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LOGIC AND THE WORLD

I. INTRODUCTION¹

Almighty and sublime, purified of any worldly infestation; our idea of logic is laden with allusions to the divine. Thus Russell:²

Pure logic aims at being true, in Leibnizian phraseology, in all possible worlds, not only in this higgledly-piggledly job-lot of a world in which chance has imprisoned us. There is a certain lordliness which the logician should preserve: he must not condescend to derive arguments from the things he sees about him.

And indeed that is how most of us, “professionals”, have come to conceive of logic: nothing chains it to the banalities of what things there are and how things are. Logic is free.

As against this lordly vision, a minority of thinkers have felt that nothing could be that free, not even logic. In the very same year in which Russell articulated the lordly vision, Wittgenstein was already asking us pointedly:

If there would be a logic even if there was no world, how could there be a logic given that there is a world?³

This question will serve as my staging point from which there is to emerge an alternative to the lordly picture. This alternative approach I call the *worldly* perspective. Its fundamental claim is that logic is as concerned with the real world just as truly as zoology, though with its more abstract and general features.

II. THE LORDLY TRADITION

Two opposite pictures then, the worldly and the lordly pictures. Before we plunge into their differences, let me stress a rare but nonetheless critical point of agreement between them. It concerns the objects of which we predicate logical truth: what ‘object’ are we describing when we describe a truth as a logical truth? The sentence expressing the

truth or the truth it expresses? A logical truth is first and foremost a logical *truth*, both sides respond. From this old-fashioned perspective, the modern, that is, post the Tarski and Carnap of 1933–34, fascination of logic with sentences and the linguistic may seem as an aberration. Thus though one may follow more recent customs and inquire which sentences are logically true, in this work, I will focus on the more traditional question as to which *truths* are logical.

Our question then is: *which* truths are logical and what *makes* any such truth into a logical one? When we come to characterize the lordly approach to these matters, three fundamental facts about it stand out.

First and foremost, the lordly tradition asserts the doctrine of *logical generalism*: only purely general propositions can serve as logical truths. Like other ‘generalistic’ doctrines, logical generalism is a *sceptical* doctrine. By nature, a generalist doubts that we can extend to *singular* propositions certain notions he has been happy to apply to general propositions. *Modal* generalism allows handsomely the application of the notion of necessity to general propositions but balks at applying it to singular propositions: to say of a singular proposition, by itself, that *it* is necessary would be meaningless. Similarly, *doxastic* generalism authorizes the notion of belief to apply to general propositions but forbids its application to singular propositions: to say that Pierre believes the singular proposition that-London-is-pretty by itself is to forget the mode-of-presentation under which he accesses that proposition. In the same vein, *logical generalism* questions our certitude that the singular truth that Quine is red or not red and the truth generated by assigning Quine to the free variable in “Red(*x*) or not Red(*x*)” are, in and of themselves, logical truths; for appearances are misleading here. It is only the *source* of these truths, the purely general proposition, *everything is red or not red*, that is a logical truth. The theorems of logic (and assuming soundness and completeness, the *truths* of logic) consist of general propositions only.⁴

This idea that only general propositions belong in logic is one facet of Russell’s above quote, i.e. the freedom from the impurities of this higgledy-piggledy job-lot of a world. The second mark of the lordly tradition is also present in Russell’s quote: the thesis that all logical truths are *necessary*.

At times, Russell, and following him Wittgenstein and Ramsey, even claimed the converse thesis, viz. that the only necessity there is is that of logical truths. Be that as it may, they surely all committed themselves to the descriptive claim that all logical truths *are* necessary; furthermore, they have asserted the normative claim that any candidate for the title *should* be necessary. The axiom of reducibility, the axiom of infinity (for individuals) and, for Russell, even the theorem of predicate logic $(\exists x)(x = x)$, all seeming very contingent, were to be banished away from the realm of logic, stigmatized by Russell as “defects in logical purity” (*op. cit.* 192–193, 203–204). Ramsey, who, as we are about to witness, wanted to keep $(\exists x)(x = x)$, $(\exists x)(\exists y)(x \neq y)$ and their ilk within the province of logic, felt that he must come up with nothing short of a ‘proof’ that would establish the necessity of such propositions, appearances to the contrary notwithstanding. Either way, Russell’s or Ramsey’s, at the end of the day, the lordly kingdom of logic was to have only necessary truths as subjects.

Insistent on the necessity of logical truths as these early logicians were, the necessity wasn’t built into their definition of the notion of logical truth. However, by the time the main modern notion of validity has emerged, the necessity has been written into the very definition of “logically true” as “true at all models” (of the relevant sort). Originally the models were just . . . models, abstract structures with no shred of modality in them. But under the guidance of Carnap, the models have been taken, more and more in the last forty years, to model the (logically) possible ways the world itself could have been. In fact, soon enough the models have even been used not only to analyze the meta-level notion of validity but also to give the truth-conditions of object-level modal locutions like “Necessarily”. When the models are made to model modal reality in this way, it becomes inevitable to think of every logical truth, i.e. a truth at all models, as modally necessary.⁵

This modal involvement on the part of the lordly logician should not obscure the third fact about this tradition, its analysis of what makes a truth into a *logical* truth. Cluttered with modal constraints as his analysis might be, the lordly logician has insisted, from time immemorial, on a non-modal, (clear and) distinct analysis of what

makes a proposition logically true, viz. that, other worlds notwithstanding, it is *actually* true solely in virtue of its structure. Even Russell, in the very same pages already quoted (*op. cit.*, 196–197) presents, very convincingly indeed, logical truth as being primarily “actual truth in virtue of the form of the proposition”. Wittgenstein seconds this with “It is the peculiar mark of logical propositions that one can recognize that they are true from the symbol alone and this fact contains in itself the whole philosophy of logic” (*Tractatus* 6.113). I should add that this analysis was not confined to philosophical expositions or introductory classes, e.g. the very first formulation of the completeness problem for (first order) logic, the one given by Hilbert in 1928, relies on this notion, asking us whether all the truths in virtue of form alone are derivable.⁶

I see in the traditional analysis of logical truth as actual truth in virtue of the form of the proposition nothing less than a recognition of the *autonomy* of logic. Logic is logic and not another thing: it is not a branch of the theory of knowledge, nor is it part of the theory of modality. Qua notion, the idea of actual truth in virtue of structure is a distinct notion; distinct from both the modal notion of truth at all *counterfactual*, merely possible, worlds and distinct from the epistemological notion of truth ascertained independently of any sensory experience. If it turns out that all the truths in actuality in virtue of structure alone are both necessary and *a priori*, fine. If not, not.

I conclude from this that we should distinguish virtues and vices in the lordly approach. It is in a disengagement of its third idea, i.e. what *makes* a truth logical, from the first two that my own proposal lies: hold on to the insight that logical truth is truth in *actuality* solely in virtue of structure, but don't follow the lordly philosopher in keeping it only for the perfected general propositions; extend it to *all* propositions.

III. TRUTH IN VIRTUE OF THE STRUCTURAL TRAITS OF THE WORLD

Consider the run-of-the-mill, singular, non-logical, truth that Quine is a philosopher:

$$P_1 = \langle \text{Quine, Philosopher} \rangle$$

Mere analysis of the “internal architecture” of P_1 wouldn’t betray its actual truth value. But though we may remain in the dark as to Quine’s philosophizing, we can extract from such an internal analysis the very existence of Quine: *his* own very lively presence inside P_1 attests for this trait of actuality: there is *that* distinct individual. Quine is. Quine exists.

This last discovery is more than a mere curiosity. For consider this truth:

$$P_2 = \langle \textit{Quine, Existence} \rangle$$

Begin with its epistemic and metaphysical categorization. If P_2 is accessed in the “standard” way, viz. via the linguistic lens of “Quine exists” or even “He exists”, it isn’t known *a priori* (subtleties aside). Perhaps Quine may so know it if he accessed it by way of “I exist”; be that as it may, you and I do not have this access to P_2 . As for its ‘modal profile’, P_2 is modally contingent: Among his imperfections, we note that Quine might not have existed.

The a posteriority and contingency of P_2 notwithstanding, P_2 is a *logical truth*. Or, so I claim. The question is: what is the general source of the difference between *Quine exists* and *Quine is a philosopher*? Between *Elizabeth II is identical to Liz* and *Elizabeth II was begotten by George VI*?

The key to the answer might seem to rest in the various propositions, but I believe the source of the difference lies at the other end, on the side of the world. Both *Quine is a philosopher* and *Quine exists* are fine propositions (sceptics about the latter have leaped, wrongly in my view, from the special *grounds* for the truth of this proposition to its alleged “incoherence”). The critical difference is that while the fact that Quine is a philosopher is an ordinary, *material* trait of the world, the existence of Quine is a very special trait of the world, what I will call a *pre-fact*. Now, what are these ‘pre-facts’?

They are *structural* traits of the world. Ordinary worldly facts are not structural; they draw blunt invidious distinctions between the existing worldly building blocks: that Quine is a philosopher is a fact; permute him with Mt. Blanc and the fact is gone. The same applies to the factivity of Mt. Blanc’s being a mountain when Quine takes over the subject position.

Structural traits, *pre-facts*, are very different in this respect. Quine exists, that is a *pre-fact*. But replace Quine in it by Church, Mt. Blanc or the moon, and you still have a *pre-fact* on your hands. That is why I believe these *pre-facts* are *structural*: they do not single out a particular constituent of the world and report that it is thus-and-such. They simply specify the very conditions for being such an arbitrary constituent of the world, “a one”, in the first place. To be “a one”, the candidate has to exist and have its distinct identity.

So much then for facts and *pre-facts*. What then makes the *proposition* that Quine exists or that Quine is distinct from Church into a logical truth?

The idea is simple — the *pre-facts* detail structural traits of the world. But if the proposition is about such structural traits, this will already be coded into its own internal ‘bowels’. What will be the relevant sign? The structurality of the *pre-fact* was reflected in its blindness to particular constituents, in the closure of *pre-facts* under permutations of their subject-constituents. My idea is that if a proposition is about these structural traits, its *truth* will also be closed under permutations of subjects (no wonder that its truth will be permutation-resistant in this way, if what *makes* it true, the *pre-fact*, is likewise permutation-resistant).

I will call such propositions, *truths in virtue of the logical structure of the world* (or, for the impatient reader, “structural truths”). I conjecture then for them the following *one-way* conditional:

[L₁] If a proposition is true in virtue of the logical structure of the world, it is permutation-resistant.

IV. PERMUTATION-RESISTANCE AS A SUFFICIENT CONDITION

I myself believe permutation-resistance is not only a necessary but also a sufficient condition for logical truth. But I wanted to separate the innocuous from the controversial: while (L₁) may seem acceptable, its converse would be out-of-the-question to many. Why then has the converse conditional been deemed unacceptable?

Two separate motivations have been operative here. I will begin with the weaker one to get it out of the way. It is often said that

there might be certain (atomic) properties P that just happen to be true of everything. Hence *Quine is P* would be permutation-resistant. And yet, it is argued, *Quine is P* is surely no logical truth, given the merely *accidental* nature of the truth of $(\forall x)Px$.

In my view, the objection commits a double error. First, if by “accidental” we mean “contingent”, I have already argued that the falsehood of a proposition in a counterfactual situation is no hold against its truth in virtue of the structural traits of this, very actual, world.

Secondly, suppose that the objector discards this modal reading of “accidental” and goes on to say that it isn’t the contingency of *Quine is P* that bothers him, it is rather its “accidental nature” in the sense that the P -ness of Quine isn’t true of him simply in virtue of his being a thing. Very well then, I concede the point: *Quine is P* isn’t true in virtue of his being a being. But, I add, it isn’t permutation-resistant either. Similarly, if the objector points to the universal truth of *If x is red then x is not green* and insists it is no logical truth, I agree but, again, point out that it isn’t permutation-resistant either.

All this is quite obvious if we remember to permute not only the individuals but the attributes too. Thus we get, e.g., *Church is British* and *If x is tall then x is fat* which would refute, in actuality, the alleged permutation-resistance of the original propositions. Of course, not all attributes (relations) can be freely permuted. As I have argued, *Identity* and *Existence* are no humdrum attributes, they ascribe logical traits. Assuming then that *Existence* is an attribute and *Identity* is a relation, they should be excluded from permutation. The logicity of *Quine exists* and *Quine is not identical to Church* attests to this.

In view of this favoritism, you may be compelled to claim that we are moving in a circle, “defining the attributes that are structural by alluding to the permutation-resistance criterion and at the same time specifying which components are not to be permuted by alluding to the criterion of structurality”.

I reject the accusation. For in the first place, I do not conceive of (L_1) , when strengthened into a biconditional, as a *definition* or an *analysis* of the notion of “truth in virtue of structural traits of the world”. I do think of it as giving necessary-and-sufficient conditions for being such a truth. But this is still a far cry from suggesting that

(L₁) “defines” or “analyzes” the notion of “structural trait”. What is more, I regard this situation, where no “definition” is forthcoming, as containing an insight rather than an embarrassment. For, in my view, the last arbiter about the notion of “structural trait of the world” is our intuition about cases. The only way to get hold of the *notion* itself is by intuitions about particular cases as to what is and what is not a trait that beings have simply qua beings; then *given* the notion, we may supply necessary-and-sufficient conditions for belonging in its *extension*. Thus, suppose one would have liked to go here beyond *existence*, *identity* and other classical candidates, e.g. follow *Tractatus* 2.0251 “space, time and color (being colored) are forms of objects”; the argument, for me, wouldn’t come primarily from the permutation-resistance of e.g. *x is in time*. It would originate with a direct observation about the allegedly special feature of this trait, showing that it is written into the very notion of a being that it be in time.

The other objection to taking permutation-resistance as sufficient for logical truth hits closer to home. The second objector argues that he cannot see why the external *fact* that every being has a property *P* simply qua being endows the *proposition* that *x is P* with an *internal* structure that is sufficient to divulge its actual truth.

Let me grant the objector his intended claim: the worldly structure of the fact that, e.g., Quine is *P* leaves us much freedom as to the parallel structure of the proposition that *Quine is P*. My answer to our objector as to why permutation resistance is nonetheless a sufficient condition for logical truth is that logic investigates the structural traits of the *world*, rather than the structural traits of propositions representing it.

A truth is made into a logical one by special traits of that which *makes* it true in the first place, i.e. the kind of trait of the *world* in virtue of which this truth is the distinct truth that it is. The structure of propositions, or for that matter of any other truth-bearing “representational” device, is irrelevant at this point. Suppose indeed that I rejected altogether the existence of propositions, let alone their endowment with internal structure. Would my above differentiation of logical from non-logical truths go by the board?

Not at all. The truth that Quine is human or not human is logical because the worldly *individual* Quine has this attribute in a *way* that is

very different from the way he bears *being-human*. If, as I would have it, the explanation of that which is special to the *logical* way of having an attribute alludes to tests like permutation-resistance, then, again, we look at nothing more than individuals in the world and the attributes they possess, i.e. we check whether the attribute is had by every being simply qua being. No mention of propositions is called upon here.

In contrast, the notion of truth in virtue of *propositional* structure focuses on the structure of the relevant representations, the propositions. Philosophers may and actually do differ widely over the structure of such representations. Some think propositions may be as structured as the sentences that express them, some think they are structureless sets of possible worlds and some think they are something in between. Some think necessarily-equivalent propositions are identical, some think logically-equivalent propositions are identical, some think only "epistemically-equivalent" propositions are identical and some (at least myself) think none of these equivalences are sufficient for propositional identity. Be all that as it may, such disputes concern the degrees to which our *representations* of the world are endowed with internal structure. This is a matter which is posterior to unfolding the structure of the world thus represented.

It may very well be the case that the discovery of a trait of the world that holds of every being qua being is not indicative of analogous internal propositional structure. But the discovery of such a trait cannot be similarly detached from the study of the structure of the world itself: To discover a trait things have simply qua things *is* to unfold the structure of the world.⁷

V. THE WORLDLINESS OF ATTRIBUTES

We have encountered above two grades of opposition to my very liberal involvement with worldly items. *First-order* generalism banished individual-involving propositions from the realm of logical truths – *Quine is red or it is not the case that Quine is red* is out, but *everything is either red or not red* is in. In its puritanism, *second-order* generalism made the latter proposition follow the former into exile; only the 'pure', schematic, *everything is F or not F* (i.e., $\forall x(Fx \vee \sim Fx)$),

everything exists (i.e., $\forall xEx$) *everything is identical to itself* (i.e., $\forall x(x = x)$) are admissible.

For my own reasons, I agree with the second-order generalists that *Red* is as unlordly as Quine. For as overtly as Quine, *Red* violates epistemological and modal desiderata the generalists posit on the notion of logical truth: their being (i) *a priori* and (ii) necessary. How does *Red* create this havoc? The second-order generalist and I agree that it may well satisfy the following epistemological and modal theses:

(E₁) That the attribute *Being-Red* exists is given *a posteriori*

(M₁) That the attribute *Being-Red* exists is merely contingent.

Why would (E₁) and (M₁) be true? The source of both (E₁) and (M₁) traces to a third, more primordial, thesis that is neither epistemic nor modal. It is an ontological thesis about the *actual* existence conditions of the attribute *Red*:

(O₁) The attribute *Being-Red* *actually* exists only if it *actually* has instances.

Call any attribute having such actual existence-conditions *worldly*. I cannot argue here for the worldliness of *Red*, though I note that my chief lordly opponent, Russell, had, in one of his other moods, admitted that “Logic doesn’t know whether there are in fact *N*-adic relations (in intension); this is an empirical question”. Be that as it may, I will ask you to meet me half way and grant here the truth of (O₁), for in the present work I merely want to establish with it a conditional point: given its truth, interesting logical truths follow.

Assume (O₁) and we are at the origin of (E₁) and (M₁): because of its ontological dependence on worldly generators, we do not know ‘without looking’ that *Being-Red* exists in actuality; also because of its dependence on such concreta, the attribute *Being-Red* is as modally imperfect as Quine: our world has been blessed with Reds but a world of Browns would not have *Being-Red* among its attributes. Whence (M₁).

We are also in a position to appreciate now how the proposition

$$P_3 = \langle \textit{Being-Red, instantiated} \rangle$$

imitates nicely the record of $\langle \textit{Quine, Existence} \rangle$. A priority is gone. There is no purely reflective guarantee that the sentence “*Being-Red* is instantiated” signifies a proposition (consider “*Being-Unicorn* is instantiated”). Modal contingency is also unavoidable: P_3 may have been generated in actuality; now that we have it we can take it and evaluate it in a counterfactual Redless world. It cannot be generated over there (cf. $\langle \textit{Quine, Existence} \rangle$ in a Quineless world); however, given its generation in *actuality*, we have *it* and *it* can be modally evaluated in counterfactual situations where it doesn’t exist.

Contingent and *a posteriori* P_3 may be; and yet, the proposition itself, P_3 , is actually true solely in virtue of the structural traits of the world, i.e. the structure (in particular, the existence conditions) of the attribute *Being-Red*. For the very existence of *Being-Red* guarantees that it bears the trait it is ascribed i.e., that it’s instantiated. This is not the case with other traits we may ascribe to it, e.g. that it is my favorite attribute or even the necessary trait that everything that has it has the same color as the actual color of fire-trucks. A separate confirmation of this point comes from the fact that the trait of *being instantiated* is permutation resistant. Any other basic attribute you might care to permute with *Being-Red* would also be actually instantiated, at least on my view of basic attributes. In contrast, permute *Being-Green* with *Being-Red*, and it wouldn’t have, in actuality, the trait of being my favorite attribute or be such that everything that bears it has the same color as the actual color of fire-trucks.

Finally, the proposition *Red is instantiated* is not only true in virtue of the structural traits of the world; the proposition is also true in virtue of the proposition’s internal structure (just like *Quine exists*). Given the assumption that it is the attribute itself that occurs in the proposition, its very presence in the proposition verifies the truth of the predication without any external evaluation. Hence the proposition, though neither *a priori* nor necessary, is true in virtue of an analysis of its own internal structure.

VI. THE WORLDLINESS OF LOGICAL NOTIONS

Aware of the unreliability of the attributes, Russell retreated to second-order logical generalism: only *purely* ‘logical’ propositions will

do. At last, a safe haven for logic. But is it? Are the propositions, e.g., *everything is self identical* and *everything exists* lordly, free of our job-lot of a world?

I doubt it very much. Without any pretense to an informed, let alone scholarly, reading of the text, I am guided here by the early Wittgenstein:

The 'experience' we need in order to understand logic is not that something or other is the state of things, but that something *is*: that however is not an experience. Logic is *prior* to every experience – that something *is so*. It is prior to the question 'How?', not prior to the question 'What?' (*Tractatus* 5.552)

So much for the motif. Here is my own version of the idea. Very much like *Red*, *Existence* and *Identity* are, on the worldly picture, subject to the ontological thesis:

(O₂) *Existence* and *Identity* actually exist, only if they have actual instances (only if at least one object actually exists).

Aware of the modal and epistemic corollaries induced by the counterpart thesis (O₁) in the case of *Red*, the lordly logician might be worried that (O₂) may engender similar repercussions:

(E₂) That *Existence* and *Identity* actually exist isn't known *a priori*.

(M₂) That *Existence* and *Identity* exist is merely contingent.

Presupposing that all logical truths are *a priori* and necessary, the lordly logician looks at (E₂) and (M₂) as the last straw; truths like $(\forall x) \textit{Exists}(x)$ and $(\forall x)(x = x)$, let alone $(\exists x)(\textit{Exists}(x))$ and $(\exists x)(x = x)$ are going to lack the desired traits and hence wouldn't, in turn, be logical truths. In a word, a *reductio* of the worldly view and of (O₂) in particular. The question is: is the lordly logician right in so arguing?

Take the most damaging case one can think of:

$$P_4 = \langle \textit{Existence}, \textit{Instantiated} \rangle$$

which is far more precarious than $(\forall x)(x = x)$ and $(\forall x) Ex$ (which may not be *a priori* but would turn out to be necessary, if the propositions actually exist in the first place).

Focusing on epistemological matters, let us “stipulate” that P_4 isn’t known *a priori*. Cartesian modes of reasoning notwithstanding, pretend that we may use the word ‘existence’ competently in the sentence “Existence has instances” and yet not know whether the attribute *Existence* has instances. Also, let us pretend that it could have been the case that no object whatsoever existed. In that apocalyptic counterfactual world, P_4 is false. Let us suppose this *is* a genuine possibility.

All this doesn’t take away the actual truth of P_4 in virtue of the structural traits of the world. For assuming that (O_2) does give the existence-conditions of *existence*, its very being *in* the world guarantees its having the trait with which it is predicated here. In contrast, its very being doesn’t guarantee that it bears traits like being Anselm’s favorite attribute or being the attribute actually discussed by Moore in “Is existence a predicate?”. What is more, assume again (O_2) , and assume that P_4 has the attribute *Existence* itself as constituent, an assumption about the proposition, not about the world itself. At this point, by a purely internal analysis of P_4 , we can establish that indeed *Existence* is instantiated. Thus, whether *we* know it *a priori* or not, whether it would still be true throughout all counterfactual situations, the proposition P_4 is *actually* true in virtue of its own internal structure. Which is also the case of $(\exists x)(x = x)$, $(\forall x)(x = x)$ and $(\forall x)(\text{Exists}(x))$.⁸

VII. LOGIC AND NECESSITY:

THE NUMBER OF INDIVIDUALS IN THE WORLD

Qua notion, in and of itself, logical truth seems distinct from the modal idea of necessity. As for the extensions of the two notions, so far my cases of contingent logical truths consisted of *singular* truths e.g., *Quine exists* and *Redness is instantiated*. There are other examples of this sort: the proposition *If a is F then some individual is F*, for every such individual existing in actuality. Every being, simply in virtue of being such, has the trait that *if it F’s something F’s*. And yet such truths might be contingent: *If Quine is possibly human then something is possibly human* is an example. Consider a Quineless counterfactual situation w in which everything is made of sand. The

antecedent *Quine is possibly human* is true at that situation, for actuality provides the required possibility. But the consequent proposition is false at w : no-thing of w , no grain of sand, could have been human. More examples of this sort can be concocted but in analyzing contingent logical truths, I would like to focus on one special case of a different sort.

The special case I have in mind originates with Ramsey: true propositions about the number of individuals in the world, e.g., *there are at least two individuals* are not mere truths but rather logical truths.

If one presupposes that all logical truths must be necessary, one might be shocked by Ramsey's view: the contingency of our proposition is after all very hard to deny. Ramsey himself avoided the "shock". How? Instead of admitting contingent logical truths, Ramsey preferred to bite the bullet and pronounce this proposition *necessary*.⁹

Ramsey's main argument (*op. cit.*, 59–61) for the logical truth of propositions stating the number of individuals in the world relies on standard first-order codings of the claims. E.g., the claim that there are at least two individuals would be coded by " $\exists x\exists y(x \neq y)$ ". But, in addition, he believed quantificational propositions are analyzable in terms of *singular* propositions: existential quantifications are identified with long disjunctions, universal quantifications are identified with long conjunctions. So analyzed, the logical truth of *At least two individuals exist* is inescapable.¹⁰

VIII. THE FLAWS IN RAMSEY'S APPROACH

The problem with Ramsey's argument is that it relies on an identification of a general proposition with its "singular expansion". This seems questionable. Of course, those who (like myself) believe that the identity of structured propositions is fixed by composition from the identity of their constituents, will reject out of hand any identification of a general proposition and a complex singular proposition. But in fact general propositions are not necessarily equivalent to their "singular expansion"; so the equivalence fails even the least demanding test of propositional identity.

Let $Dis(F)$ stand for the disjunction of propositions of the form Fa for each existing individual a . We have already seen that the conditional $Dis(F) \supset \exists xFx$ is not necessary. (Take as F , our case of *possibly-human*.) In the other direction, $\exists xFx \supset Dis(F)$ is not necessary either. How so? We have just seen that the problem for the converse of this conditional, $Dis(F) \supset \exists xFx$, came from *diminishing* domains. The counterexample to the necessity of the present conditional arises from *growing* domains. Let a counterfactual world w have a larger domain than the actual domain. Thus there is a new individual of w , for which it is true that it does not actually exist. Hence $\exists x Act(Not-Exist(x))$ is true at w . However, we can generate singular propositions only out of actual individuals (an idea to which Ramsey himself strongly adhered). But then the disjunction:

Actually (Not-exist (Church)) \vee *Actually (not-exist (Quine))* \vee *Actually (Not-exist (Mt. Blanc))* \vee . . .

is false at w . The same counterexample applies to $Conj(F) \supset \forall xFx$, with *Actually (exists)* used as the offending F . The antecedent conjunction is true at w . The consequent is false at w .¹¹

IX. A NEO-RAMSEYAN ANALYSIS

The irony of it all is that Ramsey does not need to claim that a general proposition is identical to its singular expansion. Nor does he need to claim that they are necessarily equivalent. He only needs to show they logically imply each other. To provide us with an appropriate analysis, let me hitch my wagon to a star and call my own picture “Neo-Ramseyan”. It introduces a new relation between propositions, that of *structural implication*. Say that P *structurally implies* Q iff the material conditional *If P then Q* is a truth in virtue of the structural traits of the world. The insight I extract from Ramsey’s work is that a general proposition and its singular expansion structurally imply each other. Indeed, the conditionals (1)–(2) are both logically true (and, as you can verify, permutation-resistant):

- [1] *If Dis(Happy) then some individual is happy*
- [2] *If some individual is happy then Dis(Happy)*

Assume this logical framework. Why would *at least two individuals exist* be logically true? Following Ramsey, we aim to deduce this truth from singular truths. Begin then with the conjunctive singular proposition antecedent, *John exists and Mary exists*. This is a logical truth. The conditional *If John exists and Mary exists then at least two individuals exist* is also a logical truth. Detach now its consequent and you have what Ramsey sought: the logical truth of *At least two individuals exist*. An alternative route is available to those who persist in questioning the logicity of propositions like *Mary exists*. Simply begin with *if John is distinct from Mary then there are at least two individuals*. Detach the consequent and you are home free.

What individuals there actually are (and hence, how many of them there are) may seem like a trait of the world that does not concern logic. But nothing is quite what it seems. In the way that the structural *Quine exists* differs from the material *Quine walks*, in the way that the structural *Quine is distinct from Church* differs from the material *Quine is younger than Church*, I believe that *At least two individuals exist* differs from *At least two cows exist*. The latter general fact arises from two material singular facts, e.g., *Bossie is a cow* and *Lizzy is a cow*. Two facts about the goings-on in the world, about the *state* the world is in.

Not so with *At least two individuals exist*. The latter merely reports what exists and what is identical to what. In so doing, it merely details the multiplicity of subjects the world has, not what the state of these subjects is. It tells us *what* things there are, not *how* they are.¹²

X. LOGIC AND NECESSITY – A DISENGAGEMENT

There are then contingent logical truths. The question is: what does this show? One reaction to these findings may consist in the attempt to trace the *source* of this contingency. I know of one hypothesis about this matter, due to Tony Martin (in conversation). He argued that the *source* of the contingency of the above logical truths is the contingent *existence* of their constituents (individuals, attributes). Stipulate necessary existence for these items and all logical truths would be again necessary.

Read strongly, i.e. “it is sufficient to posit necessary existence for all *actual* individuals and then . . .”, the hypothesis has already been refuted. Let all actual individuals exist necessarily. The conditional $\exists xFx \supset Dis(F)$ can be refuted in a counterfactual situation with “new”, additional, individuals. Read weakly (as Martin intended), i.e., “it is sufficient to posit necessary existence for all individuals, actual or not . . .”, the above counterexample wouldn’t work. But others would. The truth (and not only the sentence expressing it) that *if actually snow is white then snow is white* is a contingent logical truth even if all individuals existed necessarily.¹³

This first type of reaction is content to focus on the “Kantian” question “how are contingent logical truths possible?” One can go beyond this reaction by questioning the implicit presupposition of the “Kantian” formulation, viz. why should logical truths be necessary in the first place?

I believe that we find ourselves here in a situation similar to the one created by Kripke’s disengagement of the *a priori* and the necessary. Kripke argues that the latter notions seem to be distinct. Thus if anyone were to show that all *a priori* truths must be necessary, that would call for a substantive argument rather than a mere trivial observation on the meaning of “*a priori*”. And indeed such a substantive argument has been put forward: if we know about actuality a certain truth *P* “without looking” at the world itself, then surely *P* can’t depend on any contingent feature of that which we have not looked at. Hence *P* is necessary. To this Kripke would respond, even before giving his *cases*, that it is precisely because we cognize from within the actual world and *have* to so cognize that we may know of it, without looking, certain truths that would not hold of other situations.

A structurally similar situation holds in our case of the notion of logical truth. Someone may try his hand at the following “bridging argument”: if a trait is a *structural* trait of this world, then it must be an *essential* trait of it. For though we may vary the material traits of the world, the “scaffolding” must be fixed for the notion of possibility for the world to make sense. Perhaps (something like) such an argument is given in *Tractatus* 2.022, “an imagined world, however different it may be from the real one, must have *something* — a form — in common with it”.

Now I myself would respond to such an argument by pointing out that logic focuses on special, structural (“formal”), traits of actuality, traits that may very well be distinct of actuality, that may explain what the actuality of this one and only one real world boils down to. If so, there is no intrinsic reason to expect that such traits would extend to situations which, by their very nature, are not actual, are merely possible.

Be all that as it may, the real point of the present work is not so much to win this debate; it is rather to focus us on two observations which would hold no matter how the debate would unfold. The first is that the necessity of logical truths won’t come from a trivial observation on the meaning of “logical truth”; one will have to first deny my *cases* of contingent logical truths, and then propose a substantive metaphysical thesis of the order of the above quoted *Tractatus* hypothesis as to why the actual truth of the logical truths extends to all possible worlds. Secondly, such a thesis, true or false, would shift the focus of the debate. For so far we were told that the key notion was that of “logical truth”, a proper reading of which would establish automatically the necessity of such truths. But if I am right it’s the other way round: the notion of *necessity* is the critical notion here. It will have to be made to satisfy the following thesis: if the world has a given structural trait, it is a modally essential trait of it, a trait it *couldn’t* have lacked. And thus, if I am right, we shouldn’t build a modal underside to the notion of logical truth; we should rather unfold the structural underside of our modal notions.

APPENDIX: THE PERMUTATION-RESISTANCE OF LOGICAL TRUTHS

I define here the permutations with respect to which logical truths are resistant.

A. *Rules of Generation for Propositions*

(1) Let there be

- (i) The domain of existing individuals — D_i
- (ii) The domain of basic attributes — D_{BA}

- (iii) A binary relation – *identity*
 - (iv) Complex-attributes formers – *Not**, *And**, *Nec**, *Act**
 - (v) Determiners – *Every*, *Some*, *Most*, *At least one*, *At least two*, . . .
 - (vi) propositional connectives – *Not*, *And*, *Nec*, *Act*
- (2) The domain of (simple and complex) attributes, D_A , is generated by:
- If $P, Q \in D_{BA}$, then $P, Q, \text{Not}^* \langle P \rangle, \text{And}^* \langle P, Q \rangle, \text{Nec}^* \langle P \rangle, \text{Act}^* \langle P \rangle \in D_A$
- (3) The domain of propositions, D_p , is generated by:
- (i) If $x \in D_i$ and $P \in D_A$, then $\langle x, P \rangle \in D_p$
 - (ii) If $x, y \in D_i$, then $\langle \langle x, y \rangle, \text{identity} \rangle \in D_p$
 - (iii) If $P, Q \in D_A$, then $\langle \text{Every} \langle P, Q \rangle \rangle, \langle \text{Most} \langle P, Q \rangle \rangle, \langle \text{Some} \langle P, Q \rangle \rangle, \langle \text{At least one} \langle P, Q \rangle \rangle, \langle \text{At least two} \langle P, Q \rangle \rangle \in D_p$
 - (iv) if $\phi, \psi \in D_p$, then $\langle \text{Not} \langle \phi \rangle \rangle, \langle \text{And} \langle \langle \phi \rangle, \langle \psi \rangle \rangle \rangle, \langle \text{Nec} \langle \phi \rangle \rangle, \langle \text{Act} \langle \phi \rangle \rangle \in D_p$
- (4) Comments on generation rules
- (i) D_{BA} contains *Existence* and *Individual*
 - (ii) I have allowed for an “ambiguous” treatment of *not*, *and*, . . . , viz. *Not** vs. *Not*, to distinguish the two syntactical roles involved. The philosophical question as to which one is basic is not addressed
 - (iii) Similarly, reasons for treating determiners, e.g., *Every*, as *binary* relations (rather than in the Fregean monadic way) are not given here (though note that propositions allowing *Most* can be generated)

B. Permutations

- (1) We have
- (i) A permutation on individuals – Π_i , a one–one map of D_i onto D_i
 - (ii) A permutation on basic attributes – Π_{BA} , a one–one map of D_{BA} onto D_{BA} such that $\Pi_{BA}(\text{Individual}) = \text{Individual}$, $\Pi_{BA}(\text{Existence}) = \text{Existence}$.

- (2) Given the permutation Π_{BA} , a permutation Π_A is induced on D_A , satisfying:
- (i) $\Pi_A(P) = \Pi_{BA}(P)$, for $P \in D_{BA}$
 - (ii) $\Pi_A(\text{Not}^* \langle P \rangle) = \text{Not}^* \langle \Pi_A(P) \rangle$
 - (iii) $\Pi_A(\text{And}^* \langle P, Q \rangle) = \text{And}^* \langle \Pi_A(P), \Pi_A(Q) \rangle$
 - (iv) $\Pi_A(\text{Nec}^* \langle P \rangle) = \text{Nec}^* \langle \Pi_A(P) \rangle$
 - (v) $\Pi_A(\text{Act}^* \langle P \rangle) = \text{Act}^* \langle \Pi_A(P) \rangle$
- (3) Given a pair of permutations Π_i, Π_A , a permutation Π_p on propositions, a one-one map of D_p onto D_p , is induced, satisfying:
- (i) If ϕ is atomic and of the form $\langle x, P \rangle$, then $\Pi_p(\phi) = \langle \Pi_i(x), \Pi_A(P) \rangle$
 - (ii) If ϕ is atomic and of the form $\langle \langle x, y \rangle, \text{identity} \rangle$, then $\Pi_p(\phi) = \langle \langle \Pi_i(x), \Pi_i(y) \rangle, \text{identity} \rangle$
 - (iii) $\Pi_p(\langle \text{Every} \langle P, Q \rangle \rangle) = \langle \text{Every} \langle \Pi_A(P), \Pi_A(Q) \rangle \rangle$
 - (iv) $\Pi_p(\langle \text{Some} \langle P, Q \rangle \rangle) = \langle \text{Some} \langle \Pi_A(P), \Pi_A(Q) \rangle \rangle$
 - (v) $\Pi_p(\langle \text{Most} \langle P, Q \rangle \rangle) = \langle \text{Most} \langle \Pi_A(P), \Pi_A(Q) \rangle \rangle$
 - (vi) $\Pi_p(\langle \text{At least two} \langle P, Q \rangle \rangle) = \langle \text{At least two} \langle \Pi_A(P), \Pi_A(Q) \rangle \rangle$
 - (vii) $\Pi_p(\langle \text{At least one} \langle P, Q \rangle \rangle) = \langle \text{At least one} \langle \Pi_A(P), \Pi_A(Q) \rangle \rangle$
 - (viii) $\Pi_p(\langle \text{Not} \langle \phi \rangle \rangle) = \langle \text{Not} \langle \Pi_p(\phi) \rangle \rangle$
 - (ix) $\Pi_p(\langle \text{And} \langle \langle \phi \rangle, \langle \psi \rangle \rangle \rangle) = \langle \text{And} \langle \langle \Pi_p(\phi) \rangle, \langle \Pi_p(\psi) \rangle \rangle \rangle$
 - (x) $\Pi_p(\langle \text{Nec} \langle \phi \rangle \rangle) = \langle \text{Nec} \langle \Pi_p(\phi) \rangle \rangle$
 - (xi) $\Pi_p(\langle \text{Act} \langle \phi \rangle \rangle) = \langle \text{Act} \langle \Pi_p(\phi) \rangle \rangle$

C. The Thesis Formalized

(L) ϕ is logically true iff $\Pi_p(\phi)$ is true for all Π_p .

NOTES

¹ Many thanks are due to R. Albritton, K. Fine, Y. Gutgeld, M. Gehman, D. Kaplan and T. Martin. I am also indebted to the editor for his advice on matters of form and content.

² *Introduction to Mathematical Philosophy*, p. 192. All references to Russell (unless otherwise indicated) are to this work.

³ *Tractatus Logico-Philosophicus*, sec. 5.521.

⁴ Some die-hard generalists may even find *everything is red or not red* rather promiscuous: the singularity of the attribute *Red* is as offensive as that of the individual Quine. Only truly general universal closures over both subject and predicate, e.g., *everything is F or not F* would then belong in the province of logic. Call the former position *first-order generalism*, the latter *second-order-generalism* (in view of the kind of universal closures they would retreat to, the former to e.g., $(\forall x)(Red(x) \vee \sim Red(x))$, the latter to e.g., $(\forall P)(\forall x)(P(x) \vee \sim P(x))$).

One formal manifestation of this flight to (first-order) universal closures is Quine's "generality interpretation" as found e.g., in *Mathematical Logic*, pp. 80–89. The idea is stated there for open and closed *sentences*; but waving this ontological difference, it is the gambit considered here — only general propositions are asserted as theorems. The idea is anticipated, in its second-order form, in Russell's *Introduction to Mathematical Philosophy*, *op. cit.*, pp. 196–200, (especially p. 199). The earliest statement is found in the unpublished 'Necessity and Possibility', 1900, and 'Recent Work on the Principles of Mathematics', *International Monthly* 4 (1901) 83–101. See also *The Principles of Mathematics*, first edition, London, Cambridge U.P. 1903, pp. 7–9 and *Our Knowledge of the External World*, London, Allen Unwin, 1961, pp. 53–54, 67.

⁵ As hinted in the above paragraph, the notion of "truth at all models" is not, in and of itself, a modal notion. For the critical question is what do the models model. If we think of each model as reassigning, *in actuality*, semantical values to the non-formal components of the claim, or less linguistically, as permuting the constituents of the claim while preserving its form, then the notion involved is "truth in actuality no matter which ("non-logical") constituents made up this proposition", a notion that doesn't promise the necessity of a logically true proposition. Interestingly enough, a variant of such a notion of validity is given by e.g. Kripke for none other than . . . modal logic. From the original paper in 1959 through the various applications to modal and intuitionistic systems, his notion of validity was that of truth in the *actual* world of every model, not, I insist, truth at all the worlds of each model. When the language is enriched appropriately, as ours will be below, Kripke's notion of validity allows for contingent logical truths.

⁶ Hilbert speaks of "the formulas that are not refutable (*Widerlegbar*) through any definite stipulation of the suitable predicates. These formulas represent the valid logical propositions". See his 'Probleme der Grundlegung der Mathematik', *Math. Annalen* 102, pp. 1–9. See also the third footnote of K. Gödel's 'Die Vollständigkeit der Axiome der logischen Funktionalkalküls', *Monat. für Math. und Phys.* 37, 1930 (translated in *Gödel — The collected works*, Vol. 1, Oxford U.P. 1986, pp. 103–123) where Gödel uses this notion to settle the problem Hilbert posed. Furthermore and as against popular myths, this notion is contrasted by A. Tarski with the notion of truth at all set-theoretic structures and held to be the *intuitive* notion of logical validity. See his 1936 'On the Concept of Logical Consequence', in *Logic, Semantics and Metamathematics*, Clarendon Press, Oxford 1956, the bottom of p. 410 and p. 411.

⁷ So much for permutation-resistance as a sufficient condition for logical truth, i.e. truth in virtue of the structure of the *world*. I take up the question whether permutation-resistance of propositions is sufficient for truth in virtue of *internal propositional* structure in n. 12 below. A formal definition of "permutation-resistance" is given in the Appendix.

⁸ A, to me, remarkable anticipation is to be found in Wittgenstein's *Notebooks* 1914–1916 (translated from German by E. Anscombe), Blackwells 1961, p. 11e: "If we take

" $(\exists x)(x = x)$ " it might be understood to be tautological since it could not get written down at all if it were false, but here!"

⁹ See in 'The Foundations of Mathematics', in Ramsey, F. P. *The Foundations of Mathematics*, pp. 59–61. See also 'Facts and Propositions', *op. cit.*, pp. 154–155 and finally 'Critical Notice of Tractatus', *op. cit.*, pp. 280–286. Ramsey thought he had two "proofs" for the necessity of the number of individuals in the world. The main one, pp. 59–61, I discuss in the text. The additional one, pp. 154–5, can be extracted from the following passage (*op. cit.*, 155): "admit that numerical identity and difference are necessary relations, that 'There is an x such that fx ' necessarily follows from ' fa ', and that whatever follows necessarily from a necessary truth is itself necessary. If so, their position cannot be maintained; for suppose a, b, c are in fact not everything, but that there is another thing d . Then that d is not identical with a, b , or c is a necessary fact; therefore it is necessary that there is an x such that x is not identical with a, b , or c , or that a, b, c are not the only things in the world."

The passage is much too coded. I reconstruct from it two separate arguments: one for the claim that there could not have been fewer individuals than there are, the other for the claim that there could not have been *more* individuals.

For the first argument assume the *actual* existence of four individuals: a, b, c, d . Let φ be: $\varphi x \leftrightarrow ((x \neq a) \wedge (x \neq b) \wedge (x \neq c))$. We then have:

1. $\Box\varphi d$ (by necessity of distinctness).
2. $\varphi d \supset \exists x\varphi x$ (logical truth).
3. $\Box(\varphi d \supset \exists x\varphi x)$ (necessitation).
4. $\Box(\varphi d \supset \exists x\varphi x) \supset (\Box\varphi d \supset \Box\exists x\varphi x)$ (Modal logic).
5. $\Box\exists x\varphi x$ (1, 3, 4, *Modus ponens*).

Now I, of course, think Ramsey made the fallacious step of moving from the logical truth of a claim to its necessity (step 3). But on top of this, he seems to be involved here in another *faux pas*. Let Ramsey have $\Box\exists x\varphi x$. He reads it: "of necessity, there are at least four individuals". I ask: Why? All " $\Box\exists x\varphi x$ " says is that "of necessity, there exists an individual (e.g., d) that is distinct from a, b, c ". That may be true and yet other worlds would have less than four individuals (e.g., they would only have d). From the *truth* of a proposition like " d is distinct from a " at a counterfactual world w , Ramsey infers (fallaciously) the *existence* of d and a in that world. Though there cannot be *actual* truth without existence, counterfactual truth without (local) existence is fine. (cf. *Quine doesn't exist* in a Quineless world.)

The second argument I reconstruct from Ramsey proceeds as follows. Assume there are three individuals: a, b, c . Now grant that there *could* have been a 'new' individual. The argument now runs as:

- (- 1) $\Diamond\exists x\varphi x$ (by assumption)
- (0) $\Diamond\varphi d$ (by what?)
- (1) $\Box\varphi d$ (by necessity of distinctness)
- .
- .
- (5) $\Box\exists x\varphi x$ (as before)
- (6) $\exists x\varphi x$ ($\Box p \supset p$, Modal logic T)

Thus, by (6), the fourth individual, d , already exists in actuality. The procedure can be repeated for any imagined 'new' individual. Thus there can be no 'growing' domains. Note that here Ramsey doesn't merely commit the fallacy of necessitation. He also moves from $\Diamond \exists x \phi x$, a general possibility, to $\Diamond \phi d$, a singular one. He apparently thinks that just because we can evaluate propositions involving actuals in worlds where they don't exist, by symmetry, we have singular propositions involving merely possibles, i.e., d , evaluated here.

¹⁰ For we would identify $\exists x \exists y (x \neq y)$ with the following disjunction involving all pairs of actual individuals: ((Church, Quine), Distinctness) OR ((Church, Church), Distinctness). OR ((a, b), Distinctness), OR . . . By our own, "modern", lights the first disjunct is a truth in virtue of logical structure. But then so is the whole disjunction. And hence so is $\exists x \exists y (x \neq y)$.

¹¹ For Ramsey's view that general propositions are *identical* with complex singular propositions, see, *op. cit.*, pp. 34, 41 and especially 59 – 61. Now suppose that there are infinitely many individuals in the actual world. The propositions *Dis(F)* and *Conj(F)* would then involve infinitary disjunctions and conjunctions. To some this may seem unacceptable. I feel no special justification is called for this assumption in the present work if only because the assumption is made in the process of criticizing Ramsey who was *the* one to stress, thirty years before such operations became common, that infinitary logical operations are absolutely innocuous (*op. cit.*, 34, 41 and 74). I also note, and that goes beyond any discussion of Ramsey, that our operations apply to abstract structures, i.e. propositions, not to sentences. Whence the common dismay, justified or not, about "infinitely long sentences" may seem quite inapplicable.

¹² Some may agree that the above argument is fine but nonetheless insist that *At least two individuals exist* is not a logical truth. They would point out that the proposition in question is not true in virtue of *its own* internal structure and *because* of that it is no logical truth.

Regarding the general question, I have already argued that, on my view, a truth is made into a logical one by special traits of that which *makes* it true in the first place, i.e. the kind of trait of the *world* in virtue of which it is true. That is why I classify the truth *At least two individuals exist* as logical. Similarly, the question whether *modus ponens*, with the help of which we deduced *At least two individuals exist* from *John exists* and *Mary exists*, preserves logical truth should not be confused with the question whether it preserves truth-in-virtue-of-internal-propositional-structure.

Now, I also happen to believe that this proposition is true in virtue of its own structure. I take my lead from the *fact* that in order to say that there are at least two individuals, we need to appeal to two separate quantifiers, binding two different variables. (This, of course, would be trivially true on the exclusive interpretation of quantifiers that Wittgenstein seemed to have sympathy for in the *Tractatus*. But my point is that this is true for the normal inclusive interpretation as well.) Of course, I do not pretend to comment here on a whole range of delicate results in definability theory; nor do I pretend to cover the option of avoiding $\exists x \exists y (x \neq y)$ altogether and letting "two" designate directly a number, a second-order property or a set of sets. I simply note that *if* our proposition has the structure of the standard first-order coding, the use of *two* quantifiers, and more important, the use of *two* variables, is inescapable. The question is: why? Why should a quantified proposition bear a trace, not to say a scar, of the kind of singular propositions which would support its truth? Isn't the quantified proposition supposed to be *purely* general?

For me, this much is an indication that our standard assertion that the existence of the form $\exists x\exists y (x \neq y)$ "goes without saying", is very questionable. For it seems that the form codes into itself the kind of singular trait that would make it true. In fact, it seems to present in a schematic way the kind of singular trait from which it was abstracted in the first place. As such, I don't think of it as a *form*, at least not a "platonic form". I think of it as a *kind* of proposition, a kind abstracted from particular instances, a point that would have been clearer in the notation

$$\sum_{i,j} \langle x_i \neq x_j \rangle,$$

where it could be *seen* that to get the propositional form (kind) of the quantified proposition, we *operate* on already given *singular* propositions.

¹³ This is surely the case if we treat *Actually* in the way I think we should, viz. as attributing to propositions, just like *Necessarily* and *Possibly*, an irreducibly modal property (*Being actually (necessarily, possibly) true*). But even if we followed Lewis and (sometimes) Kaplan and let it introduce the world itself into the proposition, the result would still hold. For given that propositions are generable from actual building blocks only (an assumption held dear by Russell and Ramsey), the actual world is the only world that can figure in this way in propositions. Hence if a given proposition (e.g., *snow is white*) bears *true-at- W_0* , pure analysis would suffice to show that *P* is true *tout court*. And yet, the overall proposition *if snow is actually white then snow is white* is surely contingent, as witnessed by a counterfactual situation with red snow.

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