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# ON CHISHOLM'S PARADOX

## I. INTRODUCTORY

Chisholm's paradox of derived obligation stated in [1], has presented serious difficulties for deontic logic. The problem is essentially one of adequately representing certain sentences of English in deontic logic. Frequently (one might almost hazard 'generally') the conclusion drawn is that monadic deontic logics are essentially inadequate to this task. As a result a new primitive dyadic operator has often been introduced.[1] Instead of treating $O$ as a one place operator on sentences we treat it as a two place operator. Instead of sentences $OA$ (It ought to be the case that $A$) we have sentences $O(A/B)$ (It ought to be the case that $A$ given that $B$). The simple $OA$ may then be represented as $O(A/p \vee -p)$.

I wish to argue that the problems which motivated dyadic deontic logic can be adequately resolved in monadic deontic logic with a stronger conditional than material implication. I shall conclude that the conditional required is that developed by David Lewis in [8].

## II. THE PARADOX OF DERIVED OBLIGATION

I follow Hansonn's presentation in [2]. The following four sentences are seen to represent a perfectly possible situation.

(1*) Jones robs Smith.
(2*) Jones ought not to rob Smith.
(3*) It ought to be that if Jones doesn't rob Smith, he isn't punished.
(4*) If Jones does rob Smith then he ought to be punished.

We detect three adequacy conditions pertaining to any symbolic representation of (1*)–(4*).

(a1) The representation be consistent.
(a2) The entailment between (1*), (4*) and 'Jones ought to be punished' be preserved.

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(a3) The representation of 'It ought to be that if Jones doesn't rob Smith then he *is* punished' is false.

What is often called standard deontic logic (SDL) is the following system.

Subjoined to propositional calculus:

(A1) $O(A \supset B) \supset OA \supset OB$

(A2) $OA \supset -O-A$

(RD) From $\vdash A$ infer $\vdash OA$.

We note that (3*) cannot be formalised by $-p \supset O-q$ because then 'It ought to be that if Jones doesn't rob Smith he is punished' would be $-p \supset Oq$, which is true since 'Jones robs Smith' is true. This would violate (a3). Neither can (4*) be represented by $O(p \supset q)$ because then (a2) is violated ($Oq$ is not a consequence of $p$ and $O(p \supset q)$). We seem then to be left with:

(1) $p$

(2) $O-p$

(3) $O(-p \supset -q)$

(4) $p \supset Oq$

But (1)–(4) are inconsistent. This is the problem.

## III. THE SEMANTICS OF 'OUGHT'

The normal semantics for SDL are given as follows (see Hilpinen [4]): A model for SDL is a triple $\langle K, D, \phi \rangle$ where $K$ is non-empty (heuristically the 'possible worlds'), $D$ is a relation on $K$ such that $\forall w \, \exists v \; wDv$, and $\phi$ assigns to each sentence letter $P_i$ some subset $\phi(P_i)$ of $K$ (the worlds at which $P_i$ is true). The truth of a *wff* $A$ at a world $w$ in a model $M$ is defined as below:

$$\|P_i\|^M_w \text{ iff } w \in \phi(P_i)$$
$$\|-A\|^M_w \text{ iff not } \|A\|^M_w$$
$$\|A \supset B\|^M_w \text{ iff either not } \|A\|^M_w \text{ or } \|B\|^M_w$$
$$\|OA\|^M_w \text{ iff } \forall w' \; wDw' \, \|A\|^M_{w'}$$

The first step in resolving our difficulties is to examine more closely the intended interpretation of the relation $D$. Normally this is simply thought of as picking out the 'deontically ideal worlds' from a given world $w$. But this does not tell us very much.

There are clearly important differences between:

(5) Jones ought to put a pound in the Oxfam box.
(6) Someone ought to put a pound in the Oxfam box.
(7) No-one ought to be starving.

The first sentence seems to represent a (the?) central use of 'ought' in that it says that a specific person has it incumbent on him to perform a specific action. The second says that a specific action ought to be performed, but does not say that it is incumbent upon anyone in particular to perform it. The third sentence makes reference neither to any action nor, obviously, to any agent. Now among the many logical and philosophical complexities to be discovered in (5)–(7) I want to fix upon just one. Both (5) and (6) are, insofar as they say that an action should be performed, essentially forward-looking. They must be so. When we say that something to be done we never mean exactly now, for it is as impossible to set about and do things exactly now as it is to do things in the past. When we say that someone ought to do something now, we mean, rather, that they ought to do it forthwith, or at least very soon. In general, when we say that 'It ought to be the case that $S$ does $A$' we are saying that if some *future* world is going to be as it ought to be then it will have $S$ doing $A$. In our example when we say that Jones ought to put a pound in the Oxfam box we mean that the next action that Jones performs should be a putting of money in the box.

There are of course occasions where 'ought', though forward looking, is certainly not of this ought-next character. For instance 'You ought to visit your father next Tuesday' or 'When the NBTS next visits the campus you ought to give a pint.' But it is tempting to think that in all cases where we ought to do $A$ at some future time there comes a time when we ought to do $A$ next. If this is so then we can capture the sense of sentences containing temporally distant obligations by means of logic containing $O$ interpreted as 'ought-next' and appropriate tense operators. Perhaps this is too hasty. Suppose, as is no doubt true, that $S$ ought to visit his father from time to time to spend a week-end. Now if he has failed to do this for six months and his next week-end is free, then no doubt he ought to visit his father next week-end, and on Saturday morning this obligation will become immediate. But suppose that he has always been a dutiful son: he visits regularly. Then he is free on any particular week-end not to visit; in which case there never comes a time when his obligation is of the 'ought-next' variety. Do we then have a case where *S* ought to do *A* sometime in the future but there never comes a time when *S* ought to do *A* immediately? I don't think so. The whole point of the example is that *S* is dutiful enough to ensure that he is never obliged to visit, so it is not true that at some future time he ought to visit (provided he remains dutiful). It is then hardly surprising that no obligation to act immediately ever devolves upon him. What we have here is just a confusion between 'At some future time *S* ought to do *A*' and '*S* ought, at some future time, to do *A*.' But if this is a confusion something needs to be said about the sense of the second sentence (the first says that it will be incumbent upon *S* to do *A* immediately). I think that this second sentence, indeed this whole discussion raises the question of what Mill called 'general duties'. To enter into this question at any length would be beyond the scope of this paper; so I shall state what I take to be the sense of the second sentence without argument. It indicates, then, that failure to perform an action (or sort of action) for long enough gives rise to an obligation ('particular duty') to perform it. Certain actions only become obligatory if they are persistently omitted. General duties are not duties at all, but instructions about how to avoid them.

There appears then to be nothing standing in the way of maintaining that more distant duties can be adequately expressed in terms of 'ought-next' and an appropriate tensing apparatus.

We have dealt only with sentences that must use 'ought' in a forward-looking way because they say what ought to be done. But there is no reason why this forward-looking 'ought' should not modify sentences that make no reference to actions. We may truly say 'No-one ought to be starving' – that will be so when the best next-world has no one starving in it. Given an adequate tense logic we may say that no-one should be starving now: this simply by saying that it was the case immediately before this that no-one ought to be starving. Further the obvious universality of (7) can be captured by saying that it always has been, is, and always will be that no-one ought to be starving. Though 'ought-next' is tailored for actions it will fit states of affairs as well.

The standard deontic logic may thus be thought of as that fragment of the full deontic logic which deals, because of the absence of the required apparatus of tenses, only with what we ought to do (or with what ought to be) next.

The worlds $v$ such that $wDv$ are then the best next worlds from $w$. It is natural to ask, 'Best out of what?' The obvious reply is 'Best out of the bunch of possible worlds that could come after $w$'. If we accept this answer what ought-to-be is subject to the restriction of what-can-be. This seems to me to be a natural restriction, but it has a non-trivial consequence. This is that we have as many senses of ought as there are senses of 'possible'. To illustrate this let us for a moment suppose that the worlds $v$, $wDv$ are the best, selected from the *practically attainable* worlds from $w$. Since it is no doubt true that in none of these worlds is 'No-one is starving' true, it is false in $w$ that 'It ought to be that no-one is starving'. If on the other hand the possible worlds are the *logically possible* worlds then there will be one where no-one is starving, and 'It ought to be that no-one is starving' will be true at $w$. It might be thought that we could opt for logical possibility all the time when selecting the worlds $v$, $wDv$: the only consequence being that there would be many things that we ought to do next which we cannot bring off. This, it may be thought, would not be a disastrous consequence, for we could mitigate, indeed eliminate, blame wherever a person could not do what he ought to do. There would be many unfulfilled obligations but this, it may be said, is much more satisfactory than saying that 'No-one ought to be starving' is false. But the consequence of saying that $D$ selects the best worlds from the logically possible worlds is not that we have more obligations (though many of these are unfulfillable), it is rather that we have *fewer* obligations. Consider 'Jones ought to put a pound in the Oxfam box'. In the best *logically* possible next world there would be only well-fed persons on earth (due perhaps to a rain of manna) and Jones would not be required to put the cash in: necessarily so, because the best logically possible world would have the starving fed without this small sacrifice by Jones. Thus, relativising $D$ to logical possibility, we find that it is not true that Jones ought to give to Oxfam. In general, wherever fulfilling an obligation entailed some sacrifice, we would not have that obligation. This is obviously an absurd consequence. On the other hand to insist on relativising 'ought' always to the much stronger sense of possible is surely not satisfactory either. There is a sense in which 'No-one ought to be starving' is true. The answer it seems we are forced to is that there are varying senses of 'ought', and they vary just as we vary the domain of the possible.

When we consider (1*)–(4*), however, it is pretty clear that the sense of 'ought' is relative to the best attainable worlds from $w$. We shall therefore be tacitly operating with this 'ought'.

To summarise, the worlds which $D$ selects are the best of the next practically attainable worlds after $w$. A sentence of $OA$ is true at $w$ just in case $A$ is true at all these worlds. $\{v \mid wDv\}$ is a set of worlds that do not compare for moral excellence with $w$, but compete amongst themselves and with less reputable worlds for the privilege of being the next world after $w$. Formally we give the semantics of SDL in just the same way, but we understand $D$ as having the above discussed elements.

Though the interpretation of $D$ suggested above has no immediate formal consequences, it does provide two semantical insights on our problem. Firstly (1*)–(4*) are meant to portray a situation where an obligation has arisen owing to the past misdeeds of Jones. If Jones had not robbed Smith the obligation to punish would never have arisen. In this sense the obligation to punish is thought to be a derived obligation. But then (1*) and (2*) do not say this. (1*) says that Jones now robs Smith. (2*) says that he ought not *next* to rob Smith: that he shouldn't do it again. But this as it stands is quite consistent with Jones never having violated an obligation at all. To introduce this element, to say, in fact, that Jones has already done wrong, we have to say not 'Jones ought not to rob Smith' but 'Jones ought not *to have* robbed Smith'. $O-p$ is clearly inadequate as a formalisation of this latter sentence, and it is clear that SDL, which possesses no tensing apparatus, will be quite unable to provide a formalisation that captures this distinction. However, having pointed this out, it is clear that it will not save us from paradox, since 'Jones ought not to rob Smith' is true as well as 'Jones ought not to have robbed Smith'. That is we have $O-p$ even if it is not quite what we originally wanted to say.

The sentence that is not true among (1)–(4) is (3) $O(-p \supset -q)$. This asserts that the best next world that we can come to is either a $p$-world or a $-q$-world. But since Jones has in this world robbed Smith we want him next to be punished for it. So the best next world will be a $q$-world. Yet we also have no wish that Jones robs Smith *again*, which leads us to say that the best next world is a $-p$-world. Thus the best next world is a $-(-p \supset -q)$-world, and $O(-p \supset -q)$ is false. It is not surprising then that we are led to inconsistency.

But if (3) is false how are we to represent (3*)? To this question we now turn.

## IV. DEONTIC LOGIC WITH NECESSITY

We argued that $-p \supset O-q$ would not serve as a representation of (3*). The reason, to recapitulate, was that if it would so serve then $-p \supset Oq$ would have to be read 'It ought to be that if Jones doesn't rob Smith then he is punished'. Now this English sentence is certainly false: but since $p$ is true, $-p \supset Oq$ is true as well. Representing (3*) as $-p \supset O-q$ violates our third adequacy condition. But if we strengthened $-p \supset O-q$ to $\Box(-p \supset O-q)$, this difficulty is evaded. Also $\Box(-p \supset O-q)$ has about it the 'flavour' of a moral principle (albeit an inflexible one). Whenever, it tells us, Jones has not stolen, then we ought not to set about punishing him. Furthermore we can find another moral principle supporting (4*), namely $\Box(p \supset Oq)$. Thus we have two moral principles: $\Box(p \supset Oq)$ and $\Box(-p \supset O-q)$, and we represent (1*)–(4*) as:

(1′) $p$
(2′) $O-p$
(3′) $\Box(-p \supset O-q)$
(4′) $p \supset Oq$.

This method of formalising (3*) provides the chief motivation for the deontic logic with necessity we give below. However, SDLN will only serve to advance our discussion so far. It will not prove completely satisfactory.

## V. STANDARD DEONTIC LOGIC WITH NECESSITY SDLN

The language of SDLN is just that of SDL with the addition of the necessity operator $\Box$. Though we could characterise it according to any of the great variety of systems available we chose $T$ for simplicity's sake. SDLN is then given by the schemes:

(A1) $O(A \supset B) \supset OA \supset OB$

(A2) $OA \supset -O-A$
(A3) $\Box A \supset A$
(A4) $\Box(A \supset B) \supset \Box A \supset \Box B$
(A5) $\Box A \supset OA$,

and the rules RD and RL ($\vdash A$ infer $\vdash \Box A$).

The only scheme deserving of comment here is (A5). This amounts to the assertion (in some sense) that 'ought' implies 'can'. That is we have $OA \supset \Diamond A$ (form (A5) and (A2)). Reasons why this should be provable have been given (though Hintikka in [5] has argued why only the weaker $O(OA \supset \Diamond A)$ should be considered valid).

A model for SDLN is a quadruple $\langle W, D, R, \phi \rangle$ where $W$, $\phi$ are as for SDL and so is $D$, except $(\forall w')\ wDw' \supset wRw'$. $R$ is, of course, reflexive in $W$. The definition of truth at a world in a model is again as before except we add the usual clause for $\Box$, viz:

$$\|\Box A\|_w^M \quad \text{iff} \quad \forall w'\ wRw'\ \|A\|_{w'}^M .$$

SDLN is consistent and can be shown complete by the usual Henkin method. This enables us to prove that our new interpretation of (1*)–(4*) is consistent. Since it also meets the other adequacy conditions it seems that we now have an acceptable way of representing (1*)–(4*).

If this does resolve our original problem are there still any lessons to be learnt from it? That there are the following may show. Chisholm's main motivation seems to be to show that we must have a deontic logic that is sensitive to changing circumstances. That our semantic conditions capture this can be seen by the fact that there is no requirement that what is a best world from a world $w$ is also a best world from another world $u$. Thus what we ought to do in $w$ may be different from what we ought to do in $u$. But that is not to say that all deontic logics have this desirable flexibility.

For example there is a family of deontic logics obtained by defining the operator $O$ by means of the necessity operator and some propositional constant. Hilpinen describes in [4] two such logics, one presented in an unpublished paper of Kanger's and the other a modification of Anderson's logic in [6]. The idea behind Kanger's suggestion was that the obligatory is what morality prescribes, where 'prescribes' is taken to mean entails. Thus we are invited to think of morality as a set of sentences describing a presumably ideal world. Accordingly we introduce into a modal logic some constant, say $m$, which may, heuristically, be thought of as the conjunction of the sentences forming our morality. Then to say that it ought to be the case that $p$ is to say that $m$ entails $p$. That is $Op =_{df} \Box(m \supset p)$. Anderson showed that if we interpret $Op$ as $\Box(-p \supset s \;\&\; \Diamond - s)$ we can derive the system SDL from the modal system $T$. We shall follow Hilpinen and take Anderson's suggestion to be that $Op =_{df} \Box(-p \supset s)$.[2] If we then add to $T$ the axiom $\Diamond - s$, SDL can be obtained. Correspondingly, by adding $\Diamond m$ to $T$ we can derive SDL, using Kanger's definition. Now these treatments of ought carry with them an implication that what is morally desirable in any world is just what is morally desirable in all worlds. This is most clear on Kanger's definition. What $m$, a constant, *entails* will be the same in all worlds. If we think that our obligations change, then we must admit that our moral system has changed to, and with it the definition of $O$.

Given either of these definitions of $O$, the re-interpretation of (1*)–(4*) given above proves still to be inconsistent. Taking Kanger's suggestion we find that (1′)–(4′) are:

(1) $\Box(m \supset - p)$
(2) $p$
(3) $\Box(- p \supset \Box(m \supset q))$
(4) $p \supset \Box(m \supset q)$

We can now argue:

(5) $\Box(m \supset \Box(m \supset - q)$ (from (1), (3))
(6) $\Box(m \supset - q)$ (from (5) by a theorem of $T$)
(7) $\Box(m \supset - q) \supset (\Box(m \supset q) \supset \Box - m)$ (by $T$)
(8) $\Box - m$ (from (6), (2), (4))
(9) $- \Diamond m$

which contradicts the axiom $\Diamond m$. A similar argument can be constructed on Anderson's account. Incidentally it should be pointed out that Anderson in [7] abandons this way of treating $O$.

This inflexibility which is inherent in treating $O$ as above can be introduced into SDLN by adding the semantic restriction that what is desirable in this world is just what is desirable in all other worlds. That is by putting the condition on $D$ that $\forall w\, \forall w'\, \forall v\; wDv \supset w'Dv$. Corresponding to this we add the axiom scheme $OA \supset \Box OA$. We call the resulting system SDLN*. That (1′)–(4′) are inconsistent in SDLN* can be shown as follows:

| | | |
|---|---|---|
| (5) | $Oq$ | ((1′), (4′)) |
| (6) | $\Box Oq$ | (by $OA \supset \Box OA$) |
| (7) | $\Box - O - q$ | (from (A2), (A4), $R$) |
| (8) | $\Box(-p \supset - O - q)$ | |
| (9) | $\Box p$ | ((3′), (8′) by $T$) |
| (10) | $Op$ | (A5) |
| (11) | $- O - p$ | (A2) |

The lesson which we learn from Chisholm's problem is that $OA \supset \Box OA$ should not be a theorem of an adequate deontic logic with necessity. By refusing to countenance it, and the corresponding semantic restriction we allow the degree of flexibility which our ordinary moral language demands.

Unfortunately these same considerations of flexibility may lead us to doubt the adequacy of $\Box(-p \supset O-q)$ as a representation of (3*). This may be seen if we consider a world in which Jones has not robbed Smith but has murdered him instead. Here we have both $-p$ and (unless murder is a desirable thing) $Oq$. It seems that in such a world it is not true $-p \supset O-q$. But since such a world is undoubtedly *possible*, it presumably isn't true that $\Box(-p \supset O-q)$ in this world. SDLN was motivated by a desire to find a stronger connection between Jones not stealing and his not being punished, but strict implication is too strong.

One may be tempted to reply to this argument that the representation of (3*) is at fault not because of the strictness of the connection but because we have not explicitly stated other relevant circumstances. The moral principle which we tried to represent as $\Box(-p \supset O-q)$ should be represented $\Box(-p \;\&\; c \supset O-q)$ where $c$ is some *ceteris paribus* condition. Similarly one imagines that (4′) is in fact derived from a moral principle $\Box(p \;\&\; c' \supset Oq)$ which gives us $p \;\&\; c' \supset Oq$. We tacitly assume $c'$, and so write $p \supset Oq$. In fact, it may be claimed, our interpretation of (1*)–(4*) can be saved by realising that those sentences themselves presuppose a certain context, which we would do well to include in our formalisation.

The intention behind this reply seems to me right, but the reply for all that is not sound. One must look askance at a formalisation of four sentences that introduces two more; especially when those two are and must remain mere schemes of sentences. We cannot put either $c$ or $c'$ into English, for to do so would be to characterise quite generally those circumstances when the fact that $p$ introduced an obligation to do $q$. Such guessing at possible circumstances is surely not possible. The way to express the fact that $p$ only gives rise to an obligation to do $q$ given certain circumstances is not by means of strict implication and a specification of these circumstances, but by characterising an operator that reads 'In the circumstances if $p$ then $q$'. Given such an operator a deontic logic that incorporated it would (hopefully) provide us finally with an adequate representation of (1*)–(4*).

## VI. STANDARD DEONTIC LOGIC WITH CONDITIONALS: SDLC

Much work has recently been done on the semantics of conditional sentences, and in this paper we gratefully take advantage of it.[3] The basic idea has been that a sentence 'If it were that $p$ then it would be that $q$' ($p \Box\!\!\rightarrow q$) is true in a 'world' $w$ just in case in the most similar world to $w$ where $p$ is true, $q$ is true also. If there is no world remotely like ours where p is true then the conditional holds vacuously. For a detailed discussion of the philosophical background of conditional logics the reader is referred to Lewis [9]. Some general remarks are however in order here. It is clear that the similarity relation does not characterise itself any more than the relation 'possible with respect to' used in the semantics of alethic modal logics does. Various assumptions concerning it are possible. For example, are we to say that there is always a *unique* 'most similar $p$-world' to our own? If we do, then $p \Box\!\!\rightarrow q \vee p \Box\!\!\rightarrow -q$ will be a logical truth. But if we allow that there might be several most similar $p$-worlds then $q$ may hold in some but not in others. In which case neither $p \Box\!\!\rightarrow q$ nor $p \Box\!\!\rightarrow -q$ will be true. If it were the case that $p$ it might either be $q$ or $-q$, so we are justified in asserting neither conditional. Of course, independent of a decision about the similarity relation here, we will have $p \Box\!\!\rightarrow q \vee -q$. We shall not attempt to decide this question here, but simply opt for the latter alternative. The other assumptions made about similarity seem much less problematical.

## VII. SDLC

SDLC is just the standard deontic logic complicated by the addition of a logic of conditionals, in fact Lewis's conditional logic (C1) given in [8]. The language of SDLC is just the language of SDL with the operator $\Box\!\!\rightarrow$ added. The axiom schemes and rules of SDLC are those of SDL plus:

(A3) $A \Box\!\!\rightarrow A$

(A4) $A \,\&\, B \supset (A \Box\!\!\rightarrow B)$

(A5) $(A \Box\!\!\rightarrow B \,\&\, B \Box\!\!\rightarrow A) \supset (A \Box\!\!\rightarrow C \equiv B \Box\!\!\rightarrow C)$

(A6) $(A \Box\!\!\rightarrow B) \supset (A \supset B)$

(A7) $(A \vee B \Box\!\!\rightarrow B) \vee (A \vee B \Box\!\!\rightarrow A) \vee (A \vee B \Box\!\!\rightarrow C \equiv$
$\equiv (A \Box\!\!\rightarrow C \,\&\, B \Box\!\!\rightarrow C))$

(A8) $-A \Box\!\!\rightarrow A \supset OA$

(RC) From $\vdash (A \supset B) \supset (A \supset C)$ infer $\vdash (A \Box\!\!\rightarrow B) \supset (A \Box\!\!\rightarrow C)$.

## VIII. SEMANTICS

A model is a quadruple $\langle K, D, \text{Sim}, \phi\rangle$ where $K$ is non-empty (heuristically the possible worlds), $D$ is a relation on $K$ such that $\forall w\, \exists v\; wDv$, and Sim is a function such that for each sentence $A$ and $w \in K$; $\text{Sim}(Aw) \subseteq \text{K}$. $\phi(P_i) \subseteq K$, for each sentence letter $P_i$. Further Sim meets the following conditions:

(i) $\text{Sim}(A, w) \subseteq \|A\|^M (\|A\|^M = \{w \mid \|A\|^M_w)$

(ii) If $\|A\|^M_w$ then $\text{Sim}(A\, w) = \{w\}$

(iii) Either $\text{Sim}(A \vee B\, w) \subseteq \|A\|^M$ or $\text{Sim}(A \vee B\, w) \subseteq \|B\|^M$
or $\text{Sim}(A \vee B\, w) = \text{Sim}(A\, w) \cup \text{Sim}(B\, w)$

(iv) If $\text{Sim}(A\, w) \subseteq \|B\|^M$ and $\text{Sim}(B\, w) \subseteq \|A\|^M$
then $\text{Sim}(A\, w) = \text{Sim}(B\, w)$

(v) $\text{Sim}(-A\, w) = A$ then $\forall v\; wDv \supset \|A\|^M_v$

*A wff. A* is true in a model $M$ at a world as below:

$$\|P_i\|^M_W \text{ iff } w \in \phi(P_i)$$

Truth functions as usual.

$$\|OA\|^M_W \text{ iff } \forall v\; wDv\; \|A\|^M_W$$

$$\|A \,\Box\!\!\rightarrow B\|_w^M \quad \text{iff} \quad \mathrm{Sim}(A\ w) \subseteq \|B\|^M$$

Completeness and consistency results are given for SDLC by the usual Henkin method. Given SDLC, however, we can naturally represent (1*)–(4*) as:

(1″) $p$
(2″) $O-p$
(3″) $-p \,\Box\!\!\rightarrow O-q$
(4″) $p \supset Oq$.

(1″)–(4″) can be easily shown to be consistent. Further the second adequacy condition is also obviously met. The third condition is also met, for if $-p \,\Box\!\!\rightarrow O-q$ is true then the most similar $-p$-world to ours is an $O-q$ world. Thus all the desirable worlds from that world are $-q$ worlds. Thus $Oq$ is false at that world and so $-p \,\Box\!\!\rightarrow Oq$ is false.[4] We can note another desirable consequence of this way of treating (3*). We are quite at liberty to assume that behind the material conditional $p \supset Oq$ there lurks another 'moral principle' similar to (3*); and this can be expressed in just the same way as (3*); namely $p \,\Box\!\!\rightarrow Oq$ (If Jones were to rob Smith then it would be that he ought to be punished). In virtue of (A6) this enables us to derive (4″) from the moral principle in question.

We saw when considering the system SDLN that Chisholm's paradox could be restored by adding the semantic assumption that what was ethically desirable from one world was ethically desirable from all, and adding the corresponding axiom scheme: $OA \supset \Box OA$. A similar adjustment is possible with SDLC. We add the following condition on the Similarity relation:

$$\text{If } v \in \mathrm{Sim}(A\ w) \text{ for any } wff\ A, \text{ then } wDw' \equiv vDw'$$

and the three further axioms:

(1) $OA \supset (B \,\Box\!\!\rightarrow OA)$ (all our obligations remain in all worlds)
(2) $-(B \,\Box\!\!\rightarrow -B) \supset ((B \,\Box\!\!\rightarrow OA) \supset OA)$ (all possible obligations are with us now)
(3) $-(B \,\Box\!\!\rightarrow OA) \supset (B \,\Box\!\!\rightarrow -OA)$.

We then have a system SDLC* (which is consistent and complete) in which Chisholm's paradox arises, that is in which (1″)–(4″) are inconsistent. This is seen as follows. We can derive from (1″), (4″) and (1) $-p \Box\!\!\rightarrow Oq$. Thus since $\vdash O-A \supset -OA$ we have $-p \Box\!\!\rightarrow -Oq$ from (3″) and (RC). But since $\vdash (A \Box\!\!\rightarrow B) \& (A \Box\!\!\rightarrow -B) \supset (A \Box\!\!\rightarrow -A)$, we have $-p \Box\!\!\rightarrow p$. Thus by (A8): $Op$ and so $-O-p$, contradicting (2″). In the absence of (A8) – which is not universally accepted – we cannot derive any contradiction but can still get the absurd conclusion that $p \Box\!\!\rightarrow Or$, where $r$ is anything you like. (Similar remarks apply to SDLN* without axiom scheme (A5)).

## IX. CONCLUSION

It has been maintained that we are quite able to express (1*)–(4*) without the introduction of a dyadic deontic operator, provided only that we supply our standard deontic logic with a stronger conditional than material implication. The lesson learned from Chisholm's paradox has been the eminently convincing, indeed obvious, one: that what we ought to do is not determined by what is the case in some perfect world, but by what is the case in the best world we can 'get to' from this world. What we ought to do depends upon how we are circumstanced.

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## NOTES

[1] See especially G. H. Von Wright [3].
[2] $S$ is thought of as some bad thing resulting from ignoring our obligations.
[3] See in particular David Lewis [8] and [9] and Robert Stalnaker and R. Thomason, 'A Semantic Analysis of Conditional Logic', *Theoria* XXXVI (1970), 22–42. Anderson's paper [7] is also relevant.
[4] This is not true in complete generality. If there is no remotely similar $p$-world then all conditionals $p \Box\!\!\rightarrow r$ are true, including $p \Box\!\!\rightarrow Oq$ and $p \Box\!\!\rightarrow O-q$, but this does not seem an awkward consequence.

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[3] G. H. Von Wright, 'A New System of Deontic Logics' in Hilpinen, *op. cit.*
]4] Risto Hilpinen, 'Deontic Logic, an Introduction', in Hilpinen, *op. cit.*, 1–19.
[5] Jaakko Hintikka, *Models for Modalities*, Reidel, 1969, 184–213.

[6] Alan R. Anderson, 'A Reduction of Deontic Logic to Alethic Modal Logic', *Mind* 67 (1958), 100–103.
[7] Alan R. Anderson, 'Some Nasty Problems in the Formal Logic of Ethics', *Nous* 1 (1967), 345–360.
[8] David Lewis, 'Completeness and Decidability of Three Logics of Counterfactual Conditionals', *Theoria* XXXVII (1971), 74–85.
[9] David Lewis, *Counterfactuals*, Basil Blackwell, forthcoming.