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THE MODERN MATHEMATICS REFORM MOVEMENT IN BRAZIL AND ITS CONSEQUENCES FOR BRAZILIAN MATHEMATICS EDUCATION

ABSTRACT. This paper reports on the conclusions drawn from an investigation of the modern mathematics reform movement in Brazil. The focus of the study was an analysis of the mechanisms of transfer of knowledge from developed to developing countries, through a case study of Brazil and its adoption of the modern mathematics curriculum.

The report presented here focuses on the dynamics of the reform movement and on the consequences of the movement for Brazilian mathematics education. The movement is studied as to how ideas about modern mathematics reform were disseminated to the Brazilian mathematics education community and the consequences due to misinterpretations of the intended curriculum in the implementation process. The intent of the study is that future initiatives in curriculum change proceed more efficiently in light of previous experiences, without repeating the mistakes committed in the past.

INTRODUCTION

The past thirty years have been an era of great changes in mathematics education throughout the world. For the developed countries it has been a period of creative activity in curriculum development. For many developing countries it has been a period of uncritical adoption of foreign ideas about the mathematics curriculum. For other developing countries, Brazil in particular, it has been a period of adaptations of foreign ideas to national curricula and local educational policies.

This paper¹ is a summary of the conclusions drawn from an investigation of the modern mathematics reform movement in Brazil, focusing primarily, but not exclusively, on the consequences for the elementary grades (1-8). The purpose of this study was to describe the dynamics of the movement and its consequences for the Brazilian educational system. The ultimate intent is that, through a critical analysis of the process of innovation, new insights about this process will be generated. Thus, future initiatives in curriculum development will proceed in light of previous experiences, more efficiently without repeating earlier mistakes.

The conclusions presented here have been drawn from the analysis of many different sources of data. The primary sources drawn from, in order to gain some insight into the dynamics and consequences of the reform movement were the following: interviews with people involved in the movement, press reports (1960–1975), analysis of textbooks written during

the period of 1955–1985, proceedings of national and international congresses of mathematics education, documents such as curricula of preservice and in-service education programs, and relevant library acquisitions (books and journals). The data gathered was coded and triangulated, so that all the claims made in this report have been drawn from evidence from at least two sources of data and usually more.

This paper is divided in two parts. The first part describes the conclusions that are related to the dynamics of the movement. The second part deals with the specific changes in the curriculum that were found to have been a result of the movement itself. Although both of the sections are intimately related and there is much overlap between them, the ideas are separated so as to organize them for the reader.

It is beyond the scope of this paper to describe the nature of mathematics education in Brazil prior to the early sixties. For information about the situation before the reform movement the reader is referred to studies of the history of Brazilian education such as Azevedo (1943), Motejunas (1980), Ribeiro (1984), Romanelli (1985) and Roxo (1937).

I. DYNAMICS OF THE REFORM MOVEMENT

The movement in Brazil consisted of an unplanned sequence of events. In fact, the data revealed that most of the changes and decisions were circumstantial. Although many of the projects developed worldwide were research projects on curriculum development, this was not a characteristic of most of the projects in Brazil. Reform was definitely not done systematically and rarely was formal evaluation conducted.

This part focuses on the dynamics of the reform movement in Brazil. Reform occurred through the transfer of ideas from abroad and through the dissemination of ideas within the country. It is around these two processes that this section is organized.

Transfer of Ideas from Abroad

The first evidence of the presence of ideas about modern mathematics in Brazil was found in the early 50s, when the university mathematics curriculum began to be influenced by Bourbakian ideas. At the national meetings on mathematics education, held in 1957 and again in 1959, discussions about incorporating modern mathematics into the school curriculum were initiated. However, it was only in 1961, under the powerful leadership of Oswaldo Sangiorgi that the ideas were organized and took on characteristics of a reform movement intended to change the teaching of mathematics throughout Brazil.

The basis of the movement in Brazil can be described as a concoction of ideas from around the world, a synthesis of which was done by Brazilian mathematics educators themselves. Although there was obviously a heavy reliance on ideas from abroad, there was an attempt to adapt these ideas to the Brazilian educational system.

Among the major means through which ideas from abroad reached Brazilian mathematics educators three were especially prominent: the international exchange of scholars, the availability of foreign books and journals, and the participation of Brazilians in international conferences. These channels played a major role in establishing directions for the collection of data about the transfer of ideas and the dissemination process. It was from the analysis of the information gathered that the conclusions presented in this section were drawn.

The ideas having most impact on the Brazilian curriculum were those of the School Mathematics Study Group, George and Frederique Papy, Zoltan Dienes, Lucienne Felix and Caleb Gattegno. Each of the programs developed by them were based on different premises and assumptions as well as having very different focuses. The fact that these programs were combined, with little or no critical analysis, was quite detrimental to the Brazilian curriculum, for it generated a curriculum based on inconsistencies of various kinds. Some of the problems that originated from this impact of foreign ideas are raised in the following paragraphs.

Brazil has, since the early 60s, been attempting to expand the educational opportunities to all children. Many of the ideas that were borrowed had been developed to improve the mathematics and science preparation of an educational elite. For example, the U.S. reform focused on the production of a cadre of well prepared scientists and mathematicians. Many European programs also focused on the improvement of the mathematics programs for the top students. Unfortunately, the objectives and goals of the programs developed in the U.S. and Europe were quite inappropriate for the children attending the majority of the Brazilian schools. A large number of the children in the Brazilian educational system are from highly impoverished families, and the opportunity of finishing even the early grades is just an illusion. The mathematics curriculum for such a situation must have appropriate objectives and goals which should be very different from those envisioned for the industrialized world in which most children enjoy ten or more years of schooling. As an example consider the case of children attending school where the mathematics studies are programmed as building blocks for further mathematics learning, as was the essence of many modern mathematics programs proposed. A child who drops out of school early will have acquired only the first few building blocks of mathematics which may not be a complete enough foundation for use of this school learned mathematics in any real life situations. Issues of appropriateness of the proposed curriculum for the majority of the population, as described above, were never dealt with in the Brazilian reform movement.

In comparing urban and rural educational systems, Coombs (1985) concluded the following: "the highly academic, urban, and modernsector oriented curriculum of these schools did not realistically fit the learning needs and prospects of most rural youngsters" (p. 15). This statement equally describes the case of Brazil, where the foreign curricula being imported was "highly academic, urban and modern-sector oriented" and was inappropriate for the majority of the children of that country.

Resuming the discussion about the impact of the ideas from abroad, attention should be shifted to the focus of the various projects. Brazilian reform was, at first, based on the SMSG model. The focus of SMSG materials was on rethinking the content of school mathematics. Later influences on reform were that of Gattegno and Dienes, and unlike SMSG, both focused on methodology. However, although many new math advocates in Brazil defended the use of the manipulative materials suggested by these individuals, the bridge between the use of these materials and the content in the textbooks was never made for the teachers: Teachers did not know how to use the manipulative materials to bring the concepts to a more concrete level and then make the transition to the symbolic level. Many schools purchased Cuisenaire rods of Gattegno and multibase blocks of Dienes, for the value of these was being pushed by the movement leaders, but methodology was only lightly addressed in teachers' guides and in-service programs.

There was conflicting information from the various projects. While some curricular materials relied heavily on the early introduction of abstraction, other projects emphasized the use of manipulative materials to make mathematical concepts more concrete. Furthermore, each project from which Brazilians borrowed ideas was developed for very different educational structures that existed within very different socio-cultural settings. Consequently, their objectives were often quite distinct from one another as well as very different from those of the Brazilian educational system.

Dissemination of Ideas Within the Country

The Study Group for the teaching of Mathematics (referred to by its Brazilian acronym from now on - GEEM), played an important role in the diffusion of ideas about mathematics teaching. The group was founded in the city of São Paulo which at the time was economically, politically and culturally the dominant city in the country. Developments in mathematics education in São Paulo served as models for the developments in many other areas of the country. For this reason the developments in São Paulo were the major focus of this study. In the same way that many ideas borrowed from abroad were inappropriate for the Brazilian socio-cultural settings, very often the innovations made for São Paulo were inappropriate for other areas of the country.

It is also important to note that the state educational system of the state of São Paulo during the 60s educated only about 30% of the children attending schools, while the other children attended private schools. In general the private schools, with the exception of the very best ones, looked to the state schools for their model. Consequently the curriculum proposed by the state system was usually adopted by the private sector. Some of the more exclusive private schools were used as sites of curricular reform projects. These schools were looking for innovative ideas and had the money to spend on new materials. Most importantly, several of these schools had members of GEEM on their faculty.

The study group GEEM was modeled on the SMSG. The principal component that was simulated was the leadership role of such a group in the diffusion process, especially through in-service programs. However, many aspects of GEEM were different from SMSG. In particular, the membership of GEEM drew heavily from the secondary school rather than universities. In fact, the university representation was very limited. Another difference was that GEEM, as a group, did not develop any textbooks for use in the schools. Instead, GEEM members wrote guidelines for what they considered most relevant for the ideal mathematics programs to include (see GEEM, 1965). Textbook authors then followed these guidelines to develop their own series.

It is important to note that the secondary teachers who had leadership roles in GEEM were quite different from many other mathematics teachers at the time. They were the few secondary teachers who had a university degree in mathematics, thereby making their mathematics background generally more sophisticated than most of the secondary mathematics teachers in Brazil, at that time.² In fact, when GEEM began to get involved in the reform of mathematics at the elementary level the leaders of this portion of the movement were also secondary mathematics teachers with university degrees in mathematics (GEEM, 1966, 1968).

The two major sources of dissemination of ideas throughout the country were the textbooks themselves and the in-service programs offered primarily by GEEM. The group consistently offered teacher training workshops for 15 years, modeled on the National Science Foundation (NSF) Summer Institutes offered in the U.S. However, in spite of the large number of teachers reached by these courses, a majority was never trained for change. Thus, the untrained teachers were forced to rely heavily on the textbooks they had adopted in order to learn the new mathematics topics that were to be taught. Although there also was some intent to reform the methodology used for mathematics instruction, textbooks did not refer to these changes and, consequently, change in methodology was never disseminated widely. This point does not necessarily imply that incorporating different suggestions about methodological approaches into the textbooks would be the solution to the problems involved in dissemination. It is raised here only to make a statement illustrating the powerful role of the textbooks in classroom instruction.

With respect to the dissemination of ideas within the country, a comment must be made about the impact of the military take-over in 1964 on the mathematics curriculum. Surprisingly enough there was very little indication, both from the interviews and from the documents analyzed, of the extent to which this social-political movement affected the curriculum. The movement was well underway by 1964 and one would have expected such a drastic national reorganization to have influenced the course of events of such a large scale educational endeavor. Instead, the only drastic measure of oppression taken by the military government in the realm of elementary and secondary education was to close the vocational school in the state of São Paulo. These schools, so inappropriately named, were alternative schools with a unique philosophy of education for the time. They were community schools, created for the purpose of developing in the students a critical awareness ("conscientization") of their social reality and of their participation in that reality (Ribeiro, 1980). These schools were considered a threat by the military regime and were closed in the late 60s. Although the vocational schools were actively undergoing curricular reform in mathematics, there is no reason to suspect that this reform had any bearing on the reason for their closing. Instead, the philosophy of education adopted by the educators at these schools was the major concern of the military.

It would be interesting to examine whether there was an underlying intent of the military regime to support the new math movement in order to serve their interests while in power, as some Brazilian educators venture to claim. For example, the military may have chosen to ignore the reform movement since the high level of rigor and symbolism proposed was obviously not promoting creative or critical thinking in school age children and young adults. This is particularly interesting if we consider that there is evidence that in other countries the military government forbade the incorporation of new math in the curriculum and considered the attempt to be a subversive action of mathematicians and mathematics educators ("Matematica moderna incita", 1980). The examination of this issue was beyond the scope of this study but is suggested as a topic to be pursued in future research.

In the paragraphs above channels of dissemination of ideas among countries and within the country were identified. The consequences of this dissemination process for Brazilian mathematics education will be looked at in the following section.

II. CONSEQUENCES OF THE REFORM MOVEMENT

In this section the consequences of the reform movement for the Brazilian educational system are described. The section is organized in four parts. First, the changes which reflect differences between how they were intended and how they were actually implemented are described. Second, some consequences of misinterpretations of proposed changes are discussed. Third, other consequences that do not fall under either of the first two categories are summarized, and finally the lasting influences of the reform in the mathematics curriculum today are briefly described.

These sections summarize conclusions drawn from the analysis of data collected from the following sources: interviews with individuals who served as instructors in in-service programs; teachers who attended in-service programs and were expected to implement reform; individuals involved in writing the first curriculum guide for the State of São Paulo and others involved in writing the most recent guide; textbook authors; document analysis of textbooks used prior to the movement, during the movement and today; document analysis of two curriculum guides of the State of São Paulo.

Intended vs. Implemented Curriculum Changes

From the official curriculum guide of the state of São Paulo (1975) and the publications of GEEM it was possible to identify the intended curriculum

changes in São Paulo. It is important to remind the reader that the curricular developments in São Paulo served as models for curricular reform in many different areas of the country. The implemented changes, or those that actually occurred in the classroom were apparently quite different from the intended ones. These differences were identified in two ways. First, the guide and textbooks were compared. Second, the observations made by the interviewees which related to this issue were considered. The conclusions of this analysis are presented here.

The dichotomy between the curriculum which was intended and that which was implemented was not exclusive to the situation in Brazil. In fact, the proceedings of the Conference on Comparative Studies of Mathematics Curricula, Change and Stability 1960–1980, held in Osnabrück (FRG) in 1980, indicate that this is a recurring issue worldwide (Steiner, 1980). Many of the papers presented at that conference dealt with an analysis of the intended versus implemented curriculum in various countries.

Probably the strongest force working against implementation of the intended curriculum was the conservative tendency of educational systems (Beeby, 1966). This relates to the beliefs of the teachers about mathematics learning and instruction. According to Beeby:

I am sure that for three-quarters of the teachers of the world it [curriculum reconstruction] means a revolution in their teaching practice; for many, it will involve a change in their ideas of the very purpose of education. (1966, p. 40)

The Third World in general, in the promotion of curricular innovations in mathematics, ignored the fact that the successful implementation of new materials would require a change in teachers' beliefs about mathematics learning and instruction. At the time of reform most teachers were "essentialists." They basically believed that the teacher was the center of the educational process, that learning involved hard work at committing large quantities of information into memory, and that schools should promote methods of discipline in the educational process (Duck, 1981).

The proposals of the reform advocates were quite different. They were more linked to an "experimentalist" approach to education. The major premises were that the children were the center of the educational process rather than the teacher and that the teachers' role was not to direct but to advise, with discovery learning and exploration replacing the authoritarian forms of expository teaching. Finally, they believed that learning with understanding should replace inculcation of subject matter through rote memorization of large quantities of information (Duck, 1981). The teachers themselves had been taught within an essentialist framework and later had been trained for their educational roles within that same framework. Even though, with the reform movement the new mathematics advocates were promoting a more experimentalist approach to mathematics teaching, this was mostly done through an essentialist model. In other words, teachers were being told a "better" approach to teaching, but this approach was not modeled in their own training. In few instances did the teachers actually experience the benefits of a new approach. It was assumed that teachers would make this philosophical change in their belief systems as a consequence of their training through the traditional approach.

One indication that these changes in philosophy by the teachers did not occur was the evidence that teachers were still concerned primarily with finishing the textbook. Furthermore, at the secondary level teachers were still focusing their teaching on the preparation for the "vestibular" [entrance examination to the university]. Problem solving and rational understanding of mathematical concepts was only rarely the focus of instruction. Instead, memorization following formal lectures was still the principal mode of learning and instruction.

Another important problem that arose in the innovation process is suggested by Beeby in the following statement:

no change in practice, no change in the curriculum has any meaning unless the teacher understands it and accepts it \ldots . If he does not understand the new method, or if he refuses to accept it other than superficially, instructions are of no avail. At the best he will go on doing in effect what he has always done, and at the worst he will produce some travesty of modern teaching. (1966, p. 46)

The fact that new math advocates in Brazil were not sensitive to this issue is quite clear. The data indicate that during the reform period the level of education of a great number of teachers in Brazil was (and still is) quite less than adequate (Brejon, 1977). Current data reveals that in 1980 12.8% of the elementary teachers (grades 1-8) in Brazil had not completed elementary school and that another 10% of the teachers at the elementary level had only gone that far themselves (MEC, 1979/1980). This was an improvement on the indices of previous years. Consequently, the number of teachers who did not understand the changes and resorted to rote teaching of modern concepts was great.

This lack of concern with the new philosophical bases for the changes that were advocated and the low level of preparation of the teachers resulted in the misinterpretation of many items proposed by the new math supporters. The next few paragraphs will exemplify a few of the consequences of these misinterpretations.

Misinterpretations of Proposed Changes

As has already been mentioned, rote memorization continued to be a prevalent practice of both teachers and children. Both teachers and students resorted to memorization since the language of set theory was beyond the comprehension of most of them. The language was stressed to such an extent that often teachers in the elementary grades spent half a school year working exclusively with set theory, definitions and operations.

The new language incorporated into the teaching of mathematics was often incorrectly used in the textbooks. For example, the number-numeral distinction was often abused. Students were required to distinguish between numbers and numerals in lists of exercises, but in subsequent chapters in the textbooks the authors themselves ignored the distinction and confused the terms.

Translations from English generated a few misconceptions as well. For example the place-holder was translated to "quadradinho" [little square] instead of being called a box or place-holder. Consequently, its meaning and purpose of being filled by a number was totally lost. Instead it simply replaced the common representation of unknowns, such as "x."

With respect to methodology, the activities approach was misconstrued and indicated to teachers that children should learn through play. The extent of this problem was such that time for play was increased but teachers often continued to teach as they had done previously. The materials purchased and developed such as Dienes' multi-base blocks were used for unstructured play rather than for mathematics instruction. Finally, interviewees indicated that even the use of colored chalk was exaggerated in an attempt to "modernize" teaching methodology. Some teachers were convinced that they were using "modern" methodology as long as they used a lot of colored chalk.

The use of manipulatives deviated from the initial intentions of enhancing understanding. A key element for successful use of manipulatives was overlooked, the transition from work at the concrete level to work at the abstract or symbolic level. Although teachers were shown how to use manipulatives to build understanding, they were not given techniques for the transition into the symbolic levels of work which was ultimately the level which they intended to reach. Some teachers even tried to memorize how to use the manipulatives including the questions to ask and the sequencing of activities. This defeated the purpose of observing students' work and building further instruction on the students' interpretations and understanding of concepts, for this procedure did not allow for any flexibility in the learning process. The teaching of the basic facts was replaced by the teaching of the properties of the operations and the mathematical structure of the operations was a focus. The phrase, "Children must understand, not memorize," was interpreted to mean that memorization should be eliminated. Consequently, work with automatization of the basic facts was eliminated for the most part and an inordinate amount of time was usually spent on memorizing set theory instead. Although American programs were suggesting that it was essential to construct meaning before memorizing, Brazilian teachers were interpreting this to mean that constructing was more important than memorizing. However, they did not know exactly how to construct most of the topics and did neither constructing nor memorizing of many topics such as the work with the basic facts.

The following section summarizes some of the other consequences and their causes.

Other Consequences

Basic facts were not the only topic to suffer from the modifications of the curriculum. Geometry was also a victim. At first the existing geometry program, which had a strong component of Euclidean geometry, was replaced by a new approach. This new approach, so as to be done according to the principles of modern mathematics, developed geometry through the use of vectors and transformations, based on the French proposals. A strong force that molded these changes in geometry was the support given by the highly respected mathematician, Jean Dieudonné. The changes were based on linear algebra and, due to its rigor and formality, were beyond the understanding of many teachers and children. Consequently, this topic was generally relegated to the final month of the school year and was rarely dealt with properly. The development of spatial reasoning abilities through spatial problem-solving and intuitive geometry continued to be ignored. Finally, the focus of the geometry program deteriorated to the exclusive development of definitions and measurement skills. The previous axiomatic approach to geometry, which had been dropped, was never reincorporated into the curriculum and similarly, the changes proposed were never incorporated. As a result, the geometry curriculum was practically nonexistent.

The modifications also included certain changes in some traditions of mathematics teaching. The number of exercises posed to the students was greatly reduced, as was the complexity of the problem sets. Most textbook series exaggerated this reduction and problem sets were of bare minimum in difficulty level and number. The objective of doing fewer exercises with more understanding, while commendable, was never attained.

The textbooks became more colorful and much easier to read. The density of material on each page was reduced and problems were distributed throughout each chapter rather than concentrated at the end of a chapter. These were considered positive results of the movement and although often regarded as of an aesthetic nature, were judged as relevant to students' learning of mathematics. However, these modifications had negative counterparts as well. Books and workbooks became one and the same thing. The idea of a mathematics text that could be read has disappeared. This has had two major consequences for the learning of mathematics. First, as students reach higher levels of education it is assumed that they can read mathematics, thereby causing a major handicap in furthering their studies of mathematics which often requires the ability to read mathematics. The second consequence is of a social nature. Since textbooks became consumable, children enrolled in a grade must have their own text. No sharing is possible, whereas in the past the same textbook was used by many members of a family in subsequent years. In an unplanned way the cost of education rose for families, since each year they would have to purchase as many textbooks as the number of children in the family, of course this had its greatest impact on the poor majority of the population. This phenomenon was compounded by the fact that mathematics was not the only subject in which this occurred. Language, reading, science and social studies textbooks all became "modern" in this same way.

Another consequence of the movement was the identification of individuals who have assumed leadership roles in the curriculum innovation process. This leadership arose from within the secondary schools rather than from the universities. Individuals worked together to promote changes, the value of which they truly believed in. These groups of people founded different study groups of mathematics instruction throughout the country, such as GEEM, GRUEMA [Mathematics Study Group], GEPEM [Study and Research Group in Mathematics Education] and others.

In a few instances, although not as much as would have been ideal, the focus in the classrooms changed from the teacher to the student. There was more concern for the children's thinking processes. Interviewees considered this quite beneficial to the learning of mathematics.

There was also much benefit for the mathematics education community of the communication established between mathematics educators and the psychology community. Although this communication was small it set a precedence for future developments. The work of specialists in other fields has become more relevant to mathematics educators.

The changes in the conception and dynamics of in-service programs are also relevant to this discussion. The topics dealt with by these programs became more related to the teachers' work in the classrooms. In-service programs were designed to give teachers new ideas and alternatives to improve their teaching skills. It was no longer considered more important to upgrade teachers' content knowledge than to provide a support for what was happening in the classrooms themselves.³ It was realized that better understanding of higher level mathematics did not necessarily transfer into better mathematics teaching.

Another consequence of the movement that is worth mentioning was the development of the mathematics olympics for elementary children, promoted by GEEM in the state of São Paulo, and other new math study groups in other states of Brazil. Unfortunately a few teachers who intend to enter students in this contest tend to gear the classroom instruction towards those few students' performance in mathematics. The ones who are selected to partake in the event are those few outstanding students who are potential future mathematicians. The material on the examinations is quite difficult and consequently participation is limited to an intellectual elite. Mathematics teachers generally consider these students a pleasure to work with and their success will promote the specific school which they attend. This situation is not unique to the elementary schools. Secondary school teachers expend most of their energy in preparing students for the entrance examination to the university, rather than emphasizing the mathematics necessary for students' success in life; i.e., that which develops their rational understanding of mathematical concepts.

Lasting Influences

It would not be appropriate to conclude this report without making reference to a few examples of the lasting influences of the reform movement on Brazilian education.

Probably the most important influence of the movement of the 1960s on the Brazilian mathematics education community has been the value given to research projects in the field. To this day, individual mathematics educators, and teams of educators, develop many projects around the country. Curriculum development has become an area of interest to

many, and important breakthroughs in the direction of improving mathematics instruction can be attributed to this interest.

The effects of the movement of the 60s on the content of mathematics instruction in Brazil can be analyzed from two perspectives: first, that which is reflected in the current textbooks and second, from the proposal for the new curriculum guide for the state of São Paulo, which is used as a model for the curriculum guides in much of the rest of the country. The discussion here will be limited to the elementary level, grades 1-8, since that is the part of the guide that was more carefully analyzed in this study, since this is the level that this study has focused on.

Many of the textbooks in use today are in many ways similar to those used during the height of the reform movement. The most popular texts have made very few changes from that time. Set theory is still prominent in most texts, even as low as in the first and second grades. As one interviewee (textbook author and teacher educator) commented, "if the set theory chapters fall out of the curriculum, primary teachers will be at a loss about what to do with the extra time they'll have on their hands. Most of them will lose an entire semester's worth of work." With this in mind textbook authors have kept the textbooks basically the same as before. Those few daring authors who want to be innovative have not had their textbooks adopted by many teachers.

The focus on rigor has diminished. No longer is there such a great attempt to define everything according to group structures. Nevertheless, the study of relations is still a focus of the fifth grade and is followed by a study of functions.

Number theory has remained a major part of the elementary mathematics program. For example, the study of least common multiples and greatest common divisor has retained its development through the process of determining the intersection of the set of multiples or divisors of a number. Prior to the 60s children had been taught to find each of these through the use of certain algorithms.

Work with certain bases, although greatly reduced, is still in a few of the texts. A very interesting observation to make is that in the latest edition of one of the popular textbooks the chapter on bases other that 10 has been modified to include some examples of how necessary the binary system is for work with computers. For the first time the text has included an explanation for students and teachers as to why they are dealing with that topic. However, there is still much emphasis on the algorithms for changing numbers from one base to another.

On the other hand, in the proposal for the new curriculum guide the

topic has been deemphasized. The authors of the guide proposal considered it important to maintain the topic in the early grades when introducing the base ten numeration system. However the work with bases has been reduced to grouping in different bases. All algorithmic work with bases other than ten has been eliminated.

In the proposal for the new curriculum guide for the state of São Paulo, it is evident that care has been taken to incorporate some of the lessons learned from the years of reform. However, there is obviously an intentional effort to completely eliminate any remnants of set theory, both use of the language and operations. For example, in the early grades, where the first mathematics topic to be developed is classification, the word "set" has been substituted by "collection." This is a deliberate rewording that should not affect the actual mathematical concept to be developed. Classification is a very important pre-number activity that can work on logic and grouping in an informal manner. However, with this attempt not to focus on set theory all the emphasis has been placed on analyzing numerical attributes, ignoring some of the other attributes. Since the curriculum guide is written primarily for educators, the appropriate mathematical terms could be employed without jeopardizing the quality or level of instruction, as long as attention is paid to the fact that this is not being suggested for use with the children.

In the introduction to the guide the following three characteristics are emphasized in the proposed curriculum:

- a) applications of mathematics skills to real life situations,
- b) development of a conceptual basis on which other mathematics can be founded, and
- c) development of thinking skills (Secretaria, 1986).

In spite of these claims, the focus throughout the guide is on the second characteristic above, i.e. development of the bases for the later learning of mathematics, focusing primarily on developing the basic skills. Even so, the Brazilian mathematics education community considers the guide an approximation of an improved form of mathematics instruction than previous ones. It proposes a curriculum which focuses on developing understanding of mathematics, complemented by an emphasis on problem solving and applications through modeling. A curriculum that would be characterized by experiences that develop the foundations of mathematics to real life situations and that develop their thinking skills.

CONCLUSION

The critical analysis of the mathematics curricular reform movement of the 60s in Brazil, discussed in this paper, reveals several aspects of the dynamics of curriculum change that enhance our understanding of the change process in mathematics education. Some of the major findings of this study which are most relevant to future undertakings are summarized here.

Curriculum change does not occur independently of a change in teachers' beliefs about the nature of mathematics, the nature of mathematics learning and the nature of mathematics teaching. This reconceptualization by teachers is an integral part of curriculum innovation and serves as a driving force for change. Furthermore, teachers are the essence of the innovation process and should play a major role in material development and curriculum planning, as well as being involved in the evaluation of the effective-ness of new materials and methods.

Several consequences of the reform process were traced to misinterpretations of the proposed changes generating a dichotomy between the intended and the implemented curriculum. These misinterpretations can be classified as: those due to errors of translation (in cases of adoption of materials cross-culturally), those due to different philosophical frameworks between the individuals proposing change and those implementing change, and finally those due to a lack of understanding of the actual content undergoing change.

Throughout the world the reform of mathematics education continues to be a major concern of the educational community. Considerations of the issues raised by this study and other studies of this nature can shed light on paths to be followed that may assure more effective procedures for current and future attempts at curriculum change.

NOTES

¹ This paper is a summary of the conclusions drawn in the author's doctoral dissertation entitled: The Dynamics and Consequences of the Modern Mathematics Reform Movement for Brazilian Mathematics Education, Indiana University, 1987. Reference should be made to this document for information about the nature of the data collected in order for the conclusions presented in this paper to be drawn.

 $^{^2}$ For data on the qualifications of elementary and secondary teachers in Brazil during the 60s and 70s see Brejon (1977).

³ It must be made clear at this point that by no means have all in-service programs changed their attitudes towards what is important to include in the in-service process. Many programs still emphasize the mathematics content rather than equally emphasizing the methodology, or even trying to integrate both aspects.

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