SHORT COMMUNICATION

ALGEBRAIC WORD PROBLEMS: ROLE OF LINGUISTIC AND STRUCTURAL VARIABLES

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I. INTRODUCTION

No adequate theory of instruction suitable for constructing instructional programs in problem solving has yet been created. One of the reasons for this is the fact that despite intense investigation, much about the sources of problems difficulty still remains to be learned. However, there are a number of studies that examine the effect that problem variables may have on the difficulty of word problems in arithmetic. A basic assumption of this approach is that the structure of the problem itself, to a large measure determines its difficulty level. It was assumed that the relative difficulty of word problems in arithmetic could be determined by analyzing the construction of the problem statement itself. Most of such variables fall into one of the two following groups: (1) those that describe the textual statement of the problems, and (2) those that describe a standard solution algorithm for the problem. A more detailed sets of variables for arithmetic word problems are presented, among others, in Suppes, Loftus and Jerman (1969); Jerman and Rees (1972); Jerman and Mirman (1974).

The main purpose of this paper is to report the attempts made to identify and define variables that affect performance of studies on algebraic word problems.

If arithmetic word problems are solved using one or more arithmetic operations, then algebraic word problems demand the use of formulae or equations for translating the problem situation from natural language to the corresponding mathematical expression.

The variables considered were many, but the variables defined and used for the arithmetic problems in above mentioned studies were examined. It was decided to concentrate on defining variables which describe the textual statements of the problem. In order to describe a solution algorithm for the problem several addtional variables were formulated.

II. THE VARIABLES

The 31 variables used in the study were divided into two categories – linguistic and structural ones. The definitions of all the variables are given below.

Educational Studies in Mathematics **21**: 83–90, 1990.

Linguistic Variables

X1 – Literals: The number of all literals in the problem.
X2 – Letters: The number of letters in the problem.
X3 – Number of Words A: The number of single words in the problem; numbers and units were not counted.
X4 – Number of Words B: The number of words, numerals and units in the problem statement. Each numeral or unit was counted as a single work.
X5 – Mean Word Length: The ratio of X2 to X3.
X6 – Words with 6 or More Letters: The number of words with 6 or more letters. (Numerals and units were not counted.)
X7 – Words with 9 or More Letters.
X8 – Words with 12 or More Letters.
X9 – Proportion of Words with 6 or More Letters: The ratio of X6 to X3.
X10 – Proportion of Words with 9 or More Letters: The ratio of X7 to X3.
X11 – Proportion of Words with 12 or More Letters: The ratio of X8 to X3.
X12 – Sentences: The number of sentences in the problem.
X13 – Mean Sentence Length A: The ratio of X1 to X12.
X14 – Mean Sentence Length B: The ratio of X3 to X12.

Structural Variables

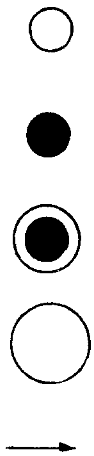
Before listing the structural variables used it is necessary to define the structural components of algebraic word problems themselves.

In a mathematical context the problem statement is nothing more than a presentation of particular quantities and a determination of their mathematical relations.

All quantities of the problem can be divided into three groups: *known*, *wanted* and *auxiliary quantities*. Known quantities – the data in the problem are the quantities whose values are presented in the problem statement. Wanted quantities – unknown quantities named in the problem statement, the determination of whose values is the aim of the solution. Auxiliary quantities – unknown quantities not named in the problem statement, but necessary for solving the problem.

The mathematical relations between quantities, determined by the formulation of the problem, find their expression in the solution of the *formulae* *used* and the *equations composed.* Using the previous elements the structure of every algebraic word problem can been analyzed.

To examine the logical structure of analyzed problems in deeper detail problem graphs were created and used. The graph representing all quantities and their relations works as the model of the structure of the problem. For constructing problem graphs the following notation was used:



– known quantities
– wanted quantities
– auxiliary quantities
} – graph node

– formulae or composed equations – graph vertex

– relations between quantities – graph arc

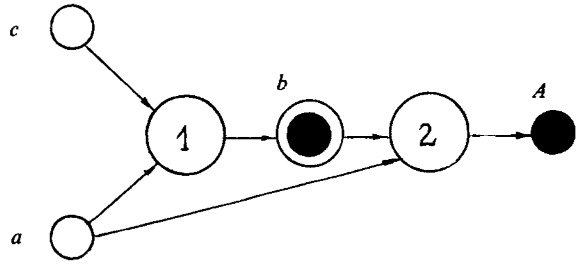
To make it clearer a sample problem together with its graph is presented below.

Sample Problem

A leg of a right triangle is $a = 4$; the hypotenuse is $c = 5$. Find the area of the triangle.

Used Quantities:

known	$a = 4$; $c = 5$
wanted	A (area)
auxiliary	b (another leg).



1. $b = \sqrt{c^2 - a^2}$

2. $A = a*b/2$

Problem Graph:

The use of graphs for modelling the logical structure of the problem enables us to define the number of variables used for describing graphs in graph theory terms.

Next all used variables of structure and their definitions are listed.

X15 – The Number of Known Quantities.
X16 – The Number of Wanted Quantities.
X17 – The Number of Auxiliary Quantities.
X18 – The Number of Unknown Quantities: X16 + X17.
X19 – The Total Number of Quantities: X15 + X18.
X20 – The Number of Formulae: The number of formulae required to apply in the solution of the problem.
X21 – The Number of Equations: The number of equations required to be composed in the solution of the problem.
X22 – The Number of Logical Operations: X20 + X21.
X23 – The Total Number of Structural Components: X19 + X22.
X24 – The Number of Relations Between Quantities: The number of graph arcs.
X25 – The Maximum Number of Relations for Each Quantity: The maximum number of arcs on the nodes of the problem graph.
X26 – The Maximum Number of Relations for Each Operation: The maximum number of arcs on the vertices of the problem graph.
X27 – Average Number of Relations for Each Quantity: The ratio of X24 to X19.
X28 – Average Number of Relations for Each Logical Operation: The ratio of X24 to X22.
X29 – The Rank of the Problem Graph: The ratio of the number of relations to the whole number of structural components: The ratio of X24 to X23.
X30 – The Number of Cycles in the Graph: The number of closed contours formed by arcs in the problem graph. For example, there exists one cycle: *a*-1-*b*-2-*a*, in the sample problem's graph presented above.

In addition to the variables listed above a variable integrating linguistic and structural features of the problem was defined:

X31 – Average Number of Words in the Problem Statement for Each Relation: The ratio of X3 to X24.

To determine the value of the listed variables a standard solution algorithm was written for each tested problem and according to it the problem graph was constructed.

III. SUBJECTS AND DESIGN

Students in five elementary schools from one class each in Grade 8 (age 13–15) were the subjects of this study. All students in each class took part in the study, the whole number of subjects was 150.

To obtain the problems for the study, 100 word problems were pilot tested in a group of 30 students (eighth graders) in order to determine the relative difficulty of the problems. It is important to note that all the problems used were similar to those presented in standard textbooks and solution methods of which had been drilled beforehand. 35 problems representative of their level of difficulty (in terms of percentage correct in the pilot study) were chosen. The chosen problems were distributed among 6 tests.

The problem tests were collectively presented on a typed sheet for students to solve individually. The problems were solved by students in a typical paper and pencil classroom setting. The numerical data in the problems were chosen so that a correct solution could easily be obtained without using a calculator. Students were instructed to do their work on the test sheets and to write down the time before starting and after finishing each problem. To record the subject's solving-time the electric digital timer was put in front of the classroom.

IV. RESULTS

Performance Measures

Two measures of performance were used to assess the solution:

1. the proportion of correct strategies
2. the average solving-time.

Correct strategies. Each solution carried out logically correctly, received one point, even if the final answer was incorrect. The aim of using such performance measure was to identify meaningful structural variables and not to concentrate on computational ones.

Solving time. This measure indicates the real solving time, the time actually spent in obtaining the solution.

Correlation Analysis

To study the influence of the defined problem variables on the proportion of correct strategies and the rate of solving, a correlation analysis was used.

Linguistic variables. As can be seen in Table I, the linguistic variables were not good predictors of the proportion correct. The single linguistic variable which reached significance was X31 – the average number of words for each relation ($r = 0.489$; $p < 0.01$). It is the variable which is describing the density of information in problem statement. Its positive correlation with the proportion correct seems to indicate the effect of a more detailed description of a problem situation on pupils' understanding of the underlying mathematical relations. According to the data of Jerman and Rees (1972) problem length variables were significant predictors of the proportion correct of arithmetic word problems. Surprisingly none of the length variables reached a significant level in this study. However, length variables proved to be significantly correlated with another performance measure – the problem solving time. As can be seen in Table I, the best predictor of solving time among the linguistic variables was X3 – the number of words in the problem statement ($r = 0.553$; $p < 0.01$). The significance level was

TABLE I

Correlation between problem variables and performance measures

No. of variable	Proportion correct	Solving time	No. of variable	Proportion correct	Solving time
X1	−0.052	0.468**	X15	0.095	0.378
X2	−0.064	0.451**	X16	0.082	0.617**
X3	−0.029	0.553**	X17	−0.544**	−0.075
X4	−0.012	0.550**	X18	−0.549**	0.320
X5	0.173	0.177	X19	−0.318	0.509**
X6	−0.163	−0.385*	X20	−0.290	−0.293
X7	0.082	−0.196	X21	−0.339*	0.514**
X8	−0.258	−0.272	X22	−0.547**	0.324
X9	−0.175	−0.427**	X23	−0.447**	0.477**
X10	0.074	−0.246	X24	−0.443**	0.515
X11	−0.163	−0.165	X25	−0.370*	0.473**
X12	−0.105	0.472**	X26	−0.074	0.575**
X13	0.051	0.247	X27	−0.402*	0.433**
X14	0.114	0.382*	X28	0.124	0.447**
X31	0.489**	−0.224	X29	−0.357*	0.507**
			X30	−0.357*	0.498**

* $p < 0.05$.
** $p < 0.01$.

also reached by the two long word variables: X7 – words with 9 or more letters ($r = -0.385$; $p < 0.05$), X10 – proportion of words with 9 or more letters ($r = -0.427$; $p < 0.01$). The observed negative correlation between long words count and solving time is an unexpected finding. Perhaps the point is that long words in the problems were principally mathematical terms (well known by students), which functioned as verbal cues for the solution and, in this way, decreased the solution time.

The results in this section can be summarized by the conclusion of Austin and Lee (1982) "an appropriate readability level would seem necessary but not sufficient for student success with world problems" (p. 285).

Structural variables. As can also be seen in Table I out of the 16 structural variables 14 reached significance at the 0.01 level. Thus, the structural variables used appeared to be good predictors of both performance measures used. The highest correlations with the proportion correct were with X22 – the total number of formulae and equations ($r = -0.547$; $p < 0.01$), also X17 – the number of auxiliary quantities needed to be used in the solution ($r = -0.544$; $p < 0.01$).

From the variables defined on the basis of the problem graph X24 – the number of relations between quantities reached significance at the 0.01 level ($r = -0.433$), while other such variables were significant at the 0.05 level (see Table I).

Solving time was positively correlated with the majority of structural variables. The best predictors were X16 – the number of wanted quantities ($r = 0.617$; $p < 0.01$), X26 – the maximum number of relations for each logical operation needed to be used ($r = 0.575$; $p < 0.01$) and X24 – the number of relations ($r = 0.515$; $p < 0.01$).

There were six variables which were good predictors for both performance measures used, namely X21 – the number of equations; X23 – the total number of quantities and operations; X24 – the number of relations; X25 – the maximum number of relations for each quantity; X29 – the rank of the problem graph; X30 – the number of cycles in the graph.

All these were structural variables, mostly defined on the basis of the problem graph.

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