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Irrigation Uniformity Related to Horizontal Extent of Root Zone

A Computational Study

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Summary. Harmonic analysis is used to derive the component waves of a given water distribution map. These components are then subjected to "smoothing" by root systems of various sizes, to obtain the *effective* variances and uniformity coefficients for these root systems. This approach helps to explain, for instance, why the effective uniformity by trickle irrigation is high, while the detailed actual distribution is very nonuniform; why the actual uniformity of under-canopy sprinkling of orchards need not be very high; or why it is usually better to have the rows of field crops parallel to the shorter spacing of the sprinklers. It is observed that ordinary distribution maps contain little variance in the shorter wave-lengths, thus suggesting a reduction in the number of collectors necessary for pattern determination. Another conclusion based on the same observation is that when plants' spacing is half the sprinkler's spacing, a spatial shift between the crop and the irrigation system may markedly affect the effective uniformity.

There are two main advantages of uniform water distribution: (1) Whenever a yield function is convex upwards (positive second derivative) in the neighborhood of the average water application, the average yield for a given field increases with increased uniformity (e.g. Zaslavsky & Buras, 1967). (2) For a given average water application depth, percolation of water to below the root zone decreases as the uniformity increases. The emphasis in the present study is on the first point, assuming that deep percolation does not occur. In the following sections, the relationship between the horizontal extent of a root zone and the *effective* uniformity of distribution will be explored.

To illustrate the problem, a one-dimensional water distribution is discussed first. Let us consider an orchard irrigated by an undercanopy sprinkling system which produces a sinusoidal pattern of wavelength L and amplitude C. In Figure 1 a, L is twice the planting spacing, D, and the relative phase is such that the minima and maxima of the distribution pattern are located *between* the trees. It is obvious from the geometry that both (all) trees receive the same amount of water. Whether they can actually utilize all the water effectively, depends on whether the water applied is confined to the root zone (both vertically and horizontally) and whether there are sufficient roots in the wetted zone. Here we shall assume that this is the case. Thus, the *effective* distribution

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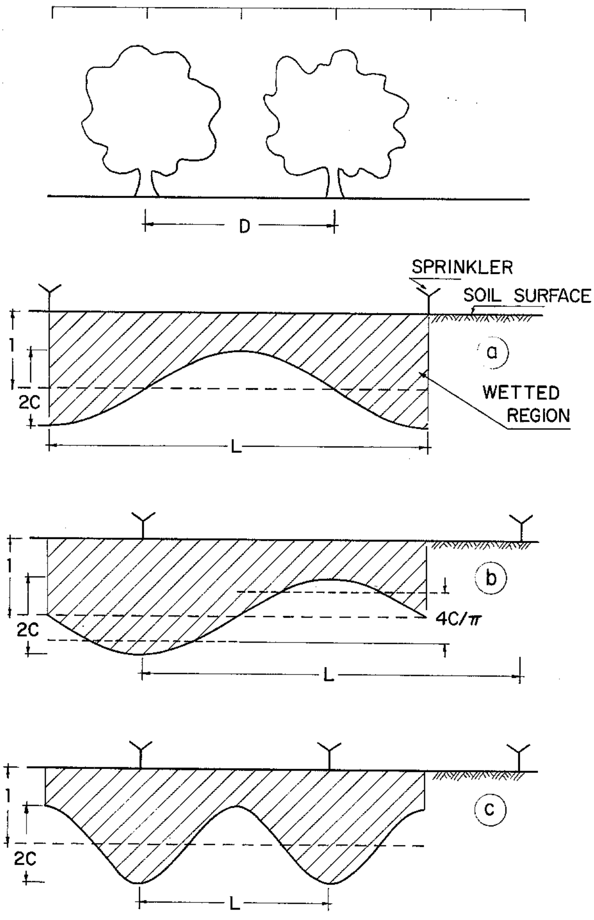

Fig. 1 a – c. Various possible water distribution patterns relative to root-zone. Coarse broken lines indicate average application over sprinkler module. Fine broken lines – over tree module

(among trees) has variance zero, while the *actual* distribution (among and within trees) has a variance $C/\sqrt{2} \cong 0.71\ C$.

Figure 1 b shows the situation when the water distribution is shifted by $D/2$ relative to the trees. Averaging first over each root-zone separately, the resulting effective (residual, among trees) variance is $2\ C/\pi \cong 0.64\ C$. Thus, when $L = 2\ D$, a phase shift may affect the effective variance very significantly.

Figure 1 c shows that when $L = D$, the water distribution is effectively uniform, independent of phase shift.

Analysis

Let us consider a repetitive water distribution patern where each module (e.g. between 4 adjacent sprinklers) occupies an area $X \times Y$. The horizontal extent of the individual root zones is assumed to be equal to the spacing between plants, namely $D_x \times D_y$. The distribution pattern can be broken down by means of harmonic analysis to its component waves of the form

$$w = C \cos 2\pi (ax + by + c) \tag{1}$$

where w is the water depth at point (x, y) and a, b and c are constants. The wavelength associated with Equation (1) is

$$L = (a^2 + b^2)^{-1/2}. \tag{2}$$

The wavelengths in the two cardinal directions are

$$L_x = 1/a \quad \text{and} \quad L_y = 1/b \tag{3}$$

so that

$$L = L_x L_y (L_x^2 + L_y^2)^{-1/2}. \tag{4}$$

Averaging w over the root-zone $D \equiv (D_x, D_y)$

$$\hat{w} = \frac{C}{D_x D_y} \int_{x'-\frac{1}{2}D_x}^{x'+\frac{1}{2}D_x} \left[\int_{y'-\frac{1}{2}D_y}^{y'+\frac{1}{2}D_y} \cos 2\pi (ax + by + c)\, dy \right] dx, \tag{5}$$

one obtains

$$\hat{w} = \frac{\sin(\pi D_x/L_x) \sin(\pi D_y/L_y)}{(\pi D_x/L_x)(\pi D_y/L_y)} \cdot w, \tag{6}$$

which is the moving average of w. Let us denote

$$f_x \equiv f\{D_x/L_x\} \equiv \sin(\pi D_x/L_x)/(\pi D_x/L_x) \tag{7}$$

and similarly for the y direction, the constants f_x and f_y being of absolute value less or equal to unity. The amplitude and variance of $\hat{w}$ are thus related to those of w by

$$A\{\hat{w}\} = f_x f_y A\{w\} \tag{8}$$

$$V\{\hat{w}\} = f_x^2 f_y^2 V\{w\} \tag{9}$$

If the distribution is composed of several sinusoidal waves, then the total depth of water and its variance are given by:

$$W = \Sigma w \tag{10}$$

$$V\{W\} = \Sigma V\{w\} \tag{11}$$

The variance of the effective distribution is

$$V\{\hat{W}\} = \Sigma f_x^2 f_y^2 V\{w\} \tag{12}$$

where the sum is over all significant waves. Any repetitive (cyclic) water distribution map can be harmonically analyzed. Thus, Equation (12) can be used to produce an effective variance (namely, uniformity coefficient) for any given water distribution and root-zone $D \equiv (D_x, D_y)$.

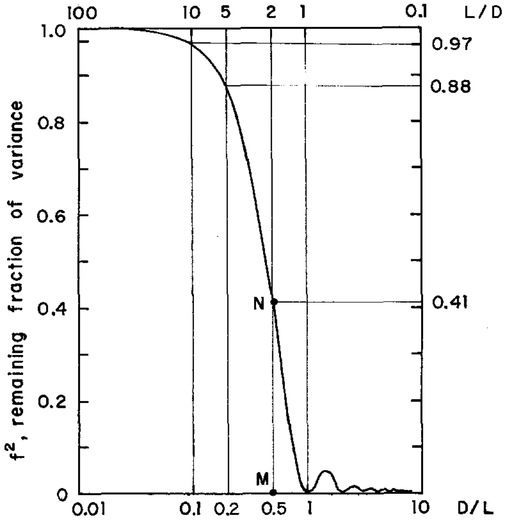

Fig. 2. Fraction of original variance of water distribution remaining effective after smoothing by root-zones of length *D*

Agricultural Significance

The function $f^2\{D/L\}$ is shown graphically in Figure 2. To illustrate its significance to irrigation problems let us consider it from two points of view.

1) Suppose that a certain crop of, say, D = 2 m is to be irrigated. Various irrigation methods may be considered, which have characteristic L values ranging over several orders of magnitudes. Any method for which $L \leqq 2$ m (e.g. drippers and porous tubes) would result in an almost uniform effective water distribution. Any method for which $L \geqq 20$ m (e.g. giant sprinklers) would result in an effective distribution very similar to the actual one. In the intermediate range (2 m $< L <$ 20 m), the effective variance is very strongly dependet on the system selected. This shows why trickle irrigation results in high *effective* uniformity, while its detailed *actual* distribution is very non-uniform.

2) Suppose that a certain sprinkling system, say small sprinklers spaced at 6 m intervals, is to be used to irrigate a range of crops. Vegetables whose root zone is smaller than 0.6 m in horizontal extent, would react to an effective water distribution similar to the actual one. Trees with root zones in excess of 6 m, would grow as if the distribution of water was uniform. This shows why under-canopy sprinkling of orchards is not required to produce a uniformity as high as is required for field crops or vegetables.

It should be noted that in principle this method of analysis applies to situations where deep seepage does not occur, where the variability of irrigation application is uniform throughout the field, and to crops whose effective root-zones cover the whole area. The latter may not be the case in certain orchards and for crops in the initial stages of their development. If, however, the irrigation is limited to the area occupied by the existing root-zones (e.g. drippers around the trunks of trees), the analysis becomes valid once again.

An interesting point concerns the effect of relative phase shift between the crop and the irrigation system on the effective results. An example has already been given in Figure 1, where a shift of phase between 1 a and 1 b produced significantly different effective results. This pair of cases can be identified in Figure 2 as points M and N respectively. (For point N (case 1 b) $(2/\pi)^2 \cong 0.41$). All intermediate cases between 1 a and 1 b fall on the line connecting M and N.

It can be shown that the dependence of the effective uniformity on phase shift is peculiar to the ratio $L/D=2$. All other ratios behave as indicated by f^2 (Figure 2). While not a widespread situation, $L/D=2$ may occur in under-canopy sprinkling of orchards. Therefore, from this point of view, it would be preferable to have the sprinklers situated in the middle of the space between trees rather than near the trees themselves.

Results and Discussion

A computer program (Seginer, 1977) was prepared to perform (on an input rectangular distribution pattern), the harmonic analysis described above. The output includes a uniformity coefficient based on the standard deviation of the effective distribution for a range of root-zones:

$$U[D]=1-S[D]/\bar{W} \tag{13}$$

where $U[D]$ is the effective uniformity for root-zone of size D, $S[D]$ is the standard deviation of the effective distribution, and $\bar{W}$ is the mean water application depth. A few sample results will now be presented.

Figure 3 shows a water distribution map whose *actual* uniformity coefficient (as determined conventionally) is $U[O]=0.51$. The harmonic analysis showed that the distribution of Figure 3 includes only 3 significant waves that together are responsi-

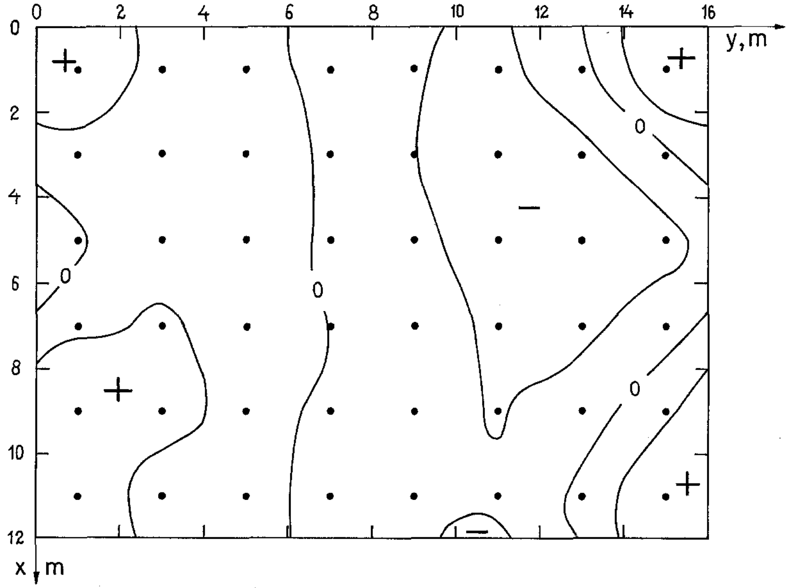

Fig. 3. A water distribution pattern of uniformity $U[0]=0.51$. The lines designated by 0 indicate the mean depth of water. The iso-lines surrounding the (–) signs indicate depth smaller than the mean by one standard deviation. Those surrounding the (+) signs are for one standard deviation above the mean. Dots indicate locations of depth-measuring collectors

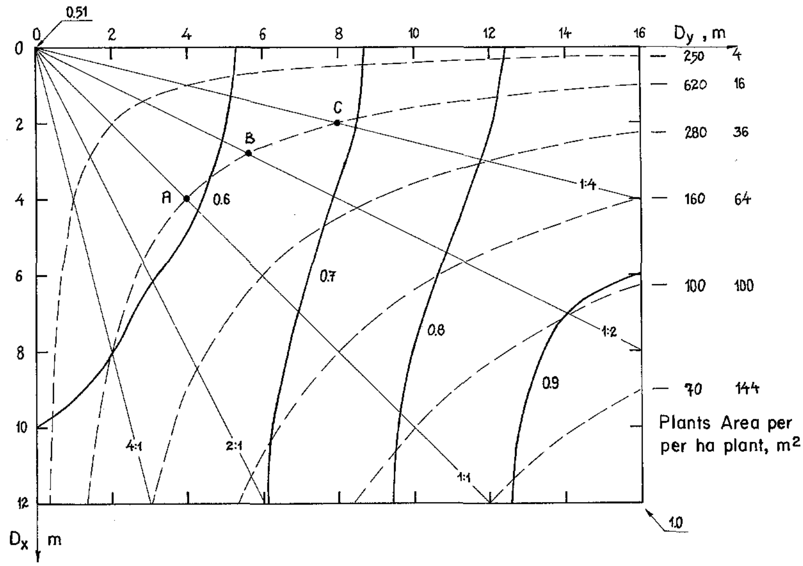

Fig. 4. Effective uniformity coefficient of map of Figure 3, as function of root-zone dimensions (D_x, D_y). Heavy lines connect equal U values, dashed lines – equal area-per-plant values, and slanting lines – equal D_x/D_y ratios. Reference to points A, B, and C is made in text

ble for 82% of the total variance. The dominant wave had its crest parallel to the x-axis and a wave-length of 16 m.

Figure 4 shows, for the map of Figure 3, the effective uniformity coefficient as a function of the root-zone dimensions (D_x, D_y). The value 1.0 at the right lower corner is the effective uniformity for plants whose root-zone covers exactly the area $12 \times 16\ m^2$. The value 0.51 at the left upper corner is the appropriate value for „point" plants, i.e. the "actual" uniformity. The figure shows that in this example the effective uniformity generally increases at the ratio D_x/D_y decreases. This is illustrated by points A, B and C, all on the $16\ m^2$ per plant (tree) curve. If the rows are parallel to the x-axis, the wider the space between rows (while the spacing along the row decreases accordingly), the better the effective uniformity. At point A ($4 \times 4\ m^2$) the uniformity is about 0.59, while at point C ($8 \times 2\ m^2$) it is about 0.69.

Analysis of a number of rectangular distribution maps indicated that the map of Figure 3 is typical in the sense that the dominant wave has a crest parallel to the shorter sprinkler spacing. Therefore, it would be generally true that a better effective distribution can be obtained with rows parallel to the shorter sprinkler spacing (x direction in Figure 3). This is normally the case, especially in field crops.

Let us now examine the proposed method of analysis from a somewhat different angle: Suppose that the same 6×8 water-depth values, as used to plot and analyze the map of Figure 3, are mixed to produce a *random* arrangement over the 6×8 matrix. The artificial distribution thus obtained can be analyzed just as if it were a real distribution. The results for the 'real' and 'random' maps are compared in Figure 5. The uniformities for point plants and $12 \times 16\ m^2$ plants are, of course, identical for both maps. For all intermediate root-zones the effective uniformity is higher for the random pattern. The reason for this is that actual distribution maps are

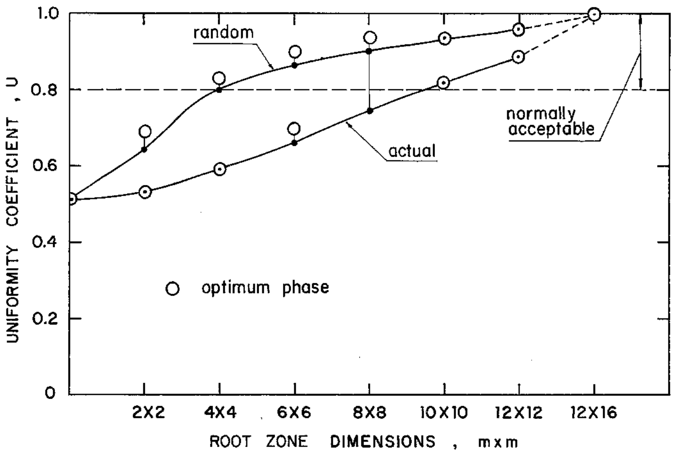

Fig. 5. Uniformity coefficient for square root-zones under two water distribution maps: That of Figure 3 and a randomized version of it

dominated by the long waves and these can only be 'smoothed out' by large root-zones. The randomized maps do not contain dominant waves. Figure 5 also shows the effect of phase shift on the uniformity. A significant effect is observed only for the actual map when $D_y = 8$, namely half the wave-length of the dominant harmonic component. Due to the roughly equal distribution of the total variance over all wave-lengths in the randomized map, the effect of phase shift is also well distributed.

The observation that most of the variance is concentrated in the longer wave-lengths, leads to the conclusion that fewer collectors may be used in the field test than is recommended, without significant effect on the calculated uniformity. The American Society of Agricultural Engineers recommends (ASAE, 1977, p. 555) that 'a minimum of 80 collectors shall receive water during a test'. If the radius of area wetted by a single sprinkler is denoted by R, the recommendation leads to spacing the collectors on a $0.2\,R \times 0.2\,R$ grid. With sprinkler spacing of about $1.5\,R \times 1.5\,R$, the number of points per distribution map would be 50 to 60, roughly as in Figure 3. Figure 5 suggests that increasing the spacing of collectors from 2×2 m^2 to 4×4 m^2 would not change significantly the calculated uniformity.

To test this assumption, 12 distribution maps (from Elhanani & Kraus, 1956) were analyzed twice: Once using the data from all the collectors, and a second time utilizing only a quarter of the data. The original maps were based on either 6×6 or 6×8 data points, and of these, 3×3 or 3×4 data points, respectively, were utilized for the second analysis. The resulting uniformity coefficients are shown in Figure 6. The uniformity coefficients based on only a quarter of the data, $U_{1/4}$, are on the average higher by about 0.5% than the more accurate values based on all the data, U_1. This slight increase is in the right direction (namely, resulting from reduced variance) and is insignificant compared to the scatter around the mean, which has a standard deviation of about 2%. This supports the above-mentioned assumption.

The error of estimate of U is composed of variances from several sources, all of which are in a sense sampling errors. The main sources of error are (1) differences between items of the same sprinkler model, (2) differences between tests with the

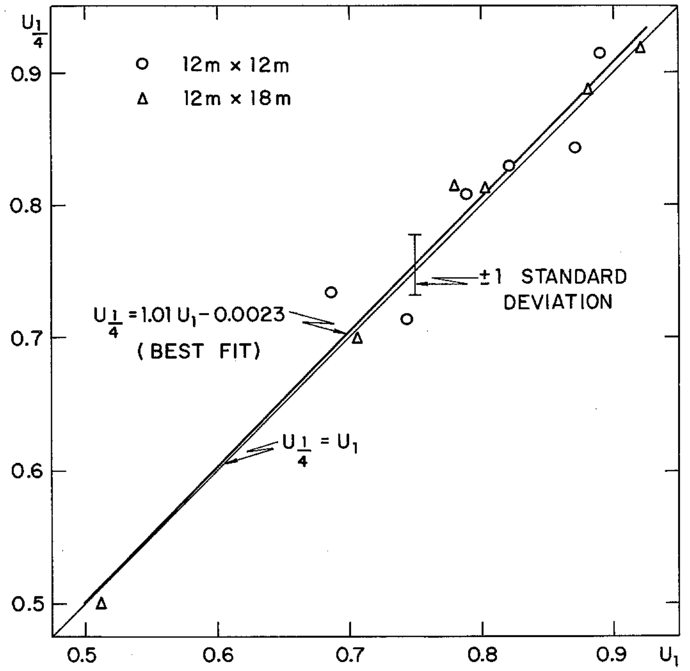

Fig. 6. The relation between the uniformity coefficients based on all collectors and on a quarter of the data

same sprinkler under presumably identical conditions and (3) insufficient spatial sampling of the distribution patterns. The latter source is the one to which Davis (1966) paid attention, but it is not necessarily the most important one. If it is, then the recommendation of Davis to increase collector density should be implemented. If not, a reduced number of collectors in conjunction with increased number of (perhaps shorter) tests, may prove to be more efficient. The results of Figure 6 may be used as a first estimate of the error involved in reduced sampling density.

In conclusion it should be emphasized that the harmonic analysis does not require any input data that are not being collected in the course of a standard sprinkler distribution test. Once a suitable computer program is available, an optional output of the effective uniformity coefficients for a range of root-zone sizes can be obtained practically free of charge.

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