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Vibration isolation using open or filled trenches

Part 1: 2-D homogeneous soil

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Abstract. The problem of structural isolation from ground transmitted vibrations by open or infilled trenches under conditions of plane strain is numerically studied. The soil medium is assumed to be linear elastic or viscoelastic, homogeneous and isotropic. Horizontally propagating Rayleigh waves or waves generated by the motion of a rigid foundation or by surface blasting are considered in this work. The formulation and solution of the problem is accomplished by the boundary element method in the frequency domain for harmonic disturbances or in conjunction with Laplace transform for transient disturbances. The proposed method, which requires a discretisation of only the trench perimeter, the soil-foundation interface and some portion of the free soil surface on either side of the trench appears to be better than either finite element or finite difference techniques. Some parametric studies are also conducted to assess the importance of the various geometrical, material and dynamic input parameters and provide useful guidelines to the design engineer.

1 Introduction

Vibration isolation of structures from ground transmitted waves generated by machines, traffic or blasting is a very important engineering problem, especially in densely populated areas or in cases where the structure houses sensitive instruments. Vibration isolation of an active or passive type is accomplished by barriers which diffract the surface waves radiated from the vibration source and sufficiently reduce their amplitude. These barriers may be trenches (open or filled with bentonite or concrete), sheet piles, or a row of solid or tubular piles.

Barkan (1962) and Dolling (1965) were the first to report on some field investigations for studying the effectiveness of wave barriers, while these authors as well as Neumeuer (1963) and McNeill, Margason and Babcock (1965) also described some successful applications of vibration isolation. The most comprehensive work on the vibration isolation problem was done by Woods (1967, 1968), Richart, Hall and Woods (1970) and Dolling (1970a, b) who performed extensive field experiments to study the effectiveness of open trenches as Rayleigh wave barriers and provide design recommendations. Some years later, Woods, Barnett and Sagesser (1974), utilizing holographic interferometry, performed model dynamic tests to study the Rayleigh wave screening effectiveness of several types of rows of cylindrical obstructions or piles. Use of piles in cases of long Rayleigh wavelengths is the only practical solution because construction of very deep trenches required for this kind of waves is impractical. Further experimental (laboratory) studies on the use of piles as isolation barriers were also done by Liao and Sangrey (1978).

The analytical treatment of the problem of vibration isolation by wave barriers in the framework of linear elastodynamics is based on the theory of wave diffraction. The text of Pao and Mow (1973) describes all the analytical work on wave diffraction up to 1972. One can mention here Hudson (1967) who obtained an approximate solution for the scattering of Rayleigh waves by a surface obstacle; Knopoff (1959a, b), Mal and Knopoff (1975), Pao and Mow (1963), Thau and Pao (1966) who studied wave diffraction by spherical, rectangular and parabolic obstacles; Trifunac (1973), Wong and Trifunac (1974), Mei and Foda (1979) who studied the scattering of SH waves in a half-space with a semicylindrical, semielliptical canyon and simple overground structures, respectively; Thau and Umek (1973, 1974), Dravinski and Thau (1976a, b) who studied the diffraction of elastic waves by rigid embedded rectangular foundations under conditions of plane strain and Luco (1969, 1976), Luco, Wong and Trifunac (1975) who studied the dynamic behaviour of cylindrical and spherical footings embedded in an elastic half-space. Recently, Mendelsohn, Achenbach and Keer (1980) and Angel and Achenbach (1984) treated the scattering of elastic P, SV and Rayleigh waves by surface-breaking cracks in a half-plane and a half-space, respectively; Avilés and Sánchez-Sesma (1983) determined the effectiveness of a row of rigid piles as barriers for elastic *SH*, *P* and *SV* waves and Lee (1982) studied the diffraction of plane elastic waves by a semispherical canyon.

Analytical solutions of wave diffraction problems are confined to simple geometries and idealized conditions. It is apparent that more realistic problems of diffraction of elastic waves in a half-space with barriers involving complex geometries can only be solved numerically. Utilising perturbation expansions and the Finite Difference Method (FDM) in both space and time domains in conjunction with a special boundary treatment Aboudi (1971, 1973) was able to obtain approximate solutions to problems of Rayleigh wave diffraction in a half-plane with a thin barrier. Recently, Fuyuki and Matsumoto (1980) employed the FDM in both the space and time domains in conjunction with nonreflecting boundaries to study in a two-dimensional context the effectiveness of trenches as Rayleigh, *P*, *SV* and *SH* wave barriers. Wass (1972) was probably the first to study in the frequency domain the screening effect of the trenches on elastic surface waves by the Finite Element Method (FEM), as an application in his thesis. He especially considered the case of *SH* waves by using special finite elements taking into account the radiation condition. Haupt (1977, 1978a–c) and Segol, Lee and Abel (1978) also employed in the frequency domain special types of the FEM in two dimensions to perform extensive numerical studies on the amplitude reduction of Rayleigh waves by open or infilled trenches. Haupt (1977, 1978a–c) utilized the FEM in conjunction with substructuring and his concept of the influence matrix boundary condition and assumed elastic, homogeneous and isotropic soil conditions, concrete filled trenches and soil material and geometrical damping. Segol, Lee and Abel (1978) developed a FEM similar to that of Wass (1972) and considered nonhomogeneous (layered) soil and open or bentonite slurry filled trenches. May and Bolt (1982) were able to study the effectiveness of trenches as wave barriers by utilizing a special time domain FEM with absorbing boundary characteristics. Their soil model was a viscoelastic layered half-plane. As it is well known, Rayleigh waves are surface waves which decrease in amplitude more slowly than body waves and for these reasons are the most important waves to consider in any isolation attempt. Most of the aforementioned references involving numerical methods as well as the present work deal with Rayleigh waves. It was found in Fuyuki and Matsumoto (1980), Haupt (1977, 1978a–c), Segol, Lee and Abel (1978), May and Bolt (1982) that trench effectiveness as a Rayleigh wave barrier increases, in general, with its depth in agreement with the experimental findings of Woods and Richart (1967), Woods (1967, 1968), Richart, Hall and Woods (1970).There is however disagreement on the importance of the trench width which is found significant in Fuyuki and Matsumoto (1980), Haupt (1977, 1978a–c) and insignificant in Woods and Richart (1967), Woods (1967, 1968), Richart, Hall and Woods (1970) and Segol, Lee and Abel (1978). Both the FDM and the FEM present a basic disadvantage in dealing with wave propagation problems in soils in that they represent a semi-infinite medium by a finite size model. Remedies to this may be either a very large, and hence uneconomical mesh, or use of complicated non-reflecting boundaries [Wass (1972), Haupt (1977, 1978a–c), Segol, Lee and Abel (1978), May and Bolt (1982)], which are sometimes applicable only when the layered soil is supported on a rigid bedrock base (Wass 1972, Segol, Lee and Abel 1978). In addition, these methods badly simulate dynamic stress concentrations in problems of wave diffraction as it has been demonstrated by Manolis and Beskos (1981, 1983). It is obvious that a more accurate, reliable and economical method for solving vibration isolation problems is needed.

The Boundary Element Method (BEM) which is employed in this paper for the study of the effectiveness of open or infilled trenches as wave barriers is free of these disadvantages as requiring only a small portion of the soil surface to be discretized and as taking automatically into account the radiation condition. Furthermore, it is a highly accurate and very efficient method, especially for vibration isolation problems where one is primarily interested in soil surface displacements. Various special forms of the BEM in the frequency domain have been employed for the solution of problems involving the scattering of elastic waves by soil surface or subsurface topography. One can mention here among others, the works of Wong and Jennings (1975), Wong, Trifunac and Westermo (1977), Wong (1982), Sánchez-Sesma and Rosenblueth (1979), Sánchez-Sesma and Esquivel (1979), Dravinski (1980, 1982a–c), Chu, Askar and Cakmak (1982) for two-dimensional and the recent work of Sánchez-Sesma (1983) for three-dimensional problems. Use of the direct BEM has also been employed for the solution of wave scattering by two- and three-dimensional rigid embedded foundations in the frequency domain by Dominguez (1978), Dominguez and Alarcon (1981) and in the time domain by Spyrakos and Beskos (1985) and Karabalis and Beskos (1985). The direct frequency domain BEM has also been utilized for the solution of wave scattering by two- and three-dimensional cavities by Kobayashi and Nishimura (1982) and Rizzo, Shippy and Rezayat (1985a, b), respectively. Very recently, an advanced BEM for both frequency and time domain elastodynamic analysis has been reported by Banerjee, Ahmad and Manolis (1986).

In this paper the problem of structural isolation from ground-transmitted vibrations by open or infilled orthogonal trenches under conditions of plane strain is numerically studied by the direct BEM in the frequency domain for harmonic waves or in conjunction with Laplace transform for the case of transient disturbances. Actually the proposed methodology can deal with trenches of any arbitrary shape but it is applied here to rectangular ones which are usually preferred in practice. The soil material is assumed to be a linear elastic or viscoelastic, homogeneous and isotropic half-plane. The methodology utilized in this work has as its starting base the type of the BEM described by Manolis and Beskos (1981). For reasons of simplicity, without, however, jeopardizing accuracy, constant boundary elements and the infinite plane Green's function are utilized in this paper. Use of infinite space Green's function requires, of course, a discretisation of the free soil surface, which is restricted however, for all practical purposes, to a small finite portion of it. For this particular problem the surface discretisation is required anyhow because information on the free surface is primarily required. The idea of employing a half-plane Greens function which requires no free soil surface discretization was abandoned in view of its high complexity (Dravinski 1980). Horizontally propagating harmonic Rayleigh waves are considered first and their scattering pattern by open or infilled trenches is determined by the proposed method and compared against other numerical methods. Two characteristic practical problems involving passive and active isolation of waves produced by machine foundations by means of trenches are also studied and parametric studies are performed in conjunction with these problems to assess the importance of the various geometric, material and dynamic input parameters and develop design recommendations. Results are also presented for the case of a passive isolation problem involving a transient disturbance. This problem is handled with the aid of Laplace transform with respect to time and its solution requires the use of numerical Laplace transform inversion as discussed in Narayanan and Beskos (1982). The very recently published work of Emad and Manolis (1985) also treats the problem of passive vibration isolation by open trenches by employing the BEM in the frequency domain under conditions of plane strain in a manner very similar to the present one. However, that work assumes the disturbance to be just a harmonically varying with time vertical concentrated force and, in general, represents a short study of a limited scope.

2 Treatment of plane elastodynamics by BEM

The formulation and solution of the general transient elastodynamics problem by the BEM in conjunction with Laplace transform has been first presented by Cruse and Rizzo (1968) and Cruse (1968) and in an improved form by Manolis and Beskos (1981). It was shown in Manolis and Beskos (1981) that one can go form Laplace transform domain to the frequency domain by simply replacing the Laplace transform parameter s by $i\omega$, where ω is the circular frequency and $i = \sqrt{-1}$. Manolis and Beskos (1981) will be the basis of the development of the present boundary element methodology both in the frequency and the Laplace transform domain.

Consider a homogeneous, isotropic and linear elastic body B with boundary S under conditions of plane strain. Its equation of motion under the assumptions of zero body forces is of the form

$$(c_1^2-c_2^2)u_{i,ij}+c_2^2u_{j,ii}=\ddot{u}_j, \tag{1}$$

where $u_i(x,y,t)$ are the components of the displacement vector, indices i and j correspond to Cartesian coordinates x and y, commas indicate differentiation with respect to these coordinates, dots indicate differentiation with respect to time t and summation over repeated indices is assumed. The c_1 and c_2 in (1) are propagation velocities of compressional (P) and shear (S) waves, respectively and are given in terms of Lamé's elastic constants λ and μ and mass density ϱ of the material by

$$c_1^2=(\lambda+2\mu)/\varrho, \qquad c_2^2=\mu/\varrho. \tag{2}$$

The constitutive equation is of the form

$$\sigma_{ij}=\varrho\,[(c_1^2-2c_2^2)u_{k,k}\delta_{ij}+c_2^2(u_{i,j}+u_{j,i})], \tag{3}$$

where $\sigma_{ij}(x,y,t)$ are components of the stress tensor and δ_{ij} is the Kronecker's delta. For the purpose of this paper, initial conditions are assumed to be zero, while the mixed type boundary conditions take the form

$$\sigma_{ij}n_j=t_{io}(x,y,t); \qquad (x,y)\in S_\sigma \tag{4}$$

$$u_i=u_{io}(x,y,t); \qquad (x,y)\in S_u \tag{5}$$

where n_j stands for the outward unit normal vector component at the boundary $S=S_\sigma+S_u$ and t_{io} and u_{io} represent prescribed boundary values for the traction and displacement vectors, respectively.

For time harmonic disturbances the solution of the problem is also time harmonic and one has

$$\sigma_{ij}=\bar{\sigma}_{ij}e^{i\omega t}, \qquad u_i=\bar{u}_ie^{i\omega t} \tag{6}$$

where ω is the circular frequency and $\bar{\sigma}_{ij}$ and $\bar{u}_i$ represent the stress and displacement amplitudes, respectively. Thus, Eqs. (1) and (3–5) take the following form in the frequency domain:

$$(c_1^2-c_2^2)\bar{u}_{i,ij}+c_2^2\bar{u}_{j,ii}+\omega^2\bar{u}_j=0, \tag{7}$$

$$\bar{\sigma}_{ij}=\varrho\,[(c_1^2-2c_2^2)\bar{u}_{k,k}\delta_{ij}+c_2^2(\bar{u}_{i,j}+\bar{u}_{j,i})], \tag{8}$$

$$\bar{\sigma}_{ij}n_j=\bar{t}_{io}(x,y,\omega); \qquad (x,y)\in S_\sigma \tag{9}$$

$$\bar{u}_i=\bar{u}_{io}(x,y,\omega); \qquad (x,y)\in S_u. \tag{10}$$

In order to solve the system of Eqs. (7–10) in the frequency domain by the BEM, use is made of the boundary integral equation (Manolis and Beskos 1981)

$$\tfrac{1}{2}\bar{u}_j(P)=-\int_S \bar{u}_i(Q)\,\bar{T}_{ji}(Q,P)\,dS(Q)+\int_S \bar{t}_i(Q)\,\bar{U}_{ji}(Q,P)\,dS(Q) \tag{11}$$

which is valid for points P and Q lying on a smooth boundary curve S as shown in Fig. 1. In Eq. (11) $\bar{U}_{ij}$ and $\bar{T}_{ij}$ are the singular influence tensors (fundamental solution or Green's functions) for the infinite plane which are given in the frequency domain by Manolis and Beskos (1981):

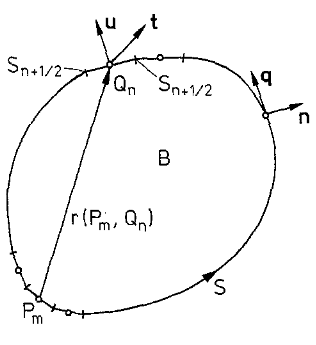

Fig. 1. Boundary surface discretisation of a body B

$$\bar{U}_{ij}=(2\pi\varrho c_2^2)^{-1}(\bar{\Psi}\delta_{ij}-\bar{\phi}r_{,i}r_{,j}) \tag{12}$$

$$\begin{aligned}\bar{T}_{ij}=&(2\pi)^{-1}\{[(\mathrm{d}\bar{\Psi}/\mathrm{d}r)-(\bar{\phi}/r)][\delta_{ij}(\partial r/\partial n)+r_{,j}n_j]\\&-(2\bar{\phi}/r)[n_j r_{,i}-2r_{,i}r_{,j}(\partial r/\partial n)]-2(\mathrm{d}\bar{\phi}/\mathrm{d}r)r_{,i}r_{,j}(\partial r/\partial n)\\&+[(c_1^2/c_2^2)-2][(\mathrm{d}\bar{\Psi}/\mathrm{d}r)-(\mathrm{d}\bar{\phi}/\mathrm{d}r)-(\bar{\phi}/r)]r_{,i}n_j\}\end{aligned} \tag{13}$$

where

$$\begin{aligned}&\bar{\Psi}=K_0(\mathrm{i}\omega r/c_2)+(c_2/\mathrm{i}\omega r)[K_1(\mathrm{i}\omega r/c_2)-(c_2/c_1)K_1(\mathrm{i}\omega r/c_1)],\\&\bar{\phi}=K_2(\mathrm{i}\omega r/c_2)-(c_2^2/c_1^2)K_2(\mathrm{i}\omega r/c_1),\end{aligned} \tag{14}$$

with $K_0(\ )$, $K_1(\ )$ and $K_2(\ )$ being the modified Bessel functions of the second kind of zero, first and second order, respectively, and r being the distance between the two boundary points P and Q.

The solution of Eq. (11) is accomplished numerically. For this purpose the boundary S of the plane body B is discretized into a number of N, in general unequal, straight segments or boundary elements over which the stress and displacement vectors are assumed to be constant as shown in Fig. 1. Constant boundary elements are chosen in this work because of their simplicity and generally good performance in a variety of boundary element applications. In plane vibration isolation problems the boundary S to be discretized consists of the perimeter of the trench, the soil-foundation interface for the case of machine foundations and the free surface of the half-plane. Use of Green's functions defined for the half-plane do not, of course, require any discretisation of the free surface but they are complicated in form (Dravinski 1980). Besides, only a small portion of the free surface around the trench and the soil-foundation interface requires discretization in order to obtain very good results (Dominguez and Alarcon 1981) when use is made of the simple Green's functions given by (12)–(14) and the discretisation is needed anyhow because information on the surface is the primary unknown in this particular problem. Equation (11) can thus reduce in its discretized form to the matrix equation

$$\overline{\mathbf{GT}}\mathbf{u}-\overline{\mathbf{GU}}\mathbf{t}=0, \tag{15}$$

where $\bar{\mathbf{u}}$ and $\bar{\mathbf{t}}$ are the displacement and traction vectors, respectively, in the frequency domain such that

$$\begin{aligned}&\bar{\mathbf{u}}=(\bar{\mathbf{u}}_1,\bar{\mathbf{u}}_2,\ldots,\bar{\mathbf{u}}_n,\ldots,\bar{\mathbf{u}}_N)^T; \quad \mathbf{u}_n=(\bar{\mathbf{u}}_n^x,\bar{\mathbf{u}}_n^y)^T\\&\bar{\mathbf{t}}=(\bar{\mathbf{t}}_1,\bar{\mathbf{t}}_2,\ldots\bar{\mathbf{t}}_n,\ldots\bar{\mathbf{t}}_N)^T; \quad \mathbf{t}_n=(\bar{\mathbf{t}}_n^x,\bar{\mathbf{t}}_n^y)^T\end{aligned} \tag{16}$$

and $\overline{\mathbf{GT}}$ and $\overline{\mathbf{GU}}$ are square influence matrices of order $N\times N$, where N is the total number of boundary elements, consisting of entries of the form

$$\begin{aligned}&\overline{\mathbf{GT}}_{ij}(m,n)=\int_{S_{n-\frac{1}{2}}}^{S_{n+\frac{1}{2}}}\bar{T}_{ij}(Q_n,P_m)\,\mathrm{d}S(Q),\\&\overline{\mathbf{GU}}_{ij}(m,n)=\int_{S_{n-\frac{1}{2}}}^{S_{n+\frac{1}{2}}}\bar{U}_{ij}(Q_n,P_m)\,\mathrm{d}S(Q),\end{aligned} \tag{17}$$

where, as shown in the Fig. 1, P_m and Q_n are the midpoints of elements m and n, respectively, $S_{n+\frac{1}{2}}$ and $S_{n-\frac{1}{2}}$ are the end points of the elements n, and m and n range from 1 to N. It should be noticed that in the vibration isolation problems treated in this paper only boundary quantities are involved and thus there is no need for the computation of internal displacements and stresses. When $Q_n \neq P_m$, i.e., when $r\neq 0$ the integrals in (17) are regular and the integration is done numerically. A 7-point-Gauss-quadrature scheme (Stroud and Secrest 1966) was used in this work. When on the other hand Q_n approaches P_m, or $r\to 0$, due to the fact that $U_{ij}=0(\ln r)$ and $T_{ij}=0(1/r)$, the integrals in (17) are singular and are evaluated analytically. In both cases 6 and 8 term expansions of the Bessel functions for the small and large argument cases, respectively, were employed (Abramowitch and Stegun 1974). It is interesting to mention that for the singular case involving straight line elements, only diagonal terms of $\overline{\mathbf{GU}}_{ij}$ are nonzero. Explicit expressions for the singular integrals of (17) can be found in Dasgupta (1986). The use of two kinds of expansion for Bessel functions and the analytic treatment of singular cases are the two major improvements over the integration methodology of Manolis and Beskos (1981), which considerably increased the accuracy of the computations.

3 Rayleigh wave diffraction by trenches

This section deals with simple problems of vibration isolation which require the determination of the effectiveness of open or infilled trenches as Rayleigh wave barriers. Plane harmonic Rayleigh waves are assumed to propagate horizontally in the half-plane and impinge on an open or infilled trench as shown in Fig. 2. One is basically interested in determining the right combination of geometrical and material properties of the trench as well as the right range of frequencies for which the presence of the trench results, through wave diffraction, in wave amplitude reduction in the region after the trench. In this problem of passive isolation the source of the Rayleigh waves is not known but it is assumed that the free-field Rayleigh wave motion is known and given in terms of its horizontal and vertical components of displacement u and v, respectively by, e.g., Richart, Hall and Woods (1970):

$$u=\bar{u}(x,y)\,\mathrm{e}^{\mathrm{i}\omega t}, \qquad v=\bar{v}(x,y)\,\mathrm{e}^{\mathrm{i}\omega t} \tag{18}$$

where

$$\begin{aligned} \bar{u}(x,y)&=A\mathrm{i}\left[-N\mathrm{e}^{-qy}+\frac{2Nqs}{s^2+N^2}\,\mathrm{e}^{-sy}\right]\mathrm{e}^{-\mathrm{i}Nx},\\ \bar{v}(x,y)&=A\left[-q\mathrm{e}^{-qy}+\frac{2qN^2}{s^2+N^2}\,\mathrm{e}^{-sy}\right]\mathrm{e}^{-\mathrm{i}Nx}, \end{aligned} \tag{19}$$

$$\begin{aligned} &q=N[1-(c_R/c_1)^2]^{1/2}, \qquad s=N[1-(c_R/c_2)^2]^{1/2},\\ &N=\omega/c_R=2\pi/L_R, \qquad c_R=Kc_2 \end{aligned} \tag{20}$$

$$K^6-8K^4+(24-16a^2)K^2+16(a^2-1)=0, \qquad a^2=\mu/(\lambda+2\mu)$$

with c_R and L_R being the Rayleigh wave velocity and wave length, respectively, and A being an arbitrary constant.

Material damping of the soil can be taken into account in the above equations with the aid of the Rayleigh wave attenuation coefficient α_R, which is related to the logarithmic decrement δ by the relationship (Richart, Hall and Woods 1970)

$$\delta=L_R\alpha_R. \tag{21}$$

Thus, the presence of material damping is reflected in Eq. (18) by replacing the exponent $\mathrm{i}\omega t$ by $(\mathrm{i}\omega t-\alpha_R x)$ or more conventionally by leaving Eqs. (18) as they are and replacing the exponent $-\mathrm{i}Nx$ in Eqs. (19) by $-(\mathrm{i}N+\alpha_R)x$.

Consider the case of the open trench first. The displacement components (19) correspond in a discretization form of the problem to the incident displacement vector field $\mathbf{u}^i$ which is connected with the total and scattered due to the presence of a trench, displacement vector fields $\mathbf{u}$ and $\mathbf{u}^s$, respectively, by the relation

$$\mathbf{u}=\mathbf{u}^i+\mathbf{u}^s \tag{22}$$

A relation similar to (22) holds true for the traction vectors along the boundary surface S, i.e.,

$$\mathbf{t}=\mathbf{t}^i+\mathbf{t}^s \tag{23}$$

where $\mathbf{t}$, $\mathbf{t}^i$ and $\mathbf{t}^s$ are the total, incident and scattered traction vectors, respectively. Formulation of the wave scattered problem in terms of the scattered field is accomplished by writing the boundary integral Eq. (15) in the form

$$\begin{pmatrix} \overline{\mathbf{GT}}{}^g_{11} & \overline{\mathbf{GT}}{}^g_{12} \\ \overline{\mathbf{GT}}{}^g_{21} & \overline{\mathbf{GT}}{}^g_{22} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}}^s_f \\ \bar{\mathbf{u}}^s_t \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{GU}}{}^g_{11} & \overline{\mathbf{GU}}{}^g_{12} \\ \overline{\mathbf{GU}}{}^g_{21} & \overline{\mathbf{GU}}{}^g_{22} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{t}}^s_f \\ \bar{\mathbf{t}}^s_t \end{pmatrix} \tag{24}$$

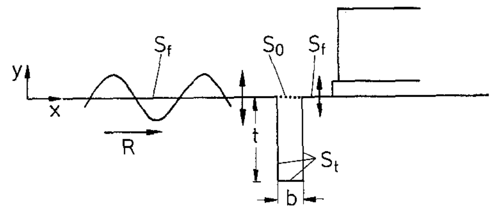

Fig. 2. Rayleigh wave diffraction by a trench

where the superscripts g and s stand for "ground" and "scattered", respectively, and the subscripts f and t correspond to quantities of the free soil surface S_f and the trench perimeter S_t, respectively, comprising the total boundary S, as shown in Fig. 2. The boundary conditions of the problem read

$$\bar{\mathbf{t}}_f^s = \mathbf{0} \quad \text{on} \quad S_f \tag{25}$$

$$\bar{\mathbf{t}} = \mathbf{0} \quad \text{on} \quad S = S_f + S_t. \tag{26}$$

Use of Eqs. (9) and (8) permits one to express $\bar{\mathbf{t}}^i$ in terms of the vector $\bar{\mathbf{u}}^i$ given by Eq. (19) as

$$\bar{\mathbf{t}}^i = \mathbf{A}\bar{\mathbf{u}}^i \tag{27}$$

where **A** is a known coefficient matrix and consequently boundary condition (26), on account of (23) and (27), becomes

$$\bar{\mathbf{t}}_t^s = -\bar{\mathbf{t}}_t^i = -\mathbf{A}\bar{\mathbf{u}}_t^i \quad \text{on} \quad S_t. \tag{28}$$

Thus, the problem consists of solving for $\bar{\mathbf{u}}_f^s$ and $\bar{\mathbf{u}}_t^s$ Eq. (24) subject to the conditions (25) and (28). Knowledge of the vectors $\bar{\mathbf{u}}^i$ and $\bar{\mathbf{u}}^s$ allows one to determine $\bar{\mathbf{u}}$ from (22) along the boundary S and from there to compute the ratio of the displacement at a boundary point after the trench over the displacement at the same point in the absence of the trench which obviously corresponds to the free field incident motion.

Consider now the case of the infilled trench for which perfect bonding is assumed between the half-plane soil medium and the infill material. Equations of equilibrium and compatibility along the trench-infill interface S_t take the form

$$\bar{\mathbf{t}}_{ct} = -\bar{\mathbf{t}}_t \tag{29}$$

$$\bar{\mathbf{u}}_{ct} = \bar{\mathbf{u}}_t \tag{30}$$

where $\bar{\mathbf{t}}_c$ and $\bar{\mathbf{u}}_c$ are the traction and displacement vectors of the infill material. The above equations can be rewritten with the aid of (22) and (23) in terms of the incident and scattered field as

$$\bar{\mathbf{t}}_{ct} = -\bar{\mathbf{t}}_t^i - \bar{\mathbf{t}}_t^s \tag{31}$$

$$\bar{\mathbf{u}}_{ct} = \bar{\mathbf{u}}_t^i + \bar{\mathbf{u}}_t^s \tag{32}$$

where $\bar{\mathbf{t}}_t^i$ and $\bar{\mathbf{u}}_t^i$ are the tractions and displacement vectors of the input motion, which can be obtained as explained in the case of the open trench. The boundary integral Eq. (15) can now be applied for both the half-plane medium and result in Eq. (24) and the infill medium to receive the equation

$$\begin{pmatrix} \overline{\mathbf{GT}}_{11}^c & \overline{\mathbf{GT}}_{12}^c \\ \overline{\mathbf{GT}}_{21}^c & \overline{\mathbf{GT}}_{22}^c \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}}_{ct} \\ \bar{\mathbf{u}}_{co} \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{GU}}_{11}^c & \overline{\mathbf{GU}}_{12}^c \\ \overline{\mathbf{GU}}_{21}^c & \overline{\mathbf{GU}}_{22}^c \end{pmatrix} \begin{pmatrix} \bar{\mathbf{t}}_{ct} \\ \bar{\mathbf{t}}_{co} \end{pmatrix} \tag{33}$$

where the index c stands for infill medium and $\bar{\mathbf{u}}_{co}$ and $\bar{\mathbf{t}}_{co}$ are displacement and traction vectors on the top boundary surface S_o of the infill, as shown in Fig. 2. The boundary conditions of this problem read

$$\bar{\mathbf{t}}_f^s = \mathbf{0}, \qquad \bar{\mathbf{t}}_{co} = \mathbf{0} \tag{34}$$

and thus use of (31), (32) and (34) reduces the system of Eqs. (24) and (33) to the single matrix equation

$$\begin{pmatrix} \overline{\mathbf{GT}}^{g}_{11} & \overline{\mathbf{GT}}^{g}_{12} & \mathbf{0} & -\overline{\mathbf{GU}}^{g}_{21} \\ \overline{\mathbf{GT}}^{g}_{21} & \overline{\mathbf{GT}}^{g}_{22} & \mathbf{0} & -\overline{\mathbf{GU}}^{g}_{22} \\ \mathbf{0} & \overline{\mathbf{GT}}^{c}_{11} & \overline{\mathbf{GT}}^{c}_{12} & \overline{\mathbf{GU}}^{c}_{11} \\ \mathbf{0} & \overline{\mathbf{GT}}^{c}_{21} & \overline{\mathbf{GT}}^{c}_{22} & \overline{\mathbf{GU}}^{c}_{21} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}}^{s}_{f} \\ \bar{\mathbf{u}}^{s}_{t} \\ \bar{\mathbf{u}}_{co} \\ \bar{\mathbf{t}}_{ct} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{K}_1 \\ \mathbf{K}_2 \end{pmatrix} \tag{35}$$

where $\mathbf{K}_1$ and $\mathbf{K}_2$ are given by

$$\mathbf{K}_1 = -\overline{\mathbf{GT}}^{c}_{11}\bar{\mathbf{u}}^{i}_{t} - \overline{\mathbf{GU}}^{c}_{11}\bar{\mathbf{t}}^{i}_{t},$$

$$\mathbf{K}_2 = -\overline{\mathbf{GT}}^{c}_{12}\bar{\mathbf{u}}^{i}_{t} - \overline{\mathbf{GU}}^{c}_{21}\bar{\mathbf{t}}^{i}_{t}. \tag{36}$$

Solution of Eq. (35) provides the displacements $\bar{\mathbf{u}}^{s}_{f}$ which are needed for assessing the screening effectiveness of the infilled trench.

4 Passive and active vibration isolation of foundations

In this section the two basic practical engineering problems of passive and active vibration isolation of rigid massive machine foundations subjected to harmonically varying with time forces are studied with the aid of the proposed methodology. The amplitude reduction of the waves generated by the motion of the foundation is accomplished again by open or infilled trenches as shown in Figs. 3 and 4.

It has been proven by Miller and Pursey (1955) that two-thirds of the total energy generated by a vertically vibrating footing on a half-space is transmitted away from the source in the form of Rayleigh waves. These waves are also characterized by a much more slower amplitude attenuation with distance than body waves (Richart, Hall and Woods 1970). These two facts indicate that Rayleigh waves are the most important ones in problems of vibration isolation of foundations. The proposed method, however, unlike many of the previous ones, takes into account all the waves generated by the moving footing and not just Rayleigh waves, exactly because it treats the foundation-trench system as a whole.

The proposed methodology assumes linear elastic material behaviour. It is well known, however, that internal or material damping in soils plays an important role in reducing the amplitude of vibrating soil media. If the soil is assumed to be a linear viscoelastic material subject to harmonic vibrations one can use the elastic formulation with the elastic moduli λ and μ being replaced by the complex moduli

$$\lambda^* = \lambda(1 + i\beta_1), \qquad \mu^* = \mu(1 + i\beta_2) \tag{37}$$

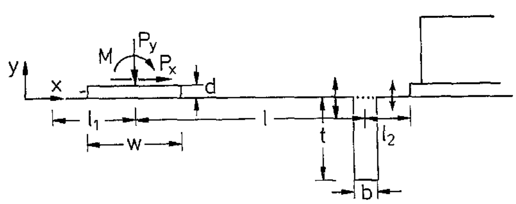

Fig. 3. Passive vibration isolation of a machine foundation by open or infilled trenches

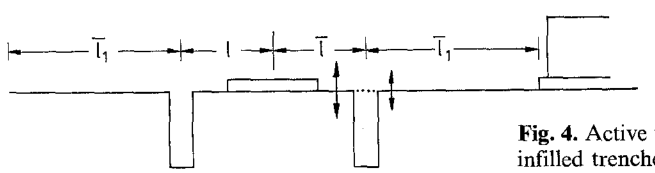

Fig. 4. Active vibration isolation of a machine foundation by open or infilled trenches

where the damping factors β_1 and β_2 are, in general, functions of frequency and are usually assumed to be equal, i.e., $\beta_1 = \beta_2 = \beta$. The logarithmic decrement δ is connected with β through the relationship (Richart, Hall and Woods 1970)

$$\delta = \pi\beta. \tag{38}$$

The damping factors β_1 and β_2 are independent of frequency for the case of the constant hysteretic soil model which is usually employed in applications.

Consider the passive vibration isolation problem associated with the open trench case, schematically described by Fig. 3. The source of disturbance is a rigid, massive, surface strip foundation in perfect bonding with the soil which is subjected to the system of forces P_x, P_y and moment M given by

$$P_x = \bar{P}_x e^{i\omega t}, \qquad P_y = \bar{P}_y e^{i\omega t}, \qquad M = \bar{M} e^{i\omega t}. \tag{39}$$

It should be noticed that the methodology developed in this section is by no means restricted to the above kind of foundation. Flexible embedded foundations with or without mass under relaxed or nonrelaxed boundary conditions can also be treated by the proposed methodology after some modifications and additions. The motion of the foundation of Fig. 3 generates surface waves which are modified in amplitude by an open trench. The present methodology is capable of determining the motion after the trench, so that the designer can be able to determine the appropriate combination of the geometrical parameters of the trench and frequency range for achieving amplitude reduction of a desirable degree.

The whole dynamic system of Fig. 3 consists of two parts, the soil half-plane with the trench and the rigid foundation, which are related to each other through the compatibility and equilibrium equations at their interface. A constant boundary element discretization is adopted for the soil, while a discretisation of the rigid foundation into a number of elements having a one-to-one correspondence with the soil elements at the contact surface, as shown in Fig. 3, is also assumed. The matrix Eq. (15) for the soil medium takes the form

$$\begin{pmatrix} \overline{\mathbf{GT}}^{g}_{11} & \overline{\mathbf{GT}}^{g}_{12} \\ \overline{\mathbf{GT}}^{g}_{21} & \overline{\mathbf{GT}}^{g}_{22} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}}_r \\ \bar{\mathbf{u}}_f \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{GU}}^{g}_{11} & \overline{\mathbf{GU}}^{g}_{12} \\ \overline{\mathbf{GU}}^{g}_{21} & \overline{\mathbf{GU}}^{g}_{22} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{t}}_r \\ \bar{\mathbf{t}}_f \end{pmatrix}, \tag{40}$$

where the subscript f refers to quantities of the free soil surface including the trench boundary, while the subscript r refers to quantities at the soil-foundation interface. Because the free soil surface is free of tractions one has

$$\bar{\mathbf{t}}_f = \mathbf{0}. \tag{41}$$

For a rigid foundation the contact area displacements, as a result of compatibility, are given by

$$\bar{u}_{rx}(x,0) = \bar{\Delta}_x, \qquad \bar{u}_{ry}(x,0) = \bar{\Delta}_y - \bar{\alpha}x, \tag{42}$$

where $\bar{\Delta}_x$ and $\bar{\Delta}_y$ correspond to the displacements at the center of the foundation, while α is the small amplitude of foundation rotation about the axis z passing through the foundation center and being perpendicular to the x-y plane. Equilibrium of forces acting on the foundation takes the form

$$\begin{aligned} \bar{P}_x &= -m\omega^2\bar{\Delta}_x + \sum_{k=1}^{n} l_k \bar{\sigma}^k_{yx} \\ \bar{P}_y &= -m\omega^2\bar{\Delta}_y + \sum_{k=1}^{n} l_k \bar{\sigma}^k_{yy} \\ \bar{M} &= -J\omega^2\bar{\alpha} + \sum_{k=1}^{n} x^k l^k \bar{\sigma}^k_{yy}, \end{aligned} \tag{43}$$

where m and J are the mass and mass moment of inertia about the z axis of the foundation, respectively, l_k represents the length of the k-th foundation element and $\bar{\sigma}^k_{yx} = \bar{t}^k_x$, $\bar{\sigma}^k_{yy} = \bar{t}^k_y$ are the interface tractions of the k-th foundation element. Equations (42) and (43) can be written in a compact matrix form as

$$\bar{\mathbf{u}}_r = \mathbf{C}\bar{\mathbf{\Delta}} \tag{44}$$

$$\bar{\mathbf{P}} = \mathbf{\Xi}\bar{\mathbf{\Delta}} + \mathbf{H}\bar{\mathbf{t}}_r. \tag{45}$$

Expanding (44) and (45) so as to include displacements and stress for all the boundary elements and taking into account (41) one can receive

$$\begin{pmatrix} \bar{\mathbf{u}}_r \\ \bar{\mathbf{u}}_f \end{pmatrix} = \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{\Delta}} \\ \bar{\mathbf{u}}_f \end{pmatrix} \tag{46}$$

$$\begin{pmatrix} \bar{\mathbf{P}} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{\Xi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{\Delta}} \\ \bar{\mathbf{u}}_f \end{pmatrix} + \begin{pmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{t}}_r \\ \bar{\mathbf{t}}_f \end{pmatrix} \tag{47}$$

where $\mathbf{I}$ is the identity matrix. Elimination of $(\bar{\mathbf{u}}_r, \bar{\mathbf{u}}_f)^T$ and $(\bar{\mathbf{t}}_r, \bar{\mathbf{t}}_f)^T$ between Eqs. (40, 46) and (47) results in the equation

$$\begin{pmatrix} \bar{\mathbf{P}} \\ \mathbf{0} \end{pmatrix} = \begin{bmatrix} \mathbf{W} \end{bmatrix} \begin{pmatrix} \bar{\mathbf{\Delta}} \\ \bar{\mathbf{u}}_f \end{pmatrix} \tag{48}$$

$$\begin{bmatrix} \mathbf{W} \end{bmatrix} = \begin{pmatrix} \mathbf{\Xi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{bmatrix} \overline{\mathbf{GU}}^g \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{GT}}^g \end{bmatrix} \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \tag{49}$$

with $\overline{\mathbf{GU}}^g$ and $\overline{\mathbf{GT}}^g$ being the complete square matrices of Eq. (40) with elements $\overline{\mathbf{GU}}_{ij}^g$ and $\overline{\mathbf{GT}}_{ij}^g$; $i,j=1,2$, respectively. Knowledge of the vector $\bar{\mathbf{u}}_f$ permits one to study the change in displacement amplitude as he considers soil surface points before and after the trench.

The problem of the active vibration isolation of a machine foundation by open trenches as depicted in Fig. 4 is mathematically the same as the passive one and can be treated in exactly the same way.

The case of the infilled trench, for both the passive and active vibration isolation problems, can be analysed by a procedure similar to the one used for the open trench in the passive isolation problem. One has simply to supplement the equations for the open trench with one additional boundary element equation for the infill material in conjunction with the appropriate equilibrium and compatibility equations at the soil-infill interface and the traction-free boundary condition at the free top surface of the infill material. For more details the interested reader is advised to consult Dasgupta (1986) and Beskos, Dasgupta and Vardoulakis (1985).

5 Isolation of waves due to transient sources of disturbance

When the ground transmitted waves that have to be isolated are produced by transient dynamic disturbances such as blasting, they are themselves transient and the previous analysis developed for harmonic waves is not capable of handling them. The vibration isolation problem involving transient surface waves is treated in this section with the BEM in conjunction with Laplace transform with respect to time by following the general procedure described in Manolis and Beskos (1981). Thus the problem is formulated in the Laplace transform domain and solved there by the BEM for a sequence of values of the Laplace transform parameter to obtain the transformed solution. The time response of the system is finally obtained by a numerical inversion of the transformed solution. If the wave generating disturbance is a complicated function of time its Laplace transform is computed numerically. Both direct and inverse numerical Laplace transforms are computed with aid of Durbin's (1974) algorithms as described in Narayanan and Beskos (1982). The formulation of the vibration isolation problem by the BEM in the Laplace transform domain under zero initial conditions can be obtained directly from the frequency domain formulations of the previous sections

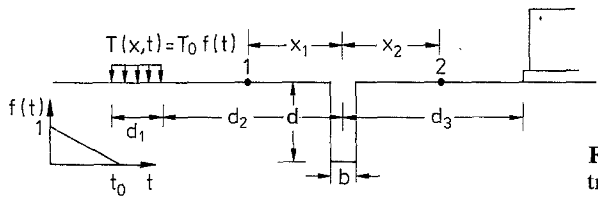

Fig. 5. Isolation of transient surface waves by open or infilled trenches

by simply replacing ω by $s/\mathrm{i} = -\mathrm{i}s$, where $\mathrm{i} = \sqrt{-1}$ and s is the Laplace transform parameter appearing in the definition

$$\bar{f}(x, y, s) = \int_0^\infty f(x, y, t)\, e^{-st}\, dt \qquad (50)$$

of the Laplace transform $\bar{f}(x, y, s)$ with respect to time of a function $f(x, y, t)$. A typical vibration isolation problem dealing with transient waves is chematically shown in Fig. 5 and is similar to one dealing with waves and depicted in Fig. 3. The basic equation here is (40) where, of course, overbars indicate now Laplace transform quantities, subject to the boundary conditions

$$\bar{\mathbf{t}}_f = \mathbf{0}$$
$$\bar{\mathbf{t}}_r = \mathbf{I} T_0 \bar{\mathbf{f}}(s) \qquad (51)$$

where $\mathbf{I}$ is the unit vector and $\bar{f}(s)$ represent the Laplace transformed time variation of the disturbance which, in general, is computed numerically. Thus, the problem consists of solving (40) for the two unknown vectors $\bar{\mathbf{u}}_f$ and $\bar{\mathbf{u}}_r$. Once $\bar{\mathbf{u}}_f$ is known for a sequence of values of s, $\mathbf{u}(t)_f$ can be obtained by numerical inversion and values of the surface displacements (usually the maximum ones) before and after the trench can be compared to assess the effectiveness of the trench as a wave barrier.

6 Numerical examples

The preceding theoretical developments are illustrated in this section by means of some numerical examples.

Example 1: The proposed methodology for the study of Rayleigh wave diffraction by trenches is employed here for the case of a semi-circular open trench for which there are numerical results available due to Wong (1982) to serve as a mean for assessing its accuracy. This is a case where the Rayleigh wave input is very close to the trench so that no material damping of the waves is taken into account. Figure 6 provides the horizontal and vertical surface amplitude u_x and u_y, respectively, as a function of the dimensionless distance x/R, where R is the radius of the trench, for the dimensionless frequency $\eta = \omega R/\pi c_2 = 0.5$ and for the case for which $G = 1$, $\varrho = 1$, $v = 1/3$ and $R = 4$. The almost excellent agreement between the results of the two methods as depicted in Fig. 6 is apparent. The boundary element discretization consisted of 64 elements and the CPU time on a CRAY-1 computer was 15.62 s.

Example 2: This is also a test example for the proposed methodology and deals with the Rayleigh wave diffraction by a rectangular trench of depth t and width b infilled with concrete for which there are numerical results available due to Haupt (1977). In this case the Rayleigh wave input is applied at a distance of 5 L_R from the trench and soil material damping is taken into account in acordance with (21). The Rayleigh wave attenuation coefficient $\alpha_R = 8.04 \cdot 10^{-3}\,\mathrm{cm}^{-1}$. The half-space soil medium is characterized by a Young's Modulus $E_s = 46.12\,\mathrm{MN/m^2}$, a Poisson's ratio $v_s = 1/4$, a weight density $\bar{\varrho}_s = 17.2\,\mathrm{KN/m^3}$ and a wavelength $L_R = 30\,\mathrm{cm}$, so that the damping logarithmic decrement $\delta_s = \alpha_R L_R$. The corresponding quantities for the concrete filling are $E_c = 34.3\,E_s$, $v_c = v_s$, $\bar{\varrho}_c = 1.37\,\bar{\varrho}_s$ and $\delta_c = 5\,\delta_s$. Figure 7 depicts the normalized vertical amplitude $A(\xi)$ at the surface of the half-plane as a function of the dimensionless distance $\xi = x/L_R$. The normalized amplitude $A(\xi)$ is defined as the

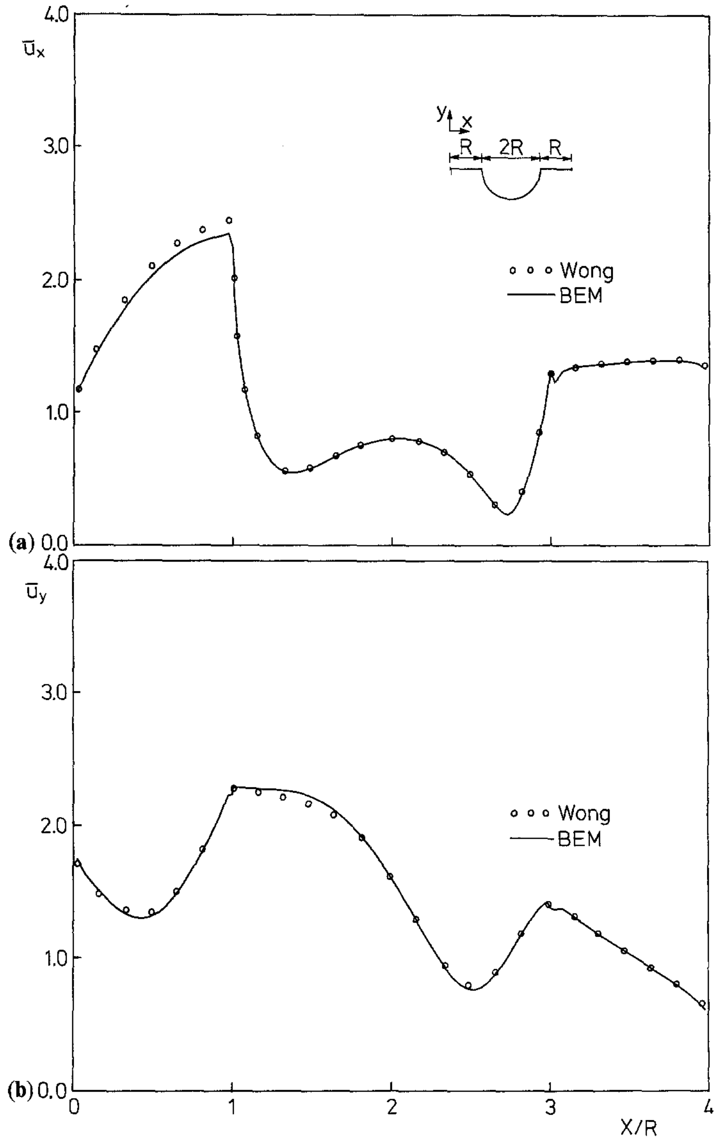

Fig. 6a and b. Diffraction of Rayleigh waves by an open semi-circular trench

ratio of the amplitude when the trench is present to the amplitude in the absence of the trench. Figure 7 covers two cases characterized by the normalized parameters $T = t/L_R$ and $B = b/L_R$ which take the values shown there. The agreement between the two methods is excellent for points to the left of the trench and just good for the points to the right of the trench. In view of the facts that the present method has given excellent results in the previous example and that Haupt's (1977) method is characterized by high complexity, one is inclined to trust the results of the present method more than those of Haupt (1977). In any case, Fig. 7 clearly shows that for the values of the parameters used there is a significant amplitude reduction due to the trench varying between $A = 0.20$ to $A = 0.40$. If one adopts values of $A \leq 0.25$ as an indication of successful isolation (Richart, Hall and Woods 1970) then the present case is very close to be considered as corresponding to a successful design. The CPU time on CRAY-1 machine and for 145 boundary elements was 26.07 s.

Example 3: The passive vibration isolation problem of Fig. 3 is studied here by the proposed methodology. A rigid footing of width w and mass m is subjected to a vertical harmonic load $P_y = P_0 e^{i\omega t}$ and its motion generates surface waves which are diffracted by a rectangular trench of depth t and width b which is at a distance l from the footing. The trench may be open or infilled with concrete. The material properties of the soil medium are as follows: shear modulus $G_s = 132$ MN/m^2, Poisson's ratio $\nu_s = 0.25$, specific weight $\bar{\varrho}_s = 17.5$ N/m^3, hysteretic damping coefficient $\beta_s = 6$ % and

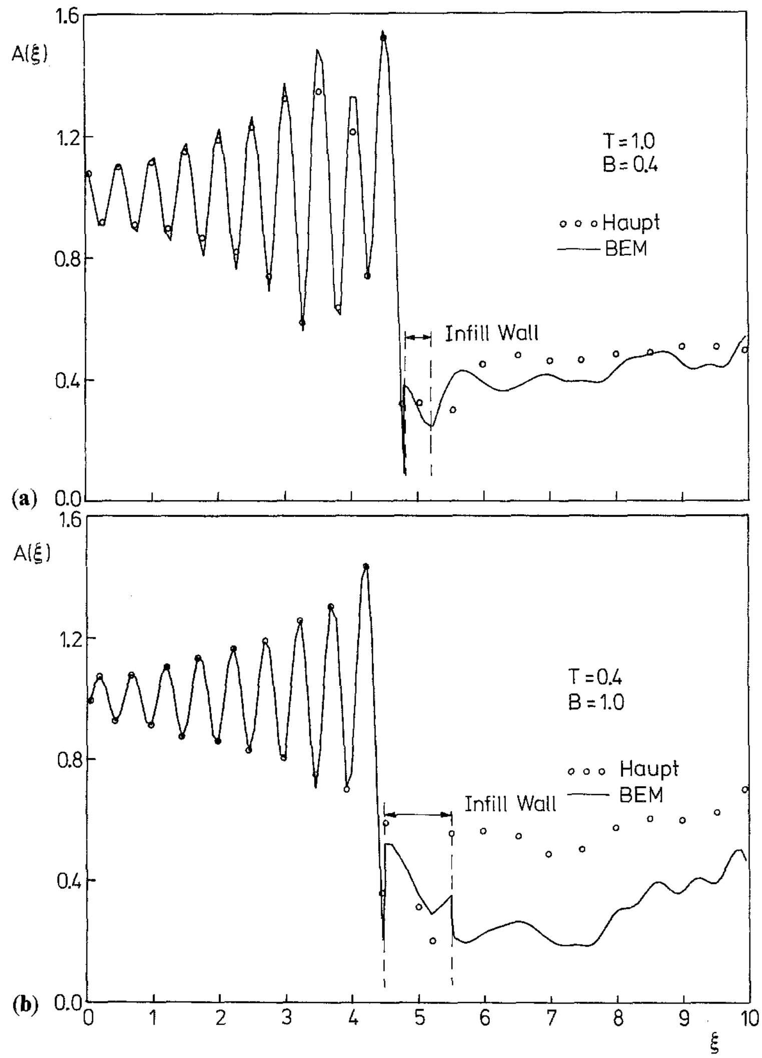

Fig. 7a and b. Diffraction of Rayleigh waves by a concrete filled rectangular trench

velocity of Rayleigh waves $c_R = 250$ m/s. The material properties of the infill medium (concrete) on the other hand are related to those of the soil as follows: shear modulus $G_c = 34.29\, G_s$, Poisson's ratio $\nu_c = \nu_s$, specific weight $\bar{\varrho}_c = 1.37\, \varrho_s$, hysteretic damping coefficient $\beta_c = 5\, \beta_s$ and Rayleigh wave velocity $c_R^c = 5\, c_R^s$. The magnitude of the applied force is $P_o = 1$ KN/m^2 and its operational frequency is 50 Hz.

The screening effectiveness of the trench is measured by the amplitude reduction factor A_R defined as the average normalized vertical surface amplitude behind the trench over the length $l_2' = l_2 - (b/2)$, i.e.,

$$A_R = (1/l_2') \int_{l_2'} A(\xi)\, d\xi. \tag{52}$$

The foundation with dimensions $w \times d = 2.50 \times 0.5$ m is made of concrete and has a weight of $W = 765$ KN/m^2. A numerical study was first conducted to assess the effect of the foundation mass on the screening effectiveness of the trench and the factor A_R was computed for zero and non zero mass for the sequence of values (2.5, 5.0, 20.0 and 50.0) of the parameter $L = l/L_R$ and for $T = t/L_R = 1.0$ and $B = b/L_R = 0.4$. The results of this study (Dasgupta 1986) indicated that the maximum difference between the values of A_R for zero and nonzero foundation mass was only 1.5 %. The presence of mass, however, has a considerable effect on the foundation displacement as the foundation is coming closer to the trench. Because the isolation effect, and not the foundation response, is the aspect of interest in this work the mass is neglected in the numerical computations

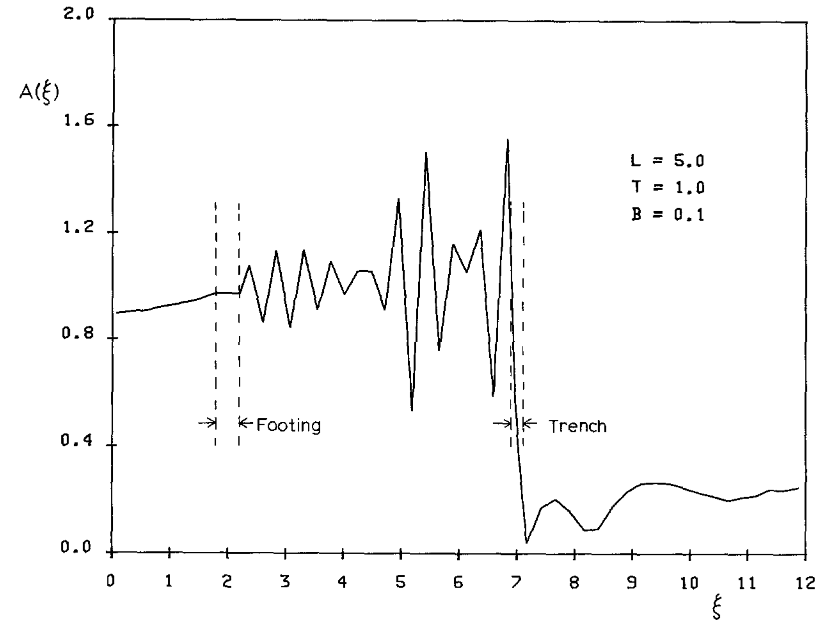

Fig. 8. Passive vibration isolation of machine foundation by an open trench: Normalized surface displacement amplitude $A = A(\xi)$

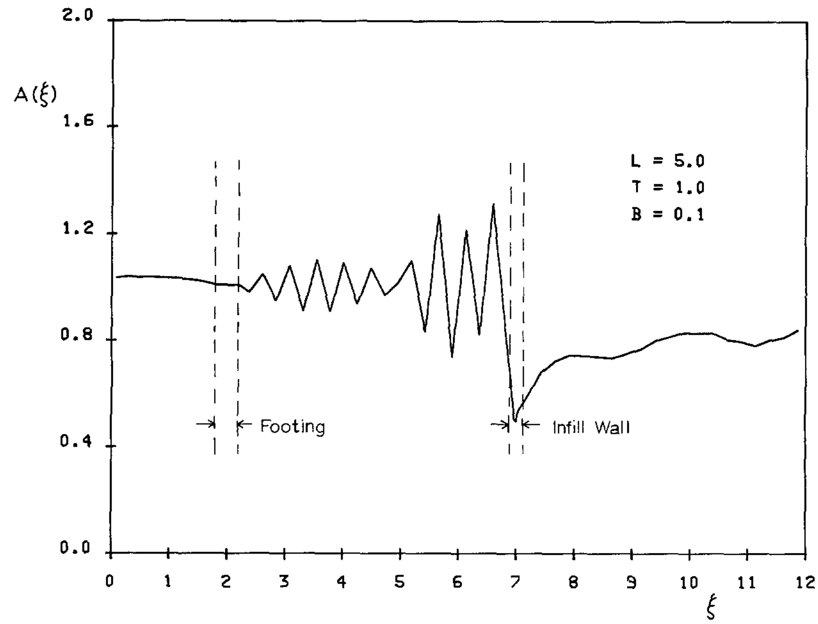

Fig. 9. Passive vibration isolation of machine foundation by a concrete filled trench: Normalized surface displacement amplitude $A = A(\xi)$

pertaining to the parametric studies. Figures 8 and 9 show the normalized vertical surface amplitude $A = A(\xi)$ where $\xi = x/L_R$ for the cases of the open trench and concrete filled trench respectively, for $T = 1.0$, $B = 0.1$, $L = 5.0$, $L_1 = l_1/L_R = 2.0$, $L_2 = l_2/L_R = 5.0$, $\Pi = w/L_R = 0.5$, $W = 765$ KN/m. In both Figs. 8 and 9 there is amplitude reduction, but while $A_R \cong 0.25$ for the open trench, $A_R \cong 0.75$ for the infilled trench for surface points after the trench. This clearly indicates that, for the parameters chosen, the open trench performs much better than the concrete filled trench.

In an effort to assess the screening effectiveness of both kinds of trenches, in a general way, parametric studies were conducted with zero foundation mass by keeping constant the parameters L_1, L_2 and Π and varying the parameters T, B and L. The results of these studies are depicted in Figs. 10–14, which provide the value of the amplitude reduction factor A_R as a function of T, B and L. It is apparent from Fig. 10 that for an open trench, A_R shows a considerable decrease for increasing values of T in a manner essentially independent of B and with a rate of decrease which is very high in the range $0.1 \leq T \leq 0.5$. Figure 11, however, corresponding to an infilled trench, shows a much slower reduction of A_R with T which considerably increases for high values of B. Thus the present findings

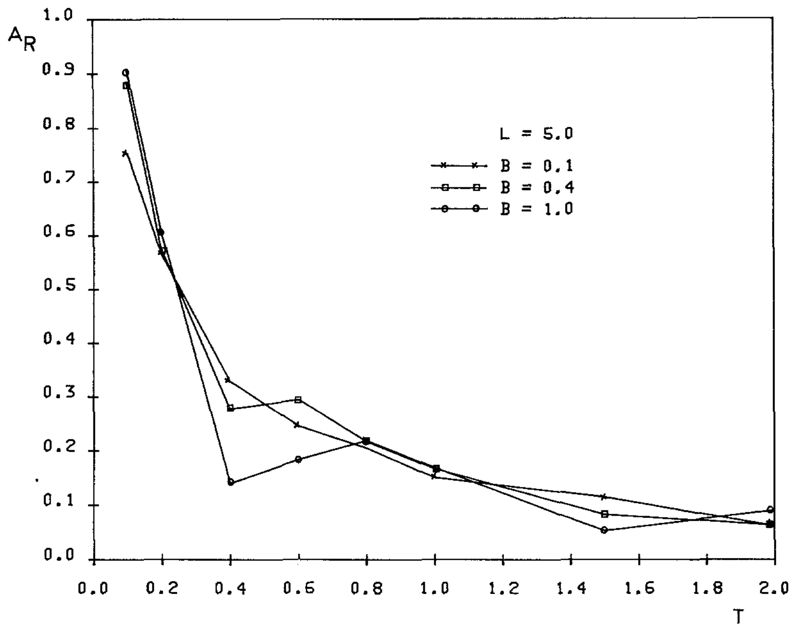

Fig. 10. Amplitude reduction A_R versus T for various B's for the open trench of example 3

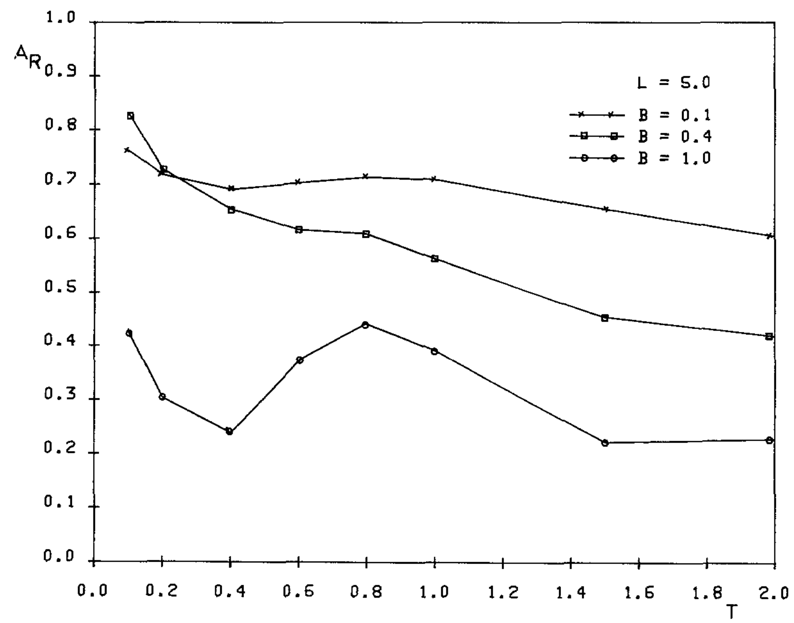

Fig. 11. Amplitude reduction A_R versus T for various B's for the concrete filled trench of example 3

resolve the problem of the significance of the trench width and prove that it strongly affects the screening of waves for infilled trenches only. Figure 12, dealing with an open trench, indicates a slight decrease of A_R with L for all values of T and this decrease becomes more significant for higher values of T. Figures 13 and 14 for the infilled trench show essentially no change of A_R with L for various values of B and T; significant reduction of A_R is noticed, however, for all values of L for increasing values of B and T, thereby confirming Haupt's (1977) results that A_R decreases with increasing values of the area of the trench BT. From the design point of view, for a successful isolation $A_R \leq 0.25$ (Richart, Hall and Woods 1970). This requirement is achieved for $T \geq 0.6$ for open and $BT \geq 1.5$ for concrete filled trenches as it can be deduced from the Figs. 10–14 and these results are in agreement with Woods (1968), Haupt (1977) and Segol, Lee and Abel (1978).

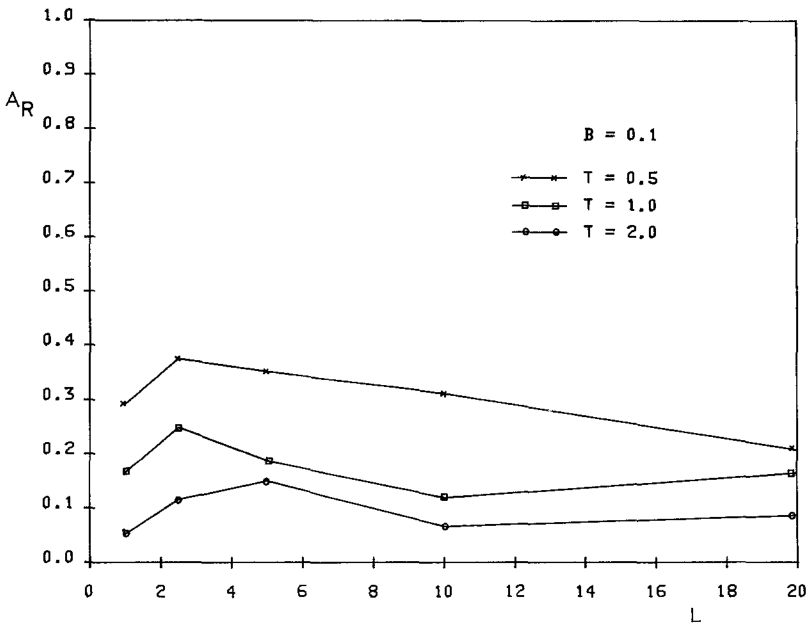

Fig. 12. Amplitude reduction A_R versus L for various T's for the open trench of example 3

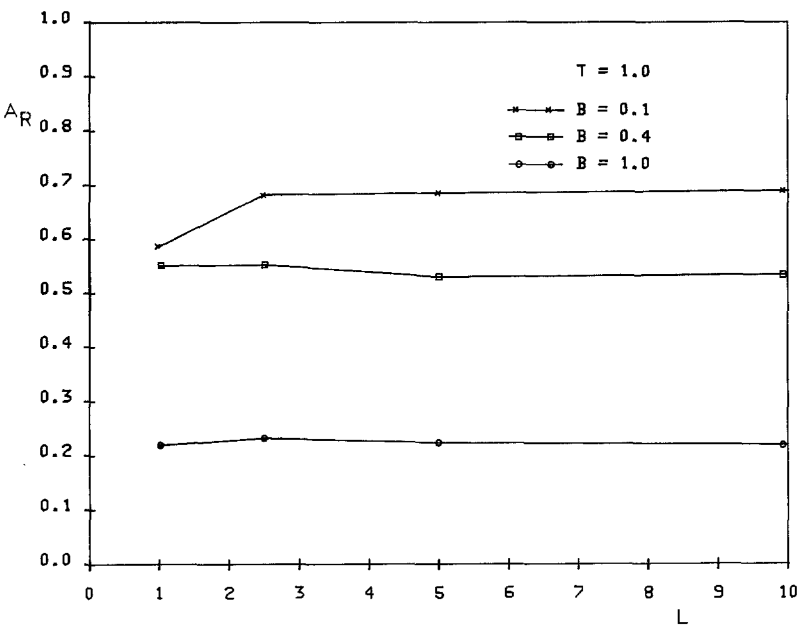

Fig. 13. Amplitude reduction A_R versus L for various B's for the concrete filled trench of example 3

Example 4: The case of active vibration isolation of a rigid foundation by means of two open trenches symmetrically placed on either side of the foundation, as shown in Fig. 5, is considered here. With the material properties and load of the previous example and with $T=1.0$, $B=0.1$, $L=l/L_R=1.0$ and $L_1=l_1/L_R=5.0$, the proposed methodology was employed to construct Fig. 15, which provides $A=A(\xi)$ and indicates that the selected parameters result in a successful vibration isolation design.

Example 5: The diffraction by an open trench of waves generated by a transient surface load as shown in Fig. 4 is studied in this example. The material properties of the soil are the same as those in example 3, while $T_o=1$ MPa, $t_o=15$ ms and initial conditions are assumed to be zero. Thus, the dynamic load $T(t)$ and its Laplace transform $\bar{T}(s)$, for $0\leq t\leq t_o$, have the form

$$T(t)=(t_o-t)/t_o, \qquad \bar{T}(s)=[(e^{-st_o}-1)t_o s^2]+(1/s). \tag{53}$$

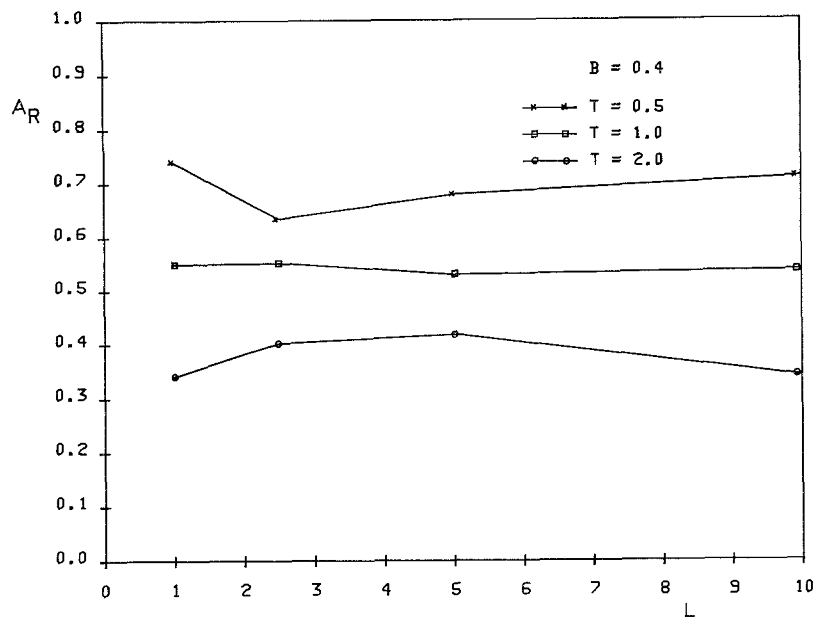

Fig. 14. Amplitude reduction A_R versus L for various T's for the concrete filled trench of example 3

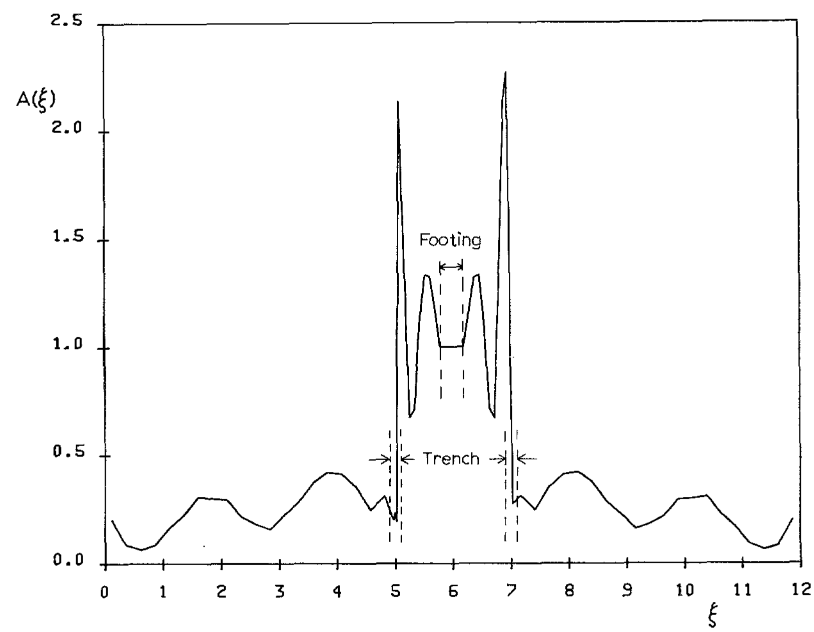

Fig. 15. Active vibration isolation of machine foundation by open trenches: Normalized surface displacement amplitude $A = A(\xi)$

The geometry of the problem, with reference Fig. 4, consists of $d_1 = 0.06$ m, $d_2 = d_3 = 2.0$ m, $b = 0.5$ m, $x_1 = 1.06$ m, $x_2 = 0.98$ m and variable d.

Figures 16, a and b, provide the vertical components of the displacement versus time at the points 1 (before the trench) and 2 (after the trench), respecitvely, for the cases without and with the trench and for three different values of the trench depth d. It is apparent that the presence of the trench slightly amplifies the maximum response at both points 1 and 2 indicating that for this particular example the vibration isolation effectiveness is slightly negative. More extensive studies involving different kinds of transient dynamic disturbances and geometry are necessary in order to reach definite conclusions concerning the trench effectiveness as a transient wave barrier. These studies are presently being conducted by the authors. For the above particular example for a total response time of 0.048 s and 60 time steps or resolutions for the numerical inversion of Laplace transform, the CP

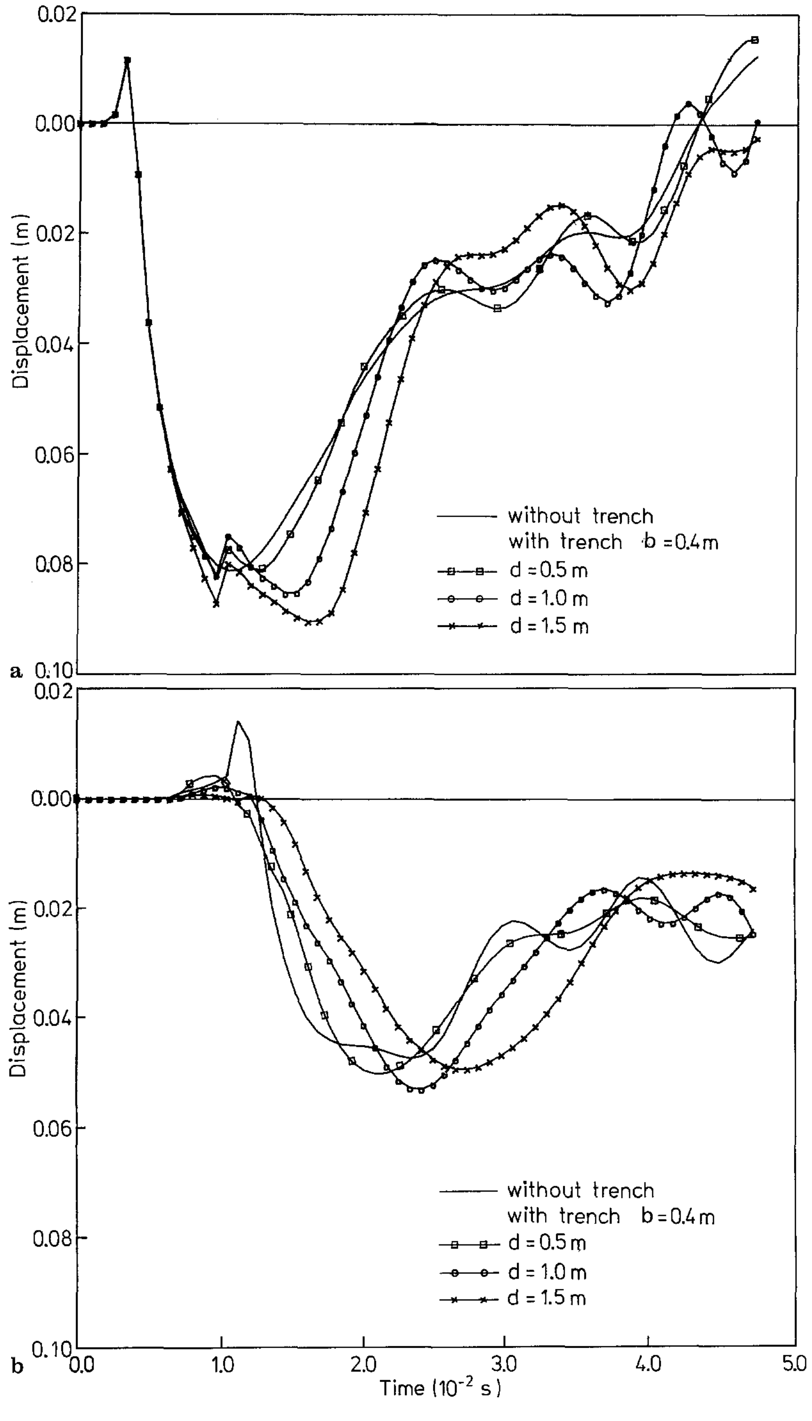

Fig. 16a and b. Vertical displacement versus time for the transient vibration isolation problem of example 5 for, **a** point 1and **b** point 2

time spent on a CRAY-1 computer was 103.87 s for the case without trench involving 35 boundary elements and 221.81 s for the case with a trench of depth 1.0 m involving 51 boundary elements.

7 Conclusions

The preceding discussions can lead to the following conclusions:

1) A numerical methodology based on the frequency domain BEM has been developed for the very accurate and efficient treatment of vibration isolation problems under conditions of plane strain and the assumption of homogeneous linear elastic or viscoelastic soil medium.

2) Both active and passive vibration isolation by open or infilled trenches in the presence of harmonic or transient dynamic disturbances have been considered and pertinent numerical examples have been solved to illustrate the proposed methodology.

3) Parametric studies have been conducted to assess the importance of the various geometrical, material and dynamic input parameters on the trench effectiveness as a wave barrier and provide some design guides to the engineer. Thus, it was found for harmonic disturbances that the screening is effective for $T \geq 0.6$ for open and $BT \geq 1.50$ for concrete filled rectangular trenches, where $T = t/L_R$ and $B = b/L_R$ with t and b being the depth and width of the trench, respectively, and L_R being the Rayleigh wave length. In general, open trenches are more effective than infilled trenches but they present wall instability problems.

4) More research work is needed in the area of vibration isolation utilizing more realistic modelling capable of taking into account problem three-dimensionality, soil layering and anisotropy. The BEM has the potential to handle in an effective manner all these problems as it will be shown by the authors in subsequent publications.

Acknowledgements

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