Social Choice and Welfare © Springer-Verlag 1984

On Dividing Justly

M. E. Yaari and M. Bar-Hillel

The Hebrew University, Jerusalem, Israel

Received September 2, 1983 / Accepted November 2, 1983

Abstract. An attempt is made to evaluate the performance of several distribution mechanisms, using experimental data on ethical judgements. Among the mechanisms examined are the competitive equilibrium with equal incomes, utilitarianism, the maximin, and several mechanisms based on bargaining. Also studied is the extent to which differences in needs, in tastes, and in beliefs may account for unequal distribution.

1. Introduction

This essay is an attempt at exploring distributive justice. The question of how a concept like distributive justice may usefully be explored is itself very much at issue, and we are fully aware of the fact that the specific methodology which will be adopted here is open to fundamental criticism. Nevertheless, it is our hope that our findings will prove to be both relevant and illuminating, thus providing, indirectly, some justification for the methodology.

Many problems of distributive justice can be cast to fit the following mold: A fixed collection of well-defined, quantitatively measurable and perfectly divisible entities is to be divided amongst certain individuals, who have no prior claims on these entities; by what rules should this distribution be carried out? That is, given that a distribution of these entities is in fact going to take place, what are the rules which ought to govern the manner in which this will be done? Problems of this type are all around us, with examples ranging from manning the rolls of the national military service to cutting a cake at a children's party. In this essay, discussion will be confined to the framework provided by this general type of distribution problem.

It is convenient to think of the entities being distributed as goods, or commodities, having the property that, other things being equal, individuals prefer having more of any of them to having less. But nothing in the formulation requires that this be the case, and the entities being distributed may well be bads rather than goods – things which people would rather have less of than more – such as unpleasant duties which must nevertheless be performed. It is possible also to treat cases where the entities

being distributed are considered goods by some and bads by others, or where they are considered goods only up to a point and bads thereafter. (The assignment of work duties in a cooperative may be a case in point.)

In the intrest of simplicity and notational austerity, we shall restrict our attention to problems of dividing a bundle between *two* individuals, subject to the requirement that the allotments to the two individuals must together exhaust the bundle exactly, without leaving anything unallotted.

Let the bundle being distributed to denoted ω , and let the two individuals between whom this bundle is to be divided be characterized by the symbols θ^1 and θ^2 respectively. We may now define a *distribution mechanism* as a function, say D, which, to each conceivable triple of the form $(\theta^1, \theta^2, \omega)$, assigns a division of the bundle ω .¹ Given a distribution mechanism D, and given a triple $(\theta^1, \theta^2, \omega)$, let us agree that $D(\theta^1, \theta^2, \omega)$ will stand for that part of the bundle ω assigned by D to the individual characterized by θ^1 , so that $\omega - D(\theta^1, \theta^2, \omega)$ will be the part assigned to the individual characterized by θ^2 . Formally, then, a distribution mechanism is a function which, given any triple $(\theta^1, \theta^2, \omega)$, determines a bundle $D(\theta^1, \theta^2, \omega)$, subject to the condition that $D(\theta^1, \theta^2, \omega) \leq \omega$. The bundle ω being distributed may be thought of as some nonnegative vector, $\omega = (\omega_1, \ldots, \omega_n)$, in *n*-dimensional space, where *n* is the number of goods (or bads) which could possibly be candidates for distribution among individuals, so that ω_i is the amount of the *i*-th good in the bundle ω being distributed. The interpretation of θ^1 and θ^2 is much more difficult and is liable, in itself, to be controversial. This issue will be taken up in Section 4, below.

2. Reflective Equilibrium

The definition of a distribution mechanism makes it possible to characterize the *rules* governing the distribution of goods and other entities to individuals as properties of such mechanisms. Thus, the question about the rules that ought to govern the manner in which goods (or bads) are distributed amongst individuals is now a question about the properties which a distribution mechanism D ought to have. A reasonable program for a study of distributive justice is, therefore, to proceed in stages, as follows: First, a list is drawn of properties which a distribution mechanism ought to have. These may be referred to as axioms. Then, the question of the existence of a distribution mechanism satisfying all of these axioms must be answered. If such a mechanism does not exist, then a revised list of axioms is drawn, with the question of existence once again to be resolved. If a mechanism satisfying the axioms can be shown to exist, then an attempt is called for to characterize all the mechanisms which do in fact satisfy the axioms. Finally, once identified, these mechanisms must be examined to discover which further properties - possibly undesirable ones - they possess, in addition to what had been postulated in the axioms. If these further properties are found to be untenable, then the whole process starts afresh until, hopefully, some sort of equilib-

¹ We deliberately refrain from considering broader – and often useful – notions of a distribution mechanism, such as mechanisms which are allowed also to transfer goods already held by individuals, or mechanisms which are allowed to use a random device to select among certain distributions

rium is reached. This iterative process of self-correction and revision is summarized in the following schematic flow chart:



Fig. 1

The study of distributive justice can be viewed as seeking a set of principles for which this flow-chart is in a state of equilibrium, in the sense that a mechanism will have been characterized for which no further revisions would be justified, i.e., further attempts to change the axioms would be deemed to be doing more harm than good. This notion, of equilibrium in a self-correcting process of exploring distributive justice, has been given the name *reflective equilibrium* by John Rawls (1971, pp. 48–51).

The notion of reflective equilibrium hinges crucially on what is meant by "tenability". When is a property of a distribution mechanism so damaging as to render the mechanism unacceptable? More generally, what is the test to which a theory of distributive justice is to be submitted? A theory of distributive justice, like any theory, is tested by how well it does when confronted with evidence. And it is our view that, in this particular case, the evidence with which the theory must be confronted consists of observed ethical judgements or moral intuitions. Thus, a distribution mechanism will be deemed untenable if its prescriptions are significantly at variance with observed ethical judgements. Now, the term "ethical judgement" is in itself an object for study and there would be but little comfort in our adding at this point that only ethical judgements made upon reflection by disinterested people, in a dispassionate way, are to be taken into account. Indeed, the very concept of reflective equilibrium has in it the notion that, at some point, the theory must be allowed to stand even if it is still in conflict with certain observed judgements and, from that point on, we must be prepared to reject the judgements for the theory. However, the study of distributive justice is still very far from this particular quandary, so it would be unproductive to dwell upon it here. The important thing is to recognize the significance of observed ethical judgements and moral intuitions, and to allow them to act as guides in the various stages of the analysis. In subsequent sections., we shall try to do just that, using as evidence certain judgements made by individuals in an experimental setting. This evidence will be used in an attempt to assess the tenability of several prominent distribution mechanisms and to point out some of the repercussions of this assessment upon the choice of axioms. Admittedly, this would be but a small step, when viewed in terms of the grand program outlined in Fig. 1.

3. Examples of Axiomatizable Mechanisms

We shall say that a distribution mechanism D is axiomatizable if there exists a set of axioms such that D is the unique mechanism characterized by these axioms. It is of course true that if D is any given mechanism, then it is trivially axiomatizable, with the definition of D serving as the appropriate axiom system. Here we shall be inter-

ested, however, in mechanisms which are *non-trivially* axiomatizable, i.e., mechanisms characterized by axioms which have a force of their own and which can reasonably be put forth as fundamental principles. Several such mechanisms have been suggested and discussed in the literature, and many others exist. Some prominent examples will now be presented, for subsequent reference. Each example depends on a specific explication of what is to be taken as the individual's characteristic, i.e., on an appropriate definition of the θ 's, which for our purposes here need not yet be stated explicitly. Also, in describing these mechanisms, we shall not pay attention to tiebreaking rules which, in some cases, must be specified in order to complete the definition of the mechanism.

(a) Competitive Equilibrium from Equal Split²

This mechanism assigns to each person that part of the bundle ω which would have been his or hers if the two persons were to engage in competitive trading, starting from an initial position where each of them owns $\frac{1}{2}\omega$.³ This mechanism has been discussed by several authors (see, for example, Varian 1975, p. 241). Recently, Dworkin (1981) has argued eloquently that this is indeed the appropriate mechanism for a theory of distributive justice.

Competitive Equilibrium from Equal Split can be axiomatized non-trivially in several ways. Here is one such axiomatization due, essentially, to Gabszewicz (1975): Consider a society consisting of a large number of agents - not just two. A coalition is any non-empty subset of these agents. Suppose that the bundle ω has been distributed among the agents in some specific way, and let S and T be two coalitions of equal size. Suppose, finally, that the members of S, when they compare the part of ω which they have received with the part awarded to the members of T, find that they would rather have the latter. That is, S finds that it could use the resources allotted to T in a way that would make all its members better off than they are with what has actually been allotted to them. In this case, we say that the given distribution of ω causes the coalition S to envy the coalition T. It can be shown (see Yaari 1982) that Competitive Equilibrium from Equal Split is the *unique* distribution mechanism having the property that it never distributes goods in a way which causes some coalition to envy some other coalition. (This axiomatization is valid only when the number of agents is sufficiently large. With a small number of agents, it is still true that Competitive Equilibrium from Equal Split never distributes goods in a way which causes one coalition to envy another, but now it is no longer unique.)

(b) Bargaining from Equal Split

Here one thinks of a distribution mechanism which assigns to each individual that part of the bundle ω which he or she would have obtained if bargaining were to take place, under the rule that, in case of failure to reach agreement, each party gets exactly $\frac{1}{2}\omega$. When John Nash presented his classical bargaining model (1950), he argued that

² The same mechanism is sometimes referred to as *Competitive Equilibrium with Equal Incomes*. Here, one thinks of giving out poker chips to agents in equal quantities, and asking them to buy commodities in competitive markets, using chips as means of payment. Prices are set at equilibrium levels, where the sum of agents' demands equals ω , which is the overall supply

³ With previously held commodities, if any, being kept out of the trading process

a mechanism of this type has moral force (and this was later criticized by Luce and Raiffa (1957), among others) and the literature on distributive justice is full of more or less explicit references to "bargaining" as a tool in the study of distributive justice (see, e.g., Braithwaite 1955).

Bargaining from Equal Split actually refers to several distinct distribution mechanisms, depending on which specific model is chosen to characterize the bargaining process. The most prominent bargaining models are, first, that proposed by Nash (1950) and, second, that sketched by Raiffa (1953) and later axiomatized by Kalai and Smorodinsky (1975). Correspondingly, we can speak of two different distribution mechanisms, namely *Bargaining from Equal Split according to Nash* and *Bargaining from Equal Split according to Raiffa-Kalai-Smorodinsky*. In either case, the axiomatization is quite straightforward: The former mechanism is axiomatized by Nash's four celebrated axioms (Symmetry, Pareto Efficiency, Invariance and Independence) plus the condition that, in case of failure to reach agreement, each party shall receive $\frac{1}{2}\omega$. The latter mechanism is axiomatized in a similar way, except that the Independence axiom is replaced by a suitable Monotonicity axiom.⁴

(c) Bargaining from Zero

Rawls (1971, p. 134 n) has argued that "... what is lacking (in the bargaining approach to distributive justice) is a suitable definition of a status quo that is acceptable from a moral point of view." In the foregoing two examples of a distribution mechanism (i.e., the two versions of Bargaining from Equal Split), the status quo has been defined to be the state in which both agents receive one-half of the total bundle being distributed. An obvious alternative is to define the status quo to be the state in which agents receive nothing at all. Among these two definitions of status quo, which one is more "acceptable from a moral point of view"? Our approach to this question is one of looking at distribution mechanisms which are based on the different definitions and attempting to compare their performance. Accordingly we now proceed to use this second definition of status quo in order to generate two more examples of a distribution mechanism. The first example is the mechanism which mimics bargaining à-la-Nash, under the threat that failure to reach agreement would lead to everybody getting nothing. This proviso, when added to Nash's four axioms, provides an axiomatization for the mechanism. The second example is the mechanism which mimics bargaining à-la Raiffa-Kalai-Smorodinsky, with the same proviso for what would happen in the case of failure to reach agreement. These two mechanisms are quite different from each other, i.e., in general they divide goods in different ways.

(d) Bargaining Over the Strong Pareto Set

A third approach to the question of how to define the status quo, for distribution mechanisms based on bargaining, is to say that bargaining should be limited only to those areas where a genuine conflict of interests exists. Here, the bargaining process takes place under the rule that, in case of failure to reach agreement, each agent is awarded that part of the bundle being distributed, if any, which can in no way benefit any other agent. The other axioms, which determine the character of the bargaining process itself, remain unchanged. This provides an axiomatization for two more

⁴ For a detailed discussion, see Roth (1979)

distribution mechanisms, namely Bargaining Over the Strong Pareto Set according to Nash and according to Raiffa-Kalai-Smorodinsky, respectively.

(e) Utilitarianism

We turn now to a distribution mechanism which would divide any given bundle ω in such a way that the resulting sum of utilities of the individuals shall be at least as great as it would be for any other division of the same bundle. It is clear, of course, that this mechanism is well-defined only if the characteristics of agents, which we have been denoting θ^1 and θ^2 , contain sufficient information about these agents' utilities. (This has been the case also in all the previous examples, but here it requires special emphasis.) To define this mechanism, it is necessary not only that individuals be characterized, among other things, by their utilities, but also that these individual utilities be measured in units which are interpersonally comparable. Whether or not such information is likely ever to be available is, of course, one of the central questions arising in any attempt to evaluate utilitarianism as a moral philosophy. Here, however, this issue will not be taken up. We shall use the term "utilitarianism" in the minimalistic sense of a distribution mechanism which, given all the necessary utility information, requires that goods be distributed so as to maximize the sum of utilities.

One way to axiomatize utilitarianism as a distribution mechanism, is via the theory of decision making under uncertainty. Given the bundle ω and given two individuals whose respective characteristics are θ^1 and θ^2 , one looks for that division of ω which a von-Neumann-Morgenstern decision maker would pick when facing the prospect of being either the θ^1 -person or the θ^2 -person, each with probability 1/2. This approach had been outlined by Vickery (1945) and was later developed by Harsanyi (1955, 1977).

(f) Maximin

The last item on our list of examples is the distribution mechanism which divides a given bundle of goods in such a way that, after the division, the position of the least advantaged individual shall be as high as possible (Rawls 1971, 1974). In order for this mechanism to be well defined, it is necessary that the term "position" be well understood and that information on the relative positions of individuals be available as part of their characteristics. Given that such information is available, one can obtain an axiomatization for this mechanism, using Arrow's social choice conditions, supplemented by a very weak equity axiom. Such axiomatizations have been proposed by Hammond (1976) and by Strasnick (1976).⁵

4. Grounds for Departure from Equality

A distribution mechanism is useless when the information which it requires is unavailable. We must therefore ask how likely it would be for this information – which is summarized in our setting by the triple $(\theta^1, \theta^2, \omega)$ – to be available. The last component, ω , is relatively uncomplicated from an informational point of view, for it is not unreasonable to suppose that the bundle to be distributed is unambiguously observable. (This would not be true for problems of re-distributing goods already held by

⁵ See also Arrow (1977)

individuals, if the possibility of concealment exists.) As regards θ^1 and θ^2 , however, the situation is much more complicated. The question here is not only one of availability of information but also of its very meaning: What do the symbols θ^1 and θ^2 stand for? Ideally, this question might be answered by saying that θ^1 is "a complete description" of the first individual and θ^2 likewise for the second individual. However, then to suppose that complete descriptions of individuals (i.e., "selves") are available for use in distribution decisions would be far-fetched, to say the least. We turn therefore to a less grandiose interpretation for the individual characteristics, θ^1 and θ^2 . In order to motivate this interpretation, let us consider the following definition:

A distribution mechanism D is said to be symmetric, if, for all $(\theta^1, \theta^2, \omega)$, the equation

 $D(\theta^1, \theta^2, \omega) + D(\theta^2, \theta^1, \omega) = \omega$

holds. (Recall that $D(\cdot, \cdot, \omega)$ is that portion of the bundle ω awarded by D to the individual whose characteristic appears in the first argument.) Symmetry of a distribution mechanism means that the order in which the two individuals are listed, i.e., whether θ^1 precedes θ^2 or the other way round, is immaterial. This kind of symmetry can be taken as one of the axioms of distributive justice, *regardless* of the interpretation given to the characteristics, θ^1 and θ^2 . Doing this, we note that a symmetric distribution mechanism always has the property that $D(\theta, \theta, \omega) = \frac{1}{2}\omega$, which is known as *equal treatment of equals*.

Under the grandiose interpretation of the θ 's as complete descriptions of individuals, the statement $D(\theta, \theta, \omega) = \frac{1}{2}\omega$ would mean that two individuals who are identical in every respect should receive equal shares. But the statement $D(\theta, \theta, \omega) = \frac{1}{2}\omega$ can also be read in a rather different way, which is much more low-key. Specifically, we can read the statement $D(\theta, \theta, \omega) = \frac{1}{2}\omega$ to mean that the difference between the two individuals is not regarded as sufficient to warrant a departure from equality, and therefore, for the purpose of distributing the goods at hand, these individuals may be taken as identical. That is, the fact that the two individuals are characterized by the same θ is not taken to mean that these individuals are truly identical, but rather that the observed differences between them are not deemed to justify a departure from equality. On this interpretation, the characteristics of individuals between whom a given bundle is to be divided are to be understood as attributes of the individuals which provide prima facie grounds for a departure from equality.

Considerations which provide a possible justification for departure from equality have been discussed extensively in philosophical writings.⁶ So far as our reading goes, these considerations can be classified in terms of the following broad categories:

- (i) differences in needs;
- (*ii*) differences in tastes, or in the capacity to enjoy various goods;
- (iii) differences in beliefs;
- (iv) differences in endowments;
- (v) differences in effort, in productivity, or in contribution;
- (iv) differences in rights or in legitimate claims.⁷

⁶ See, e.g., bibliographical listings in Rescher (1966)

⁷ Note that differences in *merit* or *desert* do not appear in this list as a separate category, because the terms "merit" and "desert" are too close to being synonyms for "attributes possibly justifying a departure from equality"

The last two categories in the foregoing list apply to distribution problems which transcend the simple fixed-bundle-and-no-prior-claims framework being considered in this essay. The fourth category – differences in endowments – is certainly quite relevant even in our simple setting, but we do not have anything to contribute, at this point, to the question of how endowments should affect distribution. Consequently, we shall concentrate entirely on considerations falling within the first three categories. The individuals among whom entities are to be distributed are thus to be viewed as *equals* in attributes falling within the last three categories. Justification for a departure from equality, if any, would have to come from attributes falling within the first three.

Accordingly, the symbols θ^1 and θ^2 will stand for descriptions of needs, tastes, or beliefs of individuals. Here we find ourselves facing the question of the availability of this kind of information. How is society to know the needs or tastes or beliefs of an individual? Clearly, much of this information must come from the individuals themselves, and many have argued ⁸ that the individual will not report this information truthfully unless it would be to his advantage to do so. Adopting this view here would force us to look for mechanisms which distribute goods in a manner so as always to elicit truthful information from purely self-seeking individuals. Such mechanisms are known to be extremely rare, and the few which do exist are deficient in many respects. For this reason, we are content to restrict our study – at least for the present time – to problems of distributive justice among individuals who regard truth-telling as one of their moral obligations.

5. Needs

Consider the following question:

- Q1: A shipment containing 12 grapefruit and 12 avocados is to be distributed between Jones and Smith. The following information is given, and is known also to the two recipients:
 - Doctors have determined that Jones's metabolism is such that his body derives 100 milligrammes of vitamin F from each grapefruit consumed, while it derives no vitamin F whatsoever from avocado.
 - Doctors have also determined that Smith's metabolism is such that his body derives 50 milligrammes of vitamin F from each grapefruit consumed and also from each avocado consumed.
 - Both persons, Jones and Smith, are interested in the consumption of grape-fruit and/or avocados only insofar as such consumption provides vitamin F
 and the more the better. All the other traits of the two fruits (such as taste, calorie content, etc.) are of no consequence to them.
 - No trades can be made after the division takes place.

How should the fruits be divided between Jones and Smith, if the division is to be just?

This is an example of a very simple distribution problem of the general type being discussed in this essay. We may summarize the data of the problem by introducing the

⁸ The reference here is to the fast-growing literature on "incentive compatibility" in economics. See, e.g., Laffont and Maskin (1981)

following notation:

 $\omega = (12, 12)$ $\theta^{J}: v_{J}(x, y) = 100 x$ $\theta^{S}: v_{S}(x, y) = 50 x + 50 y,$

where ω is the bundle of fruits to be divided between Jones and Smith, with the functions v_J and v_s describing the respective abilities of the two individuals to metabolize the fruits into vitamins. In the context of this question, the functions v_J and v_s play the role of characteristics variables, i.e., they play the role of the θ 's in the general description of a distribution mechanism. Indeed, if, in resolving this distribution problem, one is going to depart in any way from simply splitting the bundle ω down the middle, then the only way for this departure from equality to be accounted for is through an appeal to the difference between the two recipients, Jones and Smith, as expressed in the functions v_J and v_s . Also, insofar as the human organism needs vitamins, we have here a case where a departure from equality, if any, would be due to a difference in *needs*.

How would the various distribution mechanisms described in Sect. 3 resolve this problem? In order to answer this question, we must indicate, first of all, how the mechanisms described in Sect. 3 would define the *utilities* of the two persons, Jones and Smith. (Without utilities, none of the mechanisms is well defined.) But clearly, the functions v_J and v_s are in fact utilities for Jones and Smith, respectively, in the sense that they represent numerically the respective *preferences* of Jones and Smith over fruit bundles. Moreover, the physical interpretation of v_J and v_s means that these functions are also the appropriate *cardinal* utilities in this case⁹ and, furthermore, that the units of measurement (milligrammes of vitamin) are comparable across individuals. Having noted this, we can calculate how the distribution mechanisms listed in Sect. 3 would propose to divide the given bundle (12 grapefruit and 12 avocados) between Jones and Smith. Table 1 gives the result of these calculations, where we have used the notation (J: m - n, S: p - q) for the distribution where Jones gets m grapefruit and n avocados and where Smith gets p grapefruit and q avocados:

Mechanism	Prescribed distribution
Competitive equilibrium from equal split	(J: 12-0, S: 0-12)
Bargaining from equal split (Nash model)	(J:9-0, S:3-12)
Bargaining from equal split (Raiffa-Kalai-Smorodinsky model)	(J: 9-0, S: 3-12)
Bargaining from zero (Nash model)	(J: 12-0, S: 0-12)
Bargaining from zero (Raiffa-Kalai-Smorodinsky model)	(J: 8-0, S: 4-12)
Bargaining over the strong pareto set (Nash model)	(J: 6-0, S: 6-12)
Bargaining over the strong pareto set (Raiffa-Kalai-Smorodinsky model)	(J: 6-0, S: 6-12)
Utilitarianism	(J: 12-0, S: 0-12)
Maximin	(J: 8-0, S: 4-12)

⁹ If vitamin F is subject to a "law of diminishing marginal efficacy" then we may suppose that the variations allowed for in the question are small enough to make this effect negligible

It is at this point that moral intuitions must be consulted, in order to determine which distribution – among those appearing in the foregoing table – is the appropriate one, if any. In an attempt to do this, we presented two variants ¹⁰ of the question Q 1 to a total of 163 young men and women. Immediately following the statement of the question itself, five different distributions were listed. These were, in fact, the distributions appearing in Table 1 above, to which was added the straight equal-split distributions they regarded as the most just. The results are given in the following table:

Table	2
-------	---

Distribut	ion	% of resp	ondents
(J: 6-6,	S: 6-6)	8	
(J: 6-0,	S: 6-12)	0	
(J: 8-0,	S: 4-12)	82	
(J: 9-0,	S: 3-12)	8	
(J: 12-0,	S: 0-12)	2	

We are prepared to interpret the numbers contained in Table 2 as saying, for example, that the distribution (J: 8-0, S: 4-12) is much more in agreement with moral intuition than, say, the distribution (J: 12-0, S: 0-12).¹¹ And this evidence has rather drastic consequences for the evaluation of distribution mechanisms. Indeed, it would be hard to make a case for a distribution mechanism that picks the distribution (J: 12-0, S: 0-12).²¹ and this evidence has rather distribution make a case for a distribution mechanism that picks the distribution (J: 12-0, S: 0-12) – such as Competitive Equilibrium from Equal Split, Nash Bargaining from Zero, or Utilitarianism – without explaining why this distribution should fare so badly in an experimental setting designed to trace out prevailing moral intuitions.

Putting Table 1 and Table 2 next to each other, we see that two particular distribution mechanisms, amongst those listed, will resolve the problem Q1 in a manner which tends to agree with observed intuitions. One of these two is Bargaining from Zero according to the Raiffa-Kalai-Smorodinsky model, and the other is Maximin. We have here two very different mechanisms which happen to agree on how one particular distribution problem is to be resolved. In order to separate the two, all that we have to do is change the distribution problem ever so slightly, as follows: Change the *third* paragraph in Q1 to read

Q 2: - Doctors have also determined that Smith's metabolism is such that his body derives 20 milligrammes of vitamin F from each grapefruit consumed and also from each avocado consumed.

The only change, from the former version, is that the number 50 has been changed to 20, i.e., Smith's metabolism is now less proficient than formerly. All the other parts of

¹⁰ In one variant, respondents were asked to indicate how *they* would divide the shipment of fruit (on the assumption that no post-division trades can take place). In the other variant, respondents were asked to assess how the the recipients (i.e. Jones and Smith) would divide the shipment, on the assumption that both recipients are committed to looking for a just division. The difference between the distributions of responses to these two variants was negligible.

¹¹ We should like to note that response distributions almost identical to that of Table 2 were obtained also from two groups of professional economists and graduate students in economics, one in the USA and one in Israel

Q 2 are the same as in Q 1. Thus, the data of the new problem, Q 2, are given by

 $\omega = (12, 12)$ $\theta^{J}: v_{J}(x, y) = 100 x$ $\theta^{S}: v_{S}(x, y) = 20 x + 20 y.$

Applying the various distribution mechanisms appearing in Table 1 to this new problem, we find that for all but one of them, the distribution being picked out is the *same* as had been picked out for the original problem, Q 1, as listed in Table 1. The only exception is Maximin, which had previously picked out (J: 8-0, S: 4-12), and which now picks out (J: 4-0, S: 8-12). Maximin is the only mechanism, among those appearing in Table 1, which compensates poor Smith for the deterioration in his metabolism.

The new problem, Q 2, was presented to a group of 146 young men and women.¹² Their responses are summarized in Table 3.

Table 3

Distribution	% of respondents	
(J: 6-6, S: 6-6) (J: 4-0, S: 8-12) (J: 6-0, S: 6-12) (J: 8-0, S: 4-12) (J: 20, S: 4-12) (J: 12-0, S: 0-12) (J: 12-0, S	4 82 4 7 3	

In the original problem, Q 1, we had 82% of respondents picking the distribution (J: 8-0, S: 4-12). Now, for Q 2, we see the same percentage going to the distribution (J: 4-0, S: 8-12). Maximin seems to be doing very well indeed, both absolutely and in comparison with the other mechanisms. Perhaps one might have expected this to be the case, given the fact that the problem is one of distribution according to needs, with needs being so readily quantifiable. Can be say, then, that in all problems of distribution according to needs, with needs readily and unambiguously quantifiable, Maximin is likely to be the appropriate distribution mechanism? The answer is no. Even in this very special setting. Maximin has a feature that seems to bring it into conflict with moral intuition. Take another look at the two problems which we have been discussing Q1 and Q2. In the transition from the former problem to the latter, Smith's metabolism suffers a setback, the relevant coefficient having gone down from 50 to 20. Maximin now responds to this change by, among other things, cutting Jones's share in half, when Jones himself had not changed at all. Indeed, Maximin would go on mercilessly cutting Jones's share, in response to any further deterioration in Smith's metabolism. Sooner or later, this result runs the risk of becoming morally unsound. In order to check on this, we have examined yet another version of the same

¹² Each question was presented to a different group of respondents. The number 146 represents a pooling-together of responses obtained for two separate variants of the question, as explained in footnote 10, above. Some of the results to be quoted below will reflect a similar poolingtogether of responses

distribution problem, to be labeled Q 3. Everything is once again as in Q 1, except that the third paragraph now reads as follows:

Q 3: - Doctors have also determined that Smith's metabolism is such that his body derives 9.1 milligrammes of vitamin F from each grapefruit and also from each avocado consumed.

Smith's relevant coefficient is now down to 9.1, and the complete data of the problem are given by

$$\begin{split} \omega &= (12, 12) \\ \theta^{J} \colon v_{J}(x, y) = 100 \, x \\ \theta^{S} \colon v_{S}(x, y) = 9.1 \, x + 9.1 \, y \, . \end{split}$$

This problem was presented to a group of 52 respondents, with the following results.

Table 4

Distribution	% of respondents	
$\begin{array}{c} (J:6-6, S:6-6)\\ (J:2-0, S:10-12)\\ (J:6-0, S:6-12)\\ (J:8-0, S:4-12)\\ (J:12-0, S:0-12) \end{array}$	17 38 27 6 12	

For this problem, Q 3, Maximin picks out the distribution (J: 2-0, S: 10-12) which equates the vitamin intake of the two agents. As Table 4 shows, this distribution represents the modal response obtained in our sample. However, a comparison with the previous tables shows that Maximin has now lost much of its force, with a sizable majority of respondents rejecting its prescription and selecting instead a distribution which treats Jones more favorably. Even when the satisfaction of needs is unambiguously measurable and commensurable across individuals, the criterion of equalizing the satisfaction of needs, to which Maximin would lead, may not always be consistent with moral intuition. Indeed, we would expect Maximin to be abandoned altogether in distribution problems where the coefficient for Smith's metabolic capacity is reduced even further. Table 4 gives some indication that, with such reductions in the relevant coefficient, the distribution (J: 6-0, S: 6-12) may spring into prominence and with it, perhaps, the mechanism known as Bargaining over the Strong Pareto Set.

6. Tastes

As in the previous discussion, let us start out by looking at a specific question:

Q 4: A shipment containing 12 grapefruit and 12 avocados is to be distributed between Jones and Smith. The following information is given, and is known also to the two recipients:

- Jones likes grapefruit very much, and is willing to buy any number of them, provided that the price does not exceed \$ 1.00 per pound. He detests avocados, so he never buys them.
- Smith likes grapefruit and avocados equally well, and is willing to buy both grapefruit and avocado in any a number, provided that the price does not exceed \$ 0.50 per pound.
- Jones and Smith are in the same income-tax bracket.
- No trades can be made after the division takes place.

How should the fruits be divided between Jones and Smith, if the division is to be just?

The data of this question may be summarized by writing

$$\begin{split} & \omega = (12, 12) \\ & \theta^J \colon v_J(x, y) = 100 \, x \\ & \theta^S \colon v_S(x, y) = 50 \, x + 50 \, y \, , \end{split}$$

where $v_J(x, y)$ and $v_S(x, y)$ describe the most that Jones and Smith would be willing to pay for the pair (x, y), respectively.

We see immediately that this formalization is *identical* to that of the question Q 1 of the previous section. But, where previously the functions v_J and v_S carried information about the respective *needs* of Jones and Smith, now the *same* v_J and v_S carry information about their respective tastes.

Table 5 lists the results that were obtained for the question Q 4 from a group of 122 respondents:

Table 5

Distribut	ion	% of respondents
(J: 6-6, (J: 6-0, (J: 8-0, (J: 9-0, (J: 12-0,	S: 6-6) S: 6-12) S: 4-12) S: 3-12) S: 0-12)	9 4 28 24 35

A comparison of Table 5 with Table 2 reveals, first of all, that the distributions of response in the two tables are quite different from each other.¹³ This fact is in itself noteworthy because it means that the information contained in ω , v_J and v_S – i.e., in the triple $(\theta^1, \theta^2, \omega)$ – is not sufficient to characterize a distribution problem. Both Q 1 and Q 4 have the same formalization, in terms of ω , v_J and v_S . Moreover, in both questions the bundle ω is physically the same and the functions v_J and v_S common to both questions are appropriately cardinalized representations of the respective preferences of the two individuals. Yet, it is quite reasonable that these two questions should be resolved differently, as indeed they are in our samples.

Here, the reader may very well object, saying that the difficulty we have raised is but an apparent one. For what could be easier than adding to the respective data of

¹³ Using a chi-squared test, we get $\chi^2 = 112.68$, with 4 d.f., so the difference between the distributions is easily significant at the 0.01 level

Q 1 and Q 4 symbols that would stand for "Needs" and "Tastes", respectively, so that the formalizations would no longer be identical? For "synthetic" distribution problems, like Q 1 and Q 4, this argument is of course quite persuasive, and it could happen that different distribution mechanisms would be deemed appropriate for problems of distribution according to needs, distribution according to tastes, etc. In general, however, it is quite impossible to label a given distribution problem as belonging unambiguously, say, to the category of "Needs" or "Tastes". Consider, for example, the case of the sculptor who requires very expensive materials for his art (see Dworkin 1981). Can we say, unambiguously, that this requirement is a matter of needs, or of tastes, or of something altogether different, such as productivity? Indeed, it is fair to say that in *most* distribution problems, agents' utilities represent some *mixture* of needs, tastes, and other attributes.

In any case, it should be noted that all the distribution mechanisms listed in Table 1 pick the same distribution for Q 4 as they had picked for Q 1. As defined, these mechanisms are incapable of treating these two distribution problems differently which, in light of the evidence, seems to be major shortcoming.

It is interesting to see what happens when the distribution problem Q 4 is changed in a manner similar to the changes considered in Sect. 5 for Q 1. Specifically, suppose that the text of Q 4 is changed so that the third paragraph now reads as follows:

Q 5: – Smith likes grapefruit and avocado equally well, and is willing to buy both grapefruit and avocado in any number, provided that the price does not exceed \$ 0.20 per pound.

That is, Smith is now willing to pay only up to 0.20 per pound for either fruit, where previously this figure was 0.50 per pound. In all the other paragraphs, Q 5 and Q 4 are the same. The data of Q 5 are given by

$$\begin{split} & \omega = (12, 12) \\ & \theta^J \colon v_J(x, y) = 100 \, x \\ & \theta^S \colon v_S(x, y) = 20 \, x + 20 \, y \, , \end{split}$$

which coincides with the formalization of the problem Q 2 in Sect. 5, where an overwhelming majority of respondents (Table 3) had picked the distribution (J: 4–0, S: 8–12). In other words, when the formulation was one of distribution according to needs, respondents tended to *compensate* Smith for having a lower coefficient (20 instead of 50). Will this be true also in the case of distribution according to tastes? For the answer, let us consider the results obtained when 102 respondents were asked to resolve our present tastes problem, Q 5, with Smith's function, v_s , having the lower coefficient, of 20 instead of 50. These results are listed in the following table:

Table (5
---------	---

Distribution	% of respondents
(J: 6-6, S: 6-6) (J: 4-0, S: 8-12) (J: 6-0, S: 6-12) (J: 8-0, S: 4-12) (J: 12-0, S: 0-12)	12 6 7 28 47

A comparison of Tables 5 and 6 shows that the fall of Smith's utility coefficient from 50 to 20 has led to a *decline* in responses consistent with Maximin, from 28% to 6%, and to an *increase* in responses consistent with Utilitarianism, from 35% to 47%. This result should be viewed in light of the fact that Maximin is the mechanism which tends to *compensate* Smith for any decline in his utility coefficient, whilst Utilitarianism is the mechanism which tends to *penalize* Smith for incurring such a decline. However, the sharper conclusion – that the fall of Smith's coefficient from 50 to 20 leads to an unambiguous shift of resources from Smith to Jones – does not gain any clear support from Tables 5 and 6: While the percentage of respondents who say that Jones ought to get all 12 grapefruit goes up, from 35 to 47, the percentage of respondents who say that Jones ought to get at least 8 grapefruit goes down, from 87 to 75.

A comparison of Table 6 with Table 3 is also instructive, showing, once again, the profound effect of shifting from distribution according to needs to distribution according to tastes, even though the formalization of the problem in both cases is the same.

7. Beliefs

Should agents' beliefs be taken into account in the determination of how goods ought to be distributed to them? Some people would argue, on a-priori grounds, that they should not. (See, for example, Hammond 1982.) In order to shed some light on this issue, let us consider the following question:

- Q 6: A shipment containing 12 grapefruit and 12 avocados is to be distributed between Jones and Smith. The following information is given, and is known also to the two recipients:
 - Jones believes that each grapefruit contains 100 milligrammes of vitamin F and that avocado does not contain vitamin F at all.
 - Smith believes that a grapefruit and an avocado, each contains 50 milligrammes of vitamin F.
 - Information regarding the true vitamin contents of the fruits is not available.
 - Both persons, Jones and Smith, are interested in the consumption of grape-fruit and/or avocados only insofar as such consumption provides vitamin F
 and the more, the better. All the other traits of the two fruits (such as taste, calorie content, etc.) are of no consequence to them.

- No trades can be made after the division takes place.

How should the fruits be divided between Jones and Smith, if the division is to be just?

The data of this question are once again given by the formalization

$$\begin{split} & \omega = (12, 12) \\ & \theta^J \colon v_J(x, y) = 100 \, x \\ & \theta^S \colon v_S(x, y) = 50 \, x + 50 \, y, \end{split}$$

except that now v_J and v_s are functions describing *beliefs* about the vitamin contents of certain fruits, i.e., beliefs about the world. (The case of beliefs about *oneself*, which

is also of interest, may be viewed as a composite of beliefs about the world with needs, tastes, etc.)

The view that beliefs ought to be irrelevant in deciding how to distribute a given bundle of goods, may be expounded for the present context in the following way: Consider the equal-split distribution (J: 6-6, S: 6-6). What could be the grounds for a *departure* from this equal distribution? Surely, such a departure would have to rest on the difference between the two individuals, Jones and Smith, and the only information about such a difference is in the specification of the functions v_J and v_s . However, does the difference between v_J and v_s provide firm grounds for a departure from equality? Clearly not. For let $v_T(x, y)$ stand for the *true* vitamin content of a bundle composed of x grapefruit and y avocados.¹⁴ One of the clauses in Q 6 states explicitly that information regarding the form of the function v_T is unavailable. But such information is also unnecessary! *Whatever* the form of v_T , it applies equally to both individuals. Hence, the problem at hand is one of dividing the bundle ω between two identical individuals, and the equal-split distribution, (J: 6-6, S: 6-6), is appropriate.

Several objections can be raised against the foregoing conclusion. Possibly the most important objection is that implementing the equal split distribution, (J: 6-6, S: 6-6), requires coercion. Both individuals, Jones and Smith, would be glad to move away from this distribution and settle for a distribution which gives Smith all the avocados and gives Jones more than 6 – but less than 12 – grapefruit. For example, the distribution (J: 9-0, S: 3-12) would be strictly preferred by both of them to (J: 6-6, S: 6-6). Maintaining the equal-split distribution would therefore be an act which entails coercion.

To this argument – that the distribution (J: 6-6, S: 6-6) is Pareto-dominated and cannot be maintained without coercion – one can offer the following counterargument: Take Jones for instance. It is quite true that moving from (J: 6-6, S: 6-6)to, say, (J: 9-0, S: 3-12) would make him better off. However, if Jones is a moral agent, he should *refuse* this change. Why? Because according to his (Jones's) own beliefs, such a change, while certainly beneficial to himself, is harmful to his colleague Smith. (According to Jones's beliefs, the move from (J: 6-6, S: 6-6) to (J: 9-0,S: 3-12) would reduce Smith's vitamin intake from 600 to 300.) Similarly, if Smith is a moral agent, then he too will refuse the change from (J: 6-6, S: 6-6) to (J: 9-0,S: 3-12) because according to *his* beliefs, this change, while beneficial to himself, would be harmful to Jones. (Jones's vitamin intake, according to Smith's beliefs, would be going down from 600 to 450.) Thus, no coercion would be required to maintain the equal-split distribution, if both recipients are truly moral agents.

Far from being conclusive, this counter-argument is itself open to question, along lines developed in philosophical discussions of paternalism.¹⁵ Is it really Jones's moral obligation to protect Smith from what he regards as Smith's own preposterous beliefs? We shall turn shortly to a distribution question which attempts to avoid some of these vexing issues while still resting solely on differences in beliefs (Q 7, below).

The question Q 6, like all previous questions, was presented to a group of respondents, who were asked to indicate which distribution of the fruit they considered to be the most just. The total number of respondents was 145. Here is a summary of the results:

¹⁴ In Hammond's terminology (1982), v_T would be the *ex-post* utility

¹⁵ See, e.g., the discussion of paternalism in the volume edited by Wasserstrom (1971)

Table 7

Distribut	ion	% of resp	ondents
(J: 6-6,	S: 6-6)	34	
(J: 6-0,	S: 6-12)	4	
(J: 8–0,	S: 4-12)	51	
(J:9-0,	S: 3-12)	4	
(J: 12–0,	S: 0-12)	7	

Table 7 should first be compared with Tables 2 and 5, since these tables refer to distribution questions (Q 1, Q 4 and Q 6) having the same formalization. Clearly, the distribution of responses in Table 7 is markedly different from the distributions of responses in Tables 2 and 5^{16} Note that in Table 7, fully one third of respondents (as opposed to 8%-9% in Tables 2 and 5) picked the distribution (J: 6–6, S: 6–6), thus concurring with the view that "beliefs should not matter".

The fact that the distributions of responses to Q 1, Q 4 and Q 6 are all different from one another can be viewed as an expression of people's tendency to regard problems of distribution according to needs, distribution according to tastes, and distribution according to beliefs, as being very different from one another, even when their mathematical formalizations are the same. This can hardly be taken as surprising or profound. Nevertheless, the fact deserves some emphasis, because many of the frequently encountered distribution mechanisms fail to take it into account. For example, let us refer once again to the various distribution mechanisms in Table 1. Of the nine mechanisms listed there, the first seven would treat Q 1, Q 4 and Q 6 by picking the same distribution for all three problems.¹⁷ As for the remaining two – Utilitarianism and Maximin – they would still pick the same distribution for Q 1 and Q 4. But they may (or may not) go over to the equal-split distribution in the case of Q 6, depending on whether the user of these mechanisms accepts the argument that they should be applied to *ex-post* utility.

The difference between Tables 5 and 7 is all the more probelamtic, in view of the fact that beliefs and tastes cannot very easily be separated from each other. Consider the question Q 6, as it appears at the beginning of this section, and suppose that Jones values a unit of vitamin F to the same extent as does Smith. Specifically, suppose that both Jones and Smith are willing to pay up to 0.01 for each milligramme of this vitamin. Then, we can express the difference between Jones and Smith by saying that the former is willing to pay up to 1.00 per grapefruit and nothing for avocado, while the latter is willing to pay up to 0.50 per either grapefruit or avocado. While the source of this difference in willingness to pay lies in divergent *beliefs* about the vitamin contents of the fruits, the statement of the problem is now reduced to a form which is consistent also with the divergence being one of *tastes*, as in Sect. 6.

One last comment concerning Table 7. It has to do with the figure of 51 % picking the distribution (J: 8-0, S: 4-12). We have substantial evidence¹⁸ to the effect that

¹⁶ Using a chi-squared test with 4 d.f., we get $\chi^2 = 53.70$ for a comparison of Tables 7 and 2, and $\chi^2 = 77.88$ for a comparison of Tables 7 and 5. Both differences are significant at the 0.01 level

¹⁷ The only assumption required for this is that v_J and v_S correctly represent the respective preferences of Jones and Smith

 $^{1^{8}}$ Both from direct comments made by respondents and from the effect that a change in a certain coefficient had on the distribution of responses

respondents tend to pick this distribution because it provides the highest outcome for which the amount of vitamin obained by the recipients – each according to his own beliefs – is equal. This means that respondents are willing to ascribe a meaning to the equality of two numbers (800 milligrammes of vitamin according to two different sets of beliefs) even in the face of arguments which make the meaningfulness of this equality rather dubious. It is interesting to note that, when the distribution problem Q 6 was presented to a group of 44 professional economists and graduate students in economics – people who are trained to raise an eyebrow at anything remotely resembling a dubious interpretsonal comparison – they too produced a distribution of responses in which more than 50 % picked (J: 8–0, S: 4–12),¹⁹ complete with explanations on this being the distribution which gives the highest equal-vitamin outcome. Moreover, members of this group, unlike the previous respondents, had several weeks to consider the problem and weigh their answers.

We have seen that the distribution problem posed at the beginning of this section (Q 6) treats the view that "beliefs should not matter" somewhat unfairly. In the context of Q 6, taking this view leads to the Pareto dominated distribution (J: 6-6, S: 6-6) and forces the holder of the view into an argument about coercion. As these are two distinct questions – whether beliefs should matter and whether coercion upon all participants should be sanctioned – there ought to be a way to check on the first without any overtones from the second. The following distribution problem attempts to do just that.

- Q 7: Old Jacob passed away and left a will containing two provisions: first, Jacob's two beloved sons, Reuben and Simeon, shall be his sole heirs. Second, Jacob's dear friend Laban shall be the one to decide, at his sole discretion, how the estate shall be split between the two heirs. Laban proceeds to make the necessary inquiries, and comes up with the following information:
 - Old Jacob's estate is worth exactly \$ 1000.
 - Brother Reuben is absolutely convinced that the total wirth of the estate is \$ 1200.
 - Brother Simeon is equally convinced that the total worth of the estate is \$ 800.
 - Brothers Reuben and Simeon are not on speaking terms with each other, so what one of them gets will never be known to the other.
 - How should Laban split the estate?

For this question, the view that "beliefs should not matter" would lead to the distribution 500-500, whereas the opposite view – that beliefs cannot be ignored – would lead to the distribution $600-400.^{20}$ Both of these distributions are Pareto efficient, so there is no question of coercing the recipients into accepting a given distribution when both of them would rather move to a different one.

The question Q 7 was presented to a group of 62 respondents, with the following results:

One-half of the respondents now pick the distribution which ignores beliefs, as compared with one-third of respondents who had done so in the case of Q 6. To the

¹⁹ Indeed, the distribution of responses obtained from this group was not significantly different from that in Table 7. The two were very similar

²⁰ One might, perhaps, characterize 500-500 as "justice done" and 600-400 as "justice seen"

Table 8

Distribution	% of respondents
500-500	52
600-400	48

extent that such sample responses reflect moral intuitions, we can say that, in this case, moral intuition is about evenly divided as to the role of beliefs in distributive justice. Will the various distribution mechanisms be similarly undecided? Checking the nine mechanisms listed in Table 1, we find the first seven resolving the question Q 7 by picking 500-500.²¹ The remaining two mechanisms – Utilitarianism and Maximin – will either pick 500-500 or 600-400, depending on how "utility" is defined.

8. Solving in Utility Space

In the last three sections, we have seen how different interpretations of "utility" are liable to affect distributive justice. A given distribution problem will tend to be resolved differently, depending on whether "utility" is interpreted as conveying information about needs, about tastes, about beliefs, etc. The point which we would like to make in this section is that *even with the interpretation of "utility" held fixed*, two distribution problems can have the same representation in utility space and still be resolved differently, in the utility space itself.

Take the distribution mechanisms listed and discussed in Sect. 3. All these mechanisms except the first one (Competitive Equilibrium from Equal Split) solve any given distribution problem entirely in utility space. These mechanisms treat a given distribution problem by going through the following three steps: First, the mechanism translates the problem into a utility-possibilities set. That is, to each possible division of the given bundle of goods there corresponds a pair of numbers, representing the respective utility levels which the two recipients would achieve if that division were to take place. The set of all these pairs of utility numbers is known as the utility-possibilities set. Second, the mechanism operates on the utility-possibilities set, selecting from it one particular pair of utility numbers. In other words, the mechanism decides which utility distribution, among those available, is best. Third, the chosen pair of utility numbers, i.e. the utility distribution which had been designated best, is now translated back into some specific division of the original bundle of goods, corresponding to the selected pair of utility numbers. The basic act, which conveys the very nature of a distribution mechanism, that of choosing one specific distribution among those available, is carried out entirely in utility space. Mechanisms of this kind treat two distribution problems having the same utility-possibilities set as essentially identical, in the sense that the mechanism will select the same utility distribution for both problems. The rest is just a matter of translating back and forth between distributions of goods and distribution of utility.

Our aim at the present stage of the discussion may now be stated as follows: We shall present two different distribution problems having the property that their respec-

²¹ This is on the assumption that "equal split" is 500-500

tive utility-possibilities sets coincide. We shall then proceed to ask whether or not it is reasonable to expect these two problems to be resolved at the same point in utility space. In other words, suppose that the proper distribution of goods for problem A is α and that the proper distribution of goods for problem B is β , with A and B having the same utility-possibilities set. Will it necessarily be the case that α and β both correspond to the *same* distribution of utilities? A negative answer to this question would cast a shadow upon many known distribution mechanisms, including all but the first mechanism discussed in Sect. 3, above.

Let us once again recall the distribution problem Q1, as stated in Sect. 5 above. Alongside Q1, we now wish to consider a new distribution problem, as follows:

- Q 8: A shipment of 12 grapefruit and 12 avocados is to be distributed between Jones and Smith. The following information is given, and is known also to the two recipients:
 - Each grapefruit contains 100 milligrammes of vitamin F and no vitamin G.
 - Each avocado contains 100 milligrammes of vitamin G and no vitamin F.
 - Doctors have determined that, right now, Jones needs vitamin F for his health and, furthermore, that his body requires $\frac{1}{2}$ milligramme of vitamin G in order to metabolize 1 milligramme of vitamin F.
 - Doctors have also determined that Smith does not need vitamin F at the moment, but that he does need vitamin G for his health.
 - Both persons, Jones and Smith, are interested in the consumption of grapefruit and/or avocado only insofar as such consumption contributes to health through the provision of vitamin F and/or G – and the more vitamins, the better. All the other traits of the two fruits (such as taste, calorie content, etc.) are of no consequence to them.
 - No trades can be made after the division takes place.

How should the fruits be divided between Jones and Smith, if the division is to be just?

It is important to note, first of all, that in both problems, Q1 and Q8, the difference between the two individuals is one of *needs*. However, the divergence of needs between the two individuals is different in the two problems, as may be seen by comparing the formalization of Q1

$$\begin{split} & \omega = (12, 12) \\ & \theta^J \colon v_J(x, y) = 100 \ x \\ & \theta^S \colon v_S(x, y) = 50 \ x + 50 \ y \,, \end{split}$$

with the formalization of Q 8, which is given by

 $\omega = (12, 12)$ $\theta^{J}: v_{J}(x, y) = \min(100 x, 200 y)$ $\theta^{S}: v_{S}(x, y) = 100 y.$

Notwithstanding the fact that these two formalizations are indeed quite different from each other, the utility-possibilities sets for the two problems coincide. (In both cases, we take v_J and v_S to be the individuals' respective utility functions. Figure 2 shows the common utility possibilities set for the two problems:



In Fig. 2, the points marked by the Roman numerals, I through V, are precisely the utility distributions corresponding to the five distributions of goods (fruit) amongst which respondents were asked to choose for problem Q 1, as reported in Sect. 5 above. If distribution problems are in fact solved in utility space, then the distribution of choices made by respondents for problem Q 8 should not differ significantly, following translation into utility distributions, from the distribution obtained for Q 1. To check on this, we presented our new problem, Q 8, to a group of 110 respondents. Their responses, *translated into utility distributions*, are summarized in the right-hand column of Table 9.

Table	9
-------	---

Utility distribution	% of respondents	
	Q 1	Q 8
I	8	2
II	0	23
III	82	37
IV	8	_ 22
V	2	38

In order to facilitate comparison, we have also included in Table 9 (middle column) the corresponding data for Q 1, lifted from Table 2 above. The two response distributions, for Q 1, and Q 8, are obviously completely different. Although much can be said in an attempt to explain why this may not be an unreasonable outcome, we shall be content here merely to emphasize its significance: If, as Table 9 suggests, there is no

 $^{^{22}}$ Due to an error in design, the distribution corresponding to IV was not available as one of the response options for Q 8

reason to suppose that two distribution problems having the same utility-possibilities set must necessarily be resolved at the same distribution in utility space, then distribution mechanisms operating in utility space are likely to be inadequate. At first sight, this conclusion seems to lend support to the mechanism known as Competitive Equilibrium from Equal Split, being the only mechanism among those discussed in Sect. 3 which does not operate directly in utility space. Not so! It turns out that in the particular case being considered here – problems Q 1 and Q 8 – Competitive Equilibrium from Equal Split behaves like all the other mechanisms, i.e., it selects the same utility distribution for both problems.

9. Concluding Remark

In this essay, we have made an attempt to study some aspects of distributive justice, while taking account of observed ethical judgements. It is our view that the study of distributive justice cannot be oblivious to moral intuitions, but this in no way implies that we subscribe to an intuitionist point-of-view or to a theory of moral sentiments. The only general conclusion which we are prepared to draw from our work so far is that a satisfactory theory of distributive justice would have to be endowed with considerable detail and finesse. Sweeping solutions and world-embracing theories are not likely to be adequate for dealing with the intricacies inherent in the problem of How to Distribute.

Appendix

Data Sources and Experimental Design

The data reported herein are but a small subset of data collected by the authors in an extensive study of people's intuitions regarding distributive justice. Most of the respondents were applicants for admission to the Hebrew University in the years 1978 to 1980. The population of Hebrew University applicants consists of roughly equal numbers of men and women, mostly between 18 and 23 years of age. Various different socio-economic and ethnic backgrounds are represented in it. Our sample, however, consisted only of those applicants who chose to take the entrance examination in Hebrew. Since a choice of six other languages was offered (English, Arabic, Russian, French, Spanish, and a simplified Hebrew and English version), our group consisted primarily of native-born Israelis. Practically all are matriculated high-school graduates, and over half done military service.

Some of our study's results were collected from a different subject population – students and colleagues who attended various lectures and seminars on topics of distributive justice given by the first author in the years 1979-1981 in Israel and the United States. This was, typically, a more heavily male, older, and more sophisticated group, most with at least graduate level training in economics. However, except for Q 7, the results of this group are not reported here, though for identical questions they usually yielded response distributions that were very similar to the applicants group.

The (Hebrew) version of the question, as reproduced in the present text, was attached, with no instructions, to the end of our respondents' entrance examination

questionnaire. It is reasonable to assume that many of our unwitting subjects regarded the final question as part of the general examination, though it was physically and styllistically very distinct from the rest of the questionnaire (which consisted of various quiz-type aptitude and knowledge subtests). The context gives us every reason to believe that the subjects approached the question in an attentive and motivated state of mind. On the other hand, the context may have induced a problem-solving set, rather than a subjective moral-reflection one. In any case, we satisfied ourselves that responses collected in a different atmosphere (the relaxed, informal context of a lecture audience asked to volunteer answers to the questions) did not yield different response patterns.

Respondents were given five minutes to read the question and mark the answer they wanted out of the proffered set. The option of specifying some other distribution, different from those listed in the questionairre, was also available, but was hardly ever selected (except in one question where we erroneously gave an inappropriate list of possible answers, and a majority of the subjects suggested a distribution not included in our list – an event which serves as a demonstration that the small use of the "Other" option was not dictated by demand characteristics of the task). Subjects were also given space to write down any comments they might have had in justification of their choices. Only a minority chose to do so. Pilot studies indicated that the time allowance was ample, especially since the listed distributions were specified both in terms of the number of each fruit alotted to each receipient, and in terms of the amount of vitamins received (or believed to have been received) by each. Thus, little or no computational effort was required. Needless to say, the names of the various mechanisms corresponding to the different distributions (see Table 1) were not mentioned. Each subject received a single distribution problem. Problems were arbitrarily assigned to subjects.

We have no evidence that, if faced in the real world with a similar distribution problem, subjects would in fact opt for the distribution which they had picked in our questionnaire. Our results are certainly open to criticism on this count. Nonetheless, the findings have been shown to be quite robust. They replicate in different settings, and with different types of respondents. And even if there is a systematic discrepancy between responses to our hypothestical questions and behavior in real-life, a strong case can be made that it is the former that is pertinent to the present exploration, rather than the latter.

On the other hand, there is nothing as simple and straightforward as querying people directly about their intuitions in a transparent and schematic situation. The methodology has been quite popular and fruitful, for example, in the psychological study of human judgements and decision making under uncertainty (see, e.g., Kahneman et al. 1982).

Acknowledgements. We would like to thank Peter Hammond and Avishai Margalit for helpful discussions. Parts of this essay were written while Menahem Yaari was an Overseas Fellow at Churchill College, Cambridge. It is here that he wishes to record his indebtedness to Churchill College and to the SSRC Research Project on Risk at Cambridge University for their hospitality. Financial support from the U. S.-Israel Binational Science Foundation is gratefully acknowledged.

References

- Arrow KJ (1977) Extended sympathy and the possibility of social choice. Am Econ Rev 67:219-225
- Braithwaite RB (1955) Theory of games as a tool for the moral philosopher. Cambridge University Press, Cambridge
- Dworkin R (1981) What is equality? Parts 1 and 2. Philos Publ Affairs 10:185-246, 283-345
- Gabszewicz JJ (1975) Coalitional fairness of allocations in pure exchange economies. Econometrica 43:661-668
- Hammond PJ (1976) Equity, arrow's conditions, and rawl's difference principle. Econometrica 44:793-804
- Hammond PJ (1982) Utilitarianism, uncertainty and information. In: Sen AK and Williams B (eds) Utilitarianism and beyond. Cambridge University Press, Cambridge
- Harsanyi JC (1955) Cardinal Welfare, Individualistic ethics, and interpersonal comparisons of utility. J Polit Econ 63:309-321 (Reprinted in Harsanyi JC (1976) Essays in ethics, social behavior, and scientific explanation. Reidel, Dordrecht)
- Harsanyi JC (1977) Morality and the theory of rational behavior. Soc Res 44:623-656 (Reprinted in Sen AK, Williams B (eds) (1982) Utilitarianism and beyond. Cambridge University Press, Cambridge)
- Kahneman D, Slovic P, Tversky A (1982) Judgement under uncertainty: Heuristics and biases. Cambridge University Press, New York
- Kalai E, Smorodinsky M (1975) Other solutions to Nash's bargaining problem. Econometrica 43:513-518
- Laffont JJ, Maskin E (1981) Incentives in economics: An overview (mimeo.). University of Toulouse
- Luce RD, Raiffa H (1957) Games and Decisions. John Wiley & Sons, New York
- Nash JF (1950) The bargaining problem. Econometrica 18
- Raiffa H (1953) Arbitration schemes for generalized two-person games. In: Kuhn HW, Tucker AW (eds) Contributions to the theory of games II. Princeton: Princeton University Press, Princeton
- Rawls J (1971) A theory of justice. Harvard University Press, Cambridge, MA
- Rawls J (1974) Some reasons for the maximin criterion. Am Econ Rev 64:141-146
- Rescher N (1966) Distributive justice. Bobbs-Merrill, New York
- Roth AE (1979) Axiomatic models of bargaining. Springer, Berlin Heidelberg New York
- Strasnick S (1976) Social choice and the derivation of Rawl's difference principle. J Philos 63:85-99
- Varian HR (1975) Distributive justice, welfare economics, and the theory of fairness. Philosophy and Public Affairs, 4:223-247
- Vickrey WS (1945) Measuring marginal utility by reactions to risk. Econometrica 13:319-333 Wasserstrom RA (ed) (1971) Morality and the law. Wadsworth, Belmont, CA
- Yaari ME (1982) A remark on competitive equilibrium with equal incomes (mimeo.). Research Memorandum No. 49, Center for Research in Mathematical Economics, The Hebrew University, Jerusalem