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Poisson-distributed patterns of explosive eruptive activity

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Abstract. The study of patterns of eruption occurrence could lead to a better understanding of the physics behind the volcanic process. However, various attempts to find a single statistical distribution that describes the occurrences of volcanic eruptions have not been successful. Global data show that, if the energies of point events in time (eruptions) are properly accounted above a certain "noise level", the stochastic process - whose realization consists of explosive volcanic events - can be well represened by a Poisson point process, though not necessarily stationary. Many previous attempts to describe patterns of eruption occurrences were hampered by counting events with all levels of explosivity in the same category. When eruptions are separated by their sizes, the occurrence patterns of the higher magnitude eruptions become clearly Poissonian. In this study eruptions are classified by size using the Volcanic Explosivity Index (Newhall and Self 1982). Further analysis of the magnitude-characterized eruption data shows direct relations among the energy of eruptions, mean rate of occurrences and distribution of repose intervals between eruptions. An important result from the analysis of energy and mean rate of occurrence data is that, for global data, the product of those parameters is a constant. Simple load-and-discharge models provide an explanation of the random features of the volcanic processes. These considerations lead to the definition of a constinuous magnitude scale for volcanic eruptions which can consistently measure the energy and the rate-of-occurrence of eruptions over a wide range of values.

Introduction

Several attempts have been made to find periodicities, or at least some regularities, in the patterns of occurrence of eruptions in volcanoes. For instance, Wickman (1966, 1976) used stochastic principles to build up Markovian models. Such models provided a useful tool for handling the statistics of volcanic repose periods through renewal theory. The eruptive history of a volcano was expressed by two equivalent parameters: the survivor function and the age-specific eruption rate. However those models were not tested against observed records, as Wickman himself stated in his 1976 paper. Furthermore, no single survivor function fits all volcanoes when eruption data are indiscriminately used.

Certain volcanoes seem to have some periodicities. For example Björnsson et al. (1977) report several hundred years' recurrence for the Krafla region, and the last four eruptions of El Chichón appear to have a recurrence interval of about 600 years (Tilling et al. 1984). Others show random patterns, for instance the Hawaiian volcanoes (Klein 1982), or Stromboli where, over the short time scale, the eruption-triggering mechanism appears to be quite random (Settle and McGetchin 1980). Nevertheless, the vast majority of volcanoes show rather complex patterns of behavior, often departing from simple (e.g. Poissonian) models of temporal distributions of occurrences (or reposes). For instance, Reyment (1969) found that although the patterns of activity of some volcanoes approximated a Poisson model, others departed greatly from such behavior.

The present paper deals with a methodology to identify the statistical pattern of behavior that describes the temporal occurrence of eruptions. The observed patterns of eruptive activity at the global scale are then discussed.

The size of eruptions

If the eruptive events are classified by size or bigness, it becomes evident that the global statistics fit a Poisson distribution. This is so if only relatively large eruptions are considered. When minor eruptions are also introduced in the data, bimodal or multimodal patterns of the observed distributions appear (see, for instance, Fig. 2).

The problem of finding a suitable way of measuring the size of volcanic activity arises here. There are several recent attempts at quantifying the size of eruptions. Walker (1980) has proposed that five parameters are necessary to characterize the scale of explosive eruptions: *magnitude* is the total mass of ejecta; *intensity* is the rate at which magma is discharged; *dispersive power,* related to column height and therefore to intensity, is the area over which products are spread; *violence* is a measure of the kinetic energy released during explosions; and *destructive potential* is a measure of the extent of devastation.

To quantify magnitude, the Tsuya (1955) scale, which measures the volumes of different types of volcanic ejecta, has been used. Yokoyama (1957) suggested extending Tsuya's classification by volume to one by energy released by volcanic eruptions, taking into account the direct proportionality between total mass and energy. Hédervári (1963) defined an energy magnitude scale, taking into account the different forms of energy release during eruptions, and applied the same relation used in seismology to express the energy magnitude as a function of the logarithm of released energy (in ergs). Then he found a good linear correlation between the logarithm of the volume of ejecta and such an energy magnitude scale.

Similarly, several methods for the determination of intensity have been proposed, mostly from observations of the maximum eruptive column heights, the bending of eruptive plumes by cross winds, and the dispersal of pyroclastics (Settle 1978; Fedotov 1985; Carey and Sigurdsson 1989).

In 1982, Newhall and Self defined the Volcanic Explosivity Index (VEI) scale defined in terms of a composite estimate of magnitude and/or intensity and/or some estimate of the other parameters defined by Walker, depending on which data are available. One of the most significant of these parameters (for the quantification of the size of eruptions) is the volume of the ejecta, making this scale highly coincidental with the Tsuya scale. As a matter of fact, it is possible to relate the VEI scale with Tsuya's through the relation $M_{VEI} = M(\text{tsuya}) - 2$, except for the lowermost values. The other most significant parameter is the intensity estimated from eruptive column heights. An important result pointed out by Carey and Sigurdsson (1989), is that if the total erupted mass is considered, there is a positive correlation between magnitude and intensity. Therefore if the classification of some past eruptions is based on some type of intensity data, the relation with the total ejected mass is not lost.

To summarize, the size or bigness of an eruption may be measured in terms of different scales of magnitude and intensity: a mass magnitude scale (Walker 1980); a volume magnitude scale (Tsuya 1955); an energy magnitude scale (Yokoyama 1957; Hédervári 1963); magma discharge rate intensity scales, measured from lithic and pyroclastic dispersal patterns (Carey and Sigurdsson 1989), or from the height and shape of volcanic eruption clouds and plumes (Settle 1978; Fedotov 1985); and a VEI composite scale based on any of the available above parameters (Newhall and Self 1982).

The present paper attempts to analyze the behavior of global eruption patterns using available VEI data, and to use the results to define an energy magnitude scale, compatible with the VEI, but based on the global statistical patterns of eruption occurrences.

Size-rate of occurrence relationships for global data

Newhall and Self's (1982) table of numbers of reported eruptions describe the best composite estimate of the VEI magnitude of past eruptions worldwide. The table, reproduced here as Table 1, with the addition of the number of eruptions having VEI ≥ 4, and the totals for each category, covers the period 1500 to 1970. As the authors discuss, the estimate of VEI values is subject to a variety of difficulties which increase with the age of the reported activity. The difficulties become more serious when lower magnitudes are considered. The error in magnitude estimation for a minor eruption that occurred in, say, the sixteenth century is probably much larger than that of a paroxysmal event at about the same time and, consistently, an explosion reported at that time in a given place is likely to have been stronger than an explosion reported from the same place today. To compensate for this problem, Newhall and Self "corrected" their catalogued VEI values in the range 1 to 4 which occurred prior to a certain date, by upgrading them by one VEI unit. These dates vary from 1500 to 1700, depending on the region considered. When the available information about a given eruption is poor or doubtful, a default value of VEI $= 2$ is given in the catalogue, and an artificial abundance of this magnitude can thus be expected.

A simple statistical analysis of the data listed in Table 1 has been done as follows. First, the number of events (or absences of events), x, in a given VEI category are counted for each decade of the 1500–1970 sample, and the distribution of observed occurrences (using 0 for no occurrences) is represented in a histogram form. Then a Poisson distribution of the form:

$$p(x) = \lambda^x e^{-\lambda}/x! \qquad (1)$$

is obtained from the mean λ, calculated as the total number of eruptions which occurred in the given VEI category, divided by the total length of time of the studied period, and the resulting values are compared with the observed distribution. A Chi-squared goodness of fit test is then performed on the hypothesis that the observed distribution is truly Poissonian.

Table 2 shows under "OBS", the number of times that $x = 0, 1, 2, \ldots$ up to 9 eruptions per decade with VEI $= 4$ have occurred during the $N = 48$ samples, covering the period since 1500. The column $p(x)$ shows the Poisson probability calculated using a mean $\lambda = 106/48$, (since a total of 106 VEI magnitude-4 eruptions occurred during the whole sampled period). The column headed "CALC" contains the expected number of occurrences $Np(x)$.

Tables 3, 4, and 5 show the corresponding results for global data of eruptions with VEI $= 5$, VEI $= 6$, and VEI ≥ 4, respectively. Figure 1 summarizes the results in a graphic form.

Table 1. Numbers of reported eruptions by VEI category, per decade, from 1500 to 1970 (from Newhall and Self 1982, copyright the American Geophysical Union)

Decade	VEI 0	1	2	3	4	5	6	7	8	≥4
1500	0	10	12	1	0	1	0	0	0	1
1510	0	10	14	4	1	0	0	0	0	1
1520	1	10	26	5	1	0	0	0	0	1
1530	10	12	23	8	1	0	0	0	0	1
1540	11	11	7	6	0	0	0	0	0	0
1550	10	12	7	7	0	0	0	0	0	0
1560	10	21	9	11	1	0	0	0	0	1
1570	2	22	7	7	1	0	0	0	0	1
1580	1	21	18	13	1	0	0	0	0	1
1590	0	20	20	10	3	0	0	0	0	3
1600	0	23	16	7	1	0	0	0	0	1
1610	0	27	11	8	0	0	0	0	0	0
1620	0	26	4	5	0	0	0	0	0	0
1630	1	25	11	8	2	0	0	0	0	2
1640	0	28	23	14	2	1	0	0	0	3
1650	2	26	18	14	0	0	0	0	0	0
1660	0	20	21	15	2	2	0	0	0	4
1670	2	20	47	12	2	0	0	0	0	2
1680	1	22	35	15	2	0	0	0	0	2
1690	0	20	32	20	2	0	0	0	0	2
1700	5	20	29	5	1	1	0	0	0	2
1710	1	24	26	9	1	0	0	0	0	1
1720	5	23	42	7	2	0	0	0	0	2
1730	4	21	42	13	0	1	0	0	0	1
1740	1	20	47	2	2	0	0	0	0	2
1750	3	20	74	2	1	1	0	0	0	2
1760	5	12	96	6	5	0	0	0	0	5
1770	6	8	108	9	2	0	0	0	0	2
1780	7	11	111	6	4	0	0	0	0	4
1790	16	12	105	7	3	0	0	0	0	3
1800	4	18	122	5	1	0	0	0	0	1
1810	3	23	129	8	4	0	0	1	0	5
1820	12	22	154	11	2	1	0	0	0	3
1830	17	13	172	11	0	1	0	0	0	1
1840	23	13	182	11	2	0	0	0	0	2
1850	27	12	213	17	2	1	0	0	0	3
1860	30	14	207	10	1	0	0	0	0	1
1870	33	23	185	13	4	1	0	0	0	5
1880	38	29	225	18	5	1	1	0	0	7
1890	19	42	227	12	1	0	0	0	0	1
1900	34	36	306	21	8	1	1	0	0	10
1910	26	35	231	19	7	0	1	0	0	8
1920	32	63	299	13	3	0	0	0	0	3
1930	31	46	256	23	3	1	0	0	0	4
1940	25	37	261	30	3	0	0	0	0	3
1950	51	76	345	40	5	1	0	0	0	6
1960	69	76	287	81	7	0	0	0	0	7
1970	104	110	304	67	5	0	0	0	0	5
Totals	682	1245	5146	666	106	15	3	1	0	125

Regardless of the "goodness of fit analysis" discussed below, it is clear that the character of the distributions of the numbers of eruptions per sampling period is indeed approximately Poissonian for each VEI category between 4 and 6, as it is for the cumulated data for VEI≥4.

In contrast, the corresponding observed distributions for VEI values 0 to 3 do not seem to follow any clear pattern. To illustrate this, the global data for

Table 2. Observed recurrences and Poisson expectations for global data (from Table 1) with VEI = 4.
$N=48$, $\Delta t=10$ years, $\lambda=106/48$

x	$p(x)$	CALC	OBS
0	0.10988	5.27	8
1	0.24266	11.65	13
2	0.26794	12.86	12
3	0.19723	9.47	5
4	0.10889	5.23	3
5	0.04809	2.31	4
6	0.01770	0.85	0
7	0.00558	0.27	2
8	0.00154	0.07	1
9	0.00038	0.02	0

Table 3. Observed recurrences and Poisson expectations for global data (from Table 1) with VEI = 5
$N=48$, $\Delta t=10$ years, $\lambda=15/48$

x	$p(x)$	CALC	OBS
0	0.73162	35.12	34
1	0.22863	10.97	13
2	0.03572	1.71	1
3	0.00372	0.18	0

Table 4. Observed recurrences and Poisson expectations for global data (from Table 1) with VEI = 6.
$N=48$, $\Delta t=10$ years, $\lambda=3/48$

x	$p(x)$	CALC	OBS
0	0.93941	45.09	45
1	0.05871	2.82	3
2	0.00183	0.09	0

Table 5. Observed recurrences and Poisson expectations for global data (from Table 1) with VEI≥4.
$N=48$, $\Delta t=10$ years, $\lambda=125/48$

x	$p(x)$	CALC	OBS
0	0.07396	3.55	5
1	0.19262	9.25	14
2	0.25080	12.04	10
3	0.21771	10.45	7
4	0.14174	6.80	3
5	0.07382	3.54	4
6	0.03204	1.54	1
7	0.01192	0.57	2
8	0.00388	0.19	1
9	0.00112	0.05	0
10	0.00029	0.01	1
11	0.00007	0.00	0

VEI = 3 in the period 1500 to 1940 and the Poisson distribution calculated using $\lambda=10.622$, (corresponding to the 478 eruptions with that Index reported in the 45 decades of that period) are shown in Fig. 2. The last three decades (1950–1970) have been omitted from the graph since they contain very high values for the number of

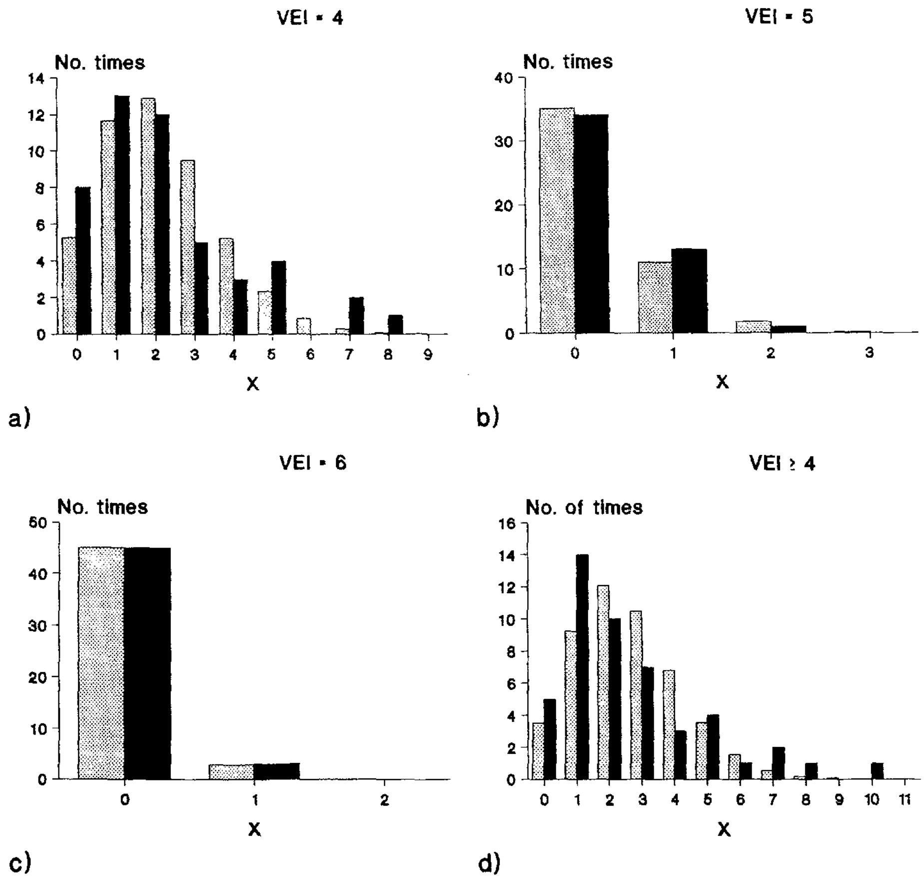


Fig. 1. Observed (■) (from Table 1) and expected (□) (Poisson) distributions of number of global eruption occurrences per decade in each VEI category, for the period 1500–1970

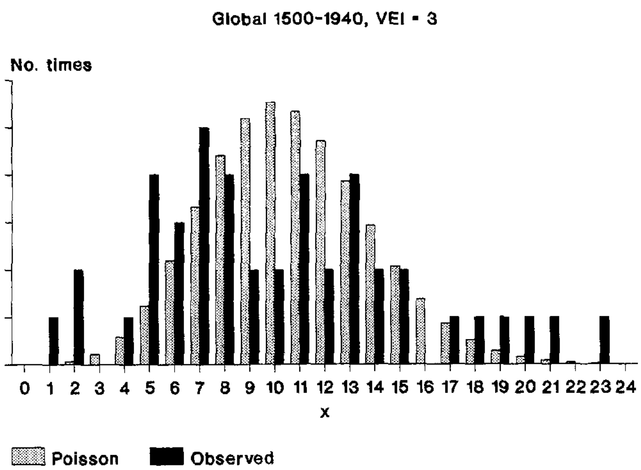


Fig. 2. Observed and expected distributions of the number of global eruption occurrences per decade in the VEI = 3 category, for the period 1500–1940

reported eruptions. It does not signify whether these high values result from incorrect sampling or another effect: their exclusion is irrelevant to the argument discussed here because their inclusion would only produce a further departure from the Poissonian curve.

The analysis for reported VEI values 0 to 2 produces similar failures when attempts are made to fit their distributions to simple Poisson patterns. However, the way in which the reported data depart from the calculated values shows some interesting features and emphasizes the difficulties in sampling low magnitudes. For instance, for the lowest VEI values (0 to 2) there is an excess of high values of the number of occurrences with respect to any expectation from the Poissonian models. However, for VEI = 3, there is also an excess of low numbers of occurrences compared with the predictions of the model.

The first excess is probably due to the combined effects of repeated reporting of low-to-null explosivity volcanic activities which are not well defined as single events in time, and the method of default assignment of Index 2 to undefined activity. The second is probably a result of lack of consistent reporting of VEI = 3 eruptions. If, during several decades, there is a failure in complete reporting of this type of eruptions, and instead only a few are reported, the data reflect an excess of decades in which only a small event number is reported at the expense of decades in which higher values should be expected. These sampling difficulties represent the main problem experienced in finding distribution patterns for the low VEI categories. However, it should be noted that, although this sampling difficulty may affect the overall fit between the reported event occurrences and the Poisson curve, the total number of events reported through the whole period may not be significantly affected (the areas of the hatched and the black bars of Fig. 2 are equal).

One of the reasons for the appearance of the abovementioned complex patterns of activity, when minor volcanic activity is considered, may be immediately attributed to the difficulties in quantifying minor volcanic activity, rather than to a statistically different behavior of the less explosive eruptions. The nature of this problem becomes clear when one looks closely at the way in which the Volcanic Explosivity Index VEI (Newhall and Self 1982), or any other scales are assigned to small events, particularly when deposits are insufficient to permit a detailed geological study, and only eyewitness reports are available. For small events, the subjective nature of eyewitness reports becomes important, and separation of individual events may be much more difficult due to the less-defined nature of non-explosive episodes as point events, particularly if they occurred long ago. In contrast, larger events are well defined in time and are probably witnessed by more people. Reports tend to converge toward increasingly accurate descriptions. Furthermore, some of the main features characterizing the intermediate and high VEI ratings (e.g. definite stratospheric injection normally associated with VEI > 3) produce phenomena that are easily recognized even in ambiguous reports, such as the total blocking of sunlight over large areas.

If one concludes that well-sampled moderate-to-large magnitude eruption sequences follow a Poisson distribution, then the basic features of Poissonian processes become fundamental in understanding the physics of volcanism. The analysis of published global data supports the notion that the occurrence of eruptions can be accurately described as a simple Poisson process.

Thus to test the reported data of patterns of volcanic activity against any statistical distribution, it is of great importance to have accurate estimates of the sizes of such episodes of activity. A reasonable assumption is that all eruption sizes in which the volcanic activity events can still be regarded as point events in time could be Poissonian, if properly sampled.

The global data show another interesting feature. If the values of $\log(\lambda)$ are plotted *versus* the VEI values greater or equal than 3 (from Table 1), an almost perfect linear fit is obtained. Figure 3 shows the line:

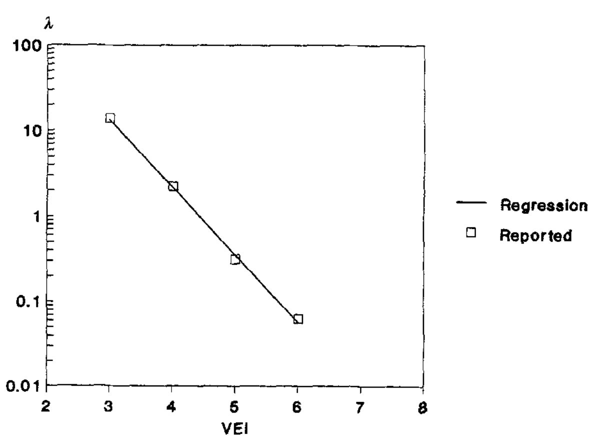


Fig. 3. Linear dependence of the logarithm of the Poisson parameter on the higher VEI magnitudes. The *squares* correspond to λ values obtained from Table 1

$$\log(\lambda) = a - bMv \tag{2}$$

where Mv is the VEI value. For $a = 3.494$ and $b = 0.789$, a linear correlation coefficient of -0.999 is obtained. Recently, McClelland et al. (1989) published similar plots for two different periods (last 200 years and 1975–1985). The best fit lines they found have essentially the same slope as (2), and only the independent term is somewhat different, depending on the size of the sampled period. The independent term value corresponding to the largest sample (48 decades) is therefore kept here.

This suggests the way in which volcanic energy is released on a global scale: the number of eruptions per unit time decays by a factor of about 6 for each successive VEI category. In addition, if one accepts that the above relation could be valid over the range of higher magnitudes, it is possible to estimate the rate of occurrence of very large eruptions and draw some conclusions regarding the hazard associated with such events, as discussed below.

Random models of eruption occurrences

The Poissonian behavior of explosive eruption occurrences at the global scale is not a surprising result, for one may expect that the eruptions of very different volcanoes are completely unrelated events. However, it seems that this is not the only reason for the random nature of global volcanism; a single volcano, if properly sampled, may also show a similar type of Poissionian behavior.

Contrary to a somewhat intuitive argument, the "memoryless" character inherent in this type of stochastic process (at least over periods greater than the sampling interval), is applicable only to major activity, involving a significant release of mass and energy by the volcanoes. One would expect that a given volcano should keep some memory of its previous large episodes of activity, but may perhaps "forget" minor events. Evidence supports the opposite. Such intuition may be derived from an incorrect interpretation of the load-and-discharge models, in which a system is loaded at a given rate until it reaches a critical, unstable condition and releases its load, returning to a certain lower level, starting the process again. Scandone (1981) considers a volcano as a thermodynamic system in dynamic equilibrium with its environment. That system will erupt when its energy level exceeds a certain stability threshold. Then a small fraction of the thermal energy of the eruption is transformed into work, which is the portion of the energy that controls the dynamics of the eruption.

This type of load-and-discharge model has been used successfully in the description of earthquake sequences in different parts of the world (Bufe et al. 1977; Shimazaki and Nakata 1980; Savage and Cockerham 1987). The global behavior of seismic time series is definitively non-Poissonian (Smalley et al. 1987; Nishenko and Buland 1987). However, Lomnitz (1988) re-

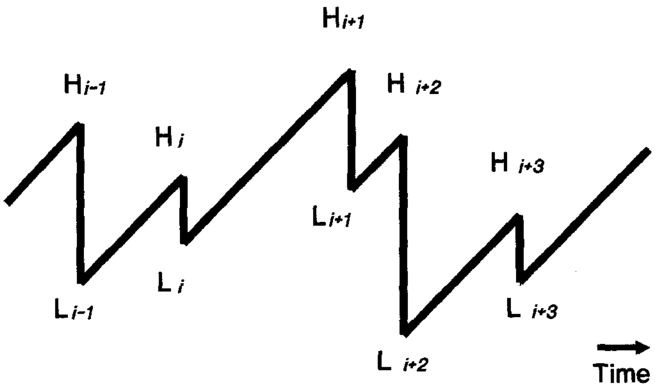


Fig. 4. A load-and-discharge eruption recurrence model with random threshold energies, H_i, and random final energies L_i

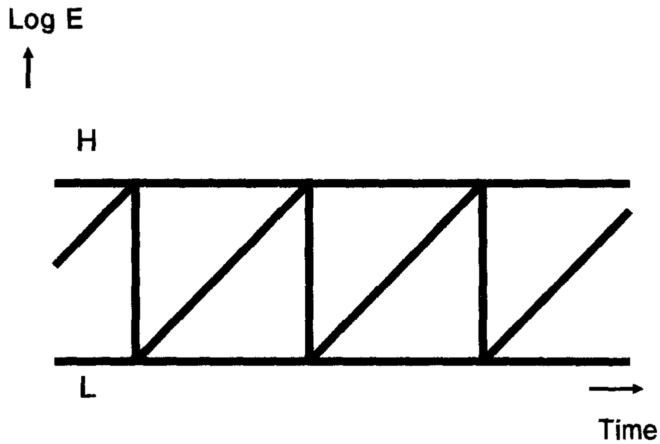


Fig. 5. A strictly periodical load-and-discharge model with constant threshold (H) and final (L) energies

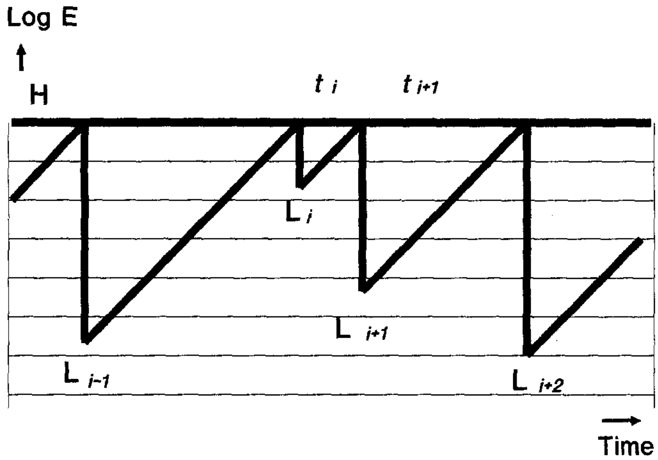


Fig. 6. A load-and-discharge model with constant threshold energy (H) and random final energies L_i. The horizontal lines represent the magnitude levels reached by the the eruption energy

cognizes a difference in the behavior of large and small earthquakes other than the scale. Therefore, some consideration regarding the possible applications of these types of load-and-discharge models to the volcanic eruptions sequences are discussed here. At this point it is convenient to review the Shimazaki and Nakata (1980) and Savage and Cockerham (1987) approach to the recurrence analysis of earthquakes, adapted to volcanoes.

An assumption can be made that the energy level (understood as the logarithm of the thermal energy of the magma reservoir) of a given volcano during a repose period increases at a constant rate σ, until it reaches a critical level H. At this point an eruption occurs and the energy of the system abruptly drops to a level L, and the process starts another cycle. The difference between the initial and the final energy levels is proportional to the logarithm of the energy released by the eruption and thus to its magnitude as discussed in the next section. This process may develop in four possible ways:

1. The initial and final energy levels are random functions of time (Fig. 4). If we denote $\xi = H - L$ the energy drop in an eruption, we have for the ith event:

$$H_i = H_1 - \sum_{j=1}^{i-1} \xi_j + \sigma(T_i - T_1)$$

thus, the time for that event will be:

$$T_i = T_1 + \left(H_i - H_1 + \sum_{j=1}^{i-1} \xi j\right)/\sigma \qquad (3)$$

and the interval between successive events:

$$t_i = T_{i+1} - T_i = (H_{i+1} - H_i + \xi_i)/\sigma \qquad (4)$$

The subsequent interval is then:

$$t_{i+1} = (H_{i+2} - H_{i+1} + \xi_{i+1})/\sigma \qquad (4')$$

In this case the repose intervals are not independent since they are correlated through the common term H_{i+1}/σ. This would produce a tendency to have short intervals followed by long ones and *vice versa*.

2. At the other extreme, the initial and final energy levels are constant and thus time-independent. Then the sequence of eruptions would represent a strictly periodic process (Fig. 5). This is contrary to most observations. However, certain volcanic systems may follow this behavior at least for limited periods.

3. Here the threshold energy H is constant, while the final energy is a random function of time (Fig. 6); then $H_i = H =$ constant, and from (3):

$$T_i = T_1 + \left(\sum_{j=1}^{i-1} \xi_j\right)/\sigma$$

Then the intervals between events:

$$t_i = \xi_i/\sigma \qquad (5)$$

become uncorrelated and depend only on the final energy state of the ith event. This type of model is called time-predictable, for the greater the energy drop, the larger is the "recharge" time.

4. Here the final energy level L is constant, while the threshold energy H is a random function of time. Then $H_i - \xi_i = L$, a constant (Fig. 7), and from (4), the intervals between eruptions:

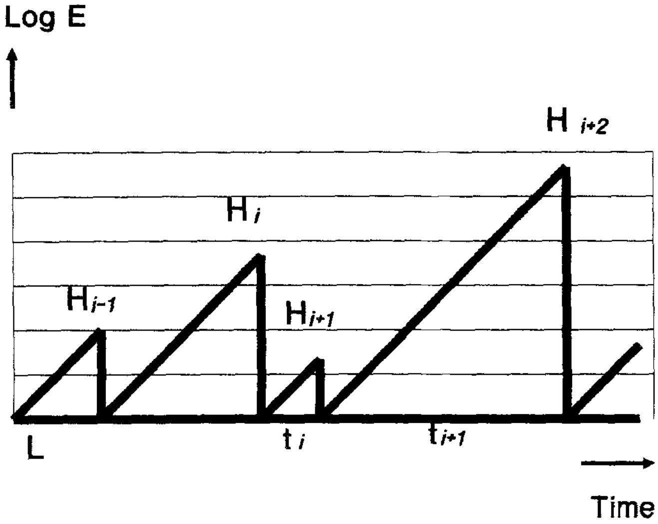


Fig. 7. A load-and-discharge model with random threshold energies, H_i, and constant final energy (ground state) L

$$t_i = (H_{i+1} - L)/\sigma \quad (6)$$

are also uncorrelated; their distribution depending solely upon the threshold energies distribution. In this situation, the longer the repose period, the larger is the energy release of the eruption that follows.

Model (1) appears to be very flexible, since the random character of the energy levels allows one to describe many possible systems. However, the successive time intervals between events in this case are correlated as shown in relations (4) and (4′), contrary to that observed in the Poissonian sequence of random events. Model (2) may be applicable to some apparently periodic volcanic systems, but certainly not to most active volcanoes as can be inferred from the global data discussed above, and from a particular case described elsewhere.

Models (3) and (4) have distributions of uncorrelated repose intervals which depend solely on the distributions of the final energy and the threshold energy respectively, as can be seen from relations (5) and (6). Either of these models could explain the observed Poissonian sequence of eruption occurrences if one makes the following considerations. Relations (5) and (6) clearly show a direct relation between the repose intervals and the energy release of events. On the other hand, global data show that the mean rate of eruption occurrences λ have a statistical tendency to decrease exponentially with size as expressed by relation (2). Thus, ideally, the probability that a given eruption reaches a certain energy level decreases exponentially as the level increases. Therefore, large events are that much less likely than small ones, and the time intervals between successive events are thus exponentially distributed with some overall mean rate, as are the events within any higher energy category with their corresponding (exponentially lower) rates.

This is equivalent to saying that if a given measure of the energy release or energy magnitude has a given mean rate of occurrence λ, the next higher energy magnitude will occur at a rate of about $10^{-b}\lambda$, where b is defined in relation (2), thus having a much less dense exponential distribution of repose times.

The Poissonian character of processes (3) or (4) could be questioned under the assertion that causality may be involved between the lengths of the repose intervals and the energy release of the eruptions that follow (case 3) or precede (case 4) them. This apparent difficulty disappears when eruptions and repose periods are separated as different sets or categories of realizations. The random flow of events, whose realizations are the eruptions, is a Poisson process, and their energy magnitudes are independent. On the other hand, the random set of repose periods is exponentially distributed. Although there may exist causal relationships among elements of both sets, there are not any among elements of the same set.

Summarizing, there are two plausible load-and-discharge models which may generate exponentially distributed repose interval sequences of eruptions, and thus follow a Poissonian pattern of occurrences. A new question now arises. Which of these models is more plausible? On physical grounds, the model (3) represents a system in which the threshold energy, at which all explosive eruptions begin, is a fixed parameter and the final energy after each eruption is a random variable following an exponential distribution. The model (4) represents a system in which the ground energy state after any eruption is a fixed parameter and the threshold energies are described by a random variable following the same type of exponential distribution. In both cases the energy drop is a measure of the size (VEI or any other, involving mechanical and thermal components), as discussed below.

At this point it seems reasonable to suggest that central volcanoes may follow a pattern resembling model (4), while the generation of monogenetic volcanic fields may behave more like model (3). To support these ideas, one may conceive of a central volcano, in a somewhat oversimplified way, as a magma reservoir fed from below at a constant rate (Wadge 1982) and erupting when a threshold level is reached. But this critical level is set by many independent external and internal factors, like conduit conditions, presence of domes, regional stresses, tectonic earthquakes, hydrogeological conditions, geochemical fluctuations of magma, etc. Thus the threshold level behaves as a random variable. However, for any threshold level, the corresponding eruption returns the system to its ground state, conceivably constant (at least over periods of time involving several eruptions).

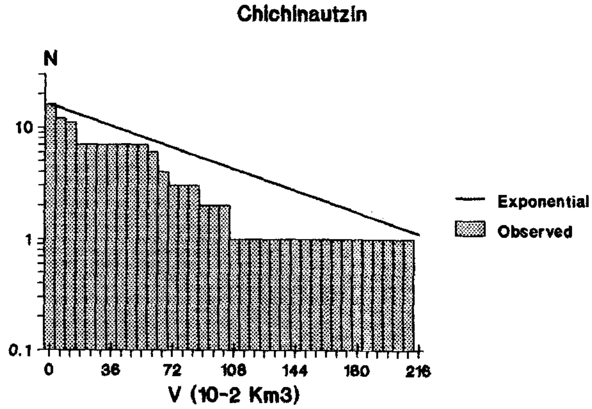
In the monogenetic volcanic field case it is reasonable to visualize a larger (with a horizontal extension comparable to the area of the field), perhaps deeper, magma reservoir, also fed at a constant rate from the depths. The threshold energy required to open the magma path through the crust and give birth to each monogenetic volcano is, in this case, roughly the same, since environmental conditions of the crust above the magma reservoir are relatively stationary compared with the changes in the reservoir itself during the eruption. The final energy level of such a system is thus a random variable of type (3) with a probability of occurrence decaying exponentially with the energy of the eruption. This is probably why small monogenetic volcanoes are more abundant than large ones.

Testing this idea is somewhat difficult due to the scarcity of information on monogenetic volcanism. However, according to the model, both the distribution of repose periods between volcano births, and the distribution of energies of monogenetic eruptions should be exponential. Since in monogenetic eruptions the kinetic energy component is usually very small compared with the thermal one, and the temperatures of different lava flows comparable, the energies of eruptions can be considered as directly proportional to the volume of their magma discharges.

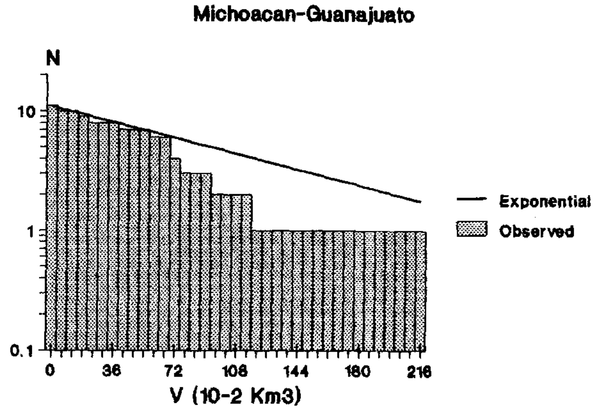
Therefore, to support the use of model (3), two examples of the distribution of lava flows in Quaternary monogenetic fields in central Mexico are briefly discussed. The first, related with the field known as the "Grupo Chichinautzin" formed by nearly 200 volcanoes, was studied in its western end by Bloomfield (1975), and the second, in the Michoacán-Guanajuato area containing over 1000 cones, has been studied by Hasenaka and Carmichael (1985).

Figure 8 shows the reported and expected (exponential) distributions of lava-flow volumes in the mentioned monogenetic fields. In the Chichinautzin case, the volumes of 16 lava flows associated with as many monogenetic cones reported by Bloomfield (1975) are compared with an exponential distribution with a mean of 16/216, since the volumes ranged from 1 to 216 ($\times 10^{-2}$) km^3, and were sampled in 36 intervals of 6×10^{-2} km^3. In the Michoacán-Guanajuato case, identical sampling was used on the cinder cone volumes of 11 volcanoes reported by Hasenaka and Carmichael (1985) (using Wood's (1980) result of a direct linear relationship between the volume of cones and their associated lava flows). Considering the small size of the samples, the fit between the reported and exponential distributions is satisfactory.

It may thus be concluded that both central volcano eruptions and monogenetic field births may be described by similar load-and-discharge models differing only in the random character of their initial or final levels of energy. To allow these models to produce the unlikely, but possible situation of having two or more large eruptions occurring in a row, one should conceive of volcanoes as "big" thermodynamic systems capable of storing amounts of energy much larger than those released by even the largest eruptions. In this way, such systems may "recover" in times as short as necessary to accomplish this situation. This concept is not difficult to support. For instance, isotopic arguments suggest that in the 1982 eruption of El Chichón, less than 3% of the available magma was ejected (De la Cruz-Reyna et al. 1985).



a



b

Fig. 8. a Reported (Bloomfield 1975) and expected (exponential) distributions of the number, N, of lava flows with volumes greater or equal than V ($\times 10^{-2}$ km^3), associated with 16 monogenetic volcanoes in Sierra Chichinautzin, Central México. **b** Reported (Hasenaka and Carmichael 1985) and expected (exponential) distributions of the number, N, of cinder cones with volumes equal or greater than V ($\times 10^{-2}$ km^3), associated with 11 monogenetic volcanoes in the Michoacán-Guanajuato area

Rates of volcanic energy release. The continuous energy magnitude

Yokoyama (1957) pointed out that in volcanic processes, the thermal energy E_{th} represents by far the largest amount of energy available, and calculates such energy for several eruptions from the relation $E_{th} = MC'\Delta\theta$, where M is the ejected mass, C' is the effective heat capacity $C' = (C_p + C_f/\Delta\theta)$ (C_p is the specific heat capacity and C_f the heat of fusion), and $\Delta\theta$ the temperature of ejecta. Other contributions that may become important, like the kinetic energy, also depend directly on the ejected mass. As Carey and Sigurdsson (1989) point out, there is a positive correlation between the intensity of the plinian fall phase and the total erupted mass (mass magnitude). Since VEI is usually calculated from intensity, an empirical relationship between energy released by eruptions and VEI can be determined from the corresponding values assigned to several important eruptions. Table 6 shows those erup-

Table 6. Logarithmic energies of some eruptions (in ergs) as calculated by Yokoyama (1957), and their corresponding VEI magnitudes as estimated by Simkin et al. (1981)

Volcano	Year	log E_t	VEI
Tambora	1815	26.92	7
Sakurazima	1914	25.66	4
Krakatoa	1883	25.0	6
Asama	1783	24.94	4
Fuji	1707	24.85	4
Sakurazima	1946	24.32	2
Mihara	1777	24.0	3
Torishima	1939	23.99	2
Mihara	1950	23.97	2
Komagatake	1929	23.75	4
Miyakeshima	1940	23.68	2
Bandaisan	1888	23.0	4
Guntur	1843	22.81	3
Asama	1935	22.68	3
Pematang Bata	1933	22.65	2
Una Una	1898	22.26	3
Adatarasan	1900	21.81	3
Asama	1938	21.60	3
Azumasan	1893	21.0	2
Mihara	1912	20.80	1
Bezymianny*	1955	25.34	5

* From Macdonald (1972)

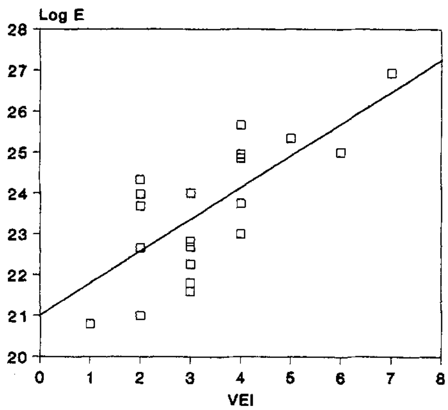


Fig. 9. Correlation between eruption energies (from Table 6) and their VEI values (from Simkin et al. 1981)

tions whose energies have been calculated (Yokoyama 1957) and separately, assigned a VEI value (Simkin et al. 1981). A least squares regression has been calculated for these values (Table 6 and Fig. 9), and the relation:

$$\log E = 0.78\,M_v + 21.02 \tag{7}$$

where E is the eruption energy expressed in ergs, and M_v the VEI value, has been found to fit the data with a correlation coefficient 0.70. A remarkable characteristic of this relation is that, regardless of the units in which the energy is expressed (which affects only the independent term), its slope is nearly orthogonal with that of relation (2), which in turn is independent of the sampling period of eruption occurrences.

A continuous energy magnitude scale for the global rate of occurrence (M_c) is thus proposed here, defined in terms of the following relations:

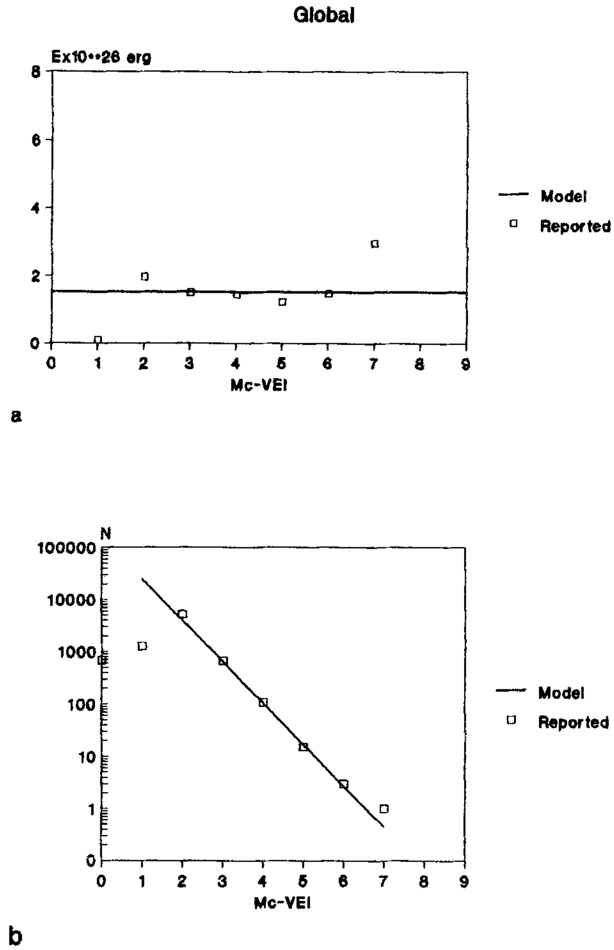


Fig. 10 a, b. Comparison of model and observed parameters. **a** Energy released in each VEI category during the period sampled in Table 1 (*squares*). The *horizontal line* represents the expected energy release in the same period. **b** Expected (*line*) and observed (*squares*) total number of global eruptions for the period reported in Table 1, as a function of magnitude

$$\log(\lambda) = -0.79\,M_c + 3.5 \tag{8}$$

$$\log E = 0.79\,M_c + 21 \tag{9}$$

where λ is expressed in eruptions per decade and E in ergs.

Eliminating M_c in (8) and (9):

$$\log(\lambda) = -\log E + 24.5.$$

from where an interesting general result is obtained:

$$(\lambda_E)\,E = k \tag{10}$$

i.e. the mean rate $(\lambda_E)\,E$ at which volcanic energy (other than mid ocean ridge activity) is being released by the Earth in the energy magnitude category corresponding to the eruption rate λ_E, is a constant which amounts to 3.16×10^{29} erg/year.

To test this result, both the expected energy release of the global volcanic activity and the expected number of eruptions as functions of VEI-energy magnitude are compared (Fig. 10) with the corresponding observed parameters, obtained from the data of Table 1. Clearly the model requires that the same amount of energy per unit time is released for each energy magnitude category (horizontal line in Fig. 10a). The observed data corroborate this for VEI values 3–6; each VEI category releases about the same energy over the sampled period. The VEI categories 0 and 1 are strongly undersampled while the VEI = 2 is somewhat oversampled, as established above. The relatively high value released in the VEI category 7 is a consequence of the occurrence of one eruption (Tambora), with this magnitude during the 48 decade sample period, when the mean global recurrence time for this energy magnitude is, from (8), 1070 years.

Figure 10b shows the expected and observed number of eruptions during the period sampled in Table 1 is a function of VEI-energy magnitude. Following similar arguments it is possible to estimate that the mean recurrence times ($1/\lambda$) for the largest eruptions are 6600 years for $M_c = 8$ and about 40 000 yr for $M_c = 9$, if such magnitudes are possible.

This takes us to the question: which is the highest possible energy magnitude for an eruption? The M_c scale as defined is open-ended; however, the Earth is probably unable to "focus" extremely large amounts of energy in single events and therefore a limit should exist.

The highest eruption energy magnitudes

Establishing the highest possible values for the energy-rate of occurrence magnitude M_c could prove to be a complex task, since it is difficult to assert if very large volcanic deposits are the result of single events or the product of multiple eruptions. Nevertheless, one may look at the reports of some large Quaternary events and estimate their energy magnitudes from the ejected mass published data. For instance, the largest eruption in Japan is probably the one which formed Aira Caldera, 22 000 B.P. (Aramaki 1969). This event produced a DRE (dense rock equivalent) magma volume of about 110 km^3 (Aramaki 1984). This corresponds to $M_c = 8.3$, as calculated from the thermal energy.

In México, several events have reached high energy magnitudes. The Toba Tala rhyolitic eruption of La Primavera, 95 000 B.P., produced nearly 40 km^3, and Los Humeros Xáltipan Ignimbrite, 460 000 B.P., about 115 km^3 (Ferriz and Mahood 1984 and 1986). These eruptions are therefore assigned continuous energy magnitudes of 7.7 and 8.3 respectively.

Under the assumption that the following giant deposits were formed by single catastrophic events, eruptions like the Tshirege member of the Bandelier Tuff, which formed Valles Caldera in the USA, yielding about 300 km^3 of magma (Wohletz et al. 1984), may be assigned $M_c = 8.8$; and the Lava Creek Tuff eruption which formed Yellowstone Caldera, producing about 10^3 km^3 of magma 630 000 B.P. (Christiansen 1984), with $M_c = 9.5$, probably sets the the far end of the energy magnitude scale. It is interesting to note that, from relation (8), such an energy magnitude should have a mean global recurrence time of $\sim 10^5$ years.

Conclusions

An analysis of global volcanic activity shows that the number of eruptions occurring per unit time follows a Poisson distribution. This result is very clear if the eruptions are separated by an appropriate size scale, and the bigger eruptions are considered. When the VEI scale is used, the Poissonian behavior of the uncorrected global data is quite evident for values VEI ≥ 4, but it is not so clear for lesser values. This is attributed to the inherent difficulties in reporting and measuring minor activity.

The Poissonian behavior of global activity is not only a consequence of describing a process involving a large number of volcanoes, considered as independent systems, but it probably also involves the Poissonian character of activity patterns of at least some individual volcanoes, as suggested by the data of Colima volcano, discussed in a separate paper. There the random behavior can be observed over a wide range of magnitudes, as a consequence of improved sampling based on the availability of more and better information.

The use of simple load-and-discharge models helps us to understand the origin of the random character of volcanic eruption sequences. Such models support the concept of volcanoes as thermodynamic systems capable of storing large amounts of thermal energy and releasing small parts of it at a constant rate through a random process of eruption occurrences. Even when this rate changes with time as a consequence of variations in either the rate of loading or the threshold (or ground) energy levels defining different regimes, the Poissonian character of the process persists.

Such thermodynamic systems are in a long-term equilibrium since, on average, the mean energy released by eruptions is probably replaced, as thermal energy, from deeper sources of magma at the same average rate over the life of the volcano. This requires that in the nonstationary models, the random variable representing the regimes has a constant mean over long periods.

The most important result of the present study comes from the analysis of two independent sets of observed data. One is the ensemble of mean rates of global eruption occurrences as a function of the VEI values (described by relations (2) or (8)), and the other is the set of correlated values of energies released in eruptions and their corresponding VEI values (described by relation (9)). The combination of those relations permits one to conclude that the product of the energy of eruptions by their corresponding mean rates of occurrence is a constant. This is a direct consequence of the Poissonian character of eruption occurrences (which requires, according to the load-and-discharge models, an exponential distribution of repose periods and therefore of energy magnitudes) and of the exponential relation between energy and magnitude. A simple relation between energy and mean rate of occurrence is thus obtained. This relation enables an energy-rate of occurrence continuous magnitude scale to be defined. This scale rates the value of the energy released by eruptions, and permits a more precise quantification of the "size" of eruptions in terms of both their energies and their "rarity".

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