Bull Volcanol (1994) 56:87–97

Bulletin of Volcanology

Simple stochastic modelling of the eruption history of a basaltic volcano: Nyamuragira, Zaire

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Received: July 20, 1993/Accepted: December 18, 1993

Abstract. Three simple models of the behaviour of a series of basaltic eruptions have been tested against the eruptive history of Nyamuragira. The data set contains the repose periods and the volumes of lava emitted in 22 eruptions since 1901. Model 1 is fully stochastic and eruptions of any volume with random repose intervals are possible. Models 2 and 3 are constrained by deterministic limits on the maximum capacity of the magma reservoir and on the lowest drainage level of the reservoir respectively. The method of testing these models involves (1) seeking change points in the time series to determine regimes of uniform magma supply rate, and (2) applying linear regression to these regimes, which for models 2 and 3 are the deterministic limits to those models. Two change points in the time series for Nyamuragira, in 1958 and 1980, were determined using a Kolmogorov-Smirnov technique. The latter change involved an increase in the magma supply rate by a factor of 2.5, from 0.55 to 1.37 $m^3 s^{-1}$. Model 2 provides the best fit to the behaviour of Nyamuragira with the ratio of variation explained by the model to total variation, R^2, being greater than 0.9 for all three regimes. This fit can be interpreted to mean that there is a deterministic limit to the elastic strength of the magma reservoir 4–8 km below the summit of the volcano.

Key words: Stochastic modelling – time series – basaltic volcano – lava flows – magma reservoir

Introduction

Volcanoes erupt variable amounts of magma at irregular intervals. A time series of eruptions from a single volcano can be treated as a stochastic process with individual eruptions as random events in time (Wickman 1966a, 1976). Statistical analyses of repose period catalogues have been performed for a large number of volcanoes (Wickman 1966b–e; Klein 1982; Mulargia et al. 1985; Ho 1990; Dubois and Cheminée 1991). Analyses of time series of eruption volumes have been fewer (e.g. Mulargia et al. 1985, 1987; De la Cruz-Reyna 1991). The ultimate purpose of such analyses should be to predict the future behaviour of the volcano or to interpret the parameters of the stochastic model that fits the observations in terms of the causative volcanological processes. Our purpose here is the latter.

In seismology, earthquake first-motion information forms a calibrated database from which frequency-magnitude statistics can be deduced and compared with physical models. In volcanology, however, there is no equivalent method of measuring energy-release statistics to provide rigorous time series catalogues. The complexities of magma interaction with surface water and atmosphere produce a great variety of eruptive phenomena that are difficult to analyse in time series form. For example, the repose periods between phreatomagmatic explosions (which may reflect the vagaries of groundwater, not magma rise) do not have the same significance as the repose periods between major plinian events. Wickman's original use of the International Association for Volcanology catalogues (Wickman 1966b–e) suffered in this regard, though he tried to address the problem by recognising different eruptive 'states' in a single eruptive history. Therefore some surface volcanic effects in event catalogues are largely a distraction from the principal process of the long-term rise of magma through the crust. What is needed is a measure of the energy budget of this process. The Volcanic Explosivity Index (Newhall and Self 1982) is a useful, though crude, surrogate for a direct measure of energy release (e.g. De la Cruz-Reyna 1991, 1993). This problem is ameliorated at basaltic volcanoes where effusive behaviour predominates over explosive activity. Estimates of basaltic lava flow volumes provide a good measure of the energy release of effusive eruptions. Also physical models of the rise of basaltic magma are simpler than those of more siliceous magmas which have a greater residence time in the crust and more complex chemical evolution.

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In this paper we present an analysis of a time series of repose periods and eruption volumes of Nyamuragira volcano in Zaire in terms of three, simple stochastic models. We emphasize the following aspects of our analysis:
(1) the data catalogue is checked to ensure uniformity;
(2) the physical significance of the models is made explicit; and
(3) the goodness-of-fit of the models to the eruption history is tested.

Nyamuragira is a good choice for stochastic modelling because it is an effusive, basaltic volcano with a detailed catalogue of recent activity. Moreover, most of that activity has been in the form of discrete flank eruptions. These are inherently easier to quantify as a series of eruption events than the quasi-continuous activity that often characterises the summit/central vent activity of some basaltic volcanoes (e.g. Etna).

Stochastic Mass Budget Models

The modelling approach we use is to treat the time series of eruptions as the consequence of a continuous rise of magma at constant mass flux perturbed by stochastic elements occurring in a magma reservoir that acts as a buffer. The depth of this reservoir needs to be within the mid to upper crust (<15 km depth) if the far-field tectonic stress needs to be invoked to explain the stochastic element. The assumption of a continuous rise of magma at a constant rate is contentious. Even for the best-studied volcano, Kilauea, the evidence is ambiguous and depends on the time-scale considered (Tilling and Dvorak 1993). The steady-state behaviour of many volcanoes suggests that the integrated (from source to surface) process of magma rise can be continuous over time-scales of 1–100 years (Wadge 1982). We return to this problem later. What is clear from volcanoes like Kilauea is that reservoirs do play a major role in buffering the process of magma rise to the surface. The time and place of rupture of the crustal reservoir are major determinants of the character of the resulting eruption. Factors controlling rupture events include: the strength of the country rocks, the variation with depth of the densities of magma and country rocks, the size, depth and shape of the reservoir, the anisotropy of the regional tectonic stress field and the local free surface and associated gravitational stress field (Parfitt et al. 1993; Gudmundsson 1986).

Assume for a given volcano that the volume of magma in a magma reservoir increases at a constant rate during a repose period until it reaches a critical value or threshold. At this point the reservoir ruptures and an eruption occurs, magma is discharged, the magma level in the reservoir falls to a fixed level and discharge stops, and the process starts another cycle. A very simple interpretation of this model could be that the magma reservoir fills to capacity, ruptures and drains to a lower level. Here the volume of magma in the reservoir both at eruption and after eruption is fixed, and hence the time between eruptions is constant and depends only upon the rate at which magma enters the reservoir and the difference between the two volumes. This model is deterministic and strictly periodic (unlike real volcanoes) but by introducing stochastic components the process can develop in different ways. Here we discuss three such situations as exemplified by the following models.

Model 1: Fully stochastic model

We assume in this, and the other models, that any component of the magma leaving the chamber and not reaching the surface as a lava flow has a constant volume and can be ignored. The time between eruptions (repose period) and lava output volume are random and thus the model does not depend on the threshold of final volumes in the reservoir. Also the reservoir can fill indefinitely and any amount of lava can be discharged. Hence the buffer capacity of the reservoir must be very large; of similar magnitude to that of the total capacity of the system.

Model 2: Pressure-cooker model

Rupture always occurs when the volume of magma in the reservoir reaches a threshold value. This volume is a function of the physical strength of the reservoir rocks and perhaps the general shape of the reservoir (Gudmundsson 1986). The volume of lava erupted each time is random, as is the final volume left in the reservoir. The source of the random nature of drainage volume could be minor changes to the reservoir shape as a result of the previous drainage event or external forcing by an independent tectonic stress history.

Model 3: Water-butt model

On eruption the magma reservoir always drains to the same level and the magma level remaining, the final volume, is constant. The threshold volume is random. A critical magma-draining pressure may explain the deterministic limit represented by the final volume. Explanations of the stochastic behaviour must also involve changes in the capacity of the reservoir or external forces. This model, unlike model 2, could accommodate periods of continuous, low effusion rate eruption, say from summit vents, when the reservoir has reached capacity.

Cumulative Volume Curves

The behaviour of the three models is best appreciated analytically by reference to curves of the cumulative volume of lava with time (Fig. 1). In models 2 and 3 the lower and upper limits to the curves represent respectively the non-stochastic elements of the two models.

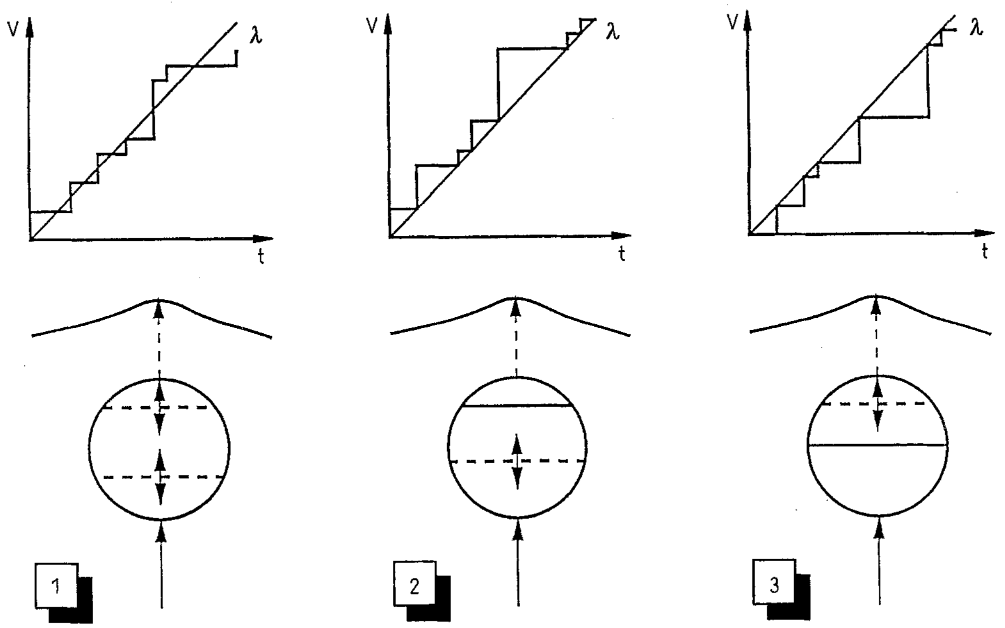

Fig. 1. Schematic sectional cartoon of three model volcano magma reservoirs (1–3). The *solid horizontal lines* in the reservoirs (*circles*) represent deterministic limits to reservoir behaviour. The *dashed horizontal lines with vertical arrows* represents stochastic limits to behaviour. The *graphs* above each model show the cumulative lava volume curves whose slope is given by the magma eruption rate which for these models represents the long-term magma supply rate (λ)

These limits are linear with a slope of λ, equal to the long-term magma supply rate. They can be described by the general equation:

$$V = \lambda t + \alpha$$

where V is cumulative volume, t is time and α is the intercept. We are interested in testing how well a time series of eruption volumes can be modelled in one of these three ways. The method of estimation of λ and α are different for each model but they can all be derived using simple regression techniques.

For a sequence of n eruptions let v_i be the observed volume of lava emitted at time t_i, $i = 1, n$ with $t_1 < t_2, \ldots < t_n$. Repose periods, or inter-event times, are defined as the time between the onset of two eruptions consistent with previous work (Klein 1982; Mulargia et al. 1987). The repose period following eruption i is given by:

$$r_i = t_{i+1} - t_i \qquad i = 1, n-1.$$

Since repose periods are times between eruptions there will always be one more volume than repose periods.

Fitting model 1. An assumption that is made for all the models is that the rate of lava being discharged is equal to the rate of magma entering the reservoir (ignoring any shallow intrusive component). A further assumption of this model is that the repose periods, r_i, are independent of the volumes, v_i. The best estimate of average output over time is then:

$$\hat{\lambda} = \frac{\sum_{i=1}^{n} v_i}{\sum_{i=1}^{n} r_i}$$

or:

$$\hat{\lambda} = \frac{\bar{v}}{\bar{r}}$$

The intercept α has no obvious physical interpretation and so an arbitrary method of estimation needs to be used. One way is to force the line through the mean values of cumulative volume, $\bar{V}$, and time, $\bar{t}$, so that:

$$\hat{\alpha} = \bar{V} - \hat{\lambda}\bar{t}$$

By using the mean values the problem of having different numbers of volumes and repose periods is overcome. The equation for estimated, or fitted, cumulative volume then becomes:

$$V_i = \hat{\lambda} t_i + \hat{\alpha}$$

Fitting model 2. For this model it is assumed that a random amount of lava is erupted. The time of the next eruption is then determined by the time required for the magma entering the reservoir to reach the threshold so that:

$$v_i = \lambda r_i + \epsilon_i \tag{1}$$

where the error term, ϵ_i, encompasses both measurement error and random variation. As in regression analysis we have assumed that the errors are occurring in the measurement of the response variable, v_i, rather than the explanatory variable, r_i. The justification for this is that the error in recording the time of the eruption is likely to be small compared to the error in the volume measurement. We have also assumed, for simplicity, that the errors are normally distributed with zero mean and constant variance σ^2, $\epsilon_i \sim N(0, \sigma^2)$. The cumulative volume, V, is given by:

$$
\begin{aligned}
V_i &= \sum_{j=1}^{i} v_j \\
&= \lambda \sum_{j=1}^{i} r_j + \sum_{j=1}^{i} \epsilon_j \\
&= \lambda t_{i+1} - \alpha + \sum_{j=1}^{i} \epsilon_j \qquad (2)
\end{aligned}
$$

where $\alpha = \lambda t_1$.

An estimate of λ can be found from (1) by fitting the regression line of v_i against r_i and forcing the fitted line through the origin. Then an estimate of α can be found from:

$$\hat{\alpha} = \hat{\lambda} t_1$$

The repose period following eruption n, r_n, is unknown since t_{n+1} is unknown. Hence the final volume, v_n, is omitted from the estimation of λ. The fitted cumulative volume can be found by substituting the estimates of α and λ into Eq. 2.

Fitting model 3. In this model the time between eruptions is random. The volume of lava erupted is then determined by the amount of magma which has entered the reservoir since the last eruption. As we assume that magma enters at a constant rate the volume of lava erupted is dependent on the length of the previous repose period:

$$v_i = \lambda r_{i-1} + \epsilon_i$$

Again, the error term is taken to be $\epsilon_i \sim N(0, \sigma^2)$ and an estimate of λ is found by regressing v_i on r_{i-1} with no intercept. The cumulative volume is given by:

$$
\begin{aligned}
V_i &= \sum_{j=1}^{i} v_j \\
&= \lambda \sum_{j=0}^{i-1} r_j + \sum_{j=1}^{i} \epsilon_j \\
&= \lambda t_i - \alpha + \sum_{j=1}^{i} \epsilon_j \qquad (3)
\end{aligned}
$$

where $\alpha = \lambda t_0$. Time t_0 is unknown but can be estimated from:

$$v_1 = \hat{\lambda} t_1 - \hat{\lambda} t_0 + \hat{\epsilon}_1$$

By assuming that $\hat{\epsilon}_1 = 0$ an estimate of t_0 is given by:

$$\hat{t}_0 = t_1 - \frac{v_1}{\hat{\lambda}}$$

and so:

$$\hat{\alpha} = \hat{\lambda} t_1 - v_1$$

Once the estimates for λ and α have been obtained, the fitted cumulative volume can be calculated from Eq. 3.

Standard error of λ

The precision of the estimate λ is given by the standard error of λ. Standard errors from models 2 and 3 are the usual standard errors of a regression coefficient. The standard error for model 1 can be calculated from:

$$se(\hat{\lambda}) = \sqrt{var(\hat{\lambda})}$$

where:

$$var(\hat{\lambda}) = var\left(\frac{\bar{v}}{\bar{r}}\right)$$

Using a formula given by Armitage and Berry (1987) this approximates to:

$$var(\hat{\lambda}) \approx \frac{var(\bar{v})}{[E(\bar{r})]^2} + \frac{[E(\bar{v})]^2}{[E(\bar{r})]^4} var(\bar{r})$$

Identification of changes in magma supply rate

If magma enters a volcano at a constant, long-term rate the volcano can be considered to be in a steady-state (Wadge 1982) and any stochastic element will mean the time series of eruptions can be treated as a homogeneous Poisson process. If this changes to a new rate there will be a change point in the cumulative volume curve between two linear segments of different gradient (Fig. 2). Alternatively, a supply rate changing continuously over time will result in a curve rather than a sudden break between linear segments and the time series would have to be treated as a non-homogeneous Poisson process (Ho 1991). We consider fully only change points and linear segments. A method of identifying such changes in supply rate is needed to fit models to the data. Visual inspection of the cumulative volume curve can be helpful in the detection of trends or changes but it does not lead to objective results and a more rigorous approach is needed.

The problem of locating a point of change in a data series is often encountered in the statistical analysis of a time series. However, as Mulargia et al. (1987) pointed out, most statistical techniques rely on simplifying assumptions such as normal distributions, large populations and a known number of changes. These assumptions may not be valid for volcanic time series

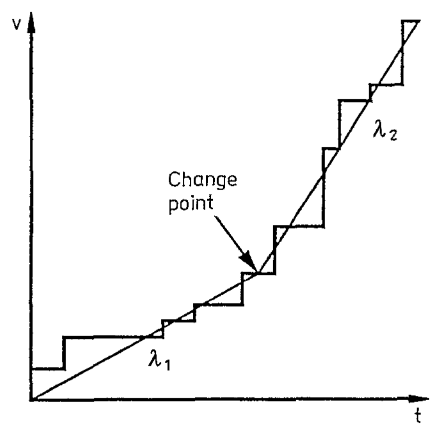

Fig. 2. Schematic cumulative volume curve showing a change point separating two values of λ

since the number of changes and the underlying distribution of the data are generally unknown and usually the sample size is small. Mulargia et al. (1987) went on to develop an algorithm, based on two-sample Kolmogorov-Smirnov statistics, which does not require such sampling assumptions. An alternative method proposed by Ho (1992) used the idea of statistical process control to signal a change in the series.

Change point analysis is complicated by the fact that there may be a change in the average repose period, or in the average volume of the lava emitted, or both. The first two changes imply a change in the input rate of magma entering the volcano. Simultaneous changes in both series may not imply such a change.

Application to Nyamuragira

Nyamuragira is one of two active basaltic volcanoes in the Virunga volcanic field of the Western arm of the East African Rift Valley. It is a shield-like volcano standing 1.5 km above the valley floor with a basal long axis of about 60 km. There have been 22 eruptions this century (Table 1) of K-rich basaltic (tephritic) lava flows mainly from flank vents. These have been studied by European and more recently by Japanese and Zairean workers (e.g. Hamaguchi 1983a).

The main evidence for a magma reservoir beneath Nyamuragira comes from detection of an aseismic zone

Table 1. Eruption catalogue for Nyamuragira (after Kasahara 1991)

Eruptive period[1] Start	End	Eruption Volume ($10^6 m^3$)	Repose period[2] (days)
1901		73	
13 May 1904		31	1047
22 Jul 1905		70	435
3 Dec 1912	Mid Mar 1913	90	2191
Mid Oct 1929	28 Jan 1938	200	6160
28 Jan 1938	Mid Jun 1940	159	3027
1 Mar 1948	Mid Jun 1948	64	3685
16 Nov 1951	Mid Jan 1952	25	1355
21 Feb 1954	28 May 1954	66	828
16 Nov 1956	17 Nov 1956	3	999
28 Dec 1957	29 Dec 1957	6	407
7 Aug 1958	21 Nov 1958	141	222
23 Apr 1967	9 May 1967	95	3181
24 Mar 1971	Mid May 1971	90	1431
23 Dec 1976	Mid Jun 1977	62	2101
30 Jan 1980	23 Feb 1980	87	1133
25 Dec 1981	14 Jan 1982	124	695
23 Feb 1984	14 Mar 1984	84	790
16 Jul 1986	20 Aug 1986	72	874
30 Dec 1987	3 Jan 1988	5	532
24 Apr 1989	Mid Aug 1989	114	481
20 Sep 1991		100[3]	879

[1] Missing dates were taken to be 15 and missing date and month was taken to be 1 July
[2] Since preceding eruption
[3] No accurate estimate yet available. Preliminary information indicates a volume of at least this figure (SEAN 1992)

4–8 km beneath the volcano's summit during the 1986 eruption (Hamaguchi 1983b). Aoki et al. (1985) argued that petrochemical evidence indicates about 30% fractionation of olivine, diopside-salite, plagioclase and titanomagnetite from an olivine basanite parent to produce the tephritic composition of the historical lavas. There is also evidence from temporal variations in the composition of historical lavas that suggests ongoing fractionation within the magma reservoir (Hayashi et al. 1989). During the 1986 eruption the composition became increasingly depleted in K_2O and more mafic, and the eruption sequence from 1976 to 1986 saw a systematic fall in K_2O from 3.8 to 3.2%.

Data catalogue

It is important to assess the quality of individual eruption entries in the data catalogue so that their statistics represent as closely as possible a single volcanic process – in this case the rupture of the crustal magma reservoir to feed lava flows on the volcano. The period of scientific reporting of the eruptions of Nyamuragira began in 1901 and there have been several compilations of information about them subsequently (Pouclet 1975; Krafft 1990; Ueki 1983; Kasahara et al. 1991). It is unlikely there are any 'missed' eruptions. We have used the Kasahara et al. (1991) catalogue to compile Table 1, with some modifications. The relatively low viscosity of the Nyamuragira lavas produces quite thin flows with less overriding of flow units in the proximal parts of the flow field than at some other volcanoes. Kasahara et al. (1991) assumed a mean thickness of 3 m for all historical flows to calculate flow volumes – a figure that probably has an uncertainty of only a few tens of percent. The endings of the 1901, 1904 and 1905 eruptions are not reported. From October 1929 to January 1938 there was intermittent lava lake activity in the summit crater which added $200 \times 10^6 m^3$ of lava [Hoier 1939; Kasahara (personal communication, 1992)]. Although this volume was a net loss from the reservoir, the manner of its release, at a much lower rate than during flank eruptions, does not represent the same process. Hence we do not use the repose period figures calculated before or after this eruption on the erupted volume in the modelling. The eruptions of 1956 and 1957 involved initial lava flows in the caldera but also lava flows outside and we treat them as flank eruptions. Kasahara et al. (1991) distinguished two eruptions in 1989 but although fissuring produced two distinct flank lava flows, effusion was essentially continuous and we interpret the activity as one rupture event of the reservoir.

Exploratory data analysis

One important consideration that relates to this data set, and many volcanic time series, is that the number of observations is small. Time-series analysis often requires that the number of observations should be large

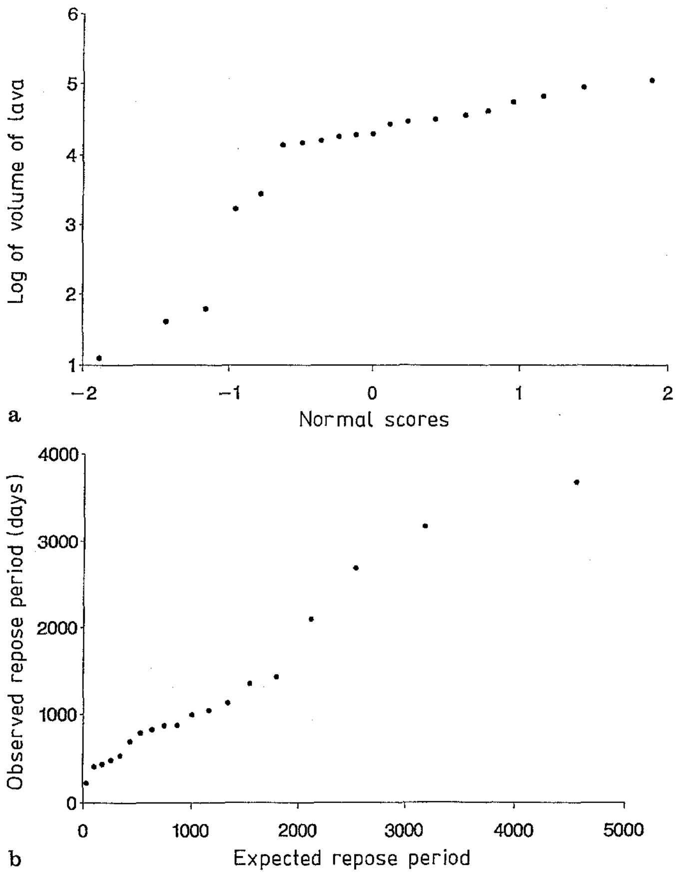

Fig. 3. a Normal probability plot of the logarithm of eruptive volume. The observations are sorted into ascending order and plotted against an approximation to their expected value, a normal score, which has been derived assuming they are normally distributed. If the plotted points fall on an approximately straight line the observations can then be assumed to have a normal distribution (Collett 1991). Note the linear trend at higher scores. **b** Plot of repose periods against those expected from an exponential distribution. This plot is similar to Fig. 3a except that the expected values have been derived assuming the observations have an exponential distribution function $F(x)=1-e^{-\mu x}$ and mean $1/\mu$

and so results from small data sets need to be interpreted with caution.

The normal probability plot of the logarithm of the volumes (Fig. 3a) suggests that the series could be a mixture of two distributions, with the dominant one possibly being a log-normal. The quantile plot of the repose periods (Fig. 3b), assuming an exponential distribution, gives an approximately straight line, if the higher values are ignored.

Each lava volume can be associated with either the following repose period, as in model 2, or the preceding repose period, as in model 3. Figure 4 indicates a strong relationship between volume and the following repose period and virtually no relationship between volume and the preceding repose period. The correlation coefficients are 0.693 (significant at the 1% level) and 0.035 respectively.

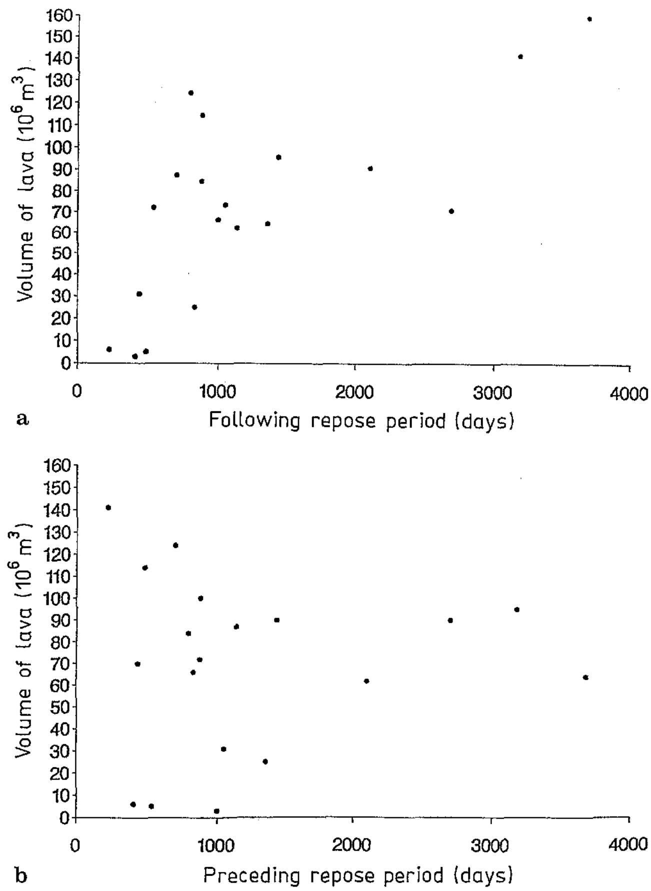

Fig. 4. a, b Plots of eruptive volume versus repose period for **a** periods following eruption, and **b** periods preceding eruption

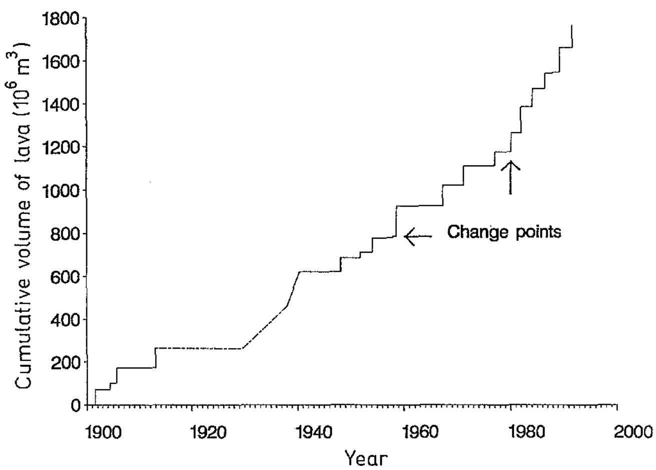

Fig. 5. Cumulative volume curves for Nyamuragira (1901–1991). Two identified change points at 1958 and 1980 are shown

Results of Modelling

The cumulative volume curve for Nyamuragira (Fig. 5) shows an obvious steepening in the latest part. Kasahara et al. (1991) suggested this change point occurred between 1977 and 1980. We now test this statistically.

1. Change points

Two approaches are used: the Kolmogorov-Smirnov test is applied to both the repose period and lava output time series and the control chart process is applied to the series of repose periods.

Kolmogorov-Smirnov test. The Kolmogorov-Smirnov (K-S) test discriminates between two data sets as belonging to two different populations based on the maximum absolute difference between the cumulative distribution functions (Ebdon 1985). The procedure developed by Mulargia et al. (1987) uses the K-S test to look at the aggregate behaviour of volcanic activity. It involves partitioning the series into two groups keeping the chronological order and applying the K-S test. Locating the partitioning which gives the two most dissimilar groups, out of all possible distinct groupings, is then given by the most significant K-S test statistic.

In the series of repose periods three regimes were identified with change points corresponding to years 1958 and 1980 at significance levels of 0.17 and 0.04 respectively. In the lava volume series one change point was found at 1958 at a significance level of 0.1. Mulargia et al. (1987) used a significance level of 0.05 to select change points. A more liberal approach was used in this work: both series identified a change point in 1958 which seemed to suggest changes were occurring prior to the more obvious change of eruption pattern in 1980 (Fig. 5).

Control chart procedure. This procedure distinguishes between random variation inherent in observed times and extraordinary variation that signals a real change. It relies on control limits so that a point lies outside the control limits almost as soon as the process enters a new regime (Ho 1992). Unlike the aggregate treatment of the K-S test this procedure is an eruption by eruption procedure, following the original order of the eruptions.

Firstly, the series including the 1929–1938 eruption was examined (Fig. 6a). Based on 90% control limits all points were between the limits, indicating an in-control process and no change point. This result was repeated when the 1929 eruption was excluded from the analysis (Fig. 6b). The conclusion drawn from these results was that the variation within the series starting at 1901 was not extraordinary. However, the plotted points are calculated using a cumulative sum and so at any time point the sum depends on all previous values. This makes it unclear whether the 1929 eruption should be included or excluded from the analysis. Tak-

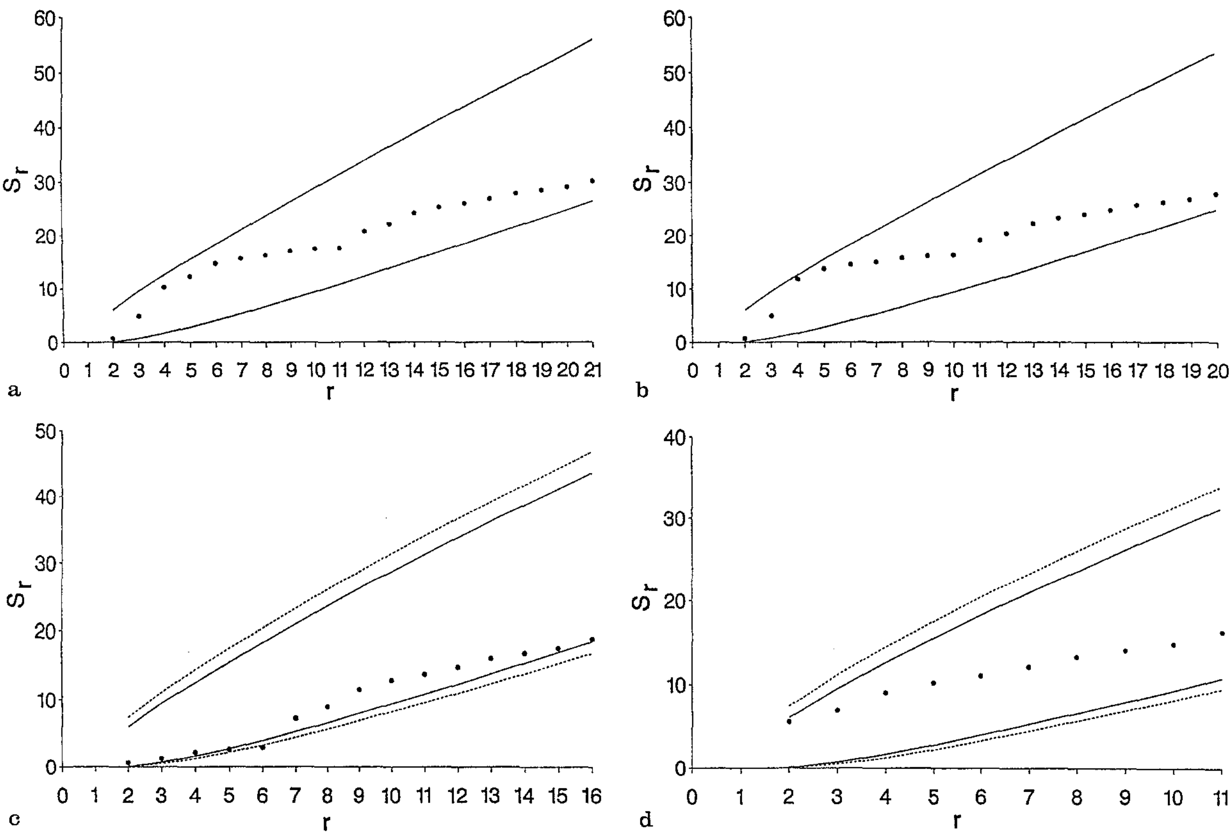

Fig. 6a–d. Control chart plots of Nyamuragira to detect time series change points using the method of Ho (1992), where S_r is the cumulative sum of the log ratios of the repose period to the preceding repose periods and r is the time sequence of reposes. Limits to the process being in 'control' are shown by the *solids lines.* **a** Time series from 1901–1991 (including the 1929–1938 eruption) with limits at 90% confidence level; **b** as for (a) excluding 1929–38 eruption; **c** time series 1938–1991 with 90% (*solid lines*) and 95% (*dashed lines*) confidence limits; and **d** time series 1957–1991 with 90% (*solid lines*) and 95% (*dashed lines*) confidence limits

ing 1938 as the start of the series avoids the uncertainties of the 1929 eruption. Figure 6c illustrates that the lower 90% control limit was crossed at time point 5 and the lower 95% control limit was crossed at time point 6. Since the evidence was stronger for a change at the later time point we assumed that a different regime started at time point 5. As a result, the change point was then identified as 1957. This eruption was then treated as the start of a new regime and a control chart starting at 1957 was plotted (Fig. 6d). All plotted points are between the control limits. Based on this technique two regimes were identified, with a change point at 1957.

These results do not match those given by the K-S test. The approach used by the two tests are fundamentally different. The K-S test is based on the whole data series and results could change with the addition of new data. The control chart procedure is based on accumulating data and results are not affected by subsequent data. However, it appears that if a trend continues for long enough then a point can lie outside the control limits. This raises the question 'Is a continuing trend indicative of a change in regime?' Perhaps the question should be 'Is a change in trend indicative of a change in regime?' Looking at the aggregated data gives a better measure of any patterns in the series and we choose to fit the models based on the change points identified by the K-S test.

2. *Parameters of the models*

Regression lines for the three models were fitted as described earlier (Fig. 7) using the three repose period regimes: 1901–1958, 1958–1980, 1980–1991 (Table 2). The underlying assumption in regression analysis is that the errors are normally distributed about the regression line. This assumption was checked using plots of residuals, the difference between the fitted and observed values, and found to be valid. Since in model 1 the estimate of λ is a ratio of two independent means it does not matter that there are a different number of

Table 2. Statistics of the different regimes with the eruption time series of Nyamuragira

Repose period (days)	Volume ($10^6 m^3$)
1901–1958 n = 9 mean = 1296.6	1901–1958 n = 10 mean = 58.7
1958–1980 n = 4 mean = 1961.5	1958–1991 n = 11 mean = 88.5
1980–1991 n = 6 mean = 708.5	
1901–1991 n = 19 mean = 1250.8	1901–1991 n = 21 mean = 74.3

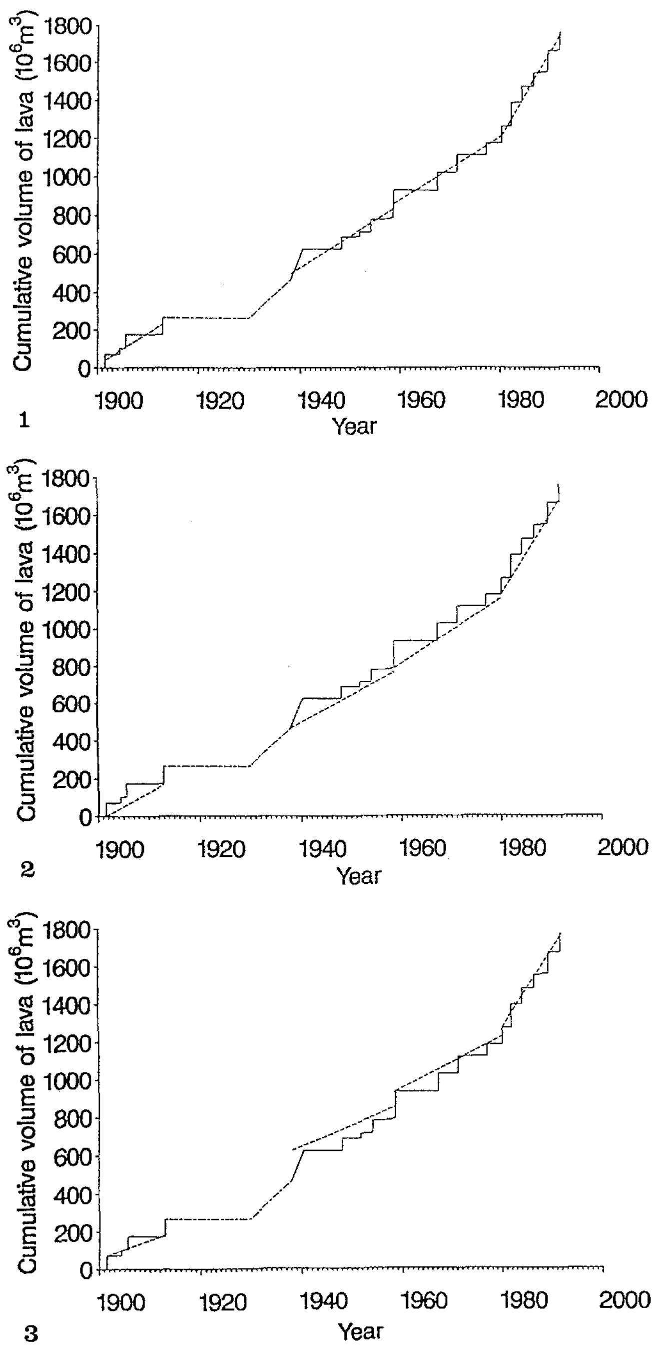

Fig. 7. Regression line fits of the three stochastic models (1–3) to the Nyamuragira cumulative volume curves. Separate line segments are fitted to the three different regimes (1901–1958; 1958–1980; 1980–1991) identified by the K-S change point analysis (*dashed lines*). For each segment the appropriate λ value is taken from Table 3 and the intercepts for models 2 and 3 are the start and end of the nearest eruption to the change point. The 1901–1958 regime is fitted by two line segments of equal slope but has two intercepts at 1901 and 1938, ignoring the 1929–1938 period of caldera lava-lake activity and the preceding repose (*long and short dashed lines*)

repose period regimes and volume regimes, provided that any change points that are located occur at the same time. The values of λ are given in Table 3. The estimates from models 1 and 2 are most similar but all

Table 3. Estimates and standard errors of λ ($m^3 s^{-1}$) and R^2 values for the three models

Regime	Model 1		Model 2			Model 3		
	λ	(S.E.)	λ	(S.E.)	R^2	λ	(S.E.)	R^2
1901–1958	0.524	(0.203)	0.466	(0.049)	0.92	0.298	(0.129)	0.40
1958–1980	0.523	(0.137)	0.547	(0.05)	0.98	0.427	(0.102)	0.85
1980–1991	1.447	(0.227)	1.366	(0.189)	0.91	1.323	(0.282)	0.81
1901–1991	0.688	(0.150)	0.573	(0.081)	0.76	0.404	(0.110)	0.46

models reflect the same pattern of λ: little or no change in the first two regimes with a marked increase in the last regime.

The standard errors for model 2 are the smallest suggesting that the estimates of λ are more precise for this model. Visually comparing the plots of the models (Fig. 7), it seems that model 2 appears to best fit the data.

3. Goodness of fit

Usual goodness of fit tests cannot be used because the set of volumes changes depending on which model is being fitted; the first volume in the series is omitted in model 2 and the last volume is omitted in model 3. Instead, the decision of which model best fits the data must be based on the merits of each model. However, since regression analysis was used to fit models 2 and 3 the ratio of the variation explained by the regression line to the total variation, R^2, can be used to give a measure of goodness of fit. The results in Table 3 show that the R^2 value for model 2 is highest in each regime. Considering also that volume is more strongly related to the following repose period than the preceding repose period (Fig. 4) then model 2 seems more appropriate than model 3.

Another way of deciding whether model 2 or model 3 is more appropriate is to include both preceding and following repose periods in the regression equation to estimate λ. The parameter with the more significant regression coefficient will be the more important in describing the time series. The following repose period was always the more important parameter. If however, the regression coefficients, the estimates of λ, had been the same for both repose periods then we would have concluded that neither repose period is more important than the other and so model 1 would be most appropriate.

Volcanological interpretation

Since 1980 the magma supply rate at Nyamuragira has increased by a factor of about 2.5 above that for the preceding 80 years. This was principally achieved by a reduction in the repose period between eruptions. The other change point in the time series data, in 1958, marked the end of a period of activity involving small to moderate volume eruptions but no significant change in the magma supply rate. Model 2, the pressure-cooker type model, fits the observed data best. From this we infer that there is a deterministic limit to the elastic strength (or holding capacity) of the magma reservoir about 4–8 km below the summit of the volcano. When this is exceeded the reservoir ruptures, creating dykes that feed flank eruptions (Fig. 8). The rate of this filling-rupture-eruption process has increased since 1980. The volume drained at each eruption is random but has not exceeded about $200 \times 10^6 m^3$ during this century. The intermittent caldera-filling activity of 1929–1938 cannot be explained by model 2, except as an interlude of model 3-type behaviour.

The measured decrease in average K_2O contents of the lavas from the 1976 to 1986 eruptions (Hayashi et al. 1989) suggests that the average composition of magma within the reservoir became more mafic during this period. With a higher rate of supply and eruption, less time is available for crystal fractionation within the reservoir and hence the magma composition has changed to one closer to the mafic composition of the deep supply.

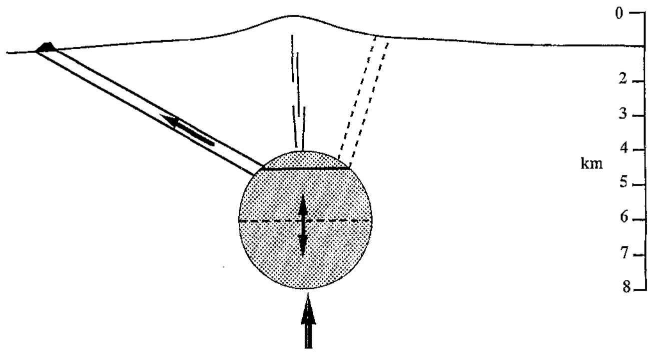

Fig. 8. True scale schematic section through Nyamuragira showing the magma reservoir (4–8 km below the summit) as a spherical body (though its actual shape is not known) behaving as a pressure-cooker type stochastic model (model 2) with a deterministic limit to reservoir strength (*upper solid control line*) and a random level of drainage after each eruption (*dashed line with arrows*). Magma leaves the reservoir along dykes (*parallel lines*) feeding flank eruptive fissures up to 20 km from the summit. The dykes are schematic only and are not meant to imply cone sheet geometry

Discussion

The fact that model 2 best fits the historical eruption record of Nyamuragira does not prove that the physical interpretation of the model is correct. More evidence could be collected to test the model. The near-field strains and surface deformation of the volcano (Kasahara et al. 1989) indicate a deformation source beneath the summit of Nyamuragira. Such measurements could be used to monitor the relative stress state of the volcano's reservoir. Model 2 predicts that eruptions should occur at the same state of tensile stress concentration. It is unlikely that petrochemical evidence could distinguish between the stochastic models. However, modelling of fractionation processes within the reservoir, given the changing rates of magma flux and the measured change in K_2O contents, could yield estimates of the reservoir volume. This in turn could be used to refine the stochastic models.

The model assumption of a constant magma supply capable of occasional, abrupt changes in rate to produce a piecewise pattern is well-supported by the Nyamuragira data. A careful examination of the regional seismicity for the period covering the 1980 change point might indicate a tectonic cause for the change in magma supply. In work not described here we have tested non-homogeneous models where λ is a function of time. Again we find that model 2 gives a better fit to the Nyamuragira data than model 3. R^2 values of 0.9 are achieved using four coefficients of a quartic function. Currently we have no volcanological criteria for deciding between the linear and non-linear models and prefer the linear model here for its simplicity. Other stochastic models, in addition to the three tested here, might be worthy of consideration. For example, a model in which an eruption of random volume can occur at any time but must occur when the volume in the reservoir reaches threshold volume and cannot drain to below a critical level is attractive. As before the deterministic elements to this model are likely to reflect fixed reservoir behaviour with the stochastic elements being derived from external forcing or complex non-linear behaviour. The cumulative volume for this curve will be bounded both above and below and the simple techniques that were used to estimate the cumulative volume curve for the previous models are not applicable. More sophisticated techniques are available, for example the EM algorithm (Dempster et al. 1977), but with so few observations the algorithm is likely to be unstable.

This emphases the principal constraint on this type of modelling: the brevity of the data catalogue. The problem is compounded by the need to subdivide the different supply rate regimes. Nyamuragira is one of the most active volcanoes in the world, though the historical record is short. The character and quality of the eruption record needs to be considered carefully before applying these models to a volcano to assess whether the data are of sufficient quality to support the inferences made.

Conclusions

1. Three simple stochastic models of basaltic volcanoes with a piecewise-varying magma supply and a reservoir provide a means of understanding their eruption histories.
2. To apply such models care must be taken to ensure the uniformity of the events in the data catalogue, test the goodness of fit of the model and be explicit about the physical significance of the elements of the models.
3. Repose period and erupted volume time series must be tested for changes in magma supply rate using Kolmogorov-Smirnov statistics or statistical process control techniques. The resultant separate regimes of the observed cumulative volume curve can then be tested for their fit to the models using regression techniques.
4. The 1901–1991 eruption history of Nyamuragira is best explained by model 2 which has a deterministic limit to the volume of magma in the reservoir prior to eruption, but the resultant eruption volume is random. Change points in the cumulative volume curve at 1958 and 1980 were detected using the Kolmogorov-Smirnov techniques. The latter eruption marked the start of a period characterised by an increase of about 2.5-fold in the magma supply rate.

Acknowledgements. We thank Prof. RN Curnow for comments on an earlier draft. Dr A Gudmundsson and an anonymous referee suggested helpful and constructive changes to the manuscript. MLB acknowledges support of a NERC research studentship (ES-/91/PES/3) and GW's work was supported by NERC Contract No. F60/G6/12.

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Editorial responsibility: H.-U. Schmincke