OLE SKOVSMOSE

MATHEMATICAL EDUCATION AND DEMOCRACY

ABSTRACT. Is it possible to develop the content and form of mathematical education in such a way that it may serve as a tool of democratization in both school and society? This question is related to two different arguments. The social argument of democratization states: (1) Mathematics has an extensive range of applications, (2) because of its applications mathematics has a "society-shaping" function, and (3) in order to carry out democratic obligations and rights it is necessary to be able to identify the main principles of the development of society. The pedagogical argument of democratization states: (1) Mathematical education has a "hidden curriculum", (2) the "hidden curriculum" of mathematical education in a traditional form implants a servile attitude towards technological questions into a large number of students, and (3) we cannot expect any development of democratic competence in school unless the teaching-learning situation is based on a dialogue and unless the curriculum is not totally determined from outside the classroom.

The social argument implies that we must aim at "empowering material" which could constitute a basis for reflective knowledge i.e. knowledge about how to evaluate and criticize a mathematical model, while the pedagogical argument implies that we must aim at "open material" leaving space for decisions to be taken in the classroom.

Will it become possible to create materials at the same time open and empowering? To answer this question we have to analyse the concept 'democratic competence', which can be related to 'reflective knowledge' characterized by a specific object of knowledge and a specific way of knowledge production. The ultimate aim will be to unify these characteristics in an epistemological theory of mathematical education.[1]

1. INTRODUCTION

Democracy not only characterizes institutional structures of society concerning distributions of rights and obligations. Democracy has also to do with existence of a competence in society, and it is some of these non-institutional aspects of democracy we want to discuss in relation to mathematical education.

We concentrate upon democratic problems in a highly technological society, i.e. a society based on a total integration of, for instance, information technology. This integration seems to imply that decisions and discussions about change and development always must be related to a technological insight. That situation brings about what I conceive as the problem of democracy in a highly technological society. That means that I, as an example, have the situation in a country like Denmark in mind.

Several times it has been underlined that mathematical education has a political dimension (see Mellin-Olsen, 1987), and this thesis could,

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naturally, become further specified. We could argue that mathematical education, in a traditional setting, will favour a certain group of students; that mathematical education will make a strong stratification of the students; or that mathematical education will serve as an introduction to an ideology characterized by rationalism and objectivism (see Bishop, 1988).

My basic perspective will be that of critical education characterized by the key terms: critical competence, critical distance, and critical engagement. The concept critical competence stresses that the students must be involved in the control of the educational process. Both the students and the teacher must establish a critical distance to the content of the education: the seemingly objective and valuefree principles for the structuring of the curriculum must be investigated and evaluated. The education must be problem oriented, that means oriented towards a situation "outside" the classroom. This orientation implies that also the dimension of critical engagement must be involved in education (see Skovsmose, 1985). From that perspective I want to relate mathematical education to the concept of democracy, focusing on the democratic problem of a highly technological society.

The main problems are: To what degree is mathematical education involved in the process of building up (or reducing) a democratic competence in society? Is it possible to develop the content and form of mathematical education in such a way that it may serve as a tool of democratization? Or has mathematical education – perhaps because of its formal and abstract nature – nothing to do with such questions? Or is the situation even worse: are undemocratic tendencies served by introducing the pupils and students to unrelated bits of knowledge putting the teacher (and the book) in a special role of authority?

2. THE SOCIAL ARGUMENT OF DEMOCRATIZATION

It is possible to divide these general remarks into two types of arguments relating mathematical education and democratization. The first one is, for example, stated in Bjørneboe and Nissen (1984), and in Niss (1984). In short we summarize it as the social argument of democratization. The next argument, the pedagogical argument of democratization, will be summarized in section 3.

The social argument tries to identify a relevant subject matter of (mathematical) education via reflections on possibilities for building up and improving democratic institutions and democratic capabilities in society by improving the content of the education.

The social argument of democratization is composed of three statements:

1. Mathematics has a very extensive range of applications. Mathematics is applied in economics (macro-economics and micro-economics), in industrial planning, in different forms of management, in marketing, as well as in all traditional fields of application in technology. However, often it is difficult in both primary and secondary school teaching to present illustrative examples of real applications; too often examples show pseudo-applications. Real applications of mathematics are normally "hidden", although extensive and important.
2. Because of its applications mathematics has a "society-shaping" function. Mathematics constitutes an integrated and unique part of society. Mathematics cannot be replaced by any other tool serving similar functions. It is impossible to imagine a development of society of the type we know without technology playing a major role and with mathematics playing a dominant role in the formation of technology. So, mathematics has important implications for development and organization of society – although those implications are difficult to identify.
3. To make it possible to carry out democratic obligations and rights, it is necessary to be able to understand the main principles in the "mechanisms" of the development of society although they may be "hidden" and difficult to identify. Especially, we must be able to understand the functions of applications of mathematics. For instance, we must understand how decisions (economical, political, . . .) are influenced by mathematical model building processes.

The social argument of democratization highlights the applications of mathematics, and the importance of mathematical model building activity is in fact often emphasized in educational literature. The basic idea in what we may call the pragmatic trend in mathematical education is: It is most important for students to learn about model building, and the best way of learning this is to build models. The pragmatic trend builds upon a philosophical assumption about mathematics, stating that an essential feature of mathematics is its usefulness (quite contrary to structuralistic and formalistic philosophy, stating that the essential feature of mathematics is its "logical architecture"). Next, the pragmatic trend incorporates the assumption that the best way of learning is by doing; and especially: the best way of learning model building is to practice a model building activity.

However, my thesis is that fundamental problems concerning the application of mathematics are not visible from "inside" the modelling process.

That means, it is not possible to develop a critical attitude towards the application of mathematics solely by improving the modelling capability of students. Fundamental problems concerning modelling cannot be formulated in the conceptual framework that students develop through practical experiences. That means: the knowledge mentioned in the social argument of democratization (understanding the functions of the application of mathematics) is not normally developed in a pragmatic educational process. Therefore, an educational practice aiming at democratization by improving the students' possibilities for criticizing model building activities cannot be solely pragmatic. It is not sufficient to become a model builder. (Later, in section 5 and 6, we will try to investigate the epistemological background for this conclusion by relating and developing the concepts of democratic competence and critical and reflective knowledge.)

However, educational practice can anticipate some of these difficulties, and let me briefly sketch the content of *Beskæftigelsesmodellen i SMEC III*, written by Mogens Niss and Kirsten Hermann, a sort of mathematical textbook with upper secondary school in mind. The intention of the book is in accordance with the social argument of democratization. The authors find it important for students to learn about "real" model building activities, – not primarily to increase the motivation of the students (motivation seen as something emotional) nor to serve as an entrance to a piece of mathematical theory, but primarily to give students opportunities to investigate different details in a model which in fact has important social implications.

The "Simulation Model of the Economic Council" (SMEC) has been used by Danish economists when advising the government and politicians concerning economic policy and its possible consequences; and SMEC III is the third version. For instance SMEC relates the number of unemployed to different economic factors, some of which are possible to influence by political decisions. That means that SMEC can be used in forcasting possible consequences of different political strategies, and SMEC has in fact had an important role to play in setting economical policy in Denmark.

It is hard to understand how the economical factors are interrelated and difficult to see how these relationships are built into the mathematical model, but this is explained in the textbook.

However, to develop a more critical attitude towards this model building we have not only to understand the mathematical construction of the model; we have also to know about its assumptions. We must be able to point out which economical ideas are hiding behind the curtain of mathematical formulas.

It is impossible to start model building without assumptions. A special choice must be made about how to conceive the economic reality. Our conception of reality must be so structured that specific patterns could be identified: we have to select elements from reality which are to be conceived as important, and we have to decide which relationships among these elements are important. These two fundamental selections constitute an interpretation of "reality". A model is not a model of "reality" as such. It is a model of a conceptual system, created by a specific interpretation, based on a more or less elaborated theoretical framework, and based on some specific interests.

To identify the "society-shaping" function of SMEC we have to point out the models' selections of elements from reality and how it interrelates these elements. In SMEC the Cobb-Douglas function of production plays a crucial role. According to this we have: $Y = f(L, I)$, i.e., the national product Y is a function of the labour force L and capital investment I.

If we are going to discuss SMEC – and not only to investigate the relationships of its different equations – we must ask questions like: What assumptions have made it possible to create the conceptual system which forms the initial step in the modelling process? Could the assumptions be verified in any independent way? What theoretical frameworks are in accordance with these assumptions? What alternative theoretical frameworks do exist? And especially: What theoretical analysis supports the assumptions built into the Cobb-Douglas function of production? To develop a critical knowledge related to SMEC we have to ask questions that lead behind the assumptions in the Cobb-Douglas function. This is a necessary condition for understanding the political function of the use of the model; we have to trace its roots back into neoclassical economy. This could indicate some of the political functions of the use of the model.

The textbook *Beskæftigelsesmodellen i SMEC III* is of great help in the formulation of these questions. However, the use of such a textbook is not without problems. It is in fact very difficult for the students to understand the details in the text. The number of simplifications cannot be too great, because the intention of the textbook is to present a real model.

Let us summarize the main features of a teaching-learning material like *Beskætigelsesmodellen i SMEC III*, i.e. teaching-learning material which tries to be in accordance with the social argument of democratization:

1. The material has to do with a real mathematical model.
2. The model has to do with important social activities in society.

3. The material does in fact develop an understanding of the mathematical content of the model, but this, more technical, knowledge is not the goal. It is to develop an insight into the assumptions integrated into the model, and by this to develop an understanding of processes (for instance processes of decisions) in society.

Teaching-learning materials characterized like this we call *empowering teaching-learning materials*. However, the material need not take the shape of a specific textbook, and more generally we could talk about *empowering teaching-learning situations*.

3. THE PEDAGOGICAL ARGUMENT OF DEMOCRATIZATION

The social argument of democratization characterizes one aspect of the relationship betwen mathematical education and democratization. A second aspect could be summarized in the *pedagogical argument of democratization*. Here, the term "pedagogical" must be understood in a very broad way. It must be interpreted in relation to the term "social" in the previous argument; but now we look into the educational process (while the social argument looked outside the process). The argument consists of the following statements:

1. Students receive a variety of impressions during an educational process. Much has to do with the curriculum in question – although often the students do not learn what the teachers expect them to learn. There exists a very big gap between the matter taught and the matter learnt. Other parts of what is learnt have not much to do with the (official) curriculum but much in common with the structure of the educational process and with the traditions and rituals connected to the subject matter in question.
2. Also mathematical education has its "hidden curriculum". Often it is stipulated that mathematical education serves important functions in relation to the students' general epistemological development. It is emphasized that mathematical studies tend to improve the students' abilities to structure and to solve logical problems. However, the rituals of mathematical education take another direction. Students learn (also) to follow explicitly stated prescriptions: "Solve the equation . . . ", "Find the length of . . . ", "Calculate the value of . . . ", etc. This has not much in common with real processes of investigations or creative ways of structuring problems. It has much more in common with instructions and regulations with which lots of people in routine-like work processes are confronted. Mathematical education socializes

(also) into quite another direction than that optimistically presumed in "official" statements about the potential epistemological functions of mathematical education. It serves, too, a function by introducing the new labour force to lots of routine work in the technological society. Education, and especially mathematical education, implants an attitude towards technology. The students learn that some people are able to manage technological problems, and that some people are not. And, consequently, the "incapable" students learn to become servile towards technological questions and to become servile towards those who can manage.

3. The possibilities for carrying out democratic obligations and rights are not only related to formal institutionalized democratic structures but also to an individually consolidated democratic attitude. Democratic actions at the macro-level must be anticipated at the micro-level. That means we cannot expect a development of a democratic attitude if the school system does not contain democratic activities as the main element. If a democratic attitude is to be developed through mathematical education, the rituals of that education cannot contain fundamentally undemocratic features. The dialogue between teacher and students has to play an important role.

The pedagogical argument stresses that mathematical education socializes in quite other directions than expected. Some reasons could be related to the structuralistic movement in mathematical education characterized by the following assumptions: The essence of mathematics can be determined by crystallizing fundamental concepts through a logical analysis of existing mathematical theories, and these fundamental concepts can be conveyed to the learner by means of suitable concretizations in accordance with the epistemological powers of the child. Therefore, basic to structuralism is the idea that the students' knowledge has to be built up in accordance with structures and contents identified independently of the students. And one of the undemocratic features of education is implied by the exclusion of students from curriculum planning.

The possibility of an "experience based" mathematical education, an education including the total experience of students both in relation to curriculum planning and in relation to subject matter content, concerns the possible relationships between our ordinary language and constructed mathematical concepts. Are they "familiar" to each other? Does a close connection exist between language games (to use a term coined by Ludwig Wittgenstein) carrying a mathematical competence and the games of ordinary language? One possibly could be stated as: the *thesis of familiarity*: A very smooth and continuous transition exists between ordinary language and the conceptual structures of (school) mathematics.

Opposite to the thesis of familiarity we have the *thesis of dichotomy*: Ordinary language and the language of mathematics constitute two quite different and independent language games. Mathematical concepts are created in a special context, and educational planning is forced to relate the two language games to each other.

The thesis of dichotomy was emphasized by structuralism in the 60's. The fundamental act of curriculum planning was – via investigations of mathematical structures – to identify fundamental concepts, and on this basis to elaborate a detailed curriculum. Jean Dieudonné, who strongly influenced the reform programmes in the 60's, has pointed out: "... if the people responsible for drawing up school curricula may be persuaded to consult professional mathematicians in order to understand the relevance of their decisions to science as it is practiced in the university and beyond, we may yet witness one day some sensible teaching of mathematics from kindergarten to graduate school" (Dieudonné, 1973, p. 19).

Much progressive work in mathematical education is, now, related to the thesis of familiarity. The intention is to place the children, their interest, their work and their experimentation in the center of the educational practice and to eliminate undesirable aspects of the hidden curriculum. We find examples in materials developed by IOWO (Instituut voor de Ontwikkeling van het Wiskunde Onderwijs). The examples of educational practice presented in *Five Years IOWO* show the openness implied by the thesis of familiarity. The basic educational process becomes mathematizing, leading from the ordinary language games towards the more regular conceptual structures of the language of mathematics. No specific road has to be planned, because of the familiarity of the language games. I find this thesis crucial in case we want to draw educational implications from the pedagogical argument of democratization.

Further reflections related to the thesis of familiarity are found in the ideas of ethno-mathematics. Especially Ubiratan D'Ambrosio has tried to develop the theoretical foundations of ethno-mathematics. A social force behind the programme is found in the educational mistakes witnessed when introducing "new math" in Third World countries. An idea about a mathematical education adapted to the cultural situation of the country grew up. D'Ambrosio characterizes the situation like this: "While Western countries have science, technology and modern development and the very concept of progress implicit in their historical evolution, Third World countries have played a subsidiary role in this evolution, and transfer has been allowed or stimulated from Western powers to Third World countries in the measure they benefit.... All the rationale of the colonial enterprise, and the apparently distinct discourse of independence and development, are pieces of a game whose rules have been and still are dictated by the developed world" (D'Ambrosio, 1984, p. 32).

An important observation is that the well-structured mathematical curriculum could become an obstruction to learning activities. The structure of the curriculum perhaps incorporates undemocratic features as stated in the pedagogical argument about democratization. D'Ambrosio states it like this: "The 'learned' matheracy eliminates the so-called 'spontaneous' matheracy. An individual who manages perfectly well numbers, operations, geometric forms and notions, when facing a completely new and formal approach to the same facts and needs creates a psychological blockade which grows as a barrier between the different modes of numerical and geometrical thought" (D'Ambrosio, 1985, p. 472).

The main ideas guiding the ethno-mathematical project are: (1) It is possible to identify a fundamental but hidden mathematical competence in all different cultural settings. This competence manifests itself in different forms, for instance in craftsmanlike abilities. (2) This hidden mathematical competence could be made explicit as ethno-mathematics. (3) It is possible to develop mathematical education based on pre-established ethno-mathematical competence. This is underlined by D'Ambrosio in the following: "We have to learn their language, their logic, their history and their evolution, their science and their technology in order to be aware of their motives and ultimate goals.... But at the same time the mathematics in schools shall be such that it facilitates knowledge, understanding, incorporation and compatibilization of known and current popular practice into the curriculum. In other words, recognition and incorporation of ethno-mathematics into the curriculum" (D'Ambrosio, 1984, p. 32).

The ethno-mathematical approach incorporates the thesis that a continuous transition exists between ordinary language and conceptual structures of mathematics. Further, this approach stresses that traditional mathematical education often socializes in ways not presupposed in official educational planning; psychological blockades could be established. If we want to break down the hidden curriculum of mathematical education and prevent students from taking up a servile attitude towards technological questions, an educational strategy could perhaps be developed from the ethno-mathematical approach characterized by the thesis of familiarity and by opening the teaching-learning situation.

More specific ideas related to the educational practice could, as mentioned, be learned from the work of IOWO, and inspiration from IOWO is found in a great variety of school materials, some produced in Denmark. The intention is not to fix the learning situation, but to create opportunities for the students to make plans for their study activities. A nice example for primary school teaching is found in *Flyv med* ("Flying") written by Peter Bollerslev *et al.* The material is to be used for a shorter period, 2–3 weeks. The book has a short introduction to teachers, students and parents stressing that it is not necessary to build paper planes in mathematical lessons, but it is possible to do so. It will face us with a great variety of practical and fundamental mathematical questions. We have to start the production of paper planes. Lines and geometrical figures on paper. Is it possible to make new constructions? We have to test and to improve our models. We have to systematize the testing results. In the whole process the students have the possibility for making decisions and for being involved in the process of educational planning (although at the micro-level).

A crucial point is the refusal to use pre-structured and ready-made materials as developed in connection to the structuralistic movement. Instead one tries to create situations which could facilitate a mathematization. The idea of concretization – to be understood as an elementarization of abstract mathematical concepts – is rejected in favour of a mathematization. And we find an important asymmetry between these two sorts of activities. To concretize is to give more abstract terms a concrete interpretation and in this way to make them more comprehensible. The activity of concretization is reserved for the planners of the curriculum (compare the remark of Dieudonné), and as such this activity is removed from the educational process. To mathematize means, in principle, to formulate, to criticize, and to develop ways of understanding. Both students and teachers must be involved in the control of this process, which, therefore, could take the form of a more democratic one.

One of the assumptions underlying these ideas is precisely the thesis of familiarity, which is based on the pedagogical argument of democratization leading to the idea of teaching-learning materials like *Flyv med*, characterized as:

1. The material has to do with a topic of subjective relevance for the students.
2. The material initiates a variety of activities, not prestructured and fully fixed.

3. Several decisions have to be taken when involved in the teaching-learning process, and the decisions normally necessitate a discussion between teacher and students.

Teaching-learning materials and situations characterized like this we call *open teaching-learning materials* and *open situations*.

4. A CONTRADICTION

An implication of the pedagogical argument of democratization is that we have to develop open situations in the educational process, i.e. situations which could take different directions depending on the results of discussions between students and students, and between students and teacher. Opening the situation means to create possibilities for educational decisions to be taken in the classroom. Students must have the possibility to form the educational process if not to become adapted to unquestionable rituals of mathematical education. Another version of this implication is that we have to develop open teaching-learning materials, which could be used in a variety of situations. Open materials must not presuppose a specific teacher-students relationship or lay down implicit teaching-learning programmes.

However, we must not forget about the social argument of democratization. The implication of this is different from the above mentioned implications. According to the social argument students have to develop not only pragmatic knowledge about how to use mathematics and how to build up (simple) models but primarily knowledge about the preconditions for the model construction, and this knowledge must be aimed at an understanding of the social functions of applications of "grown up" mathematical models.

An implication of the social argument of democratization is that we have to develop empowering teaching-learning situations and materials, i.e. situations and materials which in fact give information about real mathematical models and their functions. If a critical distance to mathematical model building has to be created, if a "curriculum critique" (in German this key term is "Fachkritik") should be realized in education, and if we try to ask questions like "Who uses mathematical models? In which situations are they used? What knowledge-constituting interests are connected with the subject matter in question?" then our teaching-learning materials must include potentials for the establishment of a critical distance.

And now we have reached what I conceive as a main problem in todays' mathematical education. What relationships do in fact exist between a mathematical education aiming at developing the experiences of the students by opening the situation, by incorporating the decisions of the students into the educational process, and by all the time relating the language of mathematics to ordinary language games, and a mathematical education trying to develop not only a pragmatic but also a critical attitude towards the use of mathematical models?

The problem is that the social and the pedagogical arguments – although both related to the concept of democratization – are pointing in two different directions. Will it become possible to create materials and situations that are at the same time open and empowering? We cannot be sure that our educational enterprise is without contradictions. We face one problem when we succeed in developing open materials (like *Flyv med*): will it be possible to build up any critical knowledge when occupied in this project? And we face another problem when we succeed in developing empowering materials (like *Beskæftigelsesmodellen i SMEC III*): will it be possible to avoid too much pre-structuring of the situation, too much lecturing, to build up that complicated stock of factual knowledge obviously needed to understand the functions of a real model? Open material could result in open and democratic educational situations – but no empowerment is guaranteed; and empowering material could result in critical understanding – but no openness is guaranteed.

This contradiction has practical implications, but we will try to look at it from an epistemological point of view. Both the social and the pedagogical argument presuppose a concept of democratization and up till now we have used this concept without any further specification. Likewise, we have talked about a democratic competence without analysing the concept.

5. DEMOCRATIC COMPETENCE

Democracy characterizes a means of social control. An unsophisticated interpretation of 'democracy' states that the 'people' must be 'ruling'. However, already in ancient Greece the concept needed further interpretation: Who are the 'people'? Not only the rich people, but definitely not the slaves, was an answer. And what is the specific meaning of 'rule'?

If the 'people' means everybody, adult women and men, and 'ruling' means actually participating in the act of taking decisions, we have the idea of 'direct democracy'. In the continuous discussion about democracy this idea often is underlined as the ultimate aim, but at the same time as an utopia. For instance, Rousseau was arguing in favour of direct democracy.

If we give up the idea of direct democracy and try to find a more feasible interpretation, we will face the problem of "transformation of sovereignty": How is it possible to combine democracy with the necessity of selecting a small group of people to actually do the ruling? How is it possible to control the "people in charge"? The necessity of the transformation of sovereignty is implied by the fact that ruling presupposes specific qualifications that are not of common nature. The people in charge must have a specific knowledge about their ruling subject. Perhaps a specific education is needed. The people in charge must possess a competence including information and knowledge.

We have to make a distinction between the competence which the people in charge must possess if they are to be able to take well-founded decisions and act in an appropriate way, and the competence which is presupposed if we have to judge if the results and consequences of the ruling are acceptable. This makes obvious what is *the basic assumption in the classical interpretation of democracy*: While the ruling-competence of the people in charge is of special nature, the judging-competence is of common nature. The latter we also call democratic competence. In other words: democratic competence is a common capacity of human beings – but perhaps only a potential capacity because only a certain attitude will stress the importance of a democratic way of social control.

An epistemological way of arguing for the basic assumption behind the classical interpretation of democracy is the following: The judgement of an act carried out by the people in charge is a moral or ethical judgement, and ethics need no foundation in facts. An ethical statement is not implied by any conjunction of factual statements. Norms do not rest on any state of affairs, neither is an understanding of norms restricted to any small group possessing certain qualifications. To argue in favour of a normative statement only rationality is needed, and rationality is of a common nature.

This classical interpretation of democratic competence we may also call rationalistic or idealistic. This has much in common with classical rationalism in philosophy that refuses every sort of dogmatism. We need not believe any statement unless we have realized that it has to be so. No statement need to be conceived as true because of what authority tells. The only authority we have to believe is our own capacity of thinking.

In opposition to the classical interpretation we put a *materialistic interpretation of democracy*, which maintains that democratic competence in no way rests upon the inner nature of man. It is not a part of a common rationality. Instead the basic assumptions are: Democratic competence has to be developed. The competence is closely related to a democratic attitude, but they are not identical. The development of a democratic competence presupposes an attitude but in addition lots of knowledge and information about the domain of the democratic processes has to be developed. The content of knowledge and information built into a democratic competence is not anything fixed. It will change in accordance with the development of society. That means that democratic competence is a socially developed characteristic of the competence which people to be ruled must possess so they can be able to judge the acts of the people in charge. This competence will vary in accordance with the structures in society.

A specific variant of the materialistic interpretation sees the problem of developing a democratic competence as an educational problem. This point of view was especially stressed in German pedagogy after World War II.

Before we go into more details about the educational variant, it must be underlined that another change in the concept has taken place. Originally democracy has to do with ruling a society. More generally, democracy describes a sort of social control of an organization. The organization could be large or small; the people in charge could be a government as well as the leaders of a minor company. However, the problems of judging the acts of the people in charge will, from a logical point of view, be the same.

I agree with the educational variant of the materialistic interpretation of democratic competence, but it needs further specifications. I find that a certain school subject plays an important role in this context – namely mathematics. (This assumption has underlain what previously has been stated in the social and pedagogical argument of democratization.)

The argumentation for the specific role of mathematics needs to be developed. The materialistic interpretation stresses that the content of a democratic competence is determined by the structure of the "domain of the ruling". The content of the competence depends on the content of the fundamental characteristics of the organization in question.

For a determination of this characteristic the concept of technology plays a crucial role. The French philosopher of technology, Jacques Ellul, in his book, *The Technological Society*, stresses that technology has replaced nature as the environment of man. Technology must be conceived as a closed circle around man. In Ellul's words: "Technique has progressively mastered all the elements of civilization . . . man himself is overpowered by technique and becomes its object. The technique which takes man for its object thus becomes the center of society: this extraordinary event . . . is often designated of technical civilization. The terminology is exact and we must fully grasp its importance. Technical civilization is constructed by technique (makes a part of civilization only what belongs to technique), for technique (in that everything in this civilization must serve a technical end), and is exclusively technique (in that it excludes whatever is not technique or reduces to technical form)" (Ellul, 1964, pp. 127–128).

Apart from finding that technology completely closes around man, Ellul finds that the evolution of technology is fully determined by technology itself, and that society and the way of living is fully determined by this evolution. Society and technology are integrated, and technology becomes the dominating feature of civilization. This idea deals with power and power relations integrated into technological structures. The consequence is that all sorts of decisions concerning society or organisations in society also concern technology.

The relationship between types of technologies and sciences varies very much. Manual tools are developed quite independently of science. The development of steam engineering takes place parallel with the theoretical understanding of thermodynamical questions, while the information technology is totally based on development in mathematics. Mathematics is of absolute importance for the development of todays' technology. That mathematics constitutes a unique part of today's technological knowledge is especially stressed in *Mathematik als Technologie* edited by Jürgen Maass and Wolfgang Schlöglmann.

A society based on advanced technology faces a specific problem of democracy. If a society is based on manual tools the idealistic interpretation of democratic competence becomes plausible; no specific technological knowledge is needed to evaluate the acts and decisions of the people in charge. Quite the contrary occurs in a highly technological society. The content of democratic competence is rapidly changing towards a tremendous complexity. On the face of it only a limited group of people seem to be able to manage this complexity. In fact this competence seems to presuppose a certain amount of technological knowledge including mathematics. The consequences seem to be that only a limited group of people can obtain a democratic competence and then become able to evaluate the actions of the people in charge. This is the problem of democracy in a highly technology society.

6. EPISTEMOLOGY

Our analysis of the problem of democracy seems to imply that technological knowledge has to be developed at all levels in the educational system. And we have to enlarge mathematical education, as an integrated part of technology. In a highly technological society mathematical competence seems to constitute a main part of democratic competence.

However, from an epistemological point of view this conclusion is problematic. First of all we have to make a distinction between: technological knowledge, and reflective knowledge. It is my thesis that the type of knowledge we have to apply in developing technology is different from the knowledge necessary in analyzing and evaluating technological constructions. Making this distinction I disagree with an epistemology operating with one sort of (scientific) knowledge, as supposed by logical positivism. Also, I disagree with the assumption that a distinction can be made between making descriptions and prescriptions, founding descriptions on facts and prescriptions on emotions and private options. Descriptive knowledge must be incorporated into a well-based normative statement, and, in contrast, a pure descriptive statement is an illusion. The distinction between technological knowledge and reflective knowledge does not recapitulate this classical distinction.

Next, it is my thesis that democratic competence to a great extent is based on reflective knowledge. That means that although technology plays a crucial role in the formation of society it is not the technological knowledge as such which constitutes the democratic competence. Therefore, the main problems are: How are technological knowledge and reflective knowledge interrelated, although different? And how is it possible to develop reflective knowledge?

In fact the integration of mathematics into technology makes an additional distinction necessary. We have to do with three different types of knowledge related to a process of mathematical modelling:

1. Mathematical knowledge itself.
2. Technological knowledge, which in this context is knowledge about how to build and how to use a mathematical model. Also, we would call it pragmatic knowledge.
3. Reflective knowledge, to be interpreted as a more general conceptual framework, or metaknowledge, for discussing the nature of models and the criteria used in their constructions, applications and evaluations.

The conceptual framework for discussing and criticizing a mathematical model cannot be reduced to the framework gained from a modelling process. In short: reflective knowledge cannot be reduced to technological knowledge. It has a different nature. Reflective knowledge does not have its epistemological basis in technological and pragmatic knowledge. Neither can technological knowledge be reduced to mathematical knowledge, an idea which is expressed in an educational context as: when you learn mathematics you also learn how to apply it – an assumption strongly criticized by the pragmatic trend in mathematical education.

We have now to get a more precise understanding of the possible sources of reflective knowledge if we want to grasp the democratic problem in a highly technological society. In Skovsmose (1988b, 1989b) an attempt is made to outline a strategy for gaining a critical distance from a mathematical model. Two different perspectives are used. A model is analysed in its "synchronic" and in its "diachronic" relationships.

The main elements in the synchronic relationships are (1) the object for the modelling process, (2) a system mediating a connection between model and object, (3) a theory giving a perspective to create the mediating system, and (4) interests illuminating an additional perspective on the process of system development. The main elements in the diachronic relationships are (1) the problem identification, initiating the process of technological investigations, (2) the structure of argumentation, including the paradigmatic way of analyzing the problem, (3) the basis for critique, conceived as the group of persons for whom it is possible to influence the process of investigation, and (4) the class of possible consequences for actions. An understanding of what is going on in a modelling process presupposes an understanding of how the elements constituting the diachronic relationships of the model are influenced during the modelling process.

This analysis is an implicit argument for making the distinction between mathematical, technological and reflective knowledge. Let us try to make it explicit. If we make a distinction between different types of knowledge, a basis for the distinction making must be specified. One possibility is to look at the *object for the knowledge*. The empiricistic doctrine, that all knowledge is of the same nature, rests on the idea that all sort of knowledge has to do with the same sort of objects, namely sense-data. What is done in analyzing the synchronic and diachronic relationships of a model is to identify an object for the reflective knowledge different from the object of the technological knowledge. The object for the reflections is the model and its relationships: How is the system development influenced by basic interests? How is the structure of argumentation influenced by the complexity of the mathematical model? etc.

Instead of the object we could use the *process of the development of knowledge* in an attempt to argue for the existence of different types of knowledge. Looking at this process we can make a distinction between a *monological* and a *dialogical* interpretation. A classical monological theory of epistemology is empiricism. To obtain knowledge is seen as an individual process; it is not necessary to interact with others. The source of knowledge is the senses. Likewise the rationalistic epistemology is monological. Knowledge is produced by "individual thinkers" – the thinking must be clear and (mono)logical.

A modern variant of monologism is the genetic theory of epistemology, developed by Piaget, combining the empiristic and the rationalistic approach into a special theory of action. Reflections on the results of the individual's operation on objects moves the individual towards mathematical insight. Mathematics gets an empirical base in the operations on objects and a rationalistic base in the reflections on these operations.

Activity theory I see as a big step towards a dialogical epistemology, first of all stressing that we have to do something to obtain knowledge, and do it in a social context. Activity is not an isolated act.

In a dialogical epistemology the dialog and the discussion come to play a crucial role. The main idea is simple: My knowledge is inadequate, it can be improved. But you are in the same situation. To improve our understanding, to move in the direction of more knowledge, we depend on each other. I cannot tell you anything true, neither can you tell me anything. But if we interact in a dialogical relationship we will be able to move in the direction of more knowledge. The condition for obtaining knowledge is not that we get additional true information but that we interact in a unique way, characterized as a dialogical relationship. In his work on communicative actions Jürgen Habermas tries to point out some basic features of an "authentic" dialogue (not to be repeated here).

I do not find a dialogical epistemology to be of general validity. Some sorts of knowledge are not primarily based on dialogue, instead a strong interaction with nature could be needed, but some sorts are. And what I want to maintain is the dialogical nature of reflective knowledge.

The dialogical element in mathematical and technological knowledge is more casual. In fact in traditional mathematical teaching the students are "informed" by the teacher. Often, no features of an authentic dialogue are to be identified. This will be underlined by the following remarks of Paulo Freire: "Through dialogue, the teacher-of-the-students and the students-of-the-teacher cease to exist and a new term emerges: teacher-student with students-teachers. The teacher is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with students, who in their turn while being taught also teach. They become jointly responsible for a process in which all grow" (Freire, 1972a, p. 53).

To summarize my main points. Reflective knowledge is characterized by its complex object outlined in the analysis of its synchronic and diachronic relationships and by its dialogical nature (which has to be developed in detail in a dialogical epistemological theory).

This analysis separates reflective knowledge from mathematical and technological knowledge, and for instance it becomes obvious that pragmatism in mathematical education will be insufficient to establish a critical distance. New features of the educational process have to be developed (because of the dialogical nature of the processes behind reflective knowledge), and also a new content has to be developed (because of the special nature of the object for reflective knowledge).

Turning back to the social argument of democratization we find it problematic. It stresses, in the right way, that democratic competence cannot be constituted by certain (democratic) attitudes alone. But no attention is paid to the dialogical aspect of knowledge production. The pedagogical argument stresses that the way of production is important, opening the teaching-learning situation to make room for a dialogue is necessary. However, no way of specifying the content of the dialogue is outlined, and reflective knowledge is not created automatically in an open dialogue. Reflective knowledge is not without an object.

It is now possible to describe the epistemological task. We have to bring about an epistemological theory integrating an analysis of the dialogical way of production of knowledge with an analysis of the complexity of the object of reflective knowledge into an educational theory. And in parallel with this: open and empowering teaching-learning materials have to be developed to get as much educational experience as possible to guide the development of theory.

NOTE

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Department of Mathematics
and Computer Sciences,
Aalborg Universitetscenter,
Strandvejen 19,
DK 9000,
Aalborg,
Denmark.