand. Since each stage of the process results in the random elimination of some photon numbers from the $p(n)$ distribution, it is clear that this decimation will generally go on until one photon number only is left.

The speed of the convergence process depends upon the "size" of the field (initial width of the $p(n)$ distribution) and upon the φ_0 value. We can define this speed as the minimal number $m_{\min}$ of atoms required to produce a Fock state. The numerical simulations show that, for a given initial field, $m_{\min}$ diverges for $\varphi_0 = 0$ (for obvious reasons) and, more generally, for values of the "phase shift per photon" such that $\varphi_0/2\pi$ is an exact rational fraction p/q, with q smaller than the total width of the initial distribution of n values. It is clear that, in this case, any two photon numbers n_1 and n_2 whose difference is an integer multiple of q will correspond to the same fringe pattern. The information obtained from the fringe observation will then be insufficient to allow us to discriminate between these numbers and the field will end up in a mixture of n states, and not in a unique Fock state. In the case of small fields, for which the initial n is known within a few units, the divergent values of $\varphi_0/2\pi$ are multiples of unity and of simple fractions such as 1/2, 1/3, 1/4 etc. If we avoid these "pathological" situations, $m_{\min}$ becomes essentially independent of the actual choice of the "phase shift per photon", provided $\varphi_0\Delta(n) \geq \pi$. For fields containing about 10 photons (which is the situation we considered in the numerical simulations and in Fig. 4a), this means that we have to choose $\varphi_0 \geq 0.3$. We then have $m_{\min}$ of the order of 50. The simulations also show that $m_{\min}$ increases logarithmically with the dispersion $\Delta(n)$ of n values in the initial field.

The Fock state reduction process desribed above has all the features of a quantum nondemolition mechanism. The photon number in the cavity is not modified by real photon absorption or emission, which are forbidden by the off-resonance condition. In fact, the photon number distribution is changed only because information is acquired on the field, in a photon-non-destructive way. Once a Fock state $|n_1\rangle$ has been produced, no more information can be obtained and this state remains obviously stable under subsequent atomic interactions, since a $\delta(n - n_1)$ distribution cannot be altered by the multiplication with any $|\Psi(n; \mathbf{r})|^2$ function. These properties are quite different from those of ordinary photon detection mechanisms based on resonant photon-atom interactions. In these processes, a Fock state is by essence unstable since it is then necessary to absorb photons in order to detect the field!

It is interesting to notice that the converging decimation process we have just analysed in quantum mechanical terms can also be justified by arguments of pure classical statistical mechanics, provided we ignore the initial coherences existing between the different Fock states. Assume that, instead of containing a coherent field, the cavity is filled with an incoherent field containing a well defined number of photons for each field realization, this number changing randomly from one realization to the next. The initial information we have on this field provides us only with the probability $p(n)$ for this number to be equal to n in any given realization. The argument would be essentially the same for a box filled with any kind of particles, supposed to be indestructible (no cavity relaxation in this problem). We then proceed to acquire more information on a given field realization by sending atoms one by one through the interferometer and registering their position in the $z = L$ plane. Each detected atom "tells" us that some photon numbers are "incompatible" with the result of this particular measurement (those corresponding to "dark fringes" at the detected position). After a few tens of such detections have been registered, only one n value remains compatible with all the collected information. The n value for this field realization has been "pinned down". It is important to notice that we do not have changed n but we have only precised its initially unperfectely known value. This statistical approach is of course the natural one when the field is initially described by a density matrix diagonal in the Fock state representation. When the field is coherent, we must a priori use the quantum approach developed above. That these two approaches yield identical results is not fortuitous. It is due to the fact that the evolution of the field energy is independent of the quantum mechanical coherences between n states, which is related to the field phase. As long as we are only interested in measuring field intensities, it is not important to know whether the field is coherent or not. All happens "as if", in each realization of the experiment, there were in the cavity a well defined number of photons "waiting to be pinned down".

We have assumed so far, to make the argument simpler, that the atoms were detected one by one. A sequence of events is indeed easier to analyze by computer- and by human mind- than an experiment in which the atoms are bunched together in the interferometer. It is, however, important to notice that the "one atom at a time" assumption is not essential at all in the above arguments. Clearly, the state reduction process we have invoked is an "instantaneous" one and the detection events in our analysis can be arbitrarily close to one another. In fact, the order of detection is irrelevant. Any two sequences of atomic detections differing only by the ordering of the **r** values has the same overall probability and results in the same final field state reduction. This follows directly from the commutative property of the products in (17) and (18). It is equally possible to consider that a measurement consists in the "simultaneous" detection of a bunch of m atoms. Just before their detection, the many-atom+field system is in the entangled state

$$|\Psi_{m\text{-atoms+field}}\rangle = \sum_n C_n^0 \iint \cdots \int \left[\prod_{1\leq i\leq m} \Psi(n;\mathbf{r}_i) \right] \times |n;\mathbf{r}_1\mathbf{r}_2\ldots\mathbf{r}_m\rangle d^3\mathbf{r}_1 d^3\mathbf{r}_2 \ldots d^3\mathbf{r}_m \qquad (19)$$

which, after measurement, "collapses" into the final state

$$|\Psi^{\text{final}}\rangle = \sum_n C_n^0 \left[\prod_{1\leq i\leq m} \Psi(n;\mathbf{r}_i) \right] |n;\mathbf{r}_1\mathbf{r}_2\ldots\mathbf{r}_m\rangle . \qquad (20)$$

Only one irreversible state reduction has to be invoked in this case, which obviously yields the same result as the "one atom at a time model". Of course the whole argument is based on the fact that the atoms do not interact mutually in the interferometer (the system state must be a tensor product of one atom wave functions; see (19)).

Let us finally make a remark about the atomic quantum statistics in this experiment. By assuming a plane wave impinging on the screen, we consider in fact atomic wave packets with a large extension, covering the whole interferometer. This means that the atoms coexist in the apparatus, which is possible only if they are physically distinguishable (for example by other degrees of freedom such as their spin state) or if they are bosons. Since the many-atom wave function given by (19) is symmetrical by atom exchange, it does describe correctly a Bose gas and there is thus no difference in our analysis between distinguishable atoms and bosons.

5 Visibility of the Matter-Wave Fringes and the Principle of Complementarity

Let us now come back to the question of the fringe visibility in the matter-wave Young experiment with a photon number-fluctuating field in the cavity. The fringes are of course reconstructed by accumulating results over a large number of single atom detection events. There are two very different ways in which this averaging may be performed.

(i) Ensemble of experiments with systems "reset" to the same initial field state: First, it is possible to send atoms one at a time through the apparatus and to perform an ensemble average on a large set of identical systems, in each of which the "retarding" cavity field is prepared in the same quantum state described by (10). A simpler way to realize practically this ensemble averaging on a single interferometer consists in sending atoms through the apparatus at a very slow rate letting enough time to reset the field to its initial value between consecutive atoms. In this case, the cumulated signal on the array of detectors reconstructs by definition the probability distribution given by (13). We thus expect to observe a superposition of fringe patterns, weighted by the $p(n)$ probabilities, each pattern being shifted along Ox by an amount proportional to n. In other words, we retrieve in this case the "naive" result described in Sect. 2, namely the fact that the fringes contrast is reduced for $\varphi_0 \Delta(n) < \pi$ and the interference pattern is fully "washed out" for $\varphi_0 \Delta(n) \geq \pi$. Figure 5 represents the patterns obtained for two φ_0 values corresponding to these two situations.

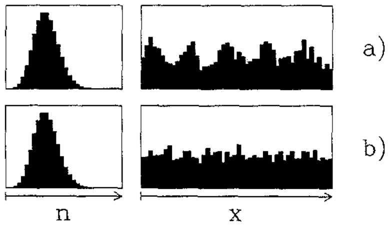

Fig. 5. Distribution of positions of detected atoms corresponding to two simulations in which the field is reset between atoms to the initial state (coherent field with $\bar{n} = 10$). The number of detected atoms is 2000 in both cases. a) Fringe pattern with reduced contrast obtained for $\varphi_0/\pi = 0.156$. b) The interferences are completely "washed out" for $\varphi_0/\pi = 0.312$

(ii) Experiment realized on a single field "history": An alternative way of accumulating points on the detector array is to send atoms through the same interferometer at a rate larger than the reciprocal of the "retarding" field damping time (it is supposed to be infinite here, but this is of course only an idealization of a real situation in which the cavity Q is finite). In this case, we perform exactly the same experiment as the one described in Sect. 4, but instead of focusing our attention on the evolution of the photon number probability law, we are interested in the atom count distribution along Ox. The computer simulation described above allows us to "follow" a single realization of the optical field "history" and to compute step by step the fringe pattern appearing on the detector array. Figure 4b shows the fringes as they appear during the field "history" followed on Fig. 4a. Each frame in the b) column corresponds to the facing photon number histogram in the a) column.

The physical interpretation of this simulation is quite clear. The first detected atom (here on the right part of the detector array) results in the decimation of several photon numbers in the initial photon number Poisson distribution. After ten atoms have been detected, we obtain a $p(n)$ distribution which is already well concentrated around a single peak (here $n = 8$) and the corresponding points on the detector array are scattered in a way which is compatible with this n value, although it is still too early to recognize a well defined fringe pattern. After 30 and 50 atoms have been detected, the field evolves with less and less uncertainty into the $n = 8$ Fock state and at the same time, an unambiguous atomic fringe pattern does emerge. This pattern remains stable thereafter, since n can no longer change. More atoms only improve the signal to noise ratio (see the histograms corresponding to $m = 100$ and 2000). The position of the bright fringes on this pattern are of course determined by the value of n obtained in this particular realization of the experiment. Another realization would lead to another Fock state, with a fringe pattern translated with respect to the previous one. Of course, the phase of the emerging pattern is impossible to be predicted beforehand. If we repeat the same experiment over and over, the sum of all the patterns finally reproduces the average pattern obtained in the experiment where the field is reset to its initial value between atoms (reduced visibility or complete washing out of the fringes depending on the $\varphi_0 \Delta(n)$ value). Finally, we should also remember that the results of the experiment are the same whether the atoms succeed each other in the apparatus or are detected in a single "snapshot". In short, we might say that all happens in this experiment "as if" the unknown photon number in the cavity were suddenly "revealed" by the appearance of the fringe pattern on the detector array, in a way quite similar to the "revelation" of the picture of a real object made by a camera when enough photons are collected on the photographic plate.

It is interesting to discuss the fringe visibility in the atomic Young interferometer in terms of the complementarity principle of quantum mechanics. We can say that in the first version of the atomic interferometry experiment described above, the atoms behave as "waves" if $\varphi_0 \Delta(n) < 1$ and as "particles" if $\varphi_0 \Delta(n) \geq 1$. On the other hand, in the second version of this experiment, they eventually behave like interfering "waves". We remember from text book discussion of wave-particle dualism that the wave aspect dominates if, once the detector has been activated, nothing in the experiment allows us to determine through "which path" the particles have propagated from the screen to the detector. The particle aspect is of course dominant in the opposite case. In other words, the fringe observation is incompatible with any information telling us unambiguously through which slit the atom has been scattered. Whether this information is actually collected or not is irrelevant. The mere fact that it is virtually possible to acquire it is enough to make the fringe visibility vanish. In our experiment, the only way to learn about the atom trajectory is to "interrogate" the field left in the cavity after the atom detection. With this basic idea in mind, let us analyse in turn the two versions of the atomic interference experiment.

In the first experiment, the field is reset in the same coherent state $|\alpha\rangle = \sum_n C_n^0 |n\rangle$ after each atom. Assume that we close slit s_2. Then the atoms have to pass through s_1, resulting in a phase shift $n\varphi_0$ for an n photon field, so that the classical state $|\alpha\rangle$ becomes $\sum_n C_n \, \mathrm{e}^{in\varphi_0}|n\rangle$ and the coherent state is merely phase shifted by the angle φ_0. If we had closed slit s_1 instead, the atoms would all have to pass through s_2 and the field in the cavity would obviously be unaltered. Thus, the complementarity argument tells us that the fringes will vanish if we can in principle recognize that the cavity field has undergone a phase shift φ_0. We thus have to compare this shift with the phase indeterminacy of a coherent state, which is of the order of $1/\Delta(n)$. If $\varphi_0 \geq 1/\Delta(n)$, the phase shift is observable and the fringes should vanish. If, on the other hand, $\varphi_0 < 1/\Delta(n)$, the phase shift is too small to be measured, it is not possible to tell through which slit the atom has passed and the fringes are observable. We retrieve here the conclusions reached by the direct argument exposed above.

In the second experiment, the main point is that the atoms reduce first the field into a Fock state. Once this state has been produced, the field in the cavity does not contain any more phase information. It thus becomes impossible to recognize through which slits subsequent atoms have passed and the fringes are always observable in this case (although the phase difference between the two paths is a random number, which makes the fringe position also random).

The complementarity arguments developed here are similar to the ones discussed in a recent paper presenting possible atomic interference experiments in which the atomic beams are crossing micromaser cavities [4]. In these experiments however, the atom-field coupling was supposed to be resonant and photons were exchanged between the atom and the field. The "which path" information was then a change in the photon number and not an alteration of the field phase. As a result, the conclusions about the fringe visibility were reversed: a Fock state in the cavity is much more sensitive to a change in n value than a coherent state superposition. Thus, fringes were more likely to be blurred with a Fock state field than with a coherent one.

6 Evolution of the "Retarding" Field Phase in the Matter-Wave Interferometer and Generation of "Schrödinger Cat" States

This study would be incomplete without a brief analysis of the field phase evolution during the atomic interferometry experiment leading to the "reduction" of the field into a Fock state. The "back action" of the field intensity measurement affects indeed the conjugate physical quantity, namely the field phase. This phase has a well defined value in the initial field state and it ends up to be completely undetermined in the final Fock state. The way in which this phase scrambling occurs is particularly simple to analyse in a "one atom at a time" experiment. After the first atom has been detected, the field is projected into the state defined by (14). Assuming that the initial field is a coherent one with a complex amplitude α and taking for $\Psi(n;\mathbf{r})$ the asymptotic form of (9), we recognize in (14) the state

$$|\Psi_{\text{field}}^{(1)}\rangle \sim \mathrm{e}^{ikx_1 a/2L}|\alpha \mathrm{e}^{i\varphi_0}\rangle + \mathrm{e}^{-ikx_1 a/2L}|\alpha\rangle \tag{21}$$

which appears as the linear quantum superposition of the initial field and another coherent field classically dephased with respect to the initial one by the angle φ_0. The relative phase difference between the two amplitudes of this quantum superposition depends upon the position where the atom happens to have been detected and is thus a random variable. If the atom has been counted near a "bright" fringe of the light-unperturbed interferometer, the two amplitudes have nearly identical phases and wet get (within an irrelevant overall phase factor) a field "reduced" into the superposition $|\alpha\rangle + |\alpha \mathrm{e}^{i\varphi_0}\rangle$. If, on the other hand, the atom is detected near a "dark" fringe of the light-unperturbed interferometer, the two amplitudes are nearly opposite and the field is reduced into the superposition $|\alpha\rangle - |\alpha \mathrm{e}^{i\varphi_0}\rangle$.

The quantum superpositions described by (21) are known as "Schrödinger cat states" [5]. They correspond to highly non-classical fields whose phase can take two "macroscopically" distinct values, the alternative between these values being of a quantum mechanical nature. As analysed in Sect. 5, each of the two possible phases of the field can be associated to a specific atom history in which it is scattered by one or the other of the slits. The mechanism producing this "Schrödinger cat" can be described as a "replication" of the atomic coherence into the field. The matter-wave is split into two mutually coherent parts which interact differently with the field, after which a non-unitary detection "collapse" of the system wave function occurs. As a result of this event, the coherence is no longer carried by the atom, but instantaneously "replicated" into the field. Similar "Schrödinger cat" generation mechanisms involving the nonlinear coupling of two fields followed by a detection process have been analysed in the literature [6]. Atomic interferometry provides a very simple way of preparing these states by making the initial coherent field interact with a single atom. The property of these states and various ways of detecting and studying them in atomic interferometry experiment involving Ramsey fringes are presented in reference [2].

The "Schrödinger cat" produced by the first atom in the cavity still contains a lot of phase information. The measurement of this phase can yield two possible values. Since this measurement will, if $\varphi_0 \Delta(n) \geq 1$, remove the trajectory ambiguity, we have seen in Sect. 5 that the first atom may indeed appear at any point along Ox. In this respect, we can say that the first atom "behaves as a particle" in the interferometer. The second atom detection at point x_2 then results in another multiplication of the C_n amplitudes by a new "fringe" function (see (17)) and the field evolves into a "second generation" Schrödinger cat state which appears now as a superposition of three coherent states with the phases $2\varphi_0, \varphi_0$ and 0:

$$|\Psi^{(2)}_{\text{field}}\rangle = \mathrm{e}^{\frac{\mathrm{i}k(x_1+x_2)a}{2L}} |\alpha \mathrm{e}^{2\mathrm{i}\varphi_0}\rangle + 2\cos\frac{k(x_1-x_2)a}{2L} |\alpha \mathrm{e}^{\mathrm{i}\varphi_0}\rangle + \mathrm{e}^{-\frac{\mathrm{i}k(x_1+x_2)a}{2L}} |\alpha\rangle. \quad (22)$$

This second generation "Schrödinger cat" contains obviously less phase information than the previous one and a measurement of the field phase will no longer tell us unambiguously through which path the second atom has travelled (there are three possible phase values and only two paths). We understand easily that, as a result of such successive multiplication processes and provided φ_0 is not a rational multiple of π, the phases components will eventually uniformly fill the whole 0–2π interval and the field phase will be totally scrambled. This is exactly what we expect to occur in the final Fock state. This analysis gives a very direct illustration of the intensity-phase uncertainty. It also explains how the "wave nature" of the atom can progressively establish itself and win over the "particle" aspect in the interferometer. As more and more atoms cross the apparatus, the field phase gets progressively "scrambled" and with it the only information which might have been used to pin down the atomic trajectory. We can now give another expression of the complementarity principle in this experiment and state "that the wave and particle aspects of the atoms and the field are mutually exclusive". If the field in the cavity behaves as a wave (i.e., has a well defined phase), then the atoms in the interferometer act as particles (they do not "interfere"). If, on the contrary, the field has a particle like character (Fock state), then the atomic wave nature manifests itself. We thus witness during the atomic interferometric measurement of the intensity of a classical coherent field a progressive exchange between the atom and field particle-wave characters. The field starts as a wave and turns into a phaseless "particle like system" whereas the atoms start as particles and progressively manifest a wave nature. Of course, the argument has been phrased here for convenience in a chronological language. What we mean in fact is that we need a large enough number of atoms (irrespective of their order) to obtain a wave like behaviour of the atoms and to "impose" a particle like behaviour to the field.

7 Conclusion: Is all this Really Feasible?

Generalizing a novel idea we had first proposed in the context of microwave experiments [1,2] we have analysed in this paper interferometric methods for manipulating optical fields with matter-waves in a Young double slit apparatus. The method is based on the fact that the volume filled by the light field behaves as a transparent medium with a position-dependent "index of refraction" in which the atomic matter-wave propagates. The analysis of the fringes produced in the Young interferometer allows us to measure this index and, consequently, the number of photons in the field. The process is a subtle one which illustrates in a striking way the postulates of quantum mechanics and the wave-particle complementarity concept. We have shown that it is in principle possible to prepare in this way Fock states of the field with no intensity fluctuations, or else macroscopic quantum superpositions of classical fields with different phases (Schrödinger cat states). Obviously, the method described here can be extended and generalized to other interferometric designs. A matter-wave Mach Zender or Michelson interferometer with a "retarding" optical field acting in one of the arms could be used instead of the Young design. An optical grating consisting of a light standing wave could also be employed to scatter an atomic beam along directions depending on the photon number in the field, as has been recently proposed in [7].

For sake of simplicity, we have discussed here an ideal experiment involving perfectly monokinetic atoms (i.e., monochromatic matter-waves) scattered by infinitely long and narrow slits. We have also assumed perfect atomic counting efficiency and disregarded the effect of unread atoms on the process. More realistic conditions can be simulated with only slight complications to the model. Longitudinal velocity dispersion of the atomic beam will result in a decrease of the fringe contrast (since k depends upon v) and will make the phase shift per photon a random variable (since φ_0 also depends upon v, see (8)). Finite slit dimensions and transverse atomic velocity dispersion will also alter the response function of the interferometer, further reducing the fringe contrast and restricting the number of visible fringes to a few units. These effects will merely increase the number of detected atoms required for the field reduction. Unread atoms, which do not bring any information on n, will have no effect on this reduction, other than lengthening further the fringe contrast build up and the practical time required to perform the experiment (note that unread atoms have on the other hand a strongly perturbing effect on the optical field phase).

The most restrictive conditions of our model are elsewhere. The optical experiments discussed above are indeed conditioned by the answer to two fundamental questions:

(i) is it practically possible to achieve a large enough "phase shift per photon" and avoid at the same time significant photon absorption in the wings of the atomic transition line?

and

(ii) is it possible to achieve cavity Q factors large enough for the photons to survive the time required to carry out a measuring sequence?

These two questions are not independent. Equation (8) shows indeed that a way to increase φ_0 without getting too close to a resonance is to decrease the cavity volume, which is obviously done at the expense of the cavity Q. Another solution is to decrease the atomic velocity but we then lengthen the duration of the experiment and put more stringent limits on an acceptable cavity Q value. The ideal experimental parameters must thus realize a compromise between contradictory requirements. With this in mind, let us dicuss in turn the phase shift per photon and the cavity damping time questions.

(i) Optimizing the phase shift per photon. This parameter can of course be increased to a point by making $|\omega - \omega_{ge}|$ small (see (8)). There is a limit to this simple solution, which is that no photon absorption can be tolerated. In fact, it is easy to show that the probability for an atom to absorb a photon during its cavity crossing time is of the order of $\bar{n}\varphi_0\Gamma/|\omega - \omega_{ge}|$ where Γ is the spontaneous emission rate on the $e \rightarrow g$ transition. Let us assume for simplicity that $\bar{n}\varphi_0 \sim 1$. If we do not tolerate more than a 10^{-3} photon absorption probability, which is a minimal "photon nondemolition" requirement for a process involving at least a few hundred atoms, the atom-cavity detuning must be not smaller than $10^3\,\Gamma$. In order to achieve not too small phase shifts per photon with such a large detuning, it will be necessary to use a beam of slow atoms (to increase the $1/v$ term in (8)) interacting with small optical Fabry Perot cavities (to maximize the $1/\mathscr{V}_{\rm cav}$ term). On the other hand, as noticed above, these parameters cannot be too small, least the cavity damping time problem becomes untractable. A good compromise might be to choose sodium atoms at $v \sim 1\,\mathrm{m/s}$ coupled to the sub-millimeter size cavities described in [8]. We then find typical φ_0 values of the order of 10^{-1} for $\Gamma/|\omega - \omega_{ge}| = 10^{-3}$ (note that this is compatible with $\bar{n}\varphi_0 \sim 1$ for fields containing about ten photons). With such φ_0 values, a reduction of a small field into a Fock state will require about 100 detected atoms in an ideal interferometer, i.e., an average absorption photon loss of about 10^{-1} per measuring sequence. Due to the system unavoidable imperfections and to limited atomic detection efficiency, it will – even in this ideal case – be very hard to avoid some photon demolition to occur.

(ii) Optimizing the cavity damping. The relaxation time of an n Fock state being $T_{\rm cav}/n$, where $T_{\rm cav} = Q/\omega$ is the cavity damping time [9, 10], it is obvious that we do not have much time to take the atomic wave "snapshot" revealing the existence of this state in the cavity. The minimal duration required for the experiment is the atom-cavity crossing time $\ell_{\rm cav}/v$, which for the typical field waist value of the small Fabry-Perot cavities considered above ($\ell_{\rm cav} = 50\,\mu\mathrm{m}$) and for an atomic velocity $v = 1\,\mathrm{m/s}$ is 5×10^{-5} s. With these parameters, the preparation of a $n = 10$ Fock state would thus theoretically require a cavity having a $T_{\rm cav}$ of the order of 10^{-5} s at least, i.e., a Q in the 10^{10} range. This is about one order of magnitude above the Q factors presently achievable in these small cavities [8].

The obvious conclusion of this discussion is that the experiments discussed in this article, although possible in principle, are still very far away from present state of the art technology. The bottom line is that we need to observe an atomic interference pattern in a very short time. Knowing that the first Young atomic interference signals have required integration times of the order of hours [3], we realize the improvements which have still to be made.

Even if the minimal requirements we have just discussed are met, it is important to notice that the field which we would have "reduced" in a Fock state would be "dead" long before we would be aware of its short term presence in the cavity (the travel time of the atoms to the detectors, typically located at a distance $L = 0.1$ m to 1 m from the slits, will be of the order of 0.1 to 1 s). In other words, it will already be very hard to generate Fock states – or Schrödinger cat states of an optical field (for which the time requirements are the same) – and it would be even harder to conserve these fields "alive" long enough to manipulate them and study their evolution in a continuous way. For example, it would be a fascinating experiment to follow in real time the photon number change in a very weakly relaxing cavity. As photons would successively disappear in the cavity walls, the fringe pattern would suddenly shift along Ox by finite steps, revealing in a striking way the field quantum jumps. These experiments would require field damping times in the fraction of a second range which is quite unrealistic in the predictible future in the optical domain.

On the other hand, such long relaxation times are quite achievable in the microwave domain with superconducting cavities. Then, the cavity size and the typical field spatial variation lengths fall in the millimeter wave domain instead of the micron one and it becomes exceedingly difficult to build matter-wave interferometers physically separating matter-wave paths at this scale. It may seem that we have traded an impossible task for another, unless we realize that interferences do not necessarily require spatially distinguishable trajectories. After all, a birefringent plate in optics is also an interfering device. A photon polarized along a linear superposition of the two plate principal polarizations propagates through the medium along the same path, but at two different phase velocities, in a quantum superposition of two distinct states. The probability of detecting this photon with an analyser at an angle with the principal polarization directions is the squared sum of two amplitudes which exhibits oscillations as a function of the plate width. In the dual version of this effect, the light beam becomes an atomic matter-wave and the photon polarization is replaced by the atom quantum state preparation. Assume we prepare the atom in a linear superposition of two energy distinct states, which undergo different phase shifts as they cross, along the same trajectory, a macroscopic superconducting cavity non resonant with the atoms and containing a microwave field. The detection of this state superposition at cavity exit will exhibit oscillations as a function of the photon number in the cavity which, in essence, are not basically different from the ones we have studied in this article. The way to prepare these states superposition and to detect them is to perform two short microwave pulsed excitations on the atoms, one at cavity entrance and the other at cavity exit, on a transition linking the two states (i.e., a transition which is not resonant with the cavity field). The probability of detecting downstream the atoms in either one of the two levels will then exhibit oscillations versus n quite analoguous to the ones obtained in a Young interferometer. We have described here a Ramsey type experiment, which is the context in which we have first discussed these atomic interference effects. All the ideas discussed in this article on the very simple Young double slit scheme can be reformulated, mutatis mutandis, in the Ramsey fringe language. A complete theory of the microwave version of these experiments can be found in [2], where we take explicitely into account finite time cavity relaxation by describing the field evolution in the density matrix formalism. This paper also contains estimates of orders of magnitudes for experiments involving beams of circular Rydberg atoms. The advantage of the microwave over the optical versions of the atomic interferometer is that experiments, in which exotic electromagnetic fields such as Fock or Schrödinger cat states could not only be generated, but also manipulated and observed over long periods of time, are no longer a distant dream but become really feasible.

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