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# A general model for Mt. Ruapehu lahars

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**Abstract.** A mathematical model of the motion of lahars is presented. Lahar flows and travel speeds are calculated using a kinematic wave model which equates gravitational accelerations to frictional losses. A Chezy- or Manning-type law of friction is assumed, in which lahar flow rate is a simple power function of lahar depth, multiplied by another simple power of the channel slope. Use of the model requires knowledge of essentially only one parameter which appears to be relatively insensitive for flows down a given channel. Variable channel slope effects are removed by a longitudinal scaling which applies to all flows down a given channel. For lahars generated by a single explosive event it is unnecessary to perform numerical calculations to predict lahar flow and travel time, but for lahars produced by multiple sources in which different lahar flows are interacting, numerical calculations appear necessary. The model is applied to all recorded lahar flows from Mt. Ruapehu, and satisfactorily described all lahar flows generated by a single explosive mechanism. Such flows depend essentially only on total lahar volume. The 1968 Mt. Ruapehu lahar, generated by a series of smaller eruptive mechanisms, was modelled as the interaction of seven point sources of fluid originating from positions mathematically extrapolated up the mountain. Good agreement was obtained between the predicted times of formation of these 1968 lahars, and the times of greatest seismic amplitude.

## Introduction

Eruptions from volcanoes containing crater lakes often produce lahar flows. The water in the crater lake can be displaced either by surges that spill over the sides of the crater; by collapse of the crater wall; or by hydrothermal or magmatic explosions that eject crater contents into the air. In all of these cases, crater water will be deposited on the side of the volcano, where it can melt nearby ice and snow, thereby increasing the flow, as well as transporting solids such as ash, mud, pumice and stones. The resulting fluid flow of liquid and solids down the volcano is called a lahar, quantification of which is the aim of this paper.

There is no clear definition in the literature of a lahar. Lahar flows are usually confined to valleys or channels, and so are primarily driven by gravity. Lahars have been defined (Pierson 1985) as 'volcanic mudflows or debris flows' comprising of 'dense, viscous slurries of poorly sorted gravel, sand, mud, and water.' Lahars are sometimes referred to (Pierson and Scott 1985) as 'volcanic debris flows'. Recent accounts of debris flows are by Pierson (1986) and Okuda et al. (1980), while mudflows have been discussed recently by Janda et al. (1981) and Cummans (1981a, 1981b). Alternatively, volcanic flows may comprise a sequence of different flows such as a landslide-debris flow-flood wave sequence (Gallino and Pierson 1985), or as an evolutionary flow from debris flow to hyperconcentrated flow (Pierson and Scott 1985).

This paper does not consider 'debris flows', which can contain up to about 20% by mass (40% by volume) of water. Debris flows have been analysed by Takahashi (1980, 1981) as a flow in which solid-solid forces produce Bagnold stresses that essentially balance the gravitational forces. The term 'lahar' used in this paper refers loosely to a hydraulic type of flow down a volcano, in which turbulent liquid (and solid) frictional forces essentially balance gravitational forces.

Lahars are a serious hazard at volcanoes, first because large and dangerous flows can develop rapidly, and second because the high liquid content enables lahars to travel long distances and present a danger for many tens of kilometres in valleys draining the volcano. During the early phase of lahar flow about the summit of the volcano, many of the heavy and large solids settle from the flow and deposit on the upper reaches of the volcano, leaving clear evidence of the lahar. Sometimes these deposits on the upper parts of the volcano are referred to as the lahar. However, in this paper we shall understand the lahar to refer to the fluid flow of

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both solids and liquids occuring down the volcano, and we shall ignore the deposition which occurs in the upper reaches of the volcano.

This paper discusses only data from the active andesitic New Zealand volcano Mt. Ruapehu, situated approximately at 39°17′S, 175°31′E. The geology of Mt. Ruapehu and its ring plain is described in a facies model by Hackett and Houghton (1989). A crater lake, fed mainly by melting summer snows, occupies the crater at the summit of Mt. Ruapehu approximately 2566 m above sea level. Lake volume has fluctuated between zero, during the eruption in 1945 when a spreading tholoid displaced the lake water, to its maximum recorded value of order $10^7$ $m^3$ between 1948 and 1953. Lake bathymetry has been described by Dibble and Christoffel (1966), and Hurst and Dibble (1981). The present size of the crater lake is of the order $7 \times 10^6$ $m^3$.

The larger eruptions from Mt. Ruapehu between 1861 and 1959 have been recorded by Gregg (1960), and later eruptions by Latter (1985). At least seven (1861, 1889, 1895, 1903, 1925, 1969, 1975) of these large eruptions produced lahars in the Whangaehu valley; three eruptions (1895, 1969, 1975) produced lahars on the northwest slopes; and outwards collapse of an ice wall generated another lahar in 1953.

The first published eye-witness observation of a Mt. Ruapehu lahar occurred on 13 December 1859, when a lahar washed out the newly constructed Whangaehu bridge. According to an observer, (O'Shea 1954). 'The flood came down at about 6 am and in little more than two hours it subsided, leaving large masses of ice, snow and mud, ... the ice and snow were remarkably compact, very black, and emitting a strong sulphurous smell'.

The largest known lahar from Mt. Ruapehu, on Christmas Eve, 1953, probably was generated by collapse of the ice wall about the crater lake and swept out the Whangaehu rail bridge, causing 151 deaths. The volume of the lahar was so large principally because more water was stored in the crater lake then than since. An eruption in 1945 had deposited an ash and debris barrier restricting lake outflow, so the depth of the crater lake built up to approximately 7 m above its usual level. The 1953 lahar is discussed in the Report of the Board of Enquiry into the Tangiwai Railway Disaster (Turner 1954) by Paterson (1980) and by Weir (1982).

The 1975 lahar was seen by Mr JB Martin, who was located at a bridge 14.3 km downstream from the Tangiwai SH 49 bridge. He reported (Paterson et al. 1976) a rapid rise in waterlevel, reaching a peak in several minutes, with a surging and turbulent flow. The water was black and oily in appearance, with a strong sulphurous smell. Assorted debris was carried along ahead of the peak flow, and large boulders could be seen being tossed around and carried partly in suspension. A small wooden farm bridge (Stracken's Bridge), 6.5 km downstream of State Highway 49, was swept away by the lahar.

Although many small lahars from Mt. Ruapehu have been discovered in hydrographic records, as well as being observed during periods of small hydrothermal eruptions in the crater lake, we shall discuss only the 13 recent and major lahars detailed in Table 1. Available information includes in most cases: the time of eruption repsonsible for lahar generation; the discharge with time of the lahar at various fixed positions about the volcano; topography of the lahar flow path; and estimates of lahar volume as inferred from changes in crater lake volume. (Changes in crater-lake volume can be less relevant to debris flows, where small amounts of water can initiate a large volume of largely solid flow.)

The Whangaehu, Whakapapa and Manganui-a-teao are the three main rivers draining Mt. Ruapehu (Fig. 1). The large loss of life caused by the 1953 lahar prompted the installation of a water-level device in the Whangaehu river channel about 12 km upstream of the rail bridge, to provide an early warning system for future lahars. Subsequently, a stage recorder was also placed downstream of the railbridge in the Whangaehu channel, and a continuous trace of river stage versus time has been available in the Whangaehu channel for all subsequent lahars. Additional stage recorders have been installed in the Whakapapa and Manganui river channels. Raingauges have been installed at Makotuku and Te Porere, yielding estimates for background channel flows, which would have occurred in the absence of any lahar flow. Seismic records for the period of interest were provided by a seismometer installed at the Chateau, and between 1968 and 1975 by another seismometer near the summit of Mt. Ruapehu (Dibble 1969).

The cross-section of lahars depends, of course, on channel cross-section. Near the summit, lahar flows of up to 8 m have been inferred from lahar deposits. O'Shea (1954) states that the lahar producing the Tangiwai disaster was 3-m deep and 30-m wide. Lahar stage is measured directly, and the associated lahar flow is then estimated by analogy with the corresponding behavior of river water at that point.

Most of the previous work on Mt. Ruapehu lahars has been of a descriptive nature, with little quantification. However most of the hydrographic records have

**Table 1.** Selected Ruapehu lahars

| Date | Channel | Volume ($m^3$) |
|---|---|---|
| 24. 12. 1953 | Whangaehu | 1900000 |
| 26. 4. 1968 | Whangaehu | 740000 |
| 22. 6. 1969 | Whangaehu | 67000 |
| 22. 6. 1969 | Manganui | 24000 |
| 22. 6. 1969 | Whakapapa | 117000 |
| 8. 5. 1971 | Whangaehu | 41400 |
| 16. 5. 1971 (0100) | Whangaehu | 72000 |
| 16. 5. 1971 (0920) | Whangaehu | 57600 |
| 19. 5. 1971 | Whangaehu | 18180 |
| 24. 4. 1975 | Whangaehu | 1800000 |
| 24. 4. 1975 | Manganui | 600000 |
| 24. 4. 1975 | Whakapapa | 900000 |
| 2. 11. 1977 | Whangaehu | 130000 |

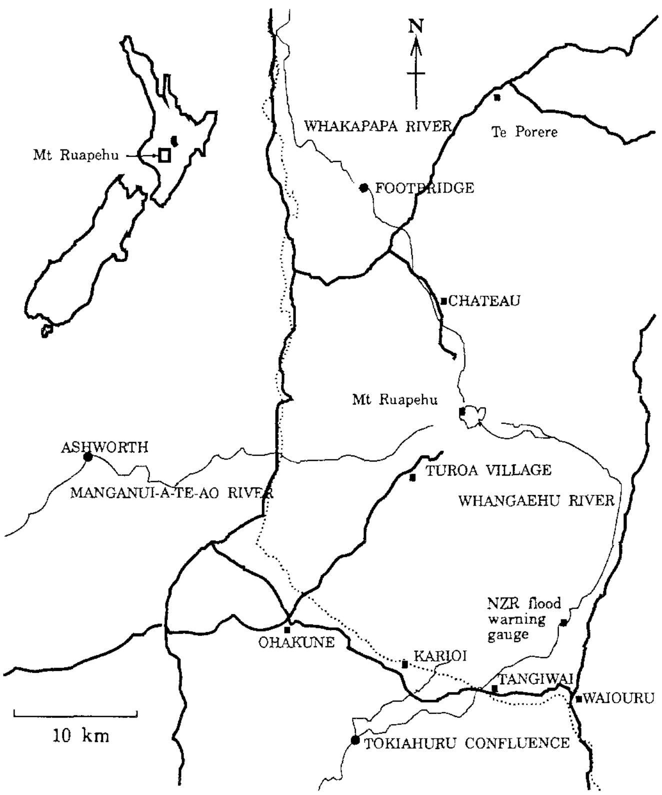


**Fig. 1.** Positions of lahar recording sites about Mt. Ruapehu. The *dotted line* denotes the main railway line north, the *heavy lines* are main roads about Mt. Ruapehu, and the *thin lines* are the main streams draining Mt. Ruapehu. Stage recorders are shown as *solid circles* at the Tokiahuru Confluence, the Whakapapa Footbridge, and at Ashworth on the Manganui-A-Te-Ao River

been combined into a theory (Weir 1982) applicable to lahars generated from a point source of fluid. The main results are given in the next section.

The main aim of this paper is to extend that theory to include cases where initial point sources of fluid are interacting. This occurred in the 1968 lahar, which was not accompanied by a large eruption. The 1968 lahar is analysed in detail in this paper so as to complete the analysis for Ruapehu lahars. In addition to being applicable to lahar flows, the theory developed in this paper will also apply to any geophysical flow of finite volume in which the kinematic wave hypothesis is approximately true. For instance, the results of this paper will apply to any isolated flow of liquid, gas or solid in which the gravitational force is approximately balanced by frictional losses in such a manner that the flow of conserved mass is a function of flow depth.

## Kinematic Wave Theory for isolated lahars

The expression 'Kinematic Wave Theory' (KWT) was coined by Lighthill and Whitham (1955) to describe types of flows in which the dominant mechanism determining propagation of the fluid was an empirical relationship connecting fluid discharge and fluid depth, which, together with conservation of fluid mass, allows the resulting flow to be calculated from a first order hyperbolic equation. Different applications of KWT have been described by Whitham (1979) and Weir (1983).

Kinematic wave theory does not provide a complete description of the various flow processes that are occurring, since it requires an empirical assumption connecting discharge and depth. Such a relationship will not hold in general and is, at best, approximate. Consequently, predictions obtained using KWT reflect, in a certain sense, only averaged information and a compromise that is attractive in geophysical applications where precise details of the flow are not often available.

The physical conceptualisation assumed for lahars in this paper, and incorporated into KWT as the lahar discharge-depth relationship, is that the gravitational force is exactly balanced by frictional forces. Accelerations of fluid are ignored, and so instantaneous steady flows are assumed. This assumption breaks down at the front of a flow wave, or lahar. In particular, the front of a lahar moves, at any given time, with almost uniform velocity, although the depth of lahar varies from zero at the front of the lahar to a maximum value some distance behind the front of the lahar. This violates the assumption in KWT that a discharge-depth relationship holds.

This difficulty is usually overcome in KWT by ignoring the forward part of the lahar, where a 'shock' is fitted. At a forward shock, lahar depth is assumed to increase instantaneously from zero to some non-zero value. This allows the discharge-depth relationship to hold in KWT. Similarly, in regions where fluid acceleration is important, it is likely that other shocks may require construction, by allowing for instantaneous rises in lahar depth. Shock construction follows naturally from the mathematical formulation, by requiring that lahar depth be single-valued, except at a shock.

The application of KWT to lahars is attractive, since the precise nature of such lahars is uncertain. In addition to the complexities associated with the flow of liquid and solid comprising the lahar, there are also difficulties resulting from the complex topography channeling the flow, and uncertainties associated with lahar formation. Such considerations suggest a compromise such as KWT which captures the main features of the flow, without introducing many unknown factors of secondary importance, and without requiring excessive computation.

The central equations of KWT for lahars state mathematically the conservation of mass,

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

and a flux law relating lahar flow $q$ to lahar depth $h$,

$$q = \alpha(x) h^k \tag{2}$$

where $x$ is distance and $t$ time (since the corresponding time of lahar formation). Flow $q$, which has units of $m^2s^{-1}$, denotes the actual flow per unit width of channel. We assume that for each lahar there is a constant value of $k$.

Equation 2 approximates the flow as a function of

**Table 2.** Scaled distances and $k$ values for Whangaehu lahars

| Date | Eruption time | $z$ | $k$ |
|---|---|---|---|
| 22. 6. 1969 | 0032 | 15816 | 1.31 |
| 8. 5. 1971 | 1555 | 15416 | 1.25 |
| 16. 5. 1971 | 0100 | 15170 | 1.24 |
| 16. 5. 1971 | 0920 | 11689 | 1.37 |
| 19. 5. 1971 | 1812 | 15571 | 1.23 |
| 24. 4. 1975 | 0359 | 15789 | 1.31 |
| 2. 11. 1977 | 1351 | 11881 | 1.47 |

flow depth and channel properties. The function selected previously, and used in this paper, is a power-law akin to the Manning-Law or Chezy-Law for hydraulic flows, and the function $\alpha(x)$ allows for such factors as longitudinal variations in channel width, roughness and channel slope. It has been shown (Weir 1983, Eq. 13) how to construct a scaled distance, $z$, in order that $\alpha(x)$ transforms into a constant. Then the characteristics become straight lines in $z-t$ space, satisfying,

$$\frac{dh}{dt}=0 \qquad (3)$$

$$\frac{dz}{dt}=\frac{dq}{dh} \qquad (4)$$

and so $h$ and $q(h)$ are constant along each characteristic.

The values of $k$ indicates how strongly flow depends on lahar depth. For water flows, $k$ is typically about 1.7; for mud-flows $k$ can be as high as 3 (Gol'din and Lyubashevsky 1966); while Table 2 shows that $k$ varies between 1.2 and 1.5 for Mt. Ruapehu lahars.

It has been proved (Weir 1983) that lahars of finite volume that are generated over a finite interval of time tend to a limiting self-similar profile with increasing time. In particular, the spatial distribution of lahar fluid tends to evolve to an essentially common, though time-dependent profile, independently of the initial distribution of lahar fluid. Self-similarity refers to the long-time mathematical property of lahar profiles, which depend essentially on the single variable $zt^{-1}$, rather than on both of the variables $z$ and $t$ separately. Self-similarity tends to develop because information concerning the initial conditions travel with the characteristic speed in Eq. 4, which is faster than the speed of the fluid. Thus the detailed nature of the initial shape of the lahar tends to be lost with time. Except for the 1968 lahar, all Mt. Ruapehu lahars considered in this paper have effectively attained such a limiting profile by the time they reach their corresponding stage recorder.

One of the main results obtained for such self-similar lahars (Weir 1983) is that the largest discharge $Q$ occurs at the front of the lahar, and satisfies,

$$Q=\frac{V}{(k-1)t} \qquad (5)$$

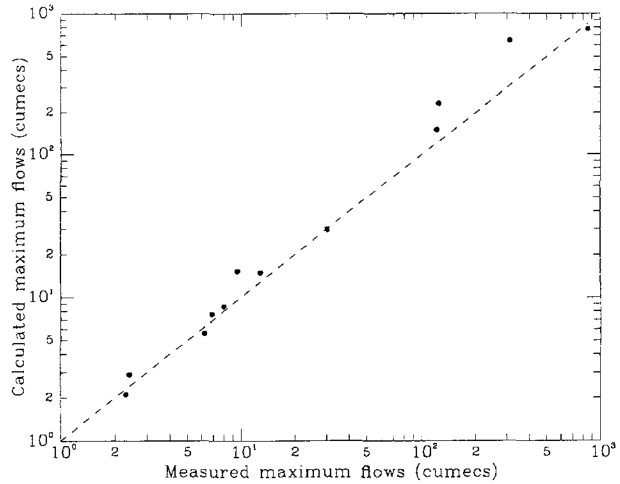


**Fig. 2.** Plot of calculated and measured laher peak flows about Mt. Ruapehu using Eq. 5. A cumec is a cubic meter per second

where at time $t$, $Q$ is the largest (shock) discharge in units of $m^3s^{-1}$, and $V$ is total lahar volume in $m^3$. A test of Eq. 5 is given in Fig. 2, using $k=1.234$ and is shown to be approximately valid over three orders of magnitude.

A second main result for such self-similar lahars (Weir 1983) is that the average travel time, since flow initiation, for lahars satisfies,

$$z=\left(\frac{A}{(k-1)}\right)^{\frac{k-1}{k}} t^{\frac{1}{k}} \qquad (6)$$

where $z$ is the scaled distance associated with the position of the stage recorder, and $A$ (in units $m^2 s^{-1}$) is total lahar volume $V$ per unit channel width. The variable $z$ is a constant that depends on the position of the stage recorder and the nature of the lahar channel. A test of Eq. 6 is given in Fig. 3, which shows that the arrival time for such lahars can be predicted reasonably accurately. Note from Eq. 6 that since $k$ is greater than unity, the average speed of lahars decrease with time.

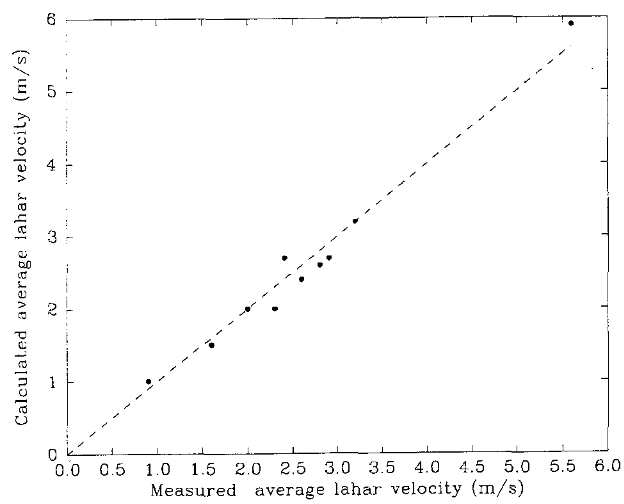


**Fig. 3.** Plot of calculated and measured average lahar velocities about Mt. Ruapehu, using Eq. 6 with $k=1.234$

Average lahar speeds are found by dividing the travel distance of the lahar by the arrival time in (6).

Equations 5 and 6 provide the main quantifications needed for such lahars - namely how large are the resulting flows, and how long after flow initiation do such flows strike. These results are particularly interesting because results involving time $t$ and flow $q$ are independent of the longitudinal scaling ($z$).

The volumetric flow per unit channel width $q$ was shown to satisfy

$$q = \left(\frac{z}{t}\right)^{\frac{k}{k-1}} \tag{7}$$

where $z$ is the scaled longitudinal distance (Weir 1982). Previous work proceeded by fitting the decay of lahar discharge at a fixed point as a function of time since flow initiation, using Eq. 7, which yielded a value of $k$ for that lahar. Then the value of $z$ was evaluated from Eq. 7. Values of $k$ and $z$ thus found are shown in Table 2. The consistency of this approach is demonstrated by the close agreement between modelled and measured shock flows and arrival times, predicted in Eqs. 5 and 6.

Finally, the mathematical transformation of $x$ into $z$ was chosen so that the position-dependent function $\alpha(x)$ transforms into a constant $\alpha^*$; namely,

$$\alpha^* = k^{-k} \tag{8}$$

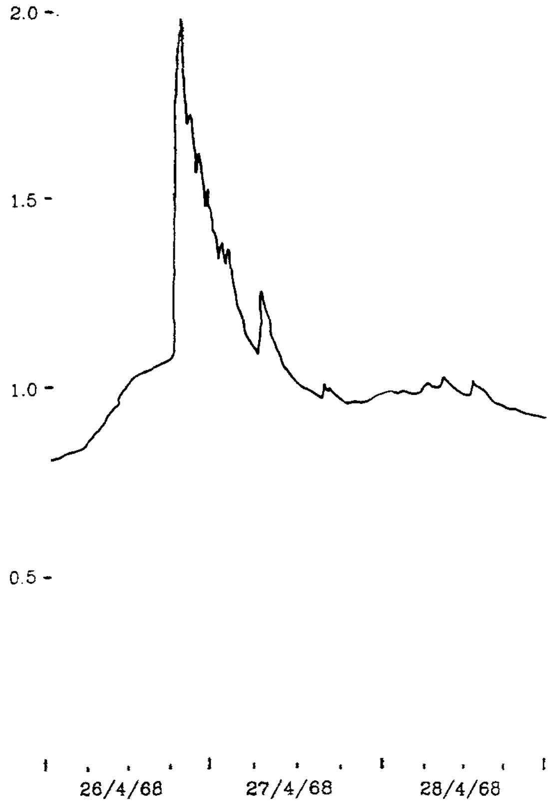


**Fig. 4.** Plot of measured lahar depths (m) at the Tokiahuru Confluence for the exceptional 1968 lahar, vs time in parts of days

## The exceptional 1968 lahar

The original hydrograph of the 26 April 1968 lahar is reproduced in Fig. 4. This lahar is exceptional in three respects. First, the initial flow took approximately one hour to reach maximum discharge. This is in contrast to the other lahars discusssed in the previous section, which, according to hydrograph records, took only several minutes to reach maximum discharge. The only eye-witness account for the rise in a Ruapehu lahar flow, by Mr. Martin (Paterson et al. 1976) observing the 1975 Whangaehu lahar, also suggests that the peak flow was reached within a few minutes of the initial onset of the lahar.

Second, the hydrograph shows that the flow does not decrease monotonically with time, as occurred for the examples discussed in the previous section, but has seven individual flow peaks within the hydrograph. This suggest that the 1968 lahar formed from either a source of fluid distributed over several hours, or else originated from a series of point sources of fluid that coalesced before arrival at the Tokiahuru Confluence.

Third, there was no single triggering event responsible for the 1968 lahar. Instead, there was seismic activity over a period of hours. A description of seismic activity at Mt. Ruapehu over this period is given in Table 3, and the seismic power chart (Dibble 1969) is reproduced in Fig. 5. Note that all of the lahars discussed in the previous section could be associated with a single formation time, the correctness of which is demonstrated by the success of the theory in the previous section.

These three exceptional properties of the 1968 lahar clearly indicate that the theory in Weir (1982), which assumed a single delta function source of fluid at a particular time, and a dry river valley, cannot be applied directly to the 1968 lahar. The aim of this section is to

**Table 3.** Seismic activity at Mt. Ruapehu 26 April 1968

| Time | Magnitude | Description |
|---|---|---|
| NZST | Richter | |
| 12:36 | 2.2–2.3 | 9 hours of strong volcanic tremor |
| 01:16 | 3.5 | Volcanic earthquake sequence, suggesting eruption |
| 07:49 | 3.5 | Volcanic earthquake sequence, suggesting eruption |
| 09:36 | 3.1 | Volcanic earthquake sequence, suggesting eruption |
| 15:30 | 2.3 | Volcanic earthquake sequence, suggesting eruption |

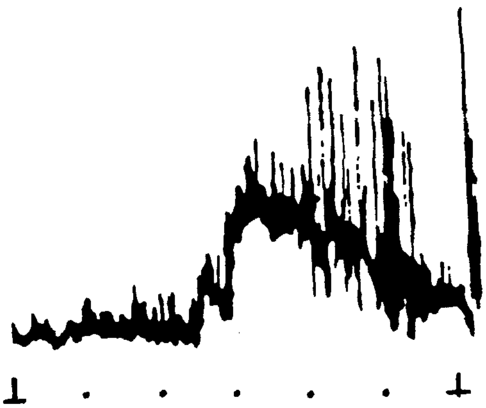

Fig. 5. Plot of measured seismic power at Mt. Ruapehu, on 26 April 1968, as a function of time (dots every 4 h), from Dibble (1969)

extend the earlier theory so as to apply to the 1968 lahar. Kinematic Wave Theory is applied to the 1968 lahar, by assuming seven delta functions of source fluid at the summit of Mt. Ruapehu. Then a fully numerical approach was used which seeks a distribution of source fluid at the summit of Mt. Ruapehu which approximately yields the measured flows at the Tokiahuru Confluence.

The analytical approach described in the previous section was successful primarily because, for all of the lahars discussed, there was an obvious seismic event responsible for the lahar, and so the time $t$ since the lahar formation was available. For the 1968 lahar, however, there was no clear time of lahar formation, and in addition the decay of flow at the Tokiahuru Confluence was non-monotonic with time, so the above procedure for determining $k$ and $z$ required modification.

The approach adopted in this paper proceeded by observing that values of $k$ and $z$ in Table 2 fall into two groups. The first and largest group has mean $z$ and $k$ values of 15552 and 1.27. This $k$ value is close to the value found in Fig. 4, as an average value for Whangaehu lahars, of 1.234. The computer run using 15552 and 1.234 was denoted by Model 1. Similarly when the second group's average values, 11785 and 1.42 were used, the corresponding run was called Model 2.

A computer program was written to solve Eqs. 1, 2 and 9. The program calculates the variation of depth at the Tokiahuru Confluence resulting from a particular initial depth profile at Crater Lake. This initial depth profile was adjusted until the final depth profile closely fitted the actual hydrograph shown in Fig. 4. The initial depth profile consisted of seven isolated sources of water of different volumes, as given in Table 4.

For mathematical convenience, the height of Mt. Ruapehu was extended, and seven isolated delta function sources of water of different volume and position were simultaneously placed on this imaginary extension. This formulation allows the 1968 lahar to evolve from a given initial condition, rather than by specifying a source of water at the rim of the crater. Physically, water is ejected from the lake as a function of time. Our

**Table 4.** Components of the 1968 Lahar

| Peak flow | Tokiahuru confluence time | Hydrograph | Equation 4 |
|---|---|---|---|
| $m^3 s^{-1}$ | 26–27 April 1968 | Volume $m^3$ | Volume $m^3$ |
| 45.6 | 1930 | 138200 (265600) | 174800 |
| 28.6 | 2039 | 109300 | 135800 |
| 23.0 | 2155 | 105100 | 111900 |
| 18.7 | 2327 | 93000 | 93000 |
| 12.0 | 0129 | 30700 | 65000 |
| 11.2 | 0221 | 71700 | 66300 |
| 7.5 | 0652 | 64800 | 41600 |

aim is to adjust the initial condition, until the water flowing down the imaginary mountainside from the seven sources approximately matches the physical source at the crater rim.

The Whangaehu channel clearly increases in width down Mt. Ruapehu. This can be allowed for by modifying the scaling function $\alpha(z)$ in Eq. 2, but since such a scaling function is not known precisely, we have made the simplifying assumption that the Whangaehu channel has a constant width of 60 m.

During the 1968 lahar flow, heavy rainfall was occurring, which is added to the lahar contribution to yield the total recorded channel flow. Since the background rainfall was almost constant during the period that the 1968 lahar reached its peak flows and then essentially decreased to zero flow, we simply subtracted the inferred contribution from rainfall, and denoted the remainder as the total contribution from the lahar. Of course, the rainfall contribution to channel flow will vary (in some unknown manner) down Mt. Ruapehu as the effective drainage area increases and depending on rainfall intensity with position, but we have ignored such effects and discussed only the lahar flow.

A discharge-depth law for the lahar was inferred for the remaining lahar flow. The discharge-depth law used by hydrologists to infer flows in the Whangaehu river at the Tokiahuru Confluence is,

$$q_t = 2.48\, H_t^{1.765} \tag{9}$$

where total flow per unit width $q_t$ has units of $m^2 s^{-1}$; channel width $w$ is 60 m; and $H_t$ is the total depth of water from both lahar and rainfall in the channel. Then, subtracting the background flow inferred for rainfall, the remaining flow per unit channel width $q$ in the channel was related to the difference $H$ in total channel flow depth minus the background depth, and yielded a relationship closely approximately by,

$$q = 0.21\, H^{1.1636} \tag{10}$$

Interestingly, the power of 1.1636 in Eq. 10 is close to the inferred Model 1 $k$ value from Fig. 3 of 1.234, which relates to an average '$k$' value obtained using travel times for the Whangaehu channel. Note that Eq. 10 relates inferred lahar flow to inferred lahar depth at the Tokiahuru Confluence, whereas Eq. 2 refers to lahar propagation betwen the summit of Mt. Ruapehu and the Tokiahuru Confluence. Further, whereas the coefficient of 0.21 in Eq. 10 contains contributions from such factors as channel slope, and is therefore position-dependent, the coefficient $\alpha^*$ in Eq. 8 has such position-dependence scaled out.

The calculations used the scaled lahar depths $h$, which are different from the dimensional depths $H$ in Eq. 10. Transformations between $h$ and $H$ are achieved through $q$, flow per unit width, which is independent of longitudinal scaling. The two main differences between the present approach and the earlier work is, first, no analytical solutions are available, necessitating numerical calculations; and second, the shock condition becomes,

$$\frac{ds}{dt} = \frac{\Delta q}{\Delta h} \quad (11)$$

where $\Delta q$ and $\Delta h$ are respectively the changes in flow and scaled lahar depth at the shock. Equation 11, together with the condition that both $q$ and $h$ are constant along characteristics, allows Eq. 1 for lahar flow to be solved.

The depth of flow at each point in space and time was found by calculating the appropriate characteristic through that point. The program follows each lahar component from its position at the starting time until it reaches the Tokiahuru Confluence. It outputs the height of water to be expected as a function of time, for comparison with the hydrograph. It also compares the mass of water present at the beginning and at the end, to check for mass conservation. The final mass is required to be within 6% of the initial mass. Note that other volcanic flows, such as debris flows, may not conserve mass, and may increase their total volume by picking up sediment by as much as a ten-fold increase.

Each lahar component was fitted sequentially, using trial and error to find the appropriate values of starting locations and starting heights. All lahar components are assumed to have originated from a delta function source of fluid, which formed 100 s before the initial time in the computer program. The 100 s delay was chosen to avoid the infinite lahar depths which follow from using delta function sources of fluid. A negative starting location means that at the starting time the lahar component had not reached the crater rim. Although the initial shape of the earliest lahar components has some influence on the shape of the corresponding final lahar component, (because the lahar component has not lost all of its memory of its original shape), for reasons of simplicity, we did not allow for variable initial shapes (by allowing the initial values of '$k$' to vary), even though this did give improved fits to the hydrograph. The later lahar components are less sensitive to their initial shape, as would be expected (since they travel the furthest). Fitting of later lahar components did not affect the fits obtained for earlier lahar components.

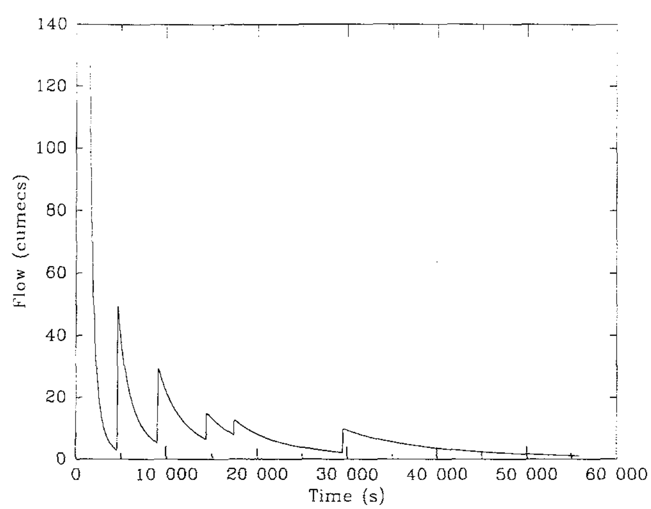


**Fig. 6.** Plot of calculated lahar flows ($m^3 s^{-1}$) at Crater Lake as a function of time ($s$) for $k$ equalling 1.42 and $z$ equalling 11785

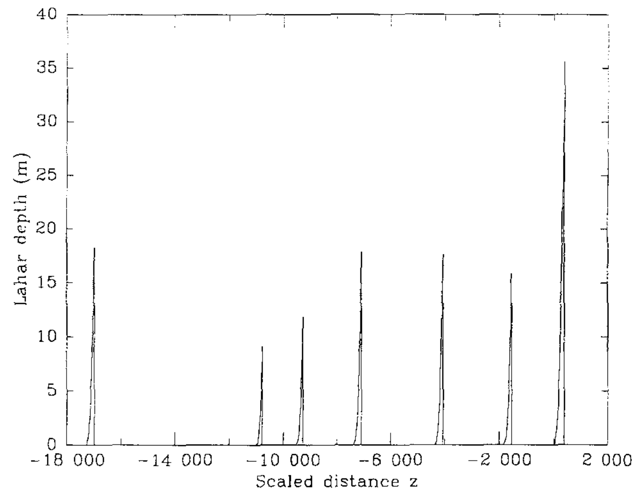


**Fig. 7.** Plot of modelled input lahar depths ($m$) above Mt. Ruapehu, at zero time, vs scaled distance for $k$ equalling 1.42 and $z$ at the Tokiahuru Confluence equalling 11785. These lahar profiles have evolved after 100 s from delta function profiles

Finally, once the kinematic wave equation has been solved, and the scaled initial lahar depths found, the initial flows and depths at the summit of Mt. Ruapehu are found.

Figure 6 plots the calculated flows at Crater Lake obtained from the numerical Model 2 using $k$ and $z$ values of respectively 1.42 and 11875. The initial depths of lahar, chosen to produce an approximate fit to flows at the Tokiahuru Confluence, are shown in Fig. 7. The negative distances indicate that these positions have been extended beyond Mt. Ruapehu.

Seismic power at Mt. Ruapehu for 26 April 1968, is indicated in Fig. 5. A description (J. H. Latter, personal communication) of Fig. 5 is given in Table 3. The onset of seismic activity at about 1236 h is clear on Fig. 5, while the subsequent individual peaks between about 1600 and 2200 h, which we suggest later are responsible for the first six lahar components in the 1968 lahar, are also clear. The final peak is suggested later as responsible for the seventh lahar component.

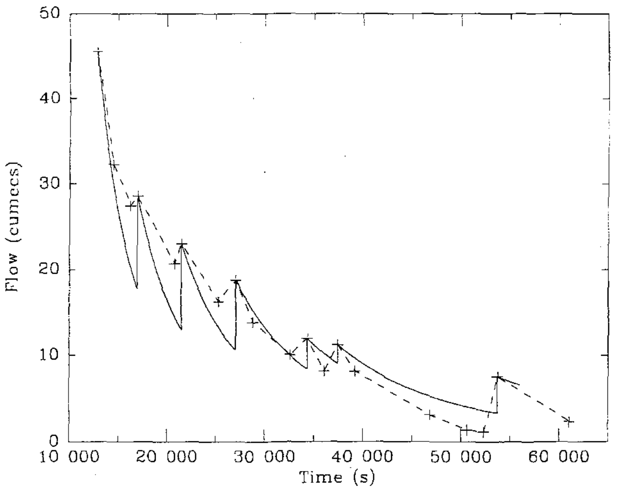


**Fig. 8.** Plot of measured and modelled lahar flows ($m^3 s^{-1}$) at the Tokiahuru Confluence, vs time in seconds. The *solid line* represents actual data; the *broken line* shows values obtained from the numerical model using $k$ equalling 1.42, and $z$ equalling 11 785

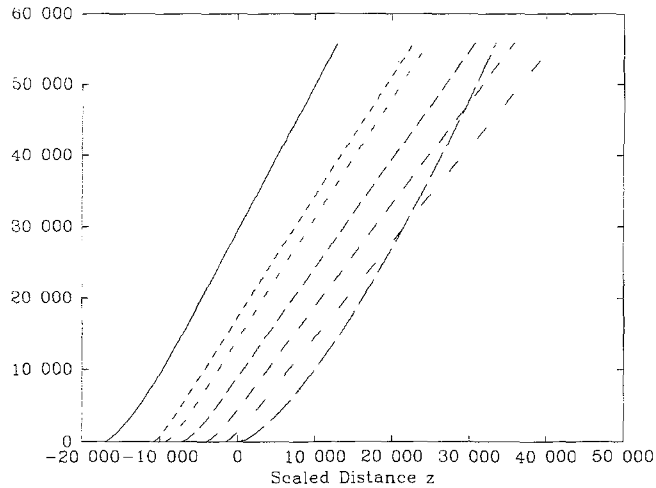


**Fig. 9.** Time ($s$) vs scaled distance plot of calculated characteristics for the seven model lahar components with $k$ equalling 1.42 and $z$ equalling 11 785

The numerically calculated flows at the Tokiahuru Confluence are shown in Fig. 8, when the Model 2 values of $k$ and $z$ (1.42 and 11 785) are used. These modelled flows were chosen by varying the initial fluid sources at the summit of Mt. Ruapehu to correctly match the measured peak lahar component flows. In general, the first three lahar components decrease with time more rapidly than the measured flows.

A plot of the characteristics in the time and scaled distance plane is shown in Fig. 9. The 'physical' range of $z$ is between 0 and 11 785 (the position of the Tokiahuru Confluence). Negative values of $z$ denote positions extended above Mt. Ruapehu, while time refers to seconds since the formation of the first lahar component at about 1600 h NZST.

The first and second characteristics intersect at a $z$ value of about 21 000. From then on a modified treatment of characteristic propagation is needed, since both lahar components will have coalesced. Indeed, if the model had been run sufficiently long, all of the lahar components would have coalesced into a single wave, with only one forward shock. This occurs because deeper fluid travels faster than shallower fluid, and so deeper fluid moves to the front of the lahar. However, such coalescences can be ignored in this paper, since these occur downstream of the Tokiahuru Confluence.

A comparison of modelled times of lahar formation at Mt. Ruapehu and times of seismic activity is given in Fig. 10 and Table 5. The modelled lahar formation time was chosen as the time the corresponding lahar shock reaches the crater rim, whereas times of seismic activity were obtained from Fig. 5. There is a good correlation between modelled source times and seismic activity for both models, but particularly for Model 2.

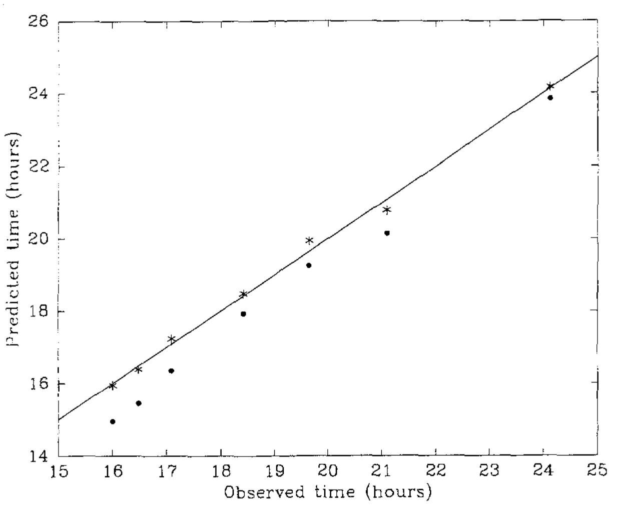


**Fig. 10.** Comparison of seismic times at Mt. Ruapehu and inferred source times at Mt. Ruapehu of lahar component formation. The $x$-axis represents time in hours of seven of the largest peaks in the seismic record. The $y$ axis represents lahar component formation times predicted by the model. The line is $y = x$, i.e. perfect correspondence. The dots show Model 1 values, while the asterisks show Model 2 values

**Table 5.** Estimated formation times (NZST) for the 26 April 1968 lahars at Crater Lake. The analytic estimates are calculated using the model values for $k$ and $z$, and assuming no interaction between lahar components. The seismic estimates have a reading error of 10 min

| Model 1 | | Model 2 | | |
|---|---|---|---|---|
| Analytic | Numerical | Analytic | Numerical | Seismic |
| 1457 | 1457 | 1557 | 1557 | 1600 |
| 1501 | 1527 | 1515 | 1623 | 1629 |
| 1608 | 1621 | 1617 | 1714 | 1705 |
| 1733 | 1756 | 1733 | 1829 | 1825 |
| 1903 | 1915 | 1818 | 1957 | 1938 |
| 1919 | 2009 | 1808 | 2048 | 2105 |
| 2418 | 2351 | 2356 | 2410 | 2407 |

**Discussion**

A general theory applicable to hydraulic lahar flows has been presented. Comparison of modelled and measured flows and travel times have been shown graphically.

The theory presented above does not involve explicit consideration of lahar depths of flow. In principle, depths can be calculated from the discharge-law in Eq. 2, but when this is done, calculated flow depths are too large. This failure represents an inconsistency in the theory. From a practical viewpoint, this problem can be avoided by using Eq. 10, which yields 'reasonable' estimates for lahar depths. Of course, while this yields consistent estimates for all lahar quantities, it does not remove the inconsistency within the theory, which is primarily due to the difference between Eq. 10, which relates to a particular point, and Eq. 2 which is an average over all points.

Lahars possibly do not propagate as cold water does, so that inferring lahar flows from lahar depths on hydrographic records using Eq. 9 could lead to inconsistencies. However, it is unlikely that estimated flows differ by an order of magnitude from actual flows, since the total lahar volumes of several of the Whangaehu lahars agree within about a factor of two with total changes in Crater Lake volume. Consequently, we have accepted Eq. 9 in the absence of an alternative.

A difficulty also arises from the slow build-up, of about one hour, to maximum flow. During this time about 127 000 $m^3$ of lahar fluid has passed the Tokiahuru Confluence, which is comparable to the volume we associated with the first lahar component. We have simply ignored this fluid, since there is no obvious way in which it can be included into our model of lahar component arrivals.

Many of the criticisms above arise largely because of lack of knowledge about lahar formation and transport. Perhaps the remarkable feature of this work is that, given so little information, the maximum lahar discharge at a given point, the subsequent decay with time of discharge at that point, and lahar travel time from Mt. Ruapehu could be described satisfactorily. This is particularly important in discussing future lahar hazards from such volcanoes, since it seems likely that the conditions surrounding their formation and transport will be extremely uncertain. Simple theories such as given above are then valuable, since they provide a robust method for estimation of discharges and travel times.

Figures 3 and 2 show the success of KWT in describing travel times and lahar discharges about Mt. Ruapehu for lahars generated by single seismic events. Modelled and actual formation times of components of the 1968 lahar, as given in Table 5 and plotted in Fig. 10, suggest that Model 2 is superior to Model 1. The near coincidence of the times for Model 2 strongly suggests that these seismic events coincided with the corresponding lahar formations.

However, Fig. 8 shows that the fit between measured and modelled flows at the Tokiahuru confluence gives good agreement for lahar-component heights and arrival times, but is rather poor in describing lahar decay, especially for the first two lahar components. An improved fit in lahar component decay could have been achieved by modifying the input function to include non-delta function sources, but this approach was not pursued.

We conclude that the early source of lahar fluid resulted from an extended period of flow at the crater. This view is supported by the slow rise in lahar flow for the first components, and also from the seismic record, which shows a period of exceptionally strong tremor with an absence of substantial seismic shocks (J. H. Latter, personal communication). It is possible that the early components of the 1968 lahar resulted from uplift or shallowing of the crater lake floor, which is known to have changed between 1965 and 1970 (Hurst and Dibble 1981) from an average depth of 47 m to 28 m.

The theory above has considered only lahars from Mt. Ruapehu. Of course, lahars are common elsewhere (Neal 1976; Legowo 1981), and the theory above may be applicable to other volcanoes. In addition, the theory need not be restricted only to lahar flows, but may also apply to compact finite volume flows, for example of poisonous gas, in which the gravitational force is essentially balanced by a frictional depth-dependent force. For gas flows, the equations corresponding to Eqs. 2 and 9 would re required.

Finally, the theory has benefited from the excellent hydrographic records of lahar flows, and the corresponding excellent seismic record from Mt. Ruapehu. How does one proceed in the absence of such records? There is probably not best approach to adopt, but since this question is likely to arise in a number of important field examples, we shall briefly address it here, although we emphasise that our tentative answer is essentially untested, and highly speculative.

If no relevant records of lahars exist, then the scaling $z$ used above will be unavailable, and it may be best to use standard distances. Also, the value of $k$ used above (1.42) will be untestable. One choice for $k$ is to guess a value based on river flow. Suppose the river hydrogaph suggests, at some point,

$$q = \alpha H^k \tag{12}$$

where $\alpha$ and $k$ are constants, $q$ is flow per unit width, and $H$ is flow depth. Then, following the approach in Weir (1983), for a finite-volume self-similar flow,

$$H = \left(\frac{x}{\alpha k t}\right)^{\frac{1}{k-1}} \tag{13}$$

where $x$ and $t$ denote (unscaled) distance and time. From Eq. 13, the first arrival time of the lahar, $T$, satisfies,

$$T = \frac{x^k}{\alpha k}\left(\frac{w(k-1)}{kV}\right)^{k-1} \tag{14}$$

while the peak flow satisfies Eq. 5.

Naturally, this approach is expected to yield a crude approximation to lahar behavior. For example, applying Eqs. 12 and 14 in an attempt to predict the maximum recorded flow from Mt. Ruapehu in 1953, and using Eq. 9 ($\alpha = 2.5$; $x = 51\,400$; $k = 1.76$; $V = 1\,900\,000$; $w = 60$) to determine the unknown parameters, yields reasonably good estimates for lahar depths (1.4 m) and travel times (8917 s), but underestimates the peak flow by about a factor of 3. This factor of three is essentially the ratio in the different $k - 1$ factors resulting from choosing 1.76 instead of 1.31 for $k$. Whether such factors of 3 are considered important will clearly depend on how well other model parameters (such as lahar volume) are known.

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