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Quantum Noise – A Limit in Photodetection

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Abstract. This paper attempts to outline the fundamental limits of the photoelectric measurement process, to give a simple picture of how their consequences can be estimated and to describe recent ideas and experimental work on how to use the current understanding of photodetection to make measurements previously thought to be beyond the quantum limit.

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The detection of light with photoelectric detectors is a central part of nearly all optical experiments. Modern photoelectric detectors, such as photodiodes and photomultipliers, approach the theoretical limit of unit quantum efficiency, corresponding to the emission of one photoelectron for every photon reaching the detector. By measuring the mean photocurrent or counting the photoelectrons, a very high sensitivity and accuracy can be achieved. For some years the technology of photoelectric detectors has been sufficiently advanced that their accuracy is limited only by fundamental quantum mechanical measurement limits.

Since more and more measurements are now reaching this quantum limit, the nature of the limitations which quantum mechanics imposes on photodetection have been studied in detail in recent years, both theoretically and experimentally [1–9]. One outcome of this research has been to show that it is possible, under certain conditions, to make measurements on light with greater accuracy than was previously thought to be allowed by quantum mechanics [10].

The paper begins with a discussion of some properties of quantum noise, and how it manifests itself in typical experiments. We then outline the principles of quantum optics, to place the discussion of quantum mechanical back action on a sound theoretical basis. The paper then concludes with an overview of some proposed and some experimentally demonstrated schemes for quantum non-demolition measurements, that is, measurements which evade the consequences of quantum mechanical back action.

1. Noise Properties of the Photoelectric Signal

For many years it has been known that even in the absence of external noise sources, the thermionically emitted current I flowing from the heated cathode to the anode of a vacuum tube exhibits fluctuations with a root-mean square (RMS) amplitude ΔI, independent of frequency, given by [11]

$$\Delta I = (2eI\Delta f)^{1/2}\,, \tag{1}$$

where Δf is the frequency interval over which the fluctuations are integrated, and e is the charge of an electron. These fluctuations were labelled shot noise, due to the sound made by audio amplifiers whose gain was such that these fluctuations were the dominant noise source. Shot noise is a consequence of the discrete nature of the electronic charge and the random manner in which electrons are ejected from the cathode and arrive at the anode. A photoelectric detector is similar to a vacuum tube, except that the hot cathode is replaced by a photocathode from which electrons are ejected due to the photoelectric effect. It therefore came as no surprise that the fluctuations in the photocurrent from a photoelectric detector always seemed to obey (1). However, the quantum theory of light, or quantum optics, has shown this to be more of a coincidence than a fundamental fact.

The primary assumption of quantum optics is that the energy transmitted by a beam of light is divided into discrete packets, or photons, much as an electric current is made up of a flow of electrons. Detailed analysis shows that the arrival times of photons from a typical laboratory light source at the photocathode of a photodetector exhibits the same Poissonian statistics as does the thermionic emission of electrons from the hot cathode of a vacuum tube. Thus the fluctuations in the photocurrent are due to the quantum nature of light, and are indistinguishable from classical shot noise, and so the resulting noise is still often called shot noise, as well as quantum noise. This noise determines the standard quantum limit (SQL), of the signal to noise ratio (SNR) which can be achieved with a photoelectric detector.

The shot noise produced by a photoelectric detector is therefore an intrinsic property of the incident light, rather than of the detector. This raises the question of whether the light itself can be manipulated to improve the signal to noise ratio of the detection process. Much attention has been devoted to this problem in recent years, and a number of methods of altering the photon statistics of light to improve detection SNR beyond the SQL have been proposed, and a smaller number have been experimentally demonstrated [1–9].

2. How to Detect Quantum Noise

Any real light source will contain a number of classical noise sources, most of which have $1/f$ frequency dependence. Consequently it is usually difficult to detect quantum noise in the dc or low frequency regions of the spectrum. However at high frequencies (typically >10 MHz) the quantum noise can dominate, and provided the intensity of the laser beam is high enough (typically >1 mW on the detector) and all the electronic components are quiet, the quantum noise is larger than the thermal noise of the detection system and it can be displayed on a spectrum analyzer.

Figure 1 shows a typical spectrum analyzer display of the electrical power generated in the case of a photodiode with a dc quantum efficiency η of about 0.6 illuminated with 1 mW of laser light followed by an amplifier with a gain of 60 dB.

3. The Signal to Noise Ratio

The signal to noise ratio SNR serves as a practical description of the accuracy of a measurement. Using (1), the noise equivalent power P_{nep}, equivalent to the quantum noise associated with a light beam of average power P_{ave} and frequency ν is given by [12–14]:

$$P_{\text{nep}} = (h\nu \Delta f P_{\text{ave}}/\eta)^{1/2}, \tag{2}$$

where h is Planck's constant. In the case of light detection the quantum noise sets an upper limit $\text{SNR}_{\max}$ for the signal to noise ratio. This can be defined as the ratio between the average power P_{ave} of the field and the power P_{nep} equivalent to the quantum noise detected

$$\text{SNR}_{\max} = P_{\text{ave}}/P_{\text{nep}} = (P_{\text{ave}}\eta/h\nu\Delta f)^{1/2}, \tag{3}$$

where η is the quantum efficiency of the detector. While $\text{SNR}_{\max}$ can be very large (for 1 mW of laser power at 600 mm, $\eta = 0.9$ and $\Delta f = 1$ kHz, $\text{SNR}_{\max} = 1.5 \times 10^6$) it nevertheless sets a limit to the smallest change in laser power which can be detected. The fact that $\text{SNR}_{\max}$ is proportional to $P_{\text{ave}}^{1/2}$ poses a problem: In order to measure a beam with power P_{ave} as accurately as possible, all the power has to illuminate the detector, and none is left to carry out an experiment. A beamsplitter which sends a fraction ε of the beam intensity off for detection while retaining $(1-\varepsilon)$ of the power for the experiment will reduce the $\text{SNR}_{\max}$ for

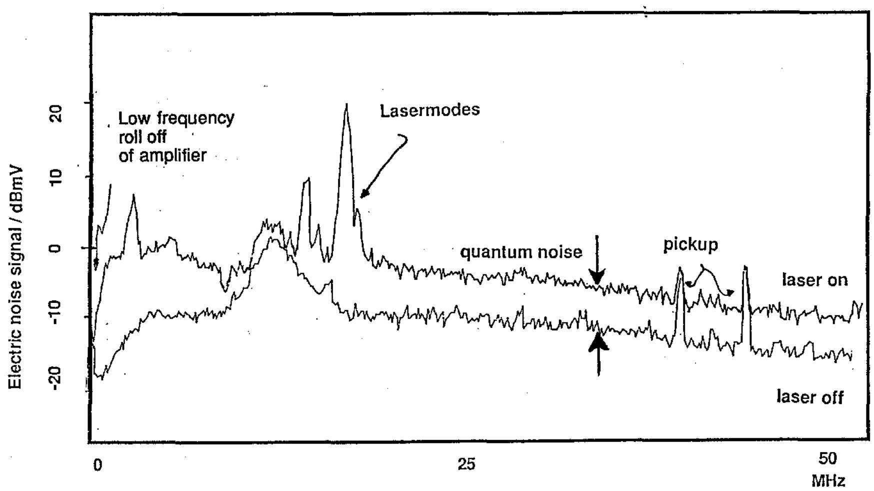

Fig. 1. The noise generated in a photodetector in the frequency range 0–50 MHz. A dye laser beam with 1 mW power illuminates a silicon photodiode. An amplifier with 60 dB gain in the range 0.5–300 MHz amplifies the signal which is detected by a spectrum analyzer. The roll off at low frequencies is due to the amplifier. Individual features include laser modes and electric pickup. Two traces are shown, with the laser on and off. The quantum noise is the frequency independent difference between the two traces remove.

both output beams [13–15]. This is a direct consequence of (3).

4. Quantum Mechanical Back Action of the Detection Process

The above example is an experimental manifestation of the quantum mechanical back action of the measurement process. A beamsplitter is required in order to obtain information about the intensity. For this purpose some of the intensity has to be destroyed on a detector and consequently this measurement reduces the accuracy, or signal to noise ratio, for subsequent measurements, in this case the detection of the intensity of the second beam leaving the beamsplitter.

According to Heisenberg's uncertainty principle, the measurement of an observable property of light, or of any physical system, necessarily disturbs the system. This quantum mechanical back action imposes limitations on optical experiments where it is desirable to know all the observable properties of a light field and simultaneously to use this light field for an experiment. However, there are practical situations where only one of the properties of the field is important. Take for example an absorption experiment [16]. The fraction of a light beam absorbed by an ensemble of atoms depends on the intensity of light, but not on the phase. In order to determine the fraction of light absorbed, the intensity has to be measured before and after the absorption process, and only the ratio of the two measurements is of interest. If, in this situation, the intensity of the light field could be measured in such a way that the quantum mechanical back action resulted only in an increased uncertainty in the phase, then for the purposes of this experiment the quantum mechanical back action of the measurement can be avoided. Such an intensity measurement is theoretically possible, and is an example of a Quantum Non-Demolition (QND) measurement [17].

It is useful to have a simple, yet reliable representation for the light in order to describe the effect of optical components such as beamsplitters, absorbers, amplifiers and nonlinear media on the fluctuations of light. Phasor diagrams, which were introduced in this context by Caves [18] and have since been used extensively in quantum optics, are such a representation. They will be used in the remainder of this paper. The actual justification for this simple representation is more involved and in the next section an attempt is made to outline the theoretical basis for this model.

5. Noise Properties of the Quantized Light Field

The electromagnetic field is quantized [19] by expressing Maxwell's equations in the form of the equation of motion for a classical harmonic oscillator, and then replacing the classical harmonic oscillator by its quantum mechanical counterpart. The energy in the quantized light field may only take discrete values, corresponding to the states of a quantum mechanical harmonic oscillator. The state of excitation of each mode, labelled by its wavevector and polarization $\mathbf{k}$, of the quantized light field can be represented by $|n_k\rangle$, where the integer n_k is the number of quanta of excitation, or photons, in the mode k. The number states $|n_k\rangle$ are eigenstates of the quantum mechanical harmonic oscillator Hamiltonian,

$$H=\hbar\omega_k(a_k^\dagger a_k+\tfrac{1}{2}), \tag{4}$$

where $\hbar\omega_k$ is the energy of a photon of the mode $\mathbf{k}$, and the creation and annihilation operators $a_k^\dagger$ and a_k are defined by:

$$a_k^\dagger|n_k\rangle=\sqrt{n_k+1}\,|n_k+1\rangle, \tag{5}$$

$$a_k|n_k\rangle=\sqrt{n_k}\,|n_k-1\rangle. \tag{6}$$

The properties of the number states are found by calculating the expectation values of certain operators. The most important of these operators is probably the number operator $\mathbf{n}_k$ defined by:

$$\mathbf{n}_k=a_k^\dagger a_k. \tag{7}$$

The number operator is diagonal on the number state basis, and has the expectation value:

$$\langle\mathbf{n}_k\rangle=\langle n_k|a_k^\dagger a_k|n_k\rangle=n_k. \tag{8}$$

It may also be shown that the photocurrent I_p generated by a photoelectric detector exposed to the mode k is proportional to $\langle\mathbf{n}_k\rangle$ [20].

Since the quantized light field was derived from a quantized form of Maxwell's equations, there also exist operators corresponding to the electric field amplitude and phase of a classical light wave. The electric field operator $\mathbf{E}_k(t)$ for the mode k at the point $\mathbf{r}$ is [21]

$$\mathbf{E}_k(t)=i\left(\frac{\hbar\omega_k}{2\varepsilon_0 V}\right)^{1/2}\{a_k\,e^{-i(\omega_k t-\mathbf{k}\cdot\mathbf{r})}-a_k^\dagger\,e^{i(\omega_k t-\mathbf{k}\cdot\mathbf{r})}\} \tag{9}$$

where ε_0 is the permittivity of free space and V is the quantization volume.

The Hermitian phase operators $\mathbf{cos\phi}$ and $\mathbf{sin\phi}$ are [21]:

$$\mathbf{cos\phi}_k=\tfrac{1}{2}\{(a_k^\dagger a_k+1)^{-1/2}a_k+a_k^\dagger(a_k^\dagger a_k+1)^{-1/2}\}, \tag{10}$$

$$\mathbf{sin\phi}_k=\frac{1}{2i}\{(a_k^\dagger a_k+1)^{-1/2}a_k-a_k^\dagger(a_k^\dagger a_k+1)^{-1/2}\}. \tag{11}$$

For the case of a number state $|n_k\rangle$ the expectation values of these operators are easily calculated, with the result:

$$\langle n_k|\mathbf{E}_k(t)|n_k\rangle=\langle n_k|\,\mathbf{cos\phi}\,|n_k\rangle=\langle n_k|\,\mathbf{sin\phi}\,|n_k\rangle=0. \tag{12}$$

Hence even in the classical limit ($n_k \gg 1$) a number state corresponds to a light field with undefined phase. An experimentalist would also be interested in the root mean square fluctuations ΔI of the photocurrent generated by a photodetector illuminated by a number state, given by;

$$\Delta I = C\sqrt{\langle n_k|(a_k^\dagger a_k)^2|n_k\rangle - (\langle n_k|a_k^\dagger a_k|n_k\rangle)^2}\,, \\ =0\,, \tag{13}$$

where C is a constant of proportionality. Therefore while the intensity of a light field in a number state could be measured without any theoretical limits in precision, no knowledge of the phase of such a light field can be obtained. A number state is clearly not a good description of the coherent light field produced by a single mode laser which typically consists of a large but fluctuating intensity, and a phase which fluctuates about a predictable mean value.

6. Phasor Diagram Representation

At this point it is convenient to introduce the phasor diagram [18] as a means of representing the state of the light field. The sinusoidally oscillating electric field component $E(t)$ of a classical light wave with amplitude E_0, frequency ω and phase ϕ may be divided into two components with amplitudes X_1 and X_2 oscillating 90° out of phase:

$$E(t) = E_0 \sin(\omega t + \phi) \\ = X_1 \sin \omega t + X_2 \cos\omega t\,, \tag{14}$$

where

$$X_1 = E_0 \cos\phi\,, \tag{15}$$

and

$$X_2 = E_0 \sin\phi\,. \tag{16}$$

Thus the amplitude and phase of a classical field is conveniently represented as a point plotted against axes representing the quadrature amplitudes X_1 and X_2 (Fig. 2a).

For the case of the quantized light field, (9) may be written in a form similar to (13), where the quadrature phase amplitudes X_1 and X_2 become operators [22]:

$$\mathbf{X}_1 = \frac{a + a^\dagger}{2}\,, \quad \mathbf{X}_2 = \frac{a - a^\dagger}{2\mathrm{i}} \tag{17}$$

Using these quadrature phase operators it may be shown that a number state is represented on a phasor diagram by an annulus [23, 24] (Fig. 2c).

For any possible state, the commutation relation for the operators $\mathbf{X}_1$ and $\mathbf{X}_2$ leads to an uncertainty relation involving their RMS deviations $\langle \Delta\mathbf{X}_1\rangle$ and $\langle \Delta\mathbf{X}_2\rangle$ [14]

$$\langle \Delta\mathbf{X}_1\rangle\langle \Delta\mathbf{X}_2\rangle \geqq \tfrac{1}{4}\,, \tag{18}$$

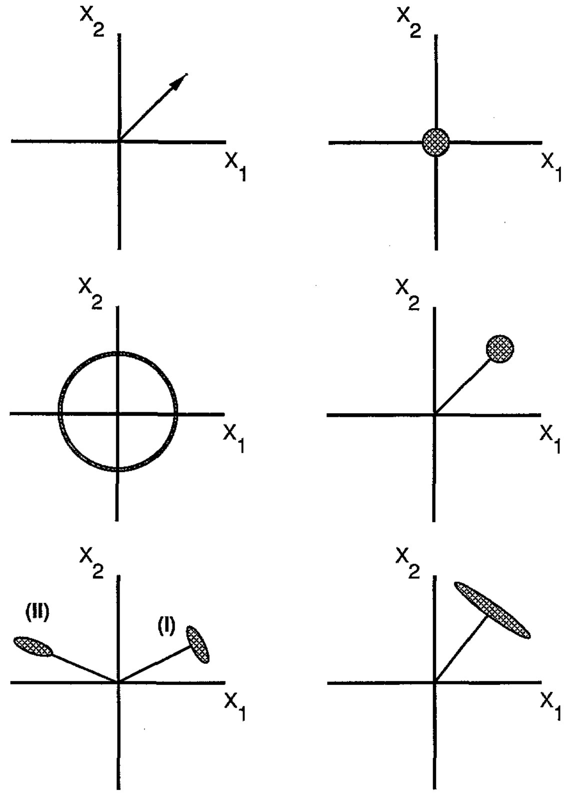

Fig. 2. **a** Classical state, **b** vacuum state, **c** number state, **d** coherent state. **e** Two squeezed states with reduced amplitude fluctuations (I) and reduced phase fluctuations (II). The size of the uncertainty area is identical to the initial coherent state. **f** Squeezed state with excess noise in one projection. Not a minimum uncertainty state

where

$$\langle \Delta\mathbf{X}_i\rangle = \sqrt{\langle(\Delta\mathbf{X}_i)^2 - (\langle \Delta\mathbf{X}_i\rangle)^2}\,, \quad i = 1,2 \tag{19}$$

and the triangular brackets indicate expectation values. The simplest case satisfying (18) is where $\langle \Delta\mathbf{X}_1\rangle = \langle \Delta\mathbf{X}_2\rangle = 1/2$. This case is commonly referred to as the standard quantum limit (SQL), and the expression (3) for the maximum signal to noise ratio $\mathrm{SNR}_{\max}$ of a measurement made on a light field assumes this symmetric distribution of the minimum allowed fluctuations in the two quadratures.

Before discussing a more useful set of basis states to represent a real light field it is interesting to investigate the properties of the vacuum state $|n_k = 0\rangle$. For a number state, the RMS deviation ΔE_k of the electric field is:

$$\Delta\mathbf{E}_k = \sqrt{\langle n_k|\mathbf{E}^2|n_k\rangle - (\langle n_k|\mathbf{E}|n_k\rangle)^2} \tag{20}$$

$$= \left(\frac{\hbar\omega}{\varepsilon_0 V}\right)^{1/2} \left(n_k + \frac{1}{2}\right)^{1/2}. \tag{21}$$

Clearly in the case of the vacuum state $|n_k = 0\rangle$ there is a finite electric field oscillating at all frequencies in every mode k with an amplitude corresponding to half a photon. The fluctuations in the quadrature phase amplitudes for the vacuum state are $\langle \Delta \mathbf{X}_1 \rangle = \langle \Delta \mathbf{X}_2 \rangle = 1/2$, thus satisfying (18). For fluctuations where $\langle \Delta \mathbf{X}_1 \rangle \langle \Delta \mathbf{X}_2 \rangle = 1/4$, the case where $\langle \Delta \mathbf{X}_1 \rangle = \langle \Delta \mathbf{X}_2 \rangle$ corresponds [25] to the minimum energy content, so it is not surprising that the vacuum fluctuations occurring in nature, for laser as well as thermal light, have this symmetric distribution of uncertainty between the quadrature phase amplitudes.

On a phasor diagram, the vacuum state is represented by a circle of uncertainty, of diameter 1/2 in appropriate units [1] (Fig. 2b). These zero point fluctuations appear in the Hamiltonian (4) as a zero point energy which, due to the continuous and infinite nature of frequency space, can be shown to be infinite, However, since it is not possible to detect a photon from the vacuum state (since $\langle 0| a^\dagger a |0\rangle = 0$), the infinite zero point energy represents a non-detectable offset to all energy measurements made on light fields.

The inability of the photon number states to describe a coherent light field led to the development of the coherent states [20] $|\alpha_k\rangle$, which are linear combinations of number states:

$$|\alpha_k\rangle = e^{-\frac{1}{2}|\alpha_k|^2} \sum_{n=0}^{\infty} \frac{(\alpha_k)^n}{\sqrt{n!}} |n_k\rangle . \qquad (22)$$

The coherent states are a good representation of the light field produced by an ideal laser operating well above threshold [20, 21]. On a phasor diagram (Fig. 2d) a coherent state is represented by a circle of uncertainty identical to that representing the vacuum fluctuations (Fig. 2b) displaced from the origin by a vector whose length represents the mean amplitude, and whose angle from the X_1 axis is $\arg\alpha$. It is worthwhile noting, that in these diagrams the size of the regions of uncertainty is independent of the intensity of the state. In the case of coherent states, and the vacuum state, the left and right hand sides of (18) are equal, and these are therefore minimum uncertainty states. Using the fact that a coherent state is an eigenstate of the destruction operator [20], we find that the mean photon number $\langle \mathbf{n}_k \rangle$ and its RMS fluctuations $\langle \Delta \mathbf{n}_k \rangle$ for a field in the state $|\alpha_k\rangle$ are

$$\langle \mathbf{n}_k \rangle = \langle \alpha_k | a_k^\dagger a_k | \alpha_k \rangle = |\alpha_k|^2 \qquad (23)$$

and

$$\langle \Delta \mathbf{n}_k \rangle = \sqrt{\langle \alpha_k | (a_k^\dagger a_k)^2 | \alpha_k \rangle - (\langle \alpha_k | a_k^\dagger a_k | \alpha_k \rangle)^2} = |\alpha_k| . \quad (24)$$

Since the photocurrent I generated by a photodetector illuminated by a light field in the state $|\alpha_k\rangle$ is proportional to $\langle \mathbf{n}_k \rangle$, it may be shown that the RMS fluctuations in the photocurrent ΔI in the frequency interval Δf are given by;

$$\Delta I = \sqrt{2eI\Delta f} \qquad (25)$$

which is identical to (1). These fluctuations, whose amplitudes are independent of frequency and proportional to the square root of the mean photocurrent, are commonly referred to as shot noise or quantum noise.

The phasor diagram provides a useful way of thinking about the quantum origin of shot noise. In the case of large field amplitudes ($|\alpha_k| \gg 1$ for a coherent state) the photocurrent is essentially proportional to the square of the total electric field amplitude $\langle \mathbf{X}_1^2 + \mathbf{X}_2^2 \rangle$. The magnitude of the vector sum of the mean electric field amplitude and the vacuum fluctuations will fluctuate, consequently so will the photocurrent, and the magnitude of these fluctuations will be proportional to the product of the vacuum noise amplitude and the mean amplitude of the coherent state. Thus shot noise may be regarded as a beating effect between the mean amplitude of the coherent state and the vacuum fluctuations. Finally, since the vacuum fluctuations have equal Fourier components at all frequencies, the photocurrent fluctuations or shot noise also have this property – they form a white noise.

7. Techniques for Evading the Standard Quantum Mechanical Limits of Measurement

As shown in the last section, the state of the quantized radiation field may be described using the two quadrature amplitudes $\langle \mathbf{X}_1 \rangle$ and $\langle \mathbf{X}_2 \rangle$. For coherent states, as generated by a laser, the fluctuations are symmetrically distributed in these two quadratures, so a detector which could measure the two quadratures independently would not show an improvement in SNR. However, the uncertainty principle (18) sets a limit only on the product $\langle \Delta \mathbf{X}_1 \rangle \langle \Delta \mathbf{X}_2 \rangle$. Asymmetric distributions of fluctuations between the two quadratures are possible. These asymmetric uncertainty distributions are the key to evading the standard quantum mechanical measurement limits, and two strategies have been demonstrated for exploiting them. Both these strategies are realized using the properties of nonlinear optical media.

7.1. Strategy 1: Squeezing

A state of the radiation field where the fluctuations in the two quadratures $\mathbf{X}_1$ and $\mathbf{X}_2$ are unequal is said to be "squeezed". On a phasor diagram the area corresponding to the fluctuations is no longer circular but is "squeezed" into an elliptical shape (Fig. 2e) [26]. In the case of a minimum uncertainty squeezed state ($\langle \Delta \mathbf{X}_1 \rangle \langle \Delta \mathbf{X}_2 \rangle = 1/2$) the fluctuations in one quadrature will be below the SQL, and measurement of this quadrature is now possible with a $\mathrm{SNR}_{\max}$ greater than that predicted by (3).

Squeezing has recently been demonstrated in several types of nonlinear media, and asymmetries or squeeze ratios of $\langle \Delta \mathbf{X}_1 \rangle / \langle \Delta \mathbf{X}_2 \rangle$ of up to 10 have been deduced from experiments [5]. In this paper we will not discuss the details of this technique and refer instead to the excellent review articles available [1, 2, 25].

Squeezing has the disadvantage that any loss process, such as absorption, diffraction or the use of beamsplitters, will reduce the squeeze ratio since vacuum fluctuations with $\langle \Delta \mathbf{X}_1 \rangle = \langle \Delta \mathbf{X}_2 \rangle = 1/2$ are mixed in [18]. Thus a squeezed state of light is very susceptible to technical problems connected with beam propagation. In addition, to date only squeezed vacuum states have been generated in experiments concerned with the propagation through nonlinear media. Only experiments with laser diodes [7] have generated light fields with non-zero mean amplitude, and intensity fluctuations below the SQL.

7.2. *Strategy 2: Quantum Non-Demolition*

If a measurement could be made on one quadrature, say $\mathbf{X}_1$, in such a way that the back action of this measurement caused increased fluctuations in $\mathbf{X}_2$ without affecting $\mathbf{X}_1$, then repeated measurements on $\mathbf{X}_1$ could be made without decreasing the SNR. This is therefore an example of a QND measurement.

Consider an interaction in a nonlinear medium between the beam to be measured, called the signal, with quadratures $(\mathbf{X}_1^i, \mathbf{X}_2^i)$, and a second beam at a different frequency, the probe, with quadratures $(\mathbf{Y}_1^i, \mathbf{Y}_2^i)$. The superscript i indicates the input into the medium, o the output. The quadratures of both the signal and the probe leaving the medium, $(\mathbf{X}_1^o, \mathbf{X}_2^o)$ and $(\mathbf{Y}_1^o, \mathbf{Y}_2^o)$, are coupled by the interaction with the nonlinear medium.

It has been shown [22] that interactions which can be described by a Hamiltonian containing a coupling term proportional to $(a^\dagger a\, b^\dagger b)$, where $a^\dagger$ and a act on the signal and $b^\dagger$ and b act on the probe, generate the following interesting output quadratures:

$$\begin{aligned}
\mathbf{X}_1^o &= A\mathbf{X}_1^i + B\mathbf{X}_2^i + C\mathbf{Y}_2^i \\
\mathbf{X}_2^o &= \mathbf{X}_2^i \\
\mathbf{Y}_1^o &= A\mathbf{Y}_1^i + B\mathbf{Y}_2^i + C\mathbf{X}_2^i \\
\mathbf{Y}_2^o &= \mathbf{Y}_2^i\,.
\end{aligned} \tag{26}$$

Here the constants A, B, and C are determined by the nonlinearity.

The two important points are that $\mathbf{X}_2^o = \mathbf{X}_2^i$, so there is no back action on this quadrature, and that $\mathbf{Y}_1^o$ contains information about $\mathbf{X}_2^i$. Thus a measurement of $\mathbf{X}_2^i$ can be made without introducing fluctuations, provided that the nonlinear coupling is sufficient which means $C > B$ and $C > A$. Note that in contrast to the case for a squeezed state, the maximum signal to noise ratio $\mathrm{SNR}_{\max}$ for the measurement of $\mathbf{X}_2^i$ is still given by (3). However, the output signal beam $(\mathbf{X}_1^o, \mathbf{X}_2^o)$ can be used for an experiment, despite the fact that $\mathbf{X}_2^o$ has been measured with the highest possible accuracy. For a direct measurement, (3) implies that to achieve this accuracy it would be necessary to annihilate all the photons on the detector. In addition it is possible to measure $\mathbf{X}_2^i$ repeatedly with the QND process with no change in $\mathrm{SNR}_{\max}$. Each consecutive QND measurement of $\mathbf{X}_2^i$ has the same signal to noise ratio.

The other quadrature of the signal has been modified in this process, since the fluctuations introduced by the measurement process have been added to it. Thus the scheme improves the measurement for one quadrature only. The uncertainty area of the signal and probe beam will be larger at the output than at the input, the output beams cannot be minimum uncertainty states (Fig. 2f).

8. Detection of One Quadrature Only – The Heterodyne Detector

Both squeezing and QND require the selective measurement of one quadrature only. This can be achieved by using a heterodyne detector as shown in Fig. 3. The signal beam (at frequency ω) is superimposed with a second laser beam (local oscillator at frequency $\omega + \Omega$) on a beamsplitter and the beat signal at frequency Ω generated by a photodiode is monitored.

Figure 3 shows a phasor diagram representation of a heterodyne detector for the simpler case where both the signal field and the local oscillator have the same frequency $(\Omega = 0)$. For typical local oscillator powers, the photocurrent is proportional to the square $|R|^2$ of the phasor representing the field incident on the detector. It can be seen from the vector addition of the signal and local oscillator phasors that

$$\begin{aligned}
|R|^2 &= \varepsilon|S|^2 + (1-\varepsilon)\,|LO|^2 + 2[\varepsilon(1-\varepsilon)]^{1/2} \\
&\quad \times |LO|\,(\cos\phi_{\mathrm{LO}}\langle \mathbf{X}_{1s}\rangle + \sin\phi_{\mathrm{LO}}\langle \mathbf{X}_{2s}\rangle)\,,
\end{aligned} \tag{27}$$

so that by adjusting the phase of the local oscillator with respect to the phase of the signal, either $\langle \mathbf{X}_{1s} \rangle$ or $\langle \mathbf{X}_{2s} \rangle$ can be measured. The generalization of the above result to the case where $\Omega \neq 0$ is straightforward.

A special case is the situation where the input into the heterodyne detector is the vacuum state. This is a common situation, a beamsplitter used to split off some intensity towards a detector is an example. The phasor diagrams can be used to represent this case elegantly and it becomes obvious that the quantum noise components of both inputs combine to the output quantum noise [18]. Thus the quantum noise of the vacuum state is the origin of the reduction of the SNR of the beam downstream of the beamsplitter. This is another interpretation of the back action of the

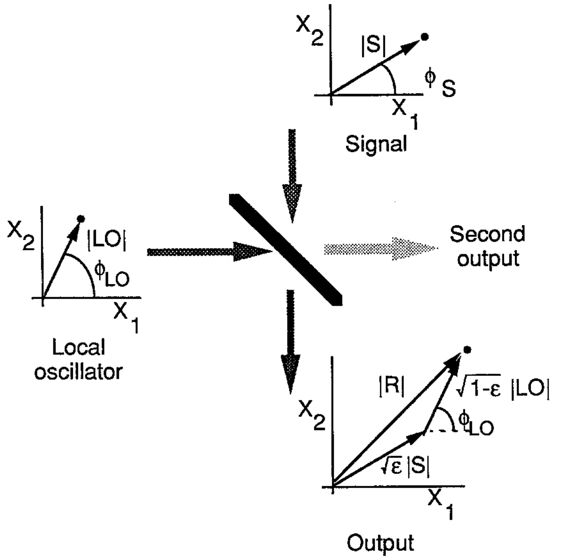

Fig. 3. The homodyne detector represented by phasor diagrams. Signal S and local oscillator LO are mixed on a beamsplitter with power reflection ε. For simplicity only classical states are shown, and the two signals have the same optical frequency. By adjusting the phase of the local oscillator, any projection of the signal onto the axes X_1 and X_2 can be selected

measurement process. However, it has been demonstrated experimentally that the noise contribution of the second port of the beamsplitter can partially be eliminated by using a squeezed vacuum state as the input [10].

9. QND Experiments

A complete experimental demonstration of an optical QND measurements has to satisfy the following three conditions:

1. The information about $\mathbf{X}_2$ appears on $\mathbf{Y}_1$. This can be demonstrated by modulating $\mathbf{X}_2$ and detecting $\mathbf{Y}_1$, thereby determining the degree of transfer between these two quadratures.

2. Not only classical modulation but also the quantum fluctuations of $\mathbf{X}_2$ must be transferred to $\mathbf{Y}_1$. This requires a measurement of the correlation between the fluctuations in $\mathbf{X}_2$ and $\mathbf{Y}_1$. This result has to be larger than the correlation which can be deduced from individual measurements of $\mathbf{X}_2$ and $\mathbf{Y}_1$ [29].

3. Successive identical QND measurements should yield the same signal to noise ratio. For this purpose several identical QND detectors have to be built and operated simultaneously.

A number of nonlinear processes have been proposed for QND measurements, amongst these are four-wave mixing in optical fibres [28, 29], a passive optical fibre tap [30], parametric down conversion [27, 31, 32], optical rectification [31] and strong field interaction in atoms with a 3-level system. Some of these experiments will now be discussed in detail.

The nonlinearity used in the first example [28, 29] is the nonlinear Kerr effect in optical fibres. It is easy to see how this process can couple the quadratures of two light fields: The refractive index of the nonlinear Kerr medium is intensity dependent, thus the intensity fluctuations of the signal field will modulate the refractive index which in turn will modulate the phase of both the signal and the probe field. However, the refractive index does not respond to changes in the phase. Thus the phase of one field does not affect the amplitude of either field. For fields where the mean electric field amplitude is much greater than its fluctuations, we can interpret the quadratures $\mathbf{X}_1$ and $\mathbf{Y}_1$ as the phase and $\mathbf{X}_2$ and $\mathbf{Y}_2$ as the amplitude, and the desired coupling between signal and probe fields is established in this nonlinear process.

Provided losses are neglected, the nonlinear Kerr interaction can be represented by $(c_1 a^\dagger a + c_2 b^\dagger b + c_3 a^\dagger a b^\dagger b)$ in the Hamiltonian and it may be shown [28] that (26) holds for this interaction term. In this case the constants A, B, and C in (26) are proportional to the third order nonlinear susceptibility χ^3 of the medium. For a fibre of about 100 m length and laser powers of 130 mW at 647 nm and 60 mW at 676 nm for signal and probe respectively, the coefficient A has a measured value of 0.3, B of 0.8 and C of 0.58 [28]. In order to model the real experiment, the loss mechanisms in the fibre and the generation of phase noise due to light scattering have to be included. The experimental results confirm this theoretical model and the condition 1 is satisfied in these experiments.

In order to increase the constants A, B, and C, higher intensities are required inside this long fibre. However, this would generate stimulated Brillouin scattering and add even more phase noise. An alternative approach is to shorten the fibre and to increase the power dramatically by forming a ring cavity out of the fibre, with finesse much larger than one. Such an experiment has been carried out [29]. Two independent laser beams at 647 nm and 676 nm generated in the same Krypton laser were transmitted into a cavity made of 13 m of polarization preserving fibre. The cavity length was locked for maximum transmission of both beams. A finesse of about 12 for the signal at 647 nm and of 7 for the probe at 676 nm was achieved. Considerable problems with the stability of the behaviour of the fibre cavity and with residual sources of phase noise were encountered. However, it was still possible to demonstrate the transfer of quantum noise from the signal to the probe fields – conditions 1 and 2 were satisfied in this experiment.

Other experiments [33] in very long fibres (500 m of polarization preserving fibre) using a Nd : YAG laser at 1.32 μm as the signal and a He Ne laser at 1.52 μm as the probe have demonstrated the transfer of modulation from signal to probe and thus have satisfied condition 1, but the losses in the fibre are still too large for a demonstration of condition 3. However, a detailed theory of QND measurements in a lossy Kerr medium has been developed [34] from which the ideal experimental parameters can be predicted.

A different group of experiments uses parametric down conversion as the nonlinear process which selectively couples the quadratures of the signal and probe fields. A combination of an optical parametric amplifier (OPA) and a Faraday Rotator can generate a QND signal [27, 32]. The amplifier is pumped with a laser beam at 526.5 nm, and nonlinearly couples the two orthogonally polarised components of an incident beam at 1053 nm which serve as the signal and probe. The parametric amplifier by itself does not satisfy the requirements for a QND detector, however in combination with a Faraday rotator set at the correct angle it allows the signal component to travel unaffected through the system while the probe component is modulated by the signal. The χ^2 nonlinearities used in this process can be much larger than the χ^3 values for the nonlinear Kerr effect. Currently available materials and lasers should provide a significant QND effect.

Experiments using quantum noise limited pulse trains from a cw mode-locked Nd:YAG laser have already demonstrated the transfer of information from the signal to the probe, and have satisfied condition 1 [27]. Further work is required in order to demonstrate the transfer of quantum fluctuations. At present, these experiments use optically non-resonant χ^2 materials, however it is expected that resonant materials, such as calcite may be advantageous due to their strongly enhanced nonlinearity [16]. Since any losses are detrimental to the experiment, a careful balance exists close to resonance between absorption and nonlinearity. Experiments in resonant solid state materials have not yet been reported.

A different proposal [35] for a QND detector, based on resonant materials, is to use the nonlinearity of an atomic 3-level system. A possible 3-level system is available in barium (Fig. 4), or in rubidium. The two dipole allowed transitions in the system are assumed to be driven by separate lasers, each tuned close to resonance. If the intensity of one field (the signal field) is sufficient to induce a light shift [36] in the middle ($6s6p\,^1P_1$) level, then any fluctuation in this intensity will cause the refractive index seen by the other field (the probe field) to fluctuate, and consequently the phase of the probe field after interaction with the atoms

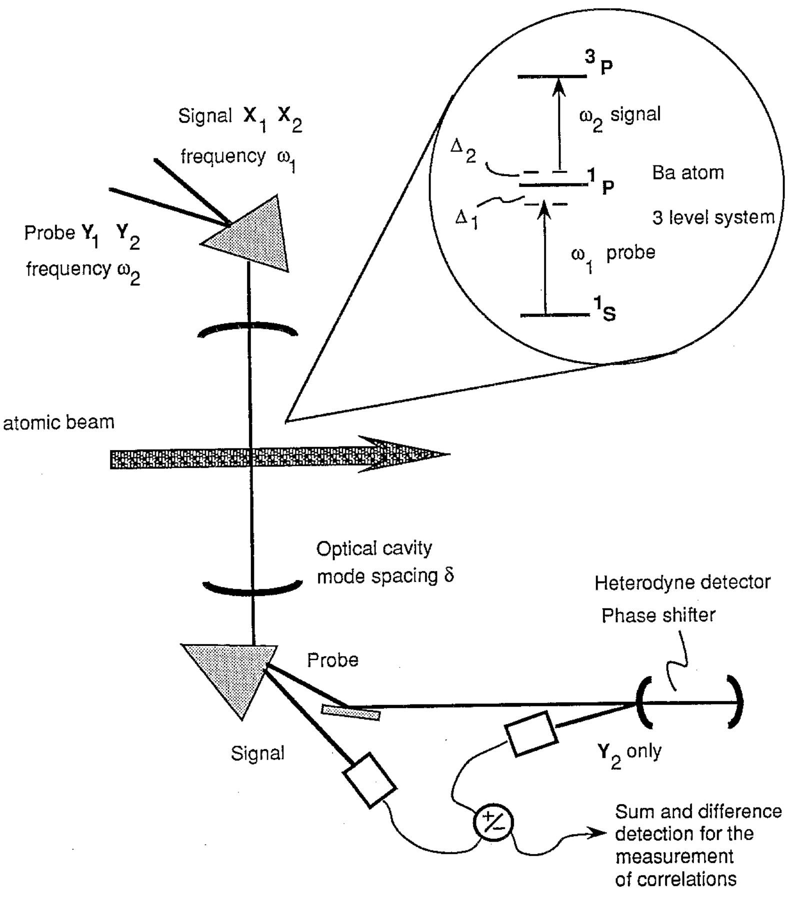

Fig. 4. Schematic diagram of resonant atomic QND experiment

will also fluctuate. This scheme is suggested as a possible QND detection scheme because there is no obvious mechanism through which the measurement of the phase of the probe field could have an effect on the intensity of the signal field.

The detunings and intensities of both fields will have to be optimized with the aim that the amplitude fluctuations of the signal couple strongly to the light shift (small signal detunings) while the light shift couples strongly to the phase of the probe (large probe detunings). Amplitude fluctuations of the probe should have no effect on the signal. This experiment will almost certainly require a sub-Doppler atomic beam. While the atom density available in an atomic beam is low in comparison to a solid, this is compensated by the large near resonant increase in χ^3. In order to achieve the required intensities, it may well be necessary to use either a cavity to enhance the intensities in cw experiments or to use quantum noise limited trains of pulses from modelocked cw lasers.

10. The QND Absorption Experiment

One direct application of a QND measurement would be the QND absorption experiment shown in Fig. 5 [16]. Before we can see the advantage of this arrangement we first have to note the fact that the sensitivity of an ordinary absorption experiment is limited by the quantum noise. In order to measure an absorption signal it is necessary to measure the intensity of the light both before and after it passes through the absorbing medium, and to record the difference between the two measurements. The intensity of the light upstream of the absorbing medium is generally determined by inserting a beamsplitter to split off an intensity reference beam. As discussed before, the use of the beamsplitter reduces the SNR for both the reference and the signal beam. Also, the reflection properties of the beamsplitter are such that the resulting fluctuations in the photocurrents from the two detectors cannot be electronically subtracted. Thus the minimum detectable absorption signal is determined by the quantum noise in the photocurrents and the SNR of the absorption measurement is given by the SNR of the smaller of the two photosignals. A 50% beamsplitter would give the best possible result. Since the absorption process is generally independent of phase, a QND intensity measurement which confined its back action to disturbing the phase could be used to replace the beamsplitter used to generate an intensity

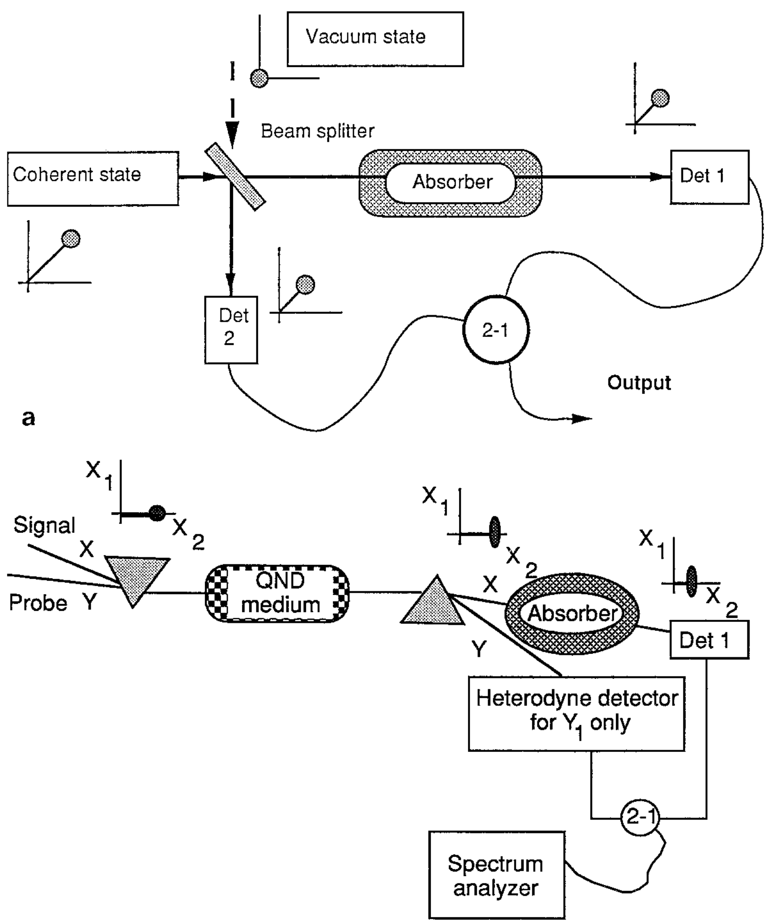

Fig. 5. a The standard absorption experiment. The beamsplitter required for the reference beam has a vacuum as the second input. The properties of the beamsplitter are such that the quantum fluctuations recorded in detectors 1 and 2 cannot be subtracted electronically. **b** The QND absorption experiment. A QND measurement of the signal amplitude is made. The electric output from the heterodyne detector which is proportional to the phase fluctuations of the probe beam (Y1) can be used as a reference and the quantum fluctuations in the signal (Det1) can be subtracted electronically. Phasor diagrams show the mean field and the fluctuations of the signal beam

reference. This has the advantage that the quantum fluctuations of the amplitude of the signal field could be measured on a separate probe beam without influencing the absorption process. This would permit the subtraction of the quantum noise from the photocurrent generated by the photodetector downstream of the absorbing medium. Thus there is no fundamental limit on the sensitivity of such an absorption experiment, a part from the noise added by the absorption process itself.

11. Conclusion

Light can be described by quantum mechanics in terms of the expectation values of two conjugate operators, or quadrature phases. Coherent light, such as the light emitted by a laser, typically has a symmetric distribution of fluctuations in these quadratures. These fluctuations give rise to quantum noise in ordinary photodetection, limiting the signal to noise ratio (SNR) achievable. To measure the intensity requires the absorption of photons on the detector, thus reducing the SNR for the remaining light, and this is a manifestation of the back action introduced by the measurement process on a quantum mechanical system. However, recent experiments have demonstrated techniques by which measurements can be made beyond the quantum noise limit as long as only one quadrature is of interest. Such QND measurements have been demonstrated in optical fibres, using the nonlinear Kerr effect, and in optical parametric amplifiers. Other experiments in solids and atoms are in preparation. So far only the transfer of modulation and of the quantum fluctuations from signal to probe have been demonstrated, and a sequence of QND measurements with undiminished SNR has not yet been demonstrated.

Apart from demonstrating the fundamental limits of measurement processes, these techniques will have increasing application in high sensitivity measurements. They will contribute to the development of ultra sensitive interferometers, for speculative projects such as the detection of gravitation waves or for technical applications such as improved navigation using more sensitive laser gyroscopes. In addition, the quantum noise limit will probably be reached in optical communications and in optical storage techniques. Thus these currently exotic measurement schemes discussed here might one day be as commonplace as lasers are now.

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