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Three Dimensional Arm Trajectories

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Abstract. Planar arm trajectories are characterized by a segmentation of the hand velocity profile and by a coupling between shape and speed. The question addressed in this paper is whether such coupling, observed in two dimensions, still holds in three dimensions. This matter was investigated experimentally by recording three dimensional "aimless" movements of the arm, particularly three dimensional scribbles, and the answer suggested by the experimental data is that only the "bending" of the trajectory is coupled with speed, whereas the "twisting" is independent of speed. The same behaviour was also found to characterize a computational model of trajectory formation which is based on the spatial composition of chains of planar strokes, overlapped in time.

1 Introduction

Arm trajectories have been studied by several authors, either in the horizontal plane or in the vertical plane (Denier van der Gon and Thuring, 1965; Kots et al., 1966; Teulings and Thomassen, 1979; Viviani and Terzuolo, 1980; Morasso, 1981; Prablanc et al., 1981; Abend et al., 1982; Morasso and Mussa Ivaldi, 1982, 1983; Lacquaniti and Soechting, 1982; Morasso, 1983). One of the most robust results which has been observed is the characteristic segmentation of the hand velocity profile and the coupling between shape and speed of the trajectories.

For planar trajectories, the notion of shape is captured by the behaviour of curvature along the trajectory: a marked bend involves a peak in the curvature profile, a straight course causes the curvature to go to zero, an inflection point is characterized by a zero-crossing of the curvature profile. For spatial trajectories, in addition to express the way in which the trajetory is "bent", we need an additional information which expresses the "twisting" of the trajectory, i.e. the way in which the local (osculating) plane of the trajectory changes its orientation: this information is conveyed by the profile of torsion. Therefore, in three dimensional space the notion of trajectory shape can be conveniently expressed by the curvature and the torsion profiles.

A question which is addressed in this paper is whether the coupling between shape and speed observed in 2D still holds in 3D. This question was investigated by recording 3D "aimless" movements, particularly three dimensional scribbles, and the answer suggested by the experimental data is that only the "bending" of the trajectory is coupled with speed, whereas the "twisting" is independent of speed.

The implications of such finding are interpreted in the framework of a computational model of trajectory formation, previously investigated for planar movements (Morasso and Mussa Ivaldi, 1982, 1983). The basic features of the model are preserved and the model is extended to the three dimensional case in a straightforward way. The behaviour of the model, tested by means of simulation, is congruent with the experimental results, with particular regard to the dissociation between "bending" and "twisting".

2 Experimental Methods

Four adult subjects (three males and one female) performed arm movements following a verbal command (such as "move forward up right", "make a downward curved movement on a transversal plane", "wrap around an horizontal axis", "perform random scribbles", etc.; on passing, simple experiments of this kind reveal the dramatic inadequacy of natural language to express movements and spatial relations).

The trajectories in space of the right had (all the subjects were right-handed) were recorded by means of two methods, a goniometric and a cinematographic method.

Recording Methods

The goniometric method uses the mechanical apparatus of Fig. 1 (top). It consists essentially of a lightweight articulated structure which mimics the human arm. The subject sits in front of the mechanical arm, holding a small support attached at the "structure's wrist", so that the mechanical arm follows the movement of the hand opposing a small inertial resistance in a working volume of about 40 cm in diameter. Friction is kept low by using ball bearings at the joints. The mechanical arm has six degree of freedom (three at the shoulder, one at the elbow, two at the wrist) but only the first four (shoulder and elbow) were monitored by means of low friction, high linearity, conductive plastic potentiometers. The redundancy of the structure helps in keeping as small as possible the constraint imposed on the subject by the recording apparatus. The angular readings coming from the four potentiometers were sampled at the rate of 50 samples per second; the trajectory of the hand in space was then reconstructed by means of trigonometric calculations on a PDP 11-24 minicomputer.

The cinematographic method has been reported elsewhere (Morasso and Tagliasco, 1983) and it is based on the use of mirrors for obtaining multiple perspective views of the moving body. Figure 1 (bottom) shows the experimental setup. The movements are recorded on film and later computer processed by projecting each frame on a digitizing tablet and by digitizing the position of the same body part (e.g. the fingertip of the right hand) coming from the different perspective views. Complex inverse perspective calculations allow to reconstruct the trajectories of the hand. The method is rather awkward to use, due to the manual digitization of points in the current implementation, but it has the great advantage of complete freedom of movement for the subjects.

In practice, the greatest part of the analyzed movements were recorded by means of the potentiometric method and the cinematographic recordings were intended to test whether or not the constraints of the mechanical arm significantly altered the computed kinematic patterns. However, no significant differences were observed between the two recordings and it can be concluded that the mechanical recording apparatus was adequate for the experimental paradigms.

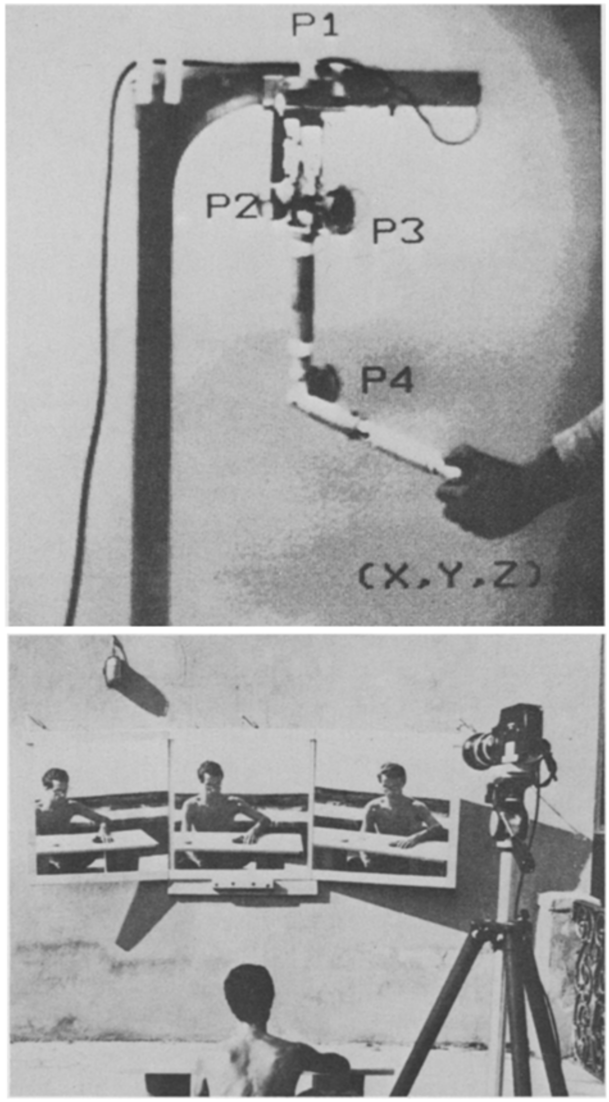

Fig. 1. Hand trajectory recording methods. *Top:* Goniometric method ($P1$, $P2$, $P3$, $P4$: potentiometers; $P1$, $P2$, $P3$ measure the Euler angles of the rotation between a Cartesian frame attached to the mechanical "humerus" and the environment frame; $P4$ measures the flexion/extension at the mechanical "elbow"; the Cartesian coordinates X, Y, Z of the handle are estimated by means of trigonometric computations). *Bottom:* Cinematographic method (multiple perspective views are obtained by means of mirrors and the movements are filmed on a 16 mm camera; trajectories of the hand are reconstructed by solving inverse perspective transformations)

Computer Graphics

The reconstructed trajectories of the hand were visualized by means of a 3D computer graphics program (3Dshow), which is based on the "synthetic camera" approach, using a raster graphic system VDS-701 with a resolution of $512 \times 512 \times 8$ pixels. The display screen is associated with the light sensitive film of a photographic camera and the center of projection is a point on the normal to the screen (which intersects its center) whose distance from the screen is equivalent to the focal length of the objective of a photographic camera. The program allows to choose the point of view (i.e. the position and orientation of the screen with respect to the coordinate system of the data) and to zoom in and out (changing the distance of the center of projection from the screen).

The samples of the trajectory are represented by spheres of constant radius ("necklace" representation), therefore the apparent size of the displayed circles encodes the distance from the observer; it is also possible to visualize horizontal or vertical networks of spheres to improve the three-dimensionality of the presentation. The visibility problem (elimination of hidden surfaces) was solved by means of the Z-buffer method, i.e. by storing, for each pixel of the image, its distance from the projecting object and by choosing, in case of overlap, the point closer to the observer.

Data Analysis

After reconstruction of the hand trajectory (represented by its Cartesian coordinates over time) the ensuing data analysis consisted of computing the time course of (i) velocity, (ii) curvature, (iii) torsion, (iv) binormal vector (represented by its Cartesian coordinates). Elementary notions of differential geometry of curves (Rektorys, 1968) are useful and, in particular, the Frenet-Serret formulae are the basis of the implemented computational procedures. A few concepts and the notation adopted in this paper are summarized here.

Let us denote by $\mathbf{r}=\mathbf{r}(t)=(x(t), y(t), z(t))$ a vector function of time in 3D Euclidean space which interpolates the recorded samples of a trajectory. The trajectory is bent and twisted in space and for each time instant a natural system of coordinates can be attached to it which consists of the tangent unit-vector $\mathbf{t}$, the normal unit-vector $\mathbf{n}$, and the binormal unit-vector $\mathbf{b}$: $\mathbf{t}$ points in the direction of movement, $\mathbf{n}$ points toward the concave side of the trajectory, and $\mathbf{b}$ is normal to the plane which locally contains the trajectory (the osculating plane). The three unit-vectors identify an orthogonal trihedron which is called the "moving trihedron" and whose motion is determined by the shape of the trajectory: it slides along $\mathbf{t}$, it rotates around $\mathbf{b}$ according to the bending of the trajectory, it rotates around $\mathbf{t}$ according to the twisting of the trajectory. These two rotation rates are measured by the curvature K_1 and by the torsion K_2, respectively (also called first and second curvatures) which have the dimension of the inverse of a length (i.e. the inverse of a radius of curvature). More concisely, the characteristics of spatial trajectories discussed above are expressed by the Serret-Frenet formulae:

$$\begin{aligned} \dot{\mathbf{t}} &= +K_1 v\mathbf{n} \\ \dot{\mathbf{n}} &= -K_1 v\mathbf{t} + K_2 v\mathbf{b} \\ \dot{\mathbf{b}} &= -K_2 v\mathbf{n}, \end{aligned} \tag{1}$$

where $v=\|\dot{\mathbf{r}}\|$ is the speed. Another set of equivalent formulae, more useful computationally, is the following one:

$$\begin{aligned} \mathbf{t} &= \dot{\mathbf{r}}/v \\ K_1\mathbf{b} &= (\dot{\mathbf{r}} \times \ddot{\mathbf{r}})/v^3 \\ \mathbf{n} &= \mathbf{b} \times \mathbf{t} \\ K_2 &= \dot{\mathbf{r}} \cdot (\ddot{\mathbf{r}} \times \dddot{\mathbf{r}})/v^6 K_1^2 . \end{aligned} \tag{2}$$

A least square polynomial approximation was used to estimate $\dot{\mathbf{r}}(t)$ from $\mathbf{r}(t)$; the same routine was also used to derive $\ddot{\mathbf{r}}(t)$ from $\dot{\mathbf{r}}(t)$, $\dddot{\mathbf{r}}(t)$ from $\ddot{\mathbf{r}}(t)$, and $\dot{\mathbf{b}}(t)$ form $\mathbf{b}(t)$. The velocity profile is obtained immediately from $\dot{\mathbf{r}}(t)$ and the curvature profile is obtained by applying the second formula of (2), which also gives the time course of $\mathbf{b}$. Finally, the torsion profile can be computed by applying either the third of (1) or the fourth of (4): both were tried with equivalent results.

It is also worth noting that the velocity, curvature, and torsion profiles ($v(t)$, $K_1(t)$, $K_2(t)$) give a complete representation of spatial curves (known as "intrinsic representation") which is invariant with respect to the system of reference. From the "intrinsic representation" ($v=v(t)$, $K_1=K_1(t)$, $K_2=K_2(t)$) it is possible to obtain the parametric representation ($x=x(t)$, $y=y(t)$, $z=z(t)$) by integrating equations (1), while the opposite transformation is accomplished by using (2).

3 Experimental Results

Point-to-point movements at natural speed produce approximately straight trajectories with bell-shaped velocity profiles, exhibiting characteristics similar to planar movements (Fig. 2, left) and such similarity also holds for voluntarily curved point-to-point movements (Fig. 2, right). In particular (i) the duration of curved movements' is consistently longer than that of straight movements (about 50% for all the subjects), (ii) the velocity profiles tend to become skewed and/or segmented, (iii) inflection points or points of minimum of the velocity profile are always associated with a simultaneous peak of curvature. With regard to the spatial behavior, point to point curved movements are essentially planar (even if the subjects were not required to do so) as it can be seen by looking at the binormal vector and by taking into account only those portions of the movements which occur at sufficiently high speed[1].

1 At slow speed, or at posture, curvature and torsion become quite noisy. However, even if we take a velocity threshold of 50% of the peak velocity, more than 2/3 of the trajectory occurs at a higher speed than the threshold

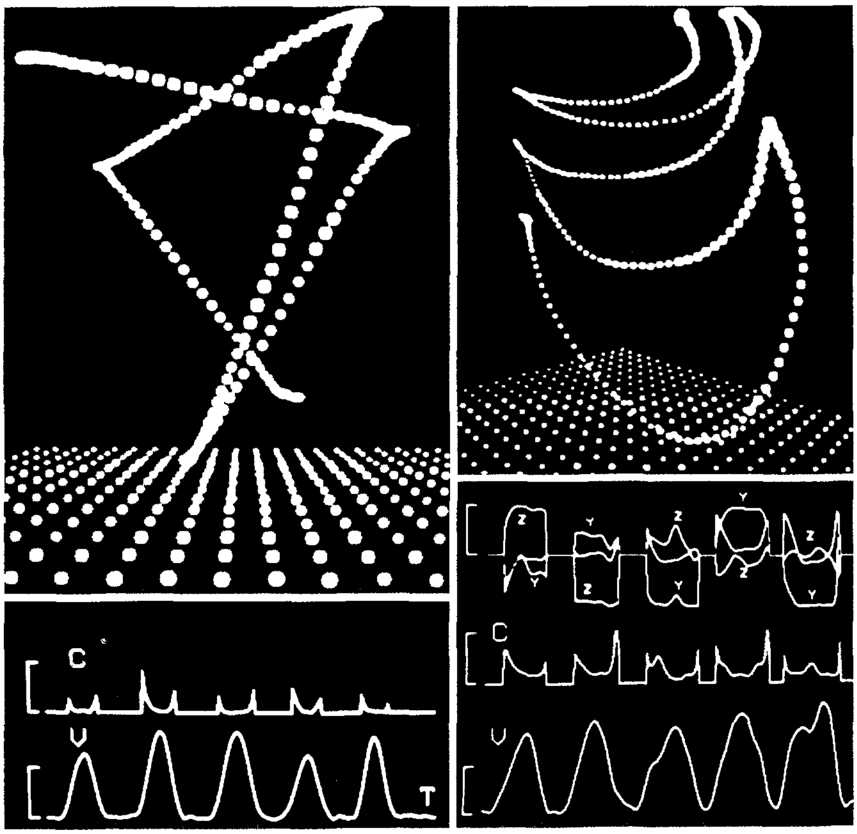

Fig. 2. *Left:* Pointing movements; *right:* curved movements. Spheres diameter: 2 mm. V: tangential velocity (calibration: 200 mm/s). C : curvature (calibration: 0.03 1/mm). x, y, z: Cartesian components of the binormal vector (calibration: 1). Duration of the displayed movements: 8 s

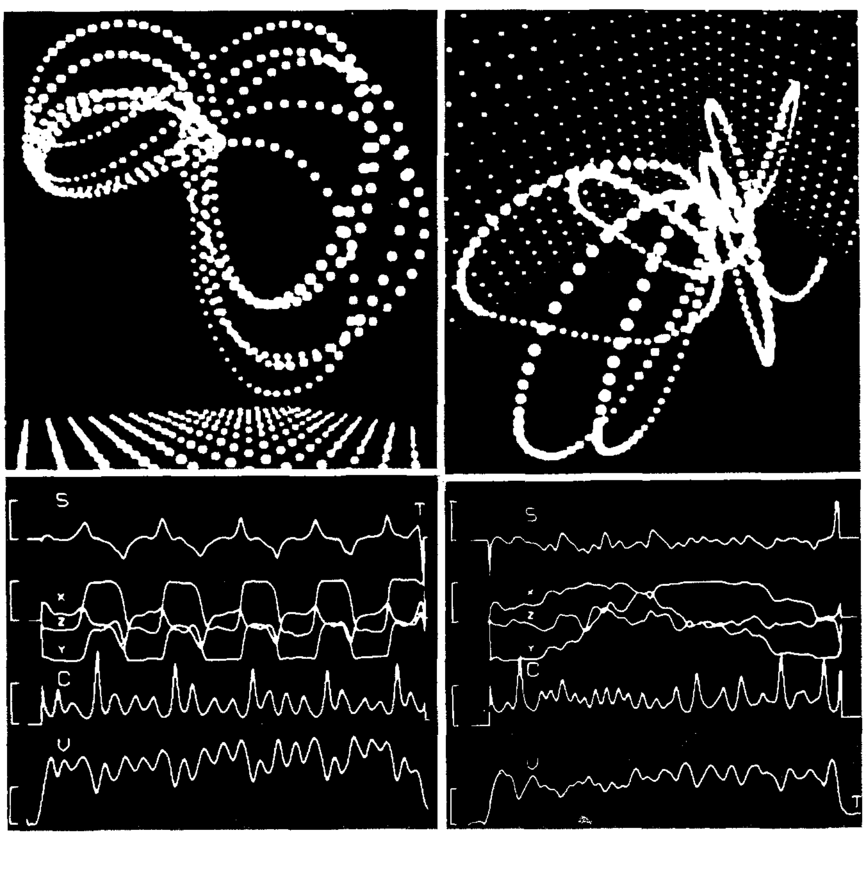

Fig. 3. Wrapping movements. Spheres diameter: 3 mm. V: tangential velocity (calibration: 200 mm/s). C: curvature. S: torsion (calibration: 0.03 1/mm). x, y, z: Cartesian components of the binormal vector (calibration: 1). Duration of the displayed movements: 8 s

For example, the first two movements of Fig. 2 (right) develop approximately in the horizontal plane (the predominant component of the binormal vector is z, its sign indicating the sense of rotation), whereas the other three movements develop approximately in a vertical plane.

More articulated movements were recorded by asking the subjects to mimic some usual manual activity. For example, Fig. 3 shows two "wrapping" movements: the former one alternates between two planes roughly at 90° and the latter is characterized by a slow change of direction. The coupling between the velocity and curvature profiles is quite consistent also for these types of movements, irrespective of the torsion profile. For the first type of movements, the trajectories are essentially planar and the twisting concentrates at the plane transitions, whereas for the slow wrapping small twisting bursts occur predominantly in one direction. In any case, the data show that the torsion profile has segmented characteristics which mirror those of velocity and curvature; however, the phase relationship of twisting with respect to bending and speed is far from obvious. In particular, these movements show how strongly the behaviour of torsion depends on the particular type of three dimensional pattern and then it seems convenient to record "general" movements in which three dimensional features occur in many directions, with different orientations, and at different size scales.

Three dimensional scribbles have such "general" characteristics and the quantitative analysis was focused on them in order to determine the relation between speed, curvature, and torsion profiles. Fifty scribbles were collected, each 10 s in duration; Fig. 4 shows one scribble for each subject. Their performance is characterized by distinctive features, such as the average speed or the ratio between average curvature and average torsion. The latter parameter is particularly significant because it expresses the degree of "three-dimensionality" of the trajectory formation process. The Table 1 lists this parameters for the four subjects:

Table 1

Subject	Average curvat./average torsion
A	2.84
B	1.82
C	1.58
D	0.84

As it can be seen, in the first three subjects bending predominates over twisting and in the last one the opposite occurs. However, more than in the inter-subject differences, which may be related to high control levels, the interest was focused on common

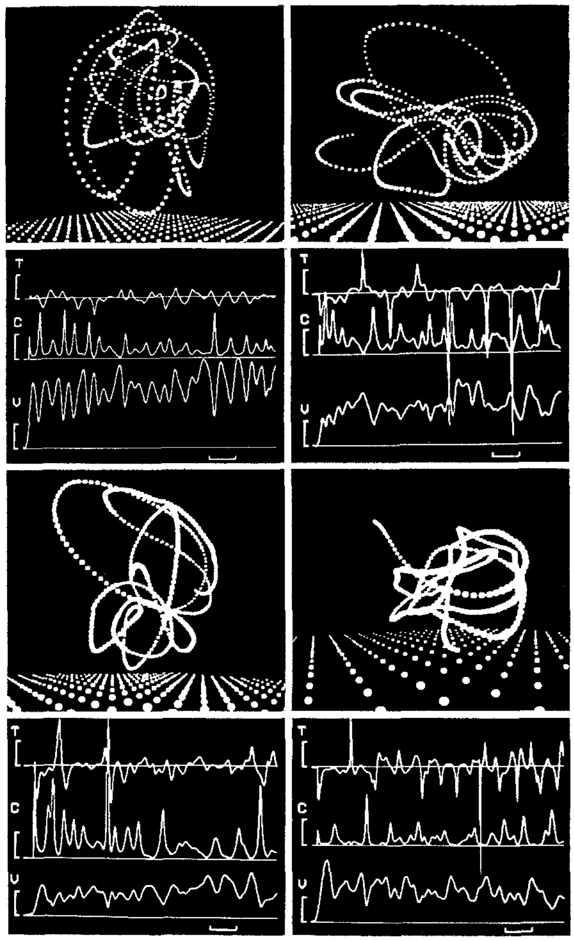

Fig. 4. Three dimensional scribbles (Subject A: top-left; Subject B: top-right; Subject C: bottom-left; Subject D: bottom-right). Spheres diameter: 3 mm (Subjects A–C), 2 mm (Subject D). V: tangential velocity (calibration: 200 mm/s Subjects A–C; 125 mm/s Subject D). C: curvature, T: torsion (calibration: 0.03 1/mm Subjects A–C; 0.1 1/mm Subject D). Horizontal calibration: 1 s

characteristics of geometric-kinematic patterns, which may reveal some basic process of trajectory formation.

In fact, the segmented nature of the velocity profile suggests that the trajectory formation process may result from a sequence of discrete motor commands and from a smoothing mechanism. If we associate a motor command with each peak of the velocity profile, one may wonder which is the distribution of delays between velocity peaks and peaks of curvature or of torsion. A normalized delay can be defined considering, for each peak of curvature or torsion, the two adjacent velocity peaks; in detail, if t_1 is the time instant of peak curvature/torsion and t_0, t_2 are the time instants of the two adjacent velocity peaks, the normalized delay d (which ranges from 0 to 1) can be defined as

$$d=(t_1-t_0)/(t_2-t_0).$$

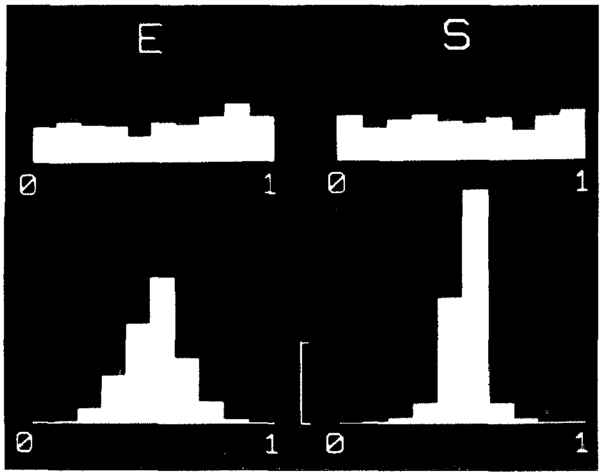

Fig. 5. Histograms of normalized curvature delays (bottom) and torsion delays (top). E: experimental data ($N=950$). S: simulated data ($N=280$). Calibration: 20%

Normalized curvature and torsion delays were computed for all the recorded scribbles. Significant inter-subject differences were not found, but a reliable difference was found between the distribution of curvature delays and the distribution of torsion delays: the curvature delays tend to cluster around the 0.5 value, whereas the torsion delays are distributed roughly in a uniform way over all the interval [see the histograms of Fig. 5 (left) which accumulate the data from all the subjects].

In other words, the times of peak curvature tend to occur midway between two consecutive velocity peaks, whereas the times of peak torsion may occur anywhere.

4 Simulation Results

The dissociation between speed and torsion, in contrast with the association between speed and curvature, is not due to any physical law and can only be related to the structure of the trajectory formation process. In a previous paper, a computational model of trajectory formation was proposed with regard to planar movements. This model can be extended to the three dimensional case and one may enquire whether or not the extended model accounts for the observed delay patterns between speeding, bending, and twisting.

The planar model is based on the notion of stroke:

$$\mathbf{s}=\mathbf{s}(t)$$

which represents, in parametric terms, a curved segment with a bell shaped velocity profile: it is characterized by geometric parameters (length, direction, curvature or angular change) and by timing parameters (duration and velocity profile). Complex trajectories can be composed by chaining sequences of strokes and by overlapping them in time:

$$\mathbf{r}=\mathbf{r}(t)=\mathbf{s}_1(t-t_1)+\mathbf{s}_2(t-t_2)+\ldots+\mathbf{s}_n(t-t_n),$$

where $\mathbf{r}(t)$ is the trajectory and $t_1, t_2, \ldots t_n$ are the activation delays of the subsequent strokes.

Such model can be extended to the three dimensional-case by keeping the same composition rule, which determines the characteristic segmentation of the velocity profile, and by allowing the same strokes, i.e. the same planar curved segments, to be oriented anywhere in space. This means that we must specify two unit-vectors for each stroke: (i) a vector which points in the direction of the movement, (ii) a vector which identifies the plane of the stroke (e.g. the binormal vector).

In order to compare the performance of the proposed model with the experimental results, simulated scribbles were computed by choosing randomly the geometric parameters (length, angular change, direction vector, binormal vector) and by collecting several simulation runs (Fig. 6).

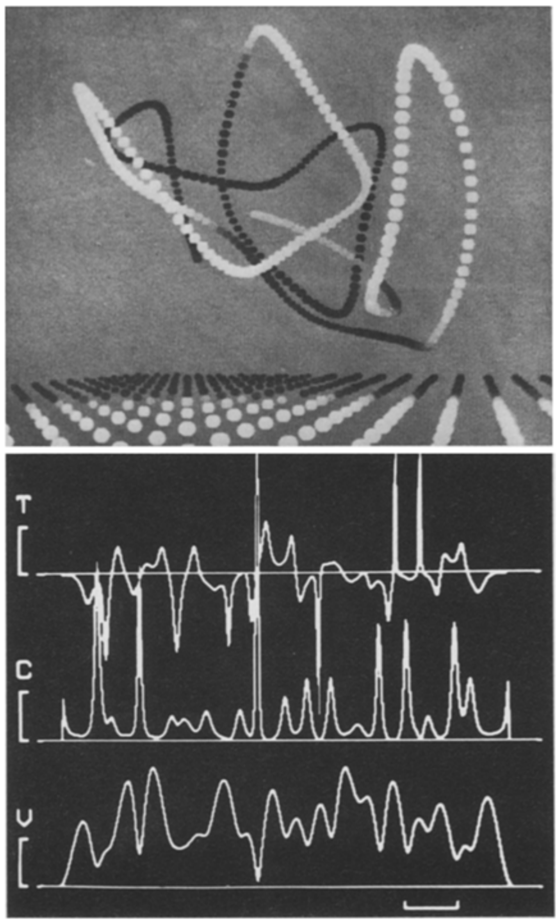

Fig. 6. Simulated three dimensional scribble. Spheres diameter: 3 mm. V: tangential velocity (calibration: 200 mm/s). C: curvature, T: torsion (calibration: 0.03 1/mm). Horizontal calibration: 1 s

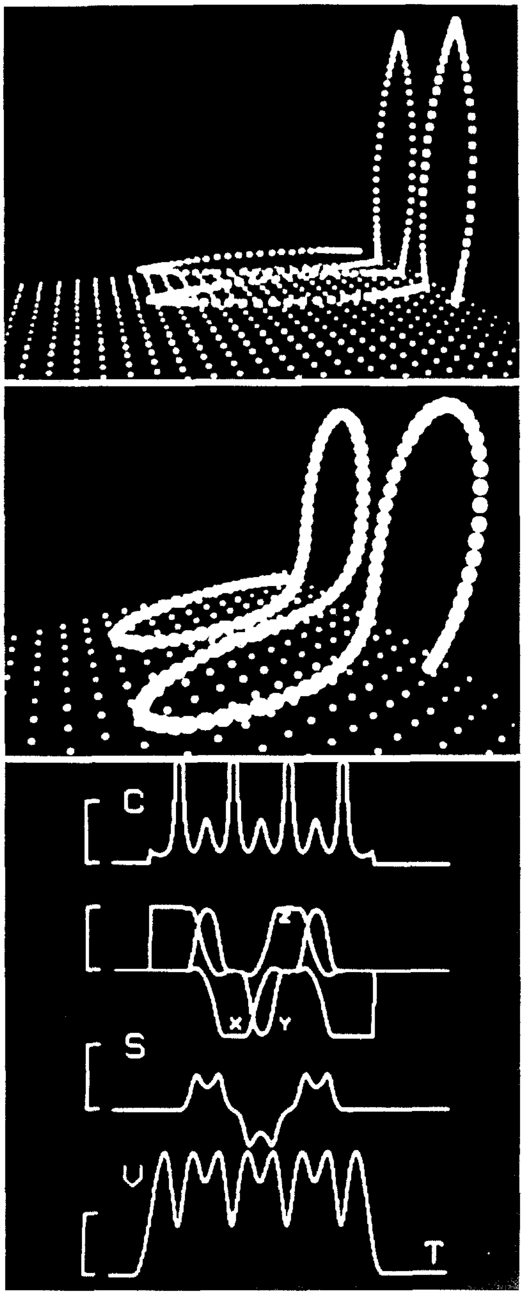

Fig. 7. Simulated regular repeating pattern. *Top:* Chain of strokes; *middle:* resulting trajectory. Spheres diameter: 3 mm. *V*: tangential velocity (calibration: 200 mm/s). *C*: curvature, *T*: torsion (calibration: 0.03 1/mm). *x*, *y*, *z*: Cartesian components of the binormal vector (calibration: 1). Horizontal calibration: 1 s

The simulated scribbles were processed in a similar way to the human-made scribbles and the normalized curvature and torsion delays were computed, deriving then the histograms of Fig. 5 (right). As it can be seen, the dissociation between speed and curvature holds for the simulated scribbles as well as for the recorded ones.

Figure 7 shows the performance of the trajectory formation model for a regular repeating pattern: the simulated trajectory is shown together with the underlying chain of strokes.

5 Discussion

If we consider general continuous spatial curves, curvature and torsion can vary independently within some range of frequencies related to the bandwidth of the generation process.

Human arm trajectories exhibit a substantial independence of curvature and torsion, which qualifies the human trajectory formation process as general. However, the stereotyped segmentation of the velocity profile and its coupling with curvature reveals the composite nature of the generation process and points to the planarity of the primitive movements (strokes), as it is confirmed by the simulation results.

Three-dimensionality is achieved, according to the model, by a three dimensional composition of planar strokes which have stereotyped kinematic characteristics (a bell-shaped velocity profile): in this situation, the curvature is bound to peak approximately midway between one stroke and the next one, whereas torsion can peak anywhere, according to the particular direction and orientation of subsequent strokes. A composition mechanism of this kind suggests then an internal representation of shape (motor shape, in this case) which, in principle, can be the basis for visuomotor integration and for spatial reasoning, either in humans or machines.

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