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The Rapid Expansion of a Turbulent Boundary Layer in a Supersonic Flow[1,2]

Douglas R. Smith and Alexander J. Smits
Princeton University, Princeton, NJ 08544, U.S.A.

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Abstract. Rapid Distortion Approximations (RDA) may be used to simplify the Reynolds stress equations in rapidly distorted flows, as suggested by Dussauge and Gaviglio (1987). These approximations neglect diffusive and dissipative terms while retaining the production and pressure terms. The retained terms are then modeled as functions of the Reynolds stress tensor and gradients of the mean flow. The models for the pressure–strain term as developed by Lumley (1978) and Shih and Lumley (1985) are evaluated by comparing the calculated results with experimental data for the case of a Mach 2.84 turbulent boundary layer in a 20° centered expansion. The agreement between computed and experimentally obtained Reynolds stresses was found to be encouraging.

1. Introduction

When a turbulent boundary layer in a supersonic flow experiences a rapid expansion the turbulence is strongly distorted under the action of the pressure gradient, in addition to significant additional strain rates due to streamline curvature and dilatation. Conventional calculation techniques cannot be expected to predict such flows very satisfactorily. However, a boundary layer in a supersonic flow can be distorted very rapidly, and it may be possible to use Rapid Distortion Approximations (RDA) to the Reynolds stress equations to predict the evolution of the Reynolds stresses in the boundary layer.

Here, we consider the response of a high Reynolds number turbulent boundary layer with a freestream Mach number of 2.84 to a 20° centered expansion. The experimental results showed that, in a region corresponding to about $3.5\delta_0$ (where δ_0 is the initial boundary layer thickness), the Mach number increased to about 3.63 and the pressure dropped by a factor of about 3.6. RDA were then used to calculate the changes in the Reynolds stress tensor, and comparisons with experiment were shown to be surprisingly good.

The analysis presented here is largely based on two earlier works: Dussauge and Gaviglio (1987), who applied this type of calculation to a boundary layer in a supersonic expansion, and Jayaram *et al.* (1989), who presented a similar calculation for a boundary layer in a supersonic flow subjected to an adverse pressure gradient.

RDA are simplifications based on the concepts of Rapid Distortion Theory (RDT). RDT was first developed by Ribner and Tucker (1952) and Batchelor and Proudman (1954). It is a method for

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calculating the turbulence in a flow subject to a rapid distortion; that is, a distortion which occurs in a time shorter than the typical turbulence time scale. The distortion is assumed to occur too rapidly for the turbulence to interact with itself and so is only affected by the history of the distortion. RDT leads to a linearization of the equations of motion. The effect of the distortion is found by an integration in wave-number space, and the Reynolds stresses are deduced from the resulting Fourier components.

Hunt (1977), Savill (1987), and Dussauge *et al.* (1989) provide excellent summaries of the developments in RDT and its current status. These reviews also highlight the procedure for using RDT ideas to simplify the Reynolds stress transport equations such that they can be integrated to give information about evolution of the Reynolds stresses in a boundary layer (RDA). For compressible flows, Dussauge and Gaviglio (1987) made additional assumptions to simplify the transport equations further and to reduce the number of terms requiring modeling.

RDA is particularly useful in flows characterized by a weak level of turbulence, $u_{\mathrm{rms}}/U_{\mathrm{ref}} \ll 1$, which interacts weakly with itself but strongly with the mean flow, and by a large turbulence Reynolds number, $u_{\mathrm{rms}}L_{\mathrm{d}}/\nu \gg 1$, such that there exists a large disparity between the energy-containing motions at low wave number and the dissipating motions at high wave number. Here, low wave-number energy-containing motions with long time scales dominate the flow dynamics.

To apply RDT and RDA concepts, the distortion must occur over a time much shorter than the characteristic eddy time scale, $T_{\mathrm{d}} \ll T_{\mathrm{t}}$. Clearly, there are regions in a boundary layer where this criterion will not be satisfied. Near the wall, turbulence intensities are high; the characteristic turbulence time scale may be small and on the order of the distortion time. At the edge of the boundary layer in an expansion, the length of the distortion is no longer short and may not be considered "rapid." Nevertheless, in the region $0.2 < y/\delta < 0.8$, the boundary layer may satisfy the necessary criteria and the distortion can be taken to be rapid.

The earlier work by Dussauge and Gaviglio (1987) and Jayaram *et al.* (1989) demonstrated the utility of this approach in supersonic flows. The purpose here is to apply the method in a flow which experiences a substantially larger change in Reynolds stress, that is, a distortion which is stronger and more severe than previously investigated. Furthermore, we attempt to evaluate the usefulness of the pressure-strain models developed by Lumley (1978) and Shih and Lumley (1985) for compressible flows.

The experiment reported here is, as far as the authors are aware, one of few which explore the behavior of a turbulent boundary layer in a supersonic expansion, some of the others being the studies conducted by Morkovin (1955) and Dussauge and Gaviglio (1987).

2. Analysis

This analysis proceeds along the same lines as given by Dussauge and Gaviglio (1987). A more complete presentation of the approximations made here can be found in their paper.

We begin with the Reynolds stress transport equation written in terms of mass-weighted variables (denoted with an overtilde) which takes the form

$$
\begin{aligned}
\frac{D}{Dt}\widetilde{u_i'u_j'} &= -\widetilde{u_i'u_k'}\frac{\partial \tilde{u}_j}{\partial x_k} - \widetilde{u_j'u_k'}\frac{\partial \tilde{u}_i}{\partial x_k} && \text{(I)}\\
&\quad + \frac{\overline{\rho' u_i'}}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x_j} + \frac{\overline{\rho' u_j'}}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x_i} && \text{(II)}\\
&\quad - \frac{1}{\bar{\rho}}\left(\overline{u_i'\frac{\partial p'}{\partial x_j}} + \overline{u_j'\frac{\partial p'}{\partial x_i}}\right) && \text{(III)}\\
&\quad - \frac{1}{\bar{\rho}}\frac{\partial}{\partial x_k}\left(\overline{\rho u_i'u_j'u_k'} - \overline{f_{ik}u_j'} - \overline{f_{jk}u_i'}\right) && \text{(IV)}\\
&\quad - \varepsilon_{ij}, && \text{(V)}
\end{aligned}
\qquad (1)
$$

where $\tilde{u}_i = \overline{\rho u_i}/\bar{\rho}$ is the mass-weighted velocity, $\widetilde{u_i' u_j'} = \overline{\rho u_i' u_j'}/\bar{\rho}$ is the mass-weighted Reynolds stress tensor, $\bar{\rho}$ is the mean density, $\bar{p}$ is the mean pressure, ε_{ij} is the dissipation rate tensor, and f_{ik} is the viscous stress tensor. For weak turbulence, the mass-weighted velocity, $\tilde{u}$, is largely indistinguishable from $\bar{u}$.

Terms (I) and (II) are production terms associated with the interaction of the turbulence with the mean velocity field and mean pressure field, respectively. Using an order-of-magnitude analysis, we can show that in a rapid distortion these terms will dominate over the diffusive and dissipative terms, (IV) and (V).

We begin by estimating the order of the production terms. Through the distortion, the changes in mean velocity and mean density will be represented as ΔU and $\Delta\rho$. These changes occur over the distortion distance, L_d. If we assume mild anisotropy, $q' = u_k' u_k'^{1/2}$ can be used as the appropriate turbulence velocity scale. We can now write an order of magnitude expression for the production due to the mean velocity gradient:

$$-\widetilde{u_i' u_k'}\frac{\partial \tilde{u}_j}{\partial x_k} - \widetilde{u_j' u_k'}\frac{\partial \tilde{u}_i}{\partial x_k} \approx q'^2 \frac{\Delta U}{L_d}. \tag{2}$$

Now the production term associated with the mean pressure gradient as it contains a turbulence mass flux, $\overline{\rho' u_i'}/\bar{\rho}$. To derive the order-of-magnitude of this term, we first rewrite it using Morkovin's (1962) strong Reynolds analogy (SRA). Briefly, the SRA says that if pressure fluctuations are negligible and the flow is adiabatic with negligible total enthalpy fluctuations, $T_0' \approx 0$, then

$$\frac{T'}{\bar{T}} = -(\gamma - 1)M^2 \frac{u'}{\bar{U}},$$

where T is the absolute temperature and M is the Mach number. This expression can be used to rewrite $\overline{\rho' u_i'}/\bar{\rho}$ as

$$\frac{\overline{\rho' u_1'}}{\bar{\rho}} \sim (\gamma - 1)M^2 \frac{u_1'^2}{\tilde{u}}.$$

The pressure gradient is represented as $\partial p/\partial x \approx \rho U \Delta U/L_d$ and the turbulence mass flux as $\overline{\rho' u'}/\bar{\rho}^2 \approx (\gamma - 1)M^2 q'^2/\rho U$. Together these terms make up the mean pressure gradient contribution to the production,

$$\frac{\overline{\rho' u_i'}}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x_j} + \frac{\overline{\rho' u_j'}}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x_i} \approx (\gamma - 1)M^2 q'^2 \frac{\Delta U}{L_d}. \tag{3}$$

For nonhypersonic flows, the production terms, (II) and (III) will be of the same order.

Now consider the diffusion term, (IV). There are two contributions to this term, viscous and nonviscous. First, it is reasonable to neglect viscous diffusion away from the wall. Second, the nonviscous part of term (IV) represents the interaction between turbulence components, that is, the interaction of turbulence with itself. Assuming weak turbulence, we can show that the turbulence diffusion term will be negligible. Dussauge and Gaviglio (1987) rewrite the turbulence diffusion term and assume that the severity of the distortion completely dampens the turbulence such that the appropriate diffusion length scale is not the integral scale as expected but rather L_d, the length of the distortion. This yields

$$\frac{\partial}{\partial x_k}\widetilde{u_i' u_j' u_k'} + \frac{\overline{u_i' u_j' u_k'}}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial x_k} \approx \frac{q'^3}{L_d} + \frac{q'^3}{\bar{\rho}}\frac{\Delta\bar{\rho}}{L_d}. \tag{4}$$

Finally, the dissipation term (V). If the turbulence is weak and interacts weakly with itself, then for the time over which the distortion is applied we can expect there to be no significant change in dissipation rate. The classical representation of the dissipation rate is adopted here; that is,

$$\varepsilon_{ij} = \frac{q'^3}{\Lambda_\varepsilon}\delta_{ij}. \tag{5}$$

Λ_ε is the energy-containing length scale since it is these eddies which determine the energy transfer (dissipation) rate.

Hence, we obtain the criteria needed to neglect dissipation and diffusion. The first is the criterion for neglecting dissipation and the next two are the criteria for neglecting diffusion:

$$\frac{q'}{\Delta U}\frac{L_d}{\Lambda_\varepsilon} \ll 1, \tag{6}$$

$$\frac{q'}{\Delta U} \ll 1, \tag{7}$$

$$\frac{q'}{\Delta U}\frac{\Delta\rho}{\rho} \ll 1.$$

3. Modeling of the Pressure-Strain Term

The pressure-strain term (III) in (1) needs to be modeled. Lumley (1978) developed a Poisson equation for p' in an incompressible flow by taking the divergence of the Navier–Stokes equations. The pressure fluctuations are influenced by two terms; a "rapid" term, which is linear in u', and a return-to-isotropy term, which is nonlinear in u'. If we adopt Rotta's hypothesis (1951), the return-to-isotropy term would be of the order of the dissipation rate, ε, and therefore negligible given (6). The pressure which is influenced only by the rapid term is called the rapid pressure. Lumley (1978) pointed out that in any distortion of isotropic turbulence, not necessarily rapid, all terms containing the rapid pressure instantly become anisotropic. Furthermore, in a true rapid distortion, the Euler equations can be linearized and the only remaining influence on p' comes from the rapid term, hence its name.

Dussauge and Gaviglio (1987) adapted Lumley's form of the linearized Euler equations for compressible flow. Terms arising from mean compressibility were shown to be negligible, and the equation with only the rapid contribution reduces to

$$-\nabla^2 p' = 2\bar{\rho}\frac{\partial u_i'}{\partial x_j}\frac{\partial \tilde{u}_j}{\partial x_i} + \frac{1}{\tilde{T}}\frac{\partial T'}{\partial x_i}\frac{\partial \bar{p}}{\partial x_i}. \tag{8}$$

This result shows that the pressure fluctuations contain contributions from the mean velocity gradient and the mean pressure gradient. Since the equation is linear, these contributions can be considered separately.

The pressure gradient term has the same form as in buoyancy problems and can be represented using a subsonic model wherein pressure–dilatation effects are neglected. For the mean presure gradient contribution, the model by Lumley (1978) was used:

$$-\frac{1}{\bar{\rho}}\left(\overline{u_i'\frac{\partial p'}{\partial x_j}} + \overline{u_j'\frac{\partial p'}{\partial x_i}}\right)_{\bar{p}} = -0.3\left(\frac{\overline{T'u_i'}}{\bar{\rho}\tilde{T}}\frac{\partial \bar{p}}{\partial x_j} + \frac{\overline{T'u_j'}}{\bar{\rho}\tilde{T}}\frac{\partial \bar{p}}{\partial x_i} - \frac{2}{3}\frac{\overline{T'u_k'}}{\bar{\rho}\tilde{T}}\frac{\partial \bar{p}}{\partial x_k}\delta_{ij}\right). \tag{9}$$

The mean velocity gradient contribution needs to be examined more rigorously for compressibility influences. By decomposing the mean and fluctuating velocity gradient tensors into isotropic (dilatation) and deviatoric (symmetric and antisymmetric) parts and assuming solenoidal velocity fluctuations under Morkovin's hypothesis, Dussauge and Gaviglio (1987) showed that the mean velocity gradient contribution to the pressure fluctuations is unaffected by compressibility. Furthermore, in a distortion involving only dilatation, it was found that the effect of compressibility could be accounted for by simply defining a new variable $T_{ij} = \widetilde{u_i'u_j'}/\bar{\rho}^{2/3}$. Since the mean compressibility does not contribute to the pressure fluctuations, the deviatoric parts of the velocity gradient tensor can be modeled as in subsonic flows where pressure–dilatation effects are neglected. For the mean velocity gradient contribution to the rapid pressure a number of models were available. Here, three models were tested due to Lumley (1978), Shih and Lumley (1985), and Launder *et al.* (1975). The general form of the model was

$$-\frac{1}{\bar{\rho}}\left(\overline{u_i'\frac{\partial p'}{\partial x_j}}+\overline{u_j'\frac{\partial p'}{\partial x_i}}\right)_{\tilde{u}}=\widetilde{q'^2}[0.4D_{ij}+K_1(a_{gj}D_{ig}+a_{gi}D_{jg}-\tfrac{2}{3}a_{gl}D_{lg}\delta_{ij})+K_2(a_{gi}R_{jg}+a_{gj}R_{ig})], \quad (10)$$

where $a_{ij}=\widetilde{u_i'u_j'}/\widetilde{q'^2}-\delta_{ij}/3$, D_{ij} is the symmetric mean rate of strain tensor, R_{ij} is the antisymmetric mean rotation rate tensor, and K_1 and K_2 are constants which vary depending on the model.

4. The Experiment

A turbulent boundary layer was developed on the floor of a 20 cm × 20 cm supersonic blowdown wind tunnel. The wind tunnel was operated at a stagnation pressure of 6.9×10^5 N/m^2 and a stagnation temperature of 270 K. The freestream Mach number was nominally 2.84, and the wall conditions were approximately adiabatic. The incoming boundary layer thickness was approximately 26 mm, and the Reynolds number based on momentum thickness was 77,600. A 20° rearward-facing ramp generated a centered expansion fan.

The mean and turbulence properties of the boundary layer were measured $1\delta_0$ upstream of the distortion and $3.5\delta_0$ downstream of the distortion.

Mean flow measurements showed that the upstream flow was typical of an equilibrium boundary layer in supersonic flow (Figures 1 and 3). Downstream measurements indicated a severely distorted boundary layer (Figures 2 and 4). Within the range of acceptable values for κ and B, no fit for the downstream velocity profile to the law of the wall was found when using different compressibility transformation. Without a logarithmic region downstream, conventional experimental techniques for determining the wall stress could not be used.

Turbulence measurements were made with a normal hotwire at the mean survey locations. The constant temperature hotwire anemometer was operated at a high overheat ratio and tuned during calibration to have a frequency response in the vicinity of 100 Khz. Data was collected at a rate of 1 Mhz. The data were analyzed following the procedure outlined in Smits *et al.* (1983), and turbulence quantities were found by using the strong Reynolds analogy. A comparison of upstream and downstream turbulence profiles are shown in Figures 5–7.

The results show that we should be cautious about using the term "relaminarization." The hotwire measurements indicate that the relative mass-flux fluctuations remained unchanged from upstream to

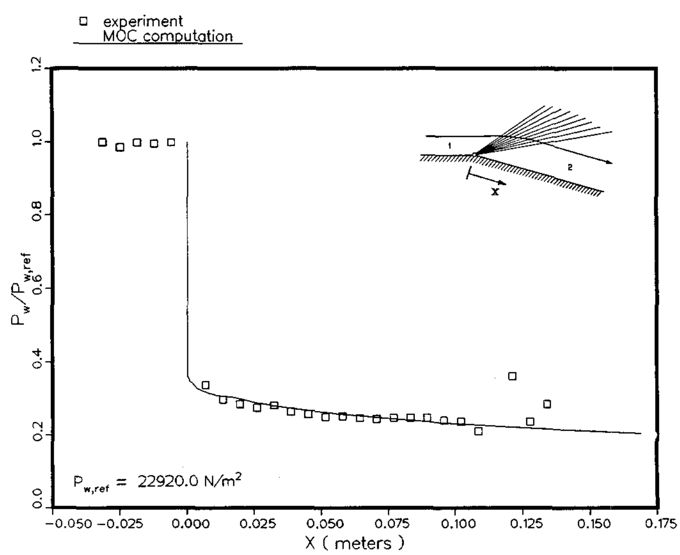

Figure 1. Comparison between the experimental wall pressure and the computed wall pressure.

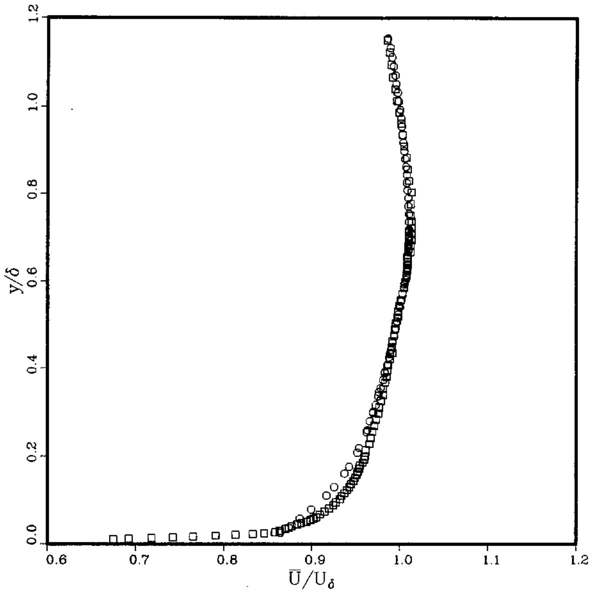

Figure 2. Comparison between the computed, ○, and the experimental, □, mean velocity profiles downstream of the distortion.

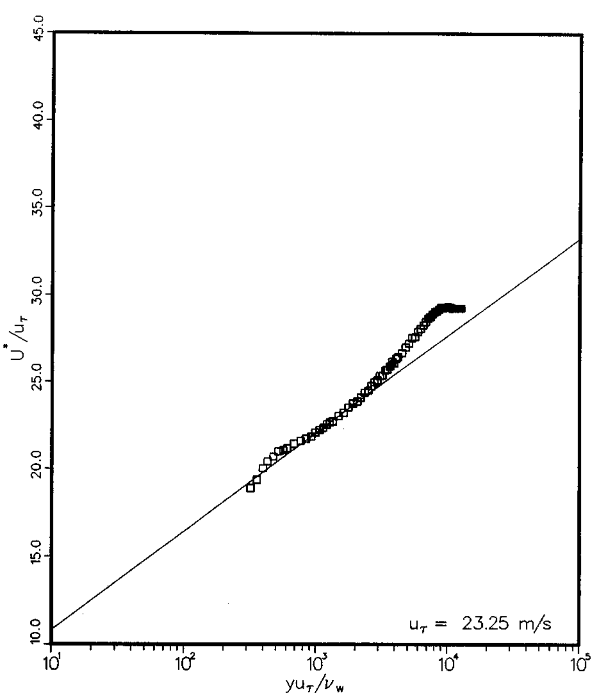

Figure 3. van Driest (1951) transformed velocity profile upstream of the distortion.

downstream (Figure 5). The downstream velocity fluctuations and Reynolds stress decreased significantly from their upstream values (Figures 6 and 7). This was also reflected in the relative velocity fluctuations. However, the SRA indicates that the relative density fluctuations do not change appreciably from their upstream values. Since the velocity fluctuations are significantly smaller than the density fluctuations, it is clear that downstream the major contribution to $(\rho U)'$ must come from ρ'. However, this result depends on the validity of the SRA downstream of the distortion. The results of

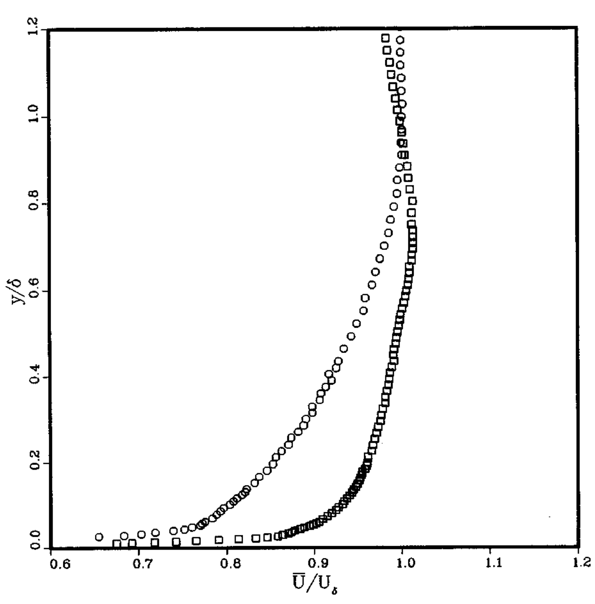

Figure 4. Comparison between the upstream, ○, and the downstream, □, mean velocity profiles.

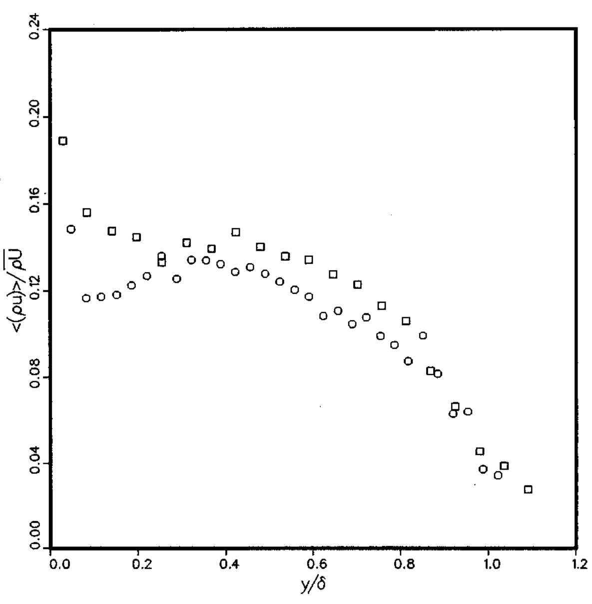

Figure 5. Comparison between the upstream, □, and the downstream, ○, mass-flux fluctuations.

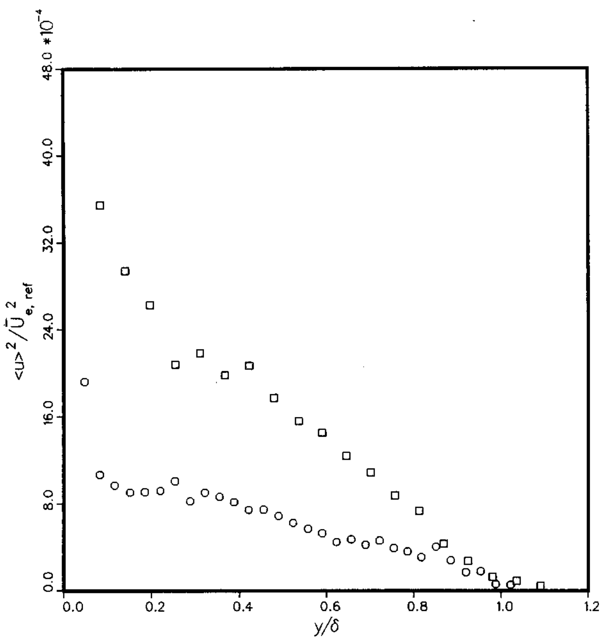

Figure 6. Comparison between the upstream, □, and the downstream, ○, turbulence intensities.

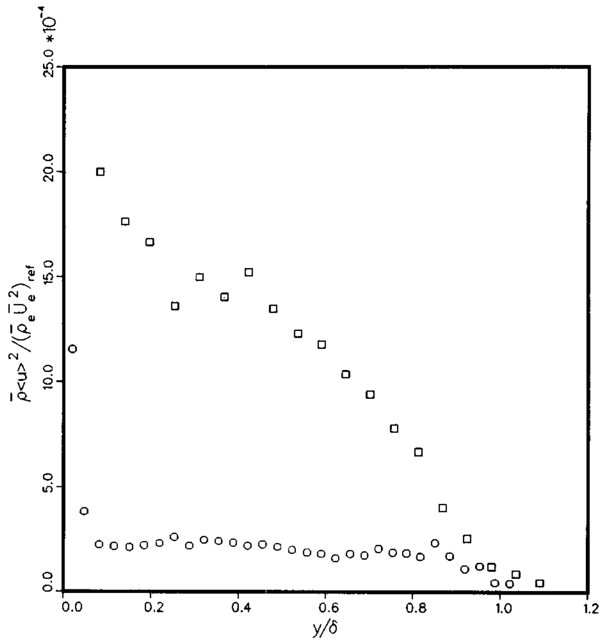

Figure 7. Comparison between the upstream, □, and the downstream, ○, longitudinal Reynolds stress.

Dussauge and Gaviglio (1987) in a less severely distorted flow satisfy the SRA, and it may continue to hold in the present flow. Further experimental work is required to verify this assumption.

5. The Computation

To make the rapid distortion calculation in the boundary layer, a complete characterization of the mean flow is necessary to evaluate gradients of mean flow quantities. The gradients are difficult to obtain accurately using experimental techniques. If instead we assume that outside the viscous region the boundary layer is a rotational perfect fluid flow, then the method of characteristics can be used to solve the Euler equations. The boundary-layer profile taken in the upstream undisturbed boundary layer was used as the input to the calculation which began at the corner.

A comparison between the computed flow and the experimental flow at $3.5\delta_0$ downstream of the distortion indicates that this method gives a good representation of the mean flow (Figures 1 and 2).

The evolution of the Reynolds stress tensor was calculated along streamlines originating at different heights in the boundary layer, where entropy was assumed constant along a streamline. To model the mean pressure gradient production terms, the SRA was used. The computation was performed with each of the pressure–strain models, and little variation was found from one model to the next. Comparisons were also made between the experimental and computed turbulence profiles. The computed profiles presented here are for the Shih and Lumley (1985) model. In the range $0.2 \leq y/\delta \leq 0.8$ the agreement was impressive (Figures 8 and 9). However, near the wall, the calculation underpredicted the Reynolds stress by as much as 75%. A calculation including only the effect of dilatation was also performed. The result was an overprediction of the Reynolds stress in the range $0.2 \leq y/\delta \leq 0.8$ but still in good agreement with the data indicating the importance of compressibility (Figure 10).

In the past, investigations of boundary layers in perturbed compressible flows revealed that the distortion had little effect on the anisotropy ratio (see, for example, Fernando and Smits (1990) for a flat plate boundary layer in an adverse pressure gradient and Donovan (1989) for a boundary layer with severe concave surface curvature). Figure 11 shows profiles of the anisotropy ratio at various locations in a boundary layer subjected to an adverse pressure gradient. Figure 12 shows the anisotropy ratio predicted by the rapid distortion calculation for the centered expansion flow studied here, indicating there appears to be little change in this case also.

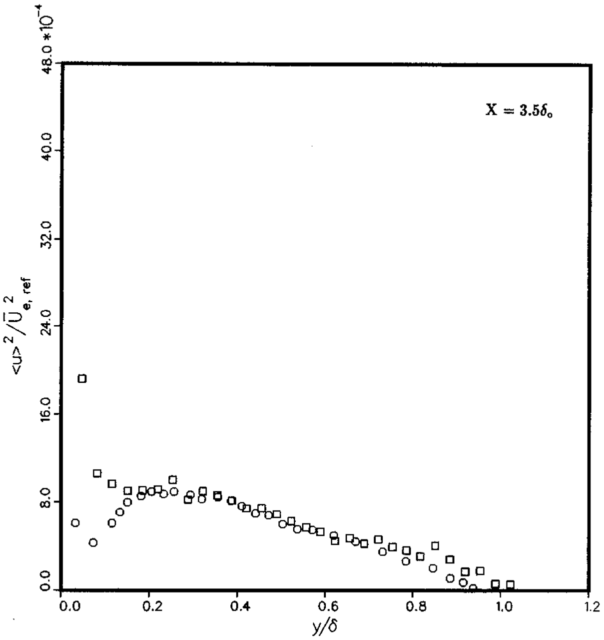

Figure 8. Comparison between the experimental, □, and the computed, ○, turbulence intensities, Shih and Lumley (1985) model.

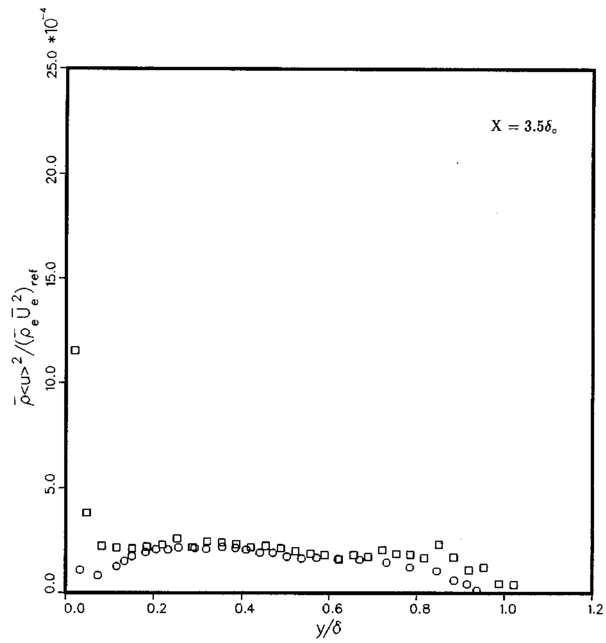

Figure 9. Comparison between the experimental, □, and the computed, ○, longitudinal Reynolds stress, Shih and Lumley (1985) model.

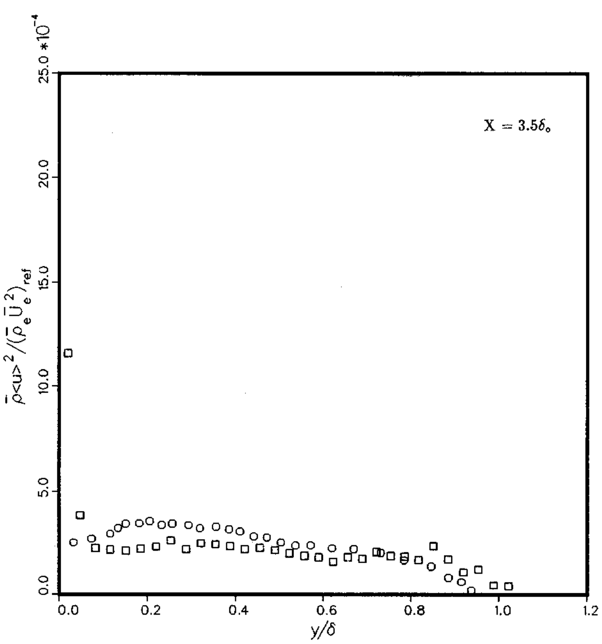

Figure 10. Comparison between the the experimental, □, and the computed, ○, longitudinal Reynolds stress, dilatation effect only.

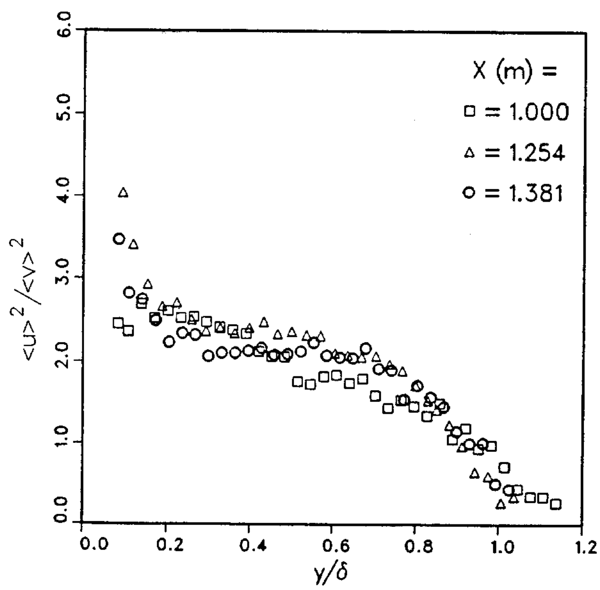

Figure 11. Streamwise variation of the anisotropy profile for a boundary layer subject to an adverse pressure gradient (Fernando and Smits, 1990).

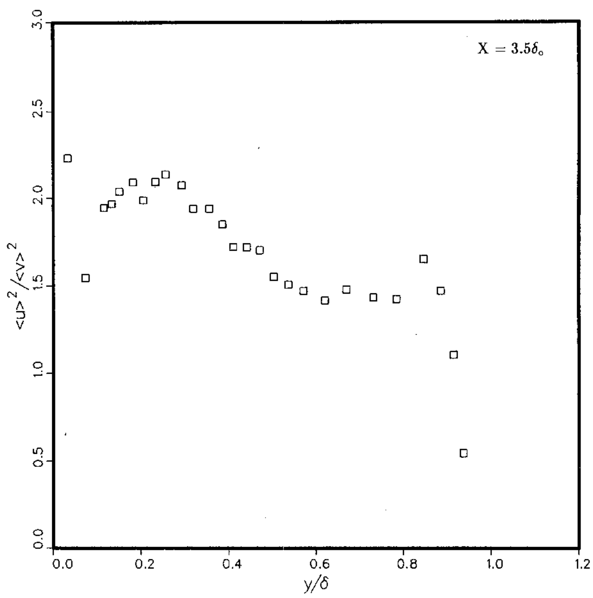

Figure 12. Predicted anisotropy profile downstream of the distortion, Shih and Lumley (1985) model.

6. Conclusions

The measurements revealed that the boundary layer downstream of the expansion distortion was dramatically different from its upstream counterpart. For example, the mean velocity profile did not possess a logarithmic region, and the velocity fluctuations decreased dramatically, while the mass-flux fluctuations remained initially unchanged. The SRA showed that the change in the mass-flux fluctuations seems to follow the change in the density fluctuations.

Within the limits of the rapid distortion approximations, the Reynolds stress equations were simplified and used to make predictions of the Reynolds stresses in this flow which were surprisingly accurate. The performance of three pressure–strain models was evaluated, and it was found that the calculation was mostly insensitive to the model chosen. The influence of dilatation alone was also investigated, and the results indicate that the dilatation has the major influence on the Reynolds stress evolution.

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