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ON THE LOGIC OF DEMONSTRATIVES

In this paper, I propose to outline briefly a few results of my investigations into the theory of demonstratives: words and phrases whose *in*tension is determined by the contexts of their use. Familiar examples of demonstratives are the nouns 'I', 'you', 'here', 'now', 'that', and the adjectives 'actual' and 'present'. It is, of course, clear that the *ex*tension of 'I' is determined by the context – if you and I both say 'I' we refer to different persons. But I would now claim that the intension is also so determined. The intension of an 'eternal' term (like 'The Queen of England in 1973') has generally been taken to be represented by a function which assigns to each possible world the Queen of England in 1973 of that world. Such functions would have been called *individual concepts* by Carnap. It has been thought by some – myself among others – that by analogy, the intension of 'I' could be represented by a function from speakers to individuals (in fact, the identity function). And similarly, that the intensions of 'here' and 'now' would be represented by (identity) functions on places and times. The role of contextual factors in determining the extension (with respect to such factors) of a demonstrative was thought of as analogous to that of a possible world in determining the extension of 'The Queen of England in 1973' (with respect to that possible world). Thus an enlarged view of an intension was derived. The intension of an expression was to be represented by a function from certain factors to the extension of the expression (with respect to those factors). Originally such factors were simply possible worlds, but as it was noticed that the so-called tense operators exhibited a structure highly analogous to that of the modal operators, the factors with respect to which an extension was to be determined were enlarged to include moments of time. When it was noticed that contextual factors were required to determine the extension of sentences containing demonstratives, a still more general notion was developed and called an 'index'. The extension of an expression was to be determined with respect to an index. The intension of an expression was that function which assigned to every index, the extension at that index. Here is a typical passage.

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The above example supplies us with a statement whose truth-value is not constant but varies as a function of $i \in I$. This situation is easily appreciated in the context of time-dependent statements; that is, in the case where I represents the instants of time. Obviously the same statement can be true at one moment and false at another. For more general situations one must not think of the $i \in I$ as anything as simple as instants of time or even possible worlds. In general we will have

$$i = (w, t, p, a, \ldots)$$

where the index i has many *coordinates*: for example, w is a *world*, t is a *time*, $p = (x, y, z)$ is a (3-dimensional) *position* in the world, a is an *agent*, etc. All these coordinates can be varied, possibly independently, and thus affect the truth values of statements which have indirect reference to these coordinates. (From the Advice of a prominent logician.)

A sentence ϕ was taken to be logically true if true at every index (in every 'structure'), and $\Box\phi$ was taken to be true at a given index (in a given structure) just in case ϕ was true at every index (in that structure).[2] Thus the familiar principle of modal generalization: if $\models \phi$, then $\models \Box\phi$, is validated.

This view, in its treatment of demonstratives, now seems to me to have been technically wrong (though perhaps correctable by minor modification) and, more importantly, conceptually misguided.

Consider the sentence

(1) I am here now.

It is obvious that for many choices of index – i.e. for many quadruples $\langle w, x, p, t\rangle$ where w is a possible world, x is a person, p is a place, and t is a time – (1) will be false. In fact, (1) is true only with respect to those indices $\langle w, x, p, t\rangle$ which are such that in the world w, x is located at p at the time t. Thus (1) fares about on a par with

(2) David Kaplan is in Los Angeles on April 21, 1973.

(2) is contingent, and so is (1).

But here we have missed something essential to our understanding of demonstratives. Intuitively, (1) is deeply, and in some sense universally, true. One need only understand the meaning of (1) to know that it cannot be uttered falsely. No such guarantees apply to (2). A *Logic of*

Demonstratives which does not reflect this intuitive difference between (1) and (2) has bypassed something essential to the logic of demonstratives.

Here is a proposed correction. Let the class of indices be narrowed to include only the *proper* ones – namely, those $\langle w, x, p, t \rangle$ such that in the world w, x *is* located at p at the time t. Such a move may have been intended originally since improper indices are like impossible worlds; no such contexts *could* exist and thus there is no interest in evaluating the extensions of expressions with respect to them. Our reform has the consequence that (1) comes out, correctly, to be logically true. Now consider

(3) $\Box$I am here now.

Since the contained sentence (namely (1)) is true at every proper index, (3) also is true at every proper index and thus also is logically true. (As would be expected by the aforementioned principle of modal generalization.)

But (3) should not be *logically* true, since it is false. It is certainly *not* necessary that I be here now. But for several contingencies, I would be working in my garden now, or even writing this in a location outside of Los Angeles.

Perhaps enough has now been said to indicate that there are difficulties in the attempt to assimilate the role of a *context* in a logic of demonstratives to that of a *possible world* in the familiar modal logics or a *moment of time* in the familiar tense logics.

I believe that the source of the difficulty lies in a conceptual confusion between two kinds of meaning. Ramifying Frege's distinction between sense and denotation, I would add two varieties of sense: content and character. The content of an expression is always taken *with respect to* a given context of use. Thus when I say

(4) I was insulted yesterday.

a specific content – *what I said* – is expressed. Your utterance of the same sentence, or mine on another day, would not express the same content. What is important to note is that it is not just the truth value that may change; what is said is itself different. Speaking today, my utterance of (4) will have a content roughly equivalent to that which

(5) David Kaplan is insulted on April 20, 1973.

would have spoken by you or anyone at any time. Since (5) contains no demonstratives, its content is the same with respect to all contexts. This content is what Carnap called an 'intension' and what, I believe, has been often referred to as a 'proposition'. So my theory is that different contexts for (4) produce not just different truth values, but different propositions.

Turning now to character, I call that component of the sense of an expression which determines how the content is determined by the context, the 'character' of an expression. Just as contents (or intensions) can be represented by functions from possible worlds to extensions, so characters can be represented by functions from contexts to contents. The character of 'I' would then be represented by *the function (or rule, if you prefer) which assigns to each context that content which is represented by the constant function from possible worlds to the agent of the context*. The latter function has been called an 'individual concept'. Note that the character of 'I' is represented by a function from contexts to individual *concepts*, not from contexts to individuals. It was the idea that a function from contexts to individuals could represent the intension of 'I' which lead to the difficulties discussed earlier.

Now what is it that a competent speaker of English knows about the word 'I'? Is it the content with respect to some particular occasion of use? No. It is the character of 'I': the rule italicized above. Competent speakers recognize that the proper use of 'I' is – loosely speaking – to refer to the speaker. Thus, that component of sense which I call 'character' is best identified with what might naturally be called 'meaning'.

To return, for a moment, to (1). The character (meaning) of (1) determines each of the following:

(a) In different contexts, an utterance of (1) expresses different contents (propositions).

(b) In most (if not all) contexts, an utterance of (1) expresses a contingent proposition.

(c) In all contexts, an utterance of (1) expresses a true proposition (i.e. a proposition which is true at the world of the context).

On the basis of (c), we might claim that (1) is analytic (i.e. it is true solely in virtue of its meaning). Although as we see from (b), (1) rarely or never expresses a necessary proposition. This separation of analyticity and necessity is made possible – even, I hope, plausible – by distinguishing the kinds of entities of which 'is analytic' and 'is necessary' are properly predicated: characters (meanings) are analytic, contents (propositions) are necessary.

The distinction between character and content was unlikely to be noticed before demonstratives came under consideration, because demonstrative-free expressions have a constant character, i.e. they express the same content in every context. Thus, character becomes an uninteresting complication in the theory.

Though I have spoken above of contexts of utterance, my primary theoretical notion of *content with respect to a context* does not require that the agent of the context utter the expression in question. I believe that there are good reasons for taking this more general notion as fundamental.

I believe that my distinction between character and content can be used to throw light on Kripke's distinction between the *a-priori* and the necessary. Although my distinction lies more purely within logic and semantics, and Kripke's distinction is of a more general epistemic metaphysical character, both seem to me to be of the same *structure*. (I leave this remark in a rather cryptic state.)

The distinction between content and character and the related analysis of demonstratives have certainly been foreshadowed in the literature (though they are original-with-me, in the sense that I did not consciously extract them from prior sources). But to my knowledge they have not previously been cultivated to meet the standards for logical and semantical theories which currently prevail. In particular, Strawson's distinction between the significance (meaningfulness) of a sentence and the statement (proposition) which is expressed in a given use is clearly related. Strawson recognizes that such sentences as 'The *present* King of France is *now* bald' may express different propositions in different utterances, and he identifies the meaningfulness of the sentence with its potential for expressing a true or false proposition in some possible utterance. Though he does not explicitly discuss *the* meaning of the sentence, it is clear that he would not identify such a meaning with any of the propositions expressed by particular utterances. Unfortunately Strawson seems to regard the fact that sentences containing demonstratives can be used to express different propositions as immunizing such sentences against treatment by 'the logician'.

In order to convince myself that it is possible to carry out a consistent analysis of the semantics of demonstratives along the above lines, I have attempted to carry through the program for a version of first order predicate logic. The result is the following Logic of Demonstratives.

If my views are correct, the introduction of demonstratives into intensional logics will require more extensive reformulation than was thought to be the case.

THE LOGIC OF DEMONSTRATIVES

The *Language* LD is based on first-order predicate logic with identity and descriptions. We deviate slightly from standard formulations in using two sorts of variables, one sort for positions and a second sort for individuals other than positions (hereafter called simply 'individuals').

Primitive Symbols for Two Sorted Predicate Logic

0. Punctuation: (,)
1. (i) An infinite set of individual variables: $\mathscr{V}_i$
   (ii) An infinite set of position variables: $\mathscr{V}_p$
2. (i) An infinite number of *m*-*n*-place predicates, for all natural numbers *m*, *n*
   (ii) The 1-0-place predicate: Exist
   (iii) The 1-1-place predicate: Located
3. (i) An infinite number of *m*-*n*-place *i*-functors (functors which form terms denoting individuals)
   (ii) An infinite number of *m*-*n*-place *p*-functors (functors which form terms denoting positions)
4. Sentential Connectives: $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$
5. Quantifiers: $\forall, \exists$
6. Definite Description Operator: the
7. Identity: =

Primitive Symbols for Modal and Tense Logic

8. Modal Operators: $\Box, \Diamond$

9. Tense Operators: F (it will be the case that)
P (it has been the case that)
G (one day ago, it was the case that)

Primitive Symbols for the Logic of Demonstratives

10. Three one place sentential operators:
N (it is now the case that)
A (it is actually the case that)
Y (yesterday, it was the case that)

11. A one place functor: dthat

12. An individual constant (0-0-place *i*-functor): I

13. A position constant (0-0-place *p*-functor): Here

The *well formed expressions* are of three kinds: formulas, position terms (*p*-terms) and individual terms (*i*-terms).

1. (i) If $\alpha \in \mathscr{V}_i$, then α is an *i*-term
(ii) If $\alpha \in \mathscr{V}_p$, then α is a *p*-term

2. If π is an *m*-*n*-place predicate, $\alpha_1 \ldots \alpha_m$ are *i*-terms, and $\beta_1 \ldots \beta_n$ are *p*-terms, then $\pi\alpha_1 \ldots \alpha_m\beta_1 \ldots \beta_n$ is a formula.

3. (i) If η is an *m*-*n*-place *i*-functor, $\alpha_1 \ldots \alpha_m, \beta_1 \ldots \beta_n$ as in 2., then $\eta\alpha_1 \ldots \alpha_m\beta_1 \ldots \beta_n$ is an *i*-term.
(ii) If η is an *m*-*n*-place *p*-functor, $\alpha_1 \ldots \alpha_m, \beta_1 \ldots \beta_n$ as in 2., then $\eta\alpha_1 \ldots \alpha_m\beta_1 \ldots \beta_n$ is a *p*-term.

4. If ϕ, ψ are formulas, then $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $\neg\phi$, $(\phi \rightarrow \psi)$, $(\phi \longleftrightarrow \psi)$ are formulas.

5. If ϕ is a formula and $\alpha \in \mathscr{V}_i \cup \mathscr{V}_p$, then $\forall\alpha\phi$, $\exists\alpha\phi$ are formulas.

6. If ϕ is a formula, then
(i) if $\alpha \in \mathscr{V}_i$, then the $\alpha\,\phi$ is an *i*-term.
(ii) if $\alpha \in \mathscr{V}_p$, then the $\alpha\,\phi$ is a *p*-term.

7. If both α, β are either *i*-terms or *p*-terms, then $\alpha = \beta$ is a formula.

8. If ϕ is a formula, then $\Box\phi$, $\Diamond\phi$ are formulas.

9. If ϕ is a formula, then $F\phi$, $P\phi$, $G\phi$ are formulas.

10. If ϕ is a formula, then $N\phi$, $A\phi$, $Y\phi$ are formulas.

11. (i) If α is an *i*-term, then dthat α is an *i*-term.

    (ii) If α is a *p*-term, then dthat α is a *p*-term.

Semantics for LD

DEFINITION. $\mathfrak{A}$ *is an LD Structure* iff there are $\mathscr{C}\,\mathscr{W}\,\mathscr{U}\,\mathscr{P}\,\mathscr{T}\,\mathscr{I}$ such that

1. $\mathfrak{A} = \langle \mathscr{C}\,\mathscr{W}\,\mathscr{U}\,\mathscr{P}\,\mathscr{T}\,\mathscr{I} \rangle$.

2. $\mathscr{C}$ is a non-empty set (the set of *contexts*, see 10 below).

3. If $c \in \mathscr{C}$, then (i) $c_A \in \mathscr{U}$ (the *agent* of c).

   (ii) $c_T \in \mathscr{T}$ (the *time* of c).

   (iii) $c_P \in \mathscr{P}$ (the *position* of c).

   (iv) $c_W \in \mathscr{W}$ (the *world* of c).

4. $\mathscr{W}$ is a non-empty set (the set of *worlds*).

5. $\mathscr{U}$ is a non-empty set (the set of all *individuals*, see 9 below).

6. $\mathscr{P}$ is a non-empty set (the set of *positions*; common to all worlds).

7. $\mathscr{T}$ is the set of integers (thought of as the *times*; common to all worlds).

8. $\mathscr{I}$ is a function which assigns to each predicate and functor an appropriate *intension* as follows:

   (i) If π is an *m-n*-place predicate, $\mathscr{I}_\pi$ is a function such that for each $t \in \mathscr{T}$ and $w \in \mathscr{W}$, $\mathscr{I}_\pi(tw) \subseteq (\mathscr{U}^m \times \mathscr{P}^n)$.

   (ii) If η is an *m-n*-place *i*-functor, $\mathscr{I}_\eta$ is a function such that for each $t \in \mathscr{T}$ and $w \in \mathscr{W}$,

$$\mathscr{I}_\eta(tw) \in (\mathscr{U} \cup \{\dagger\})^{(\mathscr{U}^m \times \mathscr{P}^n)}.$$

(*Note*: † is a completely alien entity, in neither $\mathscr{U}$ nor $\mathscr{P}$, which represents an 'undefined' value of the function. In a normal set theory we can take † to be $\{\mathscr{U}, \mathscr{P}\}$.)

(iii) If η is an m-n-place p-functor, $\mathscr{I}_\eta$ is a function such that for each $t \in \mathscr{T}$ and $w \in \mathscr{W}$, $\mathscr{I}_\eta(tw) \in (\mathscr{P} \cup \{\dagger\})^{(\mathscr{U}^m \times \mathscr{P}^n)}$

9. $i \in \mathscr{U}$ iff $\exists t \in \mathscr{T}\ \exists w \in \mathscr{W}\ \langle i \rangle \in \mathscr{I}_{\mathbf{Exists}}(tw)$.

10. If $c \in \mathscr{C}$, then $\langle c_A\ c_P \rangle \in \mathscr{I}_{\mathbf{Located}}(c_T\, c_W)$.

11. If $\langle i\ p \rangle \in \mathscr{I}_{\mathbf{Located}}(tw)$, then $\langle i \rangle \in \mathscr{I}_{\mathbf{Exists}}(tw)$.

Truth and Denotation in a Context

We write: $\models^{\mathfrak{A}}_{cftw} \phi$ for ϕ when taken in the context c (under the assignment f and in the structure $\mathfrak{A}$) *is true with respect to* the time t and the world w.

We write: $|\alpha|^{\mathfrak{A}}_{cftw}$ for *The denotation of* α when taken in the context c (under the assignment f and in the structure $\mathfrak{A}$) *with respect to* the time t and the world w.

In general we will omit the superscript '$\mathfrak{A}$', and we will assume that the structure $\mathfrak{A}$ is $\langle \mathscr{C}\mathscr{W}\mathscr{U}\mathscr{P}\mathscr{T}\mathscr{I} \rangle$.

DEFINITION. *f is an assignment* (with respect to $\langle \mathscr{C}\mathscr{W}\mathscr{U}\mathscr{P}\mathscr{T}\mathscr{I} \rangle$) iff

$$\exists f_1 f_2 (f_1 \in \mathscr{U}^{\mathscr{V}_i}\ \&\ f_2 \in \mathscr{P}^{\mathscr{V}_p}\ \&\ f = f_1 \cup f_2).$$

DEFINITION. $f^\alpha_x = (f \sim \{\langle \alpha\, f(\alpha) \rangle\}) \cup \{\langle \alpha\, x \rangle\}$ (i.e. the assignment which is just like f except that it assigns x to α).

For the following recursive definitions, assume that $c \in \mathscr{C}$, f is an assignment, $t \in \mathscr{T}$, and $w \in \mathscr{W}$.

1. If α is a variable, $|\alpha|_{cftw} = f(\alpha)$.

2. $\models_{cftw} \pi\alpha_1 \ldots \alpha_m \beta_1 \ldots \beta_n$ iff $\langle |\alpha_1|_{cftw} \ldots |\beta_n|_{cftw} \rangle \in \mathscr{I}_\pi(tw)$.

3. If η is neither I nor Here (see 12, 13 below), then

$$|\eta\alpha_1 \ldots \alpha_m\beta_1 \ldots \beta_n|_{cftw} = \begin{cases} \mathscr{I}_\eta(tw)(\langle|\alpha_1|_{cftw} \ldots |\beta_n|_{cftw}\rangle), \text{ if none of } |\alpha_j|_{cftw}\,|\beta_k|_{cftw} \text{ are } \dagger \\ \dagger, \text{ otherwise.} \end{cases}$$

4. (i) $\models_{cftw} (\phi \wedge \psi)$ iff $\models_{cftw} \phi \;\&\; \models_{cftw} \psi$.

(ii) $\models_{cftw} \neg\phi$ iff $\sim \models_{cftw} \phi$.

etc.

5. (i) If $\alpha \in \mathscr{V}_i$, then $\models_{cftw} \forall\alpha\phi$ iff $\forall_i \in \mathscr{U} \models_{cf_i^\alpha tw} \phi$.

(ii) If $\alpha \in \mathscr{V}_p$, then $\models_{cftw} \forall\alpha\phi$ iff $\forall_p \in \mathscr{P} \models_{cf_p^\alpha tw} \phi$.

Similarly for $\exists\alpha\phi$.

6. (i) If $\alpha \in \mathscr{V}_i$, then $|\text{the } \alpha\, \phi|_{cftw}$

$$= \begin{cases} \text{the unique } i \in \mathscr{U} \text{ such that } \models_{cf_i^\alpha tw} \phi, \text{ if there is such} \\ \dagger, \text{ otherwise.} \end{cases}$$

(ii) Similarly for $\alpha \in \mathscr{V}_p$.

7. $\models_{cftw} \alpha = \beta$ iff $|\alpha|_{cftw} = |\beta|_{cftw}$.

8. (i) $\models_{cftw} \Box\phi$ iff $\forall w' \in \mathscr{W} \models_{cftw'} \phi$.

(ii) $\models_{cftw} \Diamond\phi$ iff $\exists w' \in \mathscr{W} \models_{cftw'} \phi$.

9. (i) $\models_{cftw} F\phi$ iff $\exists t' \in \mathscr{T}$ such that $t' > t$ and $\models_{cft'w} \phi$.

(ii) $\models_{cftw} P\phi$ iff $\exists t' \in \mathscr{T}$ such that $t' < t$ and $\models_{cft'w} \phi$.

(iii) $\models_{cftw} G\phi$ iff $\models_{cf(t-1)w} \phi$.

10. (i) $\models_{cftw} N\phi$ iff $\models_{cfc_T w} \phi$.

(ii) $\models_{cftw} A\phi$ iff $\models_{cftc_W} \phi$.

(ii) $\models_{cftw} Y\phi$ iff $\models_{cf(c_T - 1)w} \phi$.

11. $|\text{dthat } \alpha|_{cftw} = |\alpha|_{cfc_T c_W}$.

12. $|\text{I}|_{cftw} = c_A$.

13. $|\text{Here}|_{cftw} = c_P$.

Remark 1. Expressions containing demonstratives will, in general, express different concepts in different contexts. We call the concept expressed in a given context, the *Content* of the expression in that context. The Content of a sentence in a context is, roughly, the proposition the sentence would express if uttered in that context. This description is not quite accurate on two counts. First, it is important to distinguish an *utterance* from a *sentence-in-a-context*. The former notion is from the theory of speech acts, the latter from semantics. Utterances take time, and utterances of distinct sentences can not be simultaneous (i.e. in the same context). But in order to develop a logic of demonstratives it seems most natural to be able to evaluate several premisses and a conclusion all in the same context. Thus, the notion of ϕ being true in c and $\mathfrak{A}$ does not require an utterance of ϕ. In particular, c_A need not be uttering ϕ in c_W at c_T. Second, the truth of a proposition is not usually thought of as dependent on a time as well as a possible world. The time is though of as fixed by the context. If ϕ is a sentence, the more usual notion of the proposition expressed by ϕ-in-c is what is here called the Content of $N\phi$ in c.

Where Γ is either a term or a formula, we write: $\{\Gamma\}^{\mathfrak{A}}_{cf}$ for

the Content of Γ in the context c (under the assignment f and in the structure $\mathfrak{A}$).

DEFINITION. (i) If ϕ is a formula, $\{\phi\}_{cf}$ = that function which assigns to each $t \in \mathscr{T}$ and $w \in \mathscr{W}$, Truth if $\models^{\mathfrak{A}}_{cftw} \phi$, and Falsehood otherwise.

(ii) If α is a term, $\{\alpha\}_{cf}$ = that function which assigns to each $t \in \mathscr{T}$ and $w \in \mathscr{W}$, $|\alpha|^{\mathfrak{A}}_{cftw}$.

Remark 2. $\models^{\mathfrak{A}}_{cftw} \phi$ iff $\{\phi\}^{\mathfrak{A}}_{cf}(tw)$ = Truth. Roughly speaking, the sentence ϕ taken in the context c is *true with respect to t* and w iff the proposition expressed by ϕ-in-the-context-c would be true at the time t if w were the actual world. In the formal development of pages 89 and 90 it was smoother to ignore the conceptual break marked by the notion of

Content in a context and to directly define *truth in a context with respect to a possible time and world*. The important conceptual role of the notion of Content is partially indicated by the following two definitions.

DEFINITION. ϕ *is true in the context c* (in the structure $\mathfrak{A}$) iff for every assignment f, $\{\phi\}_{cf}^{\mathfrak{A}}(c_T, c_W) = \text{Truth}$.

DEFINITION. ϕ is valid in LD ($\models\phi$) iff for every LD structure $\mathfrak{A}$, and every context c of $\mathfrak{A}$, ϕ is true in c (in $\mathfrak{A}$).

Remark 3. $\models(\alpha = \text{dthat } \alpha)$, $\models$N (Located I, Here), $\models$Exist I, $\sim\models\Box(\alpha = \text{dthat } \alpha)$, $\sim\models\Box$N(Located I, Here), $\sim\models\Box$(Exist I). In the converse direction we have the usual results in view of the fact that $\models(\Box\phi \rightarrow \phi)$.

DEFINITION. If $\alpha_1 \ldots \alpha_n$ are all the free variables of ϕ in alphabetical order, then *the closure of* ϕ = AN$\forall\alpha_1 \ldots \alpha_n\phi$.

DEFINITION. ϕ *is closed* iff ϕ is equivalent to its closure (in the sense of Remark 12, below).

Remark 4. If ϕ is closed, then ϕ is true in c (and $\mathfrak{A}$) iff for every assignment f, time t, and world $w \models_{cftw}^{\mathfrak{A}} \phi$.

DEFINITION. Where Γ is either a term or a formula, *the Content of* Γ *in the context c (in the structure* $\mathfrak{A}$*) is stable* iff for every assignment f, $\{\Gamma\}_{cf}^{\mathfrak{A}}$ is a constant function. (i.e., $\{\Gamma\}_{cf}^{\mathfrak{A}}(tw) = \{\Gamma\}_{cf}^{\mathfrak{A}}(t'w')$, for all t, t', w, w' in $\mathfrak{A}$).

Remark 5. Where ϕ is a formula, α is a term, and β is a variable, each of the following has a stable Content in every context (in every structure): ANϕ, dthat α, β, I, Here.

If we were to extend the notion of Content to apply to operators, we would see that all demonstratives have a stable Content in every context. The same is true of the familiar logical constants although it does not hold for the modal and tense operators (not, at least, according to the foregoing development).

Remark 6. That aspect of the meaning of an expression which determines what its Content will be in each context, we call the *Character* of the expression. Although a lack of knowledge about the context (or perhaps about the structure) may cause one to mistake the Content of a given utterance, the Character of each well formed expression is determined by rules of the language (such as 1–13 pages 89–91 above) which are presumably known to all competent speakers. Our notation '$\{\phi\}^{\mathfrak{A}}_{cf}$' for the Content of an expression gives a natural notation for the Character of an expression, namely '$\{\phi\}$'.

DEFINITION. Where Γ is either a term or a formula the *Character of* Γ is that function which assigns to each structure $\mathfrak{A}$, assignment f, and context c of $\mathfrak{A}$, $\{\Gamma\}^{\mathfrak{A}}_{cf}$.

DEFINITION. Where Γ is either a term or a formula, *the Character of* Γ *is stable* iff for every structure $\mathfrak{A}$, and assignment f the Character of Γ (under f in $\mathfrak{A}$) is a constant function. (i.e. $\{\Gamma\}^{\mathfrak{A}}_{cf} = \{\Gamma\}^{\mathfrak{A}}_{c'f}$ for all c, c' in $\mathfrak{A}$).

Remark 7. A formula or term has a stable Character iff it has the same Content in every context (for each $\mathfrak{A}, f$).

Remark 8. A formula or term has a stable Character iff it contains no essential occurrence of a demonstrative.

Remark 9. The logic of demonstratives determines a sub-logic of those formulas of LD which contain no demonstratives. These formulas (and their equivalents which contain inessential occurrences of demonstratives) are exactly the formulas with a stable Character. The logic of demonstratives brings a new perspective even to formulas such as these. The sub-logic of LD which concerns only formulas of stable Character is not identical with traditional logic. Even for such formulas, the familiar Principle of Necessitation: if $\models\phi$, then $\models\Box\phi$, fails. And so does its tense logic counterpart: if $\models\phi$, then $\models (\neg P \neg\phi \wedge \neg F \neg\phi \wedge \phi)$. From the perspective of LD, validity is truth in every possible *context*. For traditional logic, validity is truth in every possible *circumstance*. Each possible context determines a possible circumstance, but, it is not the case that each possible circumstance is part of a possible context. In particular, the fact that each possible context has an agent implies that any possible circumstance in which no individuals exist will not form a part of any possible context. Within LD, a possible context is represented by $\langle\mathfrak{A}, c\rangle$ and a possible circumstance by $\langle\mathfrak{A}, t, w\rangle$. To any $\langle\mathfrak{A}, c\rangle$, there corresponds $\langle\mathfrak{A}, c_T, c_W\rangle$. But it is not the case that to every $\langle\mathfrak{A}, t, w\rangle$ there exists a context c of $\mathfrak{A}$ such that $t = c_T$ and $w = c_W$. The result is that in LD such sentences as

$\exists x$ Exist x, and $\exists x \exists p$ Located x, p are valid, although they would not be so regarded in traditional logic. At least not in the neo-traditional logic that countenances empty worlds. Using the semantical developments of pages 88–91, we can define this traditional sense of validity (for formulas which do not contain demonstratives) as follows. First note that by Remark 7, if ϕ has a stable Character

$$\models^{\mathfrak{A}}_{cftw} \phi \quad \text{iff} \quad \models^{\mathfrak{A}}_{c'ftw} \phi.$$

Thus for such formulas we can define,

ϕ *is true at tw (in* $\mathfrak{A}$*)* iff for every assignment f and every context c $\models^{\mathfrak{A}}_{cftw} \phi$.

The neo-traditional sense of validity is now definable as follows,

$\models_{T} \phi$ iff for all structures $\mathfrak{A}$, times t, and worlds w, ϕ is true at tw (in $\mathfrak{A}$).

(Properly speaking, what I have called the neo-traditional sense of validity is the notion of validity now common for a quantified S5 modal tense logic with individual variables ranging over possible individuals and a predicate of existence.) Adding the subscript 'LD' for explicitness, we can now state some results.

(i) If ϕ contains no demonstratives, if $\models_{T} \phi$, then $\models_{LD} \phi$.

(ii) $\models_{LD} \exists x$ Exist x, but $\sim \models_{T} \exists x$ Exist x.

Of course $\Box \exists x$ Exist x is not valid even in LD. Nor are its counterparts, $\neg F \neg \exists x$ Exist x and $\neg P \neg \exists x$ Exist x.

This suggests that we can transcend the context oriented perspective of LD by generalizing over times and worlds so as to capture those possible circumstances $\langle \mathfrak{A}, t, w \rangle$ which do not correspond to any possible contexts $\langle \mathfrak{A}, c \rangle$. We have the following result.

(iii) If ϕ contains no demonstratives
$\models_{T} \phi$ iff $\models_{LD} \Box(\neg P \neg \phi \wedge \neg F \neg \phi \wedge \phi)$.

Although our definition of the neo-traditional sense of validity was motivated by consideration of demonstrative-free formulas, we could apply it also to formulas containing essential occurrences of demonstratives.

To do so would nullify the most interesting features of the logic of demonstratives. But it raises the question, can we express our new sense of validity in terms of the neo-traditional sense. This can be done:

(iv) $\models_{\text{LD}} \phi$ iff $\models_{\text{T}} \text{AN}\phi$.

Remark 10. Rigid designators (in the sense of Kripke) are terms with a stable Content. Since Kripke does not discuss demonstratives, his examples all have, in addition, a stable Character (by Remark 8). Kripke claims that for proper names α, β it may happen that $\alpha = \beta$, though not *a-priori*, is nevertheless necessary. This, in spite of the fact that the names α, β may be introduced by means of descriptions α', β' for which $\alpha' = \beta'$ is not necessary. An analogous situation holds in LD. Let α', β' be definite descriptions (without free variables) such that $\alpha' = \beta'$ is not *a-priori*, and consider the rigid terms dthat α' and dthat β' which are formed from them. We know that $\models (\text{dthat } \alpha' = \text{dthat } \beta' \leftrightarrow \alpha' = \beta')$. Thus, if $\alpha' = \beta'$ is not *a-priori*, neither is dthat α' = dthat β'. But, since $\models [\text{dthat } \alpha' = \text{dthat } \beta' \rightarrow \Box(\text{dthat } \alpha' = \text{dthat } \beta')]$, it may happen that dthat α' = dthat β' is necessary. The converse situation can also be illustrated in LD. Since $(\alpha = \text{dthat } \alpha)$ is valid (see Remark 3), it is surely capable of being known *a-priori*. But if α lacks a stable Content (in some context c), $\Box(\alpha = \text{dthat } \alpha)$ will be false.

Remark 11. Our *o-o*-place *i*-functors are not proper names, in the sense of Kripke, since they do not have a stable Content. But they can easily be converted by means of the stabilizing influence of dthat. Even dthat α lacks a stable Character. The process by which such expressions are converted into expressions with a stable Character is 'dubbing' – a form of definition in which context may play an essential role. The means to deal with such context indexed definitions is not available in our object language.

There would, of course, be no difficulty in supplementing our language with a syntactically distinctive set of *o-o*-place *i*-functors whose semantics requires them to have both a stable Character and a stable Content in every context. Variables already behave this way, what is wanted is a class of constants that behave, in these respects, like variables.

The difficulty comes in expressing the definition. My thought is that when a name, like 'Bozo', is introduced by someone saying, in some context c^*, 'Let's call the Governor, "Bozo"', we have a context indexed definition of the form: $A =_{c^*} \alpha$, where A is a new constant (here, 'Bozo') and α is some term whose denotation depends on context (here, 'the Governor'). The intention of such a dubbing is, presumably, to induce the semantical clause: for all c, $\{A\}^{\mathfrak{A}}_{cf} = \{\alpha\}^{\mathfrak{A}}_{c^*f}$. Such a clause gives A a stable Character. The context indexing is required by the fact that the Content of α (the 'definiens') may vary from context to context. Thus the same semantical clause is not induced by taking either $A = \alpha$ or even $A = \text{dthat } \alpha$ as an axiom;

I think it likely that such definitions play a practically (and perhaps theoretically) indispensable role in the growth of language, allowing us to introduce a vast stock of names on the basis of a meager stock of demonstratives and some ingenuity in the staging of demonstrations.

Perhaps such introductions should not be called 'definitions' at all, since they essentially enrich the expressive power of the language. What a nameless man may express by 'I am hungry' may be inexpressible in remote contexts. But once he says 'Let's call me "Bozo"' his Content is accessible to us all.

Remark 12. The strongest form of logical equivalence between two formulas ϕ and ϕ' is sameness of Character, $\{\phi\} = \{\phi'\}$. This form of synonymy is expressible in terms of validity.

$$\{\phi\} = \{\phi'\} \quad \text{iff} \quad \models \Box[\neg P \neg(\phi \leftrightarrow \phi') \wedge \neg F \neg(\phi \leftrightarrow \phi') \wedge (\phi \leftrightarrow \phi')].$$

[Using Remark 9 (iii) and dropping the condition, which was stated only to express the intended range of applicability of $\models_{\mathrm{T}}$, we have: $\{\phi\} = \{\phi'\}$ iff $\models_{\mathrm{T}} (\phi \leftrightarrow \phi')$.] Since definitions of the usual kind (as opposed to dubbings) are intended to introduce a short expression as a mere abbreviation of a longer one, the Character of the defined sign should be the same as the Character of the definiens. Thus, with LD, definitional axioms must take the form indicated above.

Remark 13. If β is a variable of the same sort as the term α but is not free in α, then $\{\text{dthat } \alpha\} = \{\text{the } \beta \text{ AN}(\beta = \alpha)\}$. Thus for every formula ϕ, there can be constructed a formula ϕ' such that ϕ' contains no occurrence of dthat and $\{\phi\} = \{\phi'\}$.

Remark 14. Y (yesterday) and G (one day ago) superficially resemble one another in view of the fact that $\models (Y\phi \longleftrightarrow G\phi)$. But the former is a demonstrative whereas the latter is an iterative temporal operator. 'One day ago it was the case that one day ago it was the case that John yawned' means that John yawned the day before yesterday. But 'Yesterday it was the case that yesterday it was the case that John yawned' is only a stutter.

POSSIBLE REFINEMENTS

(1) The primitive predicates and functors of first-order predicate logic are all taken to be extensional. Alternatives are possible.

(2) Many conditions might be added on $\mathscr{P}$; many alternatives might be chosen for $\mathscr{T}$. If the elements of $\mathscr{T}$ do not have a natural relation to play the role of $<$, such a relation must be added to the structure.

(3) When K is a set of LD formulas, $K \models \phi$ is easily defined in any of the usual ways.

(4) Aspects of the contexts other than c_A, c_P, c_T, and c_W would be used if new demonstratives (e.g. pointings, 'You', etc.) were added to the language. (Note that the subscripts A, P, T, W are external parameters. They may be thought of as functions applying to contexts, with c_A being the value of A for the context c.

(5) Special continuity conditions through time might be added for the predicate Exists.

(6) If individuals lacking positions are admitted as agents of contexts, 3(iii) of page 88 should be weakened to $c_P \in \mathscr{P} \cup \{\dagger\}$. It would no longer be the case that $\models$ Located I, Here. If individuals also lacking temporal location (disembodied minds?) are admitted as agents of contexts, a similar weakening is required of 3(ii). In any case it would still be true that $\models$Exist I.

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NOTES

[1] This paper was originally composed in two parts. The formal Logic of Demonstratives was first presented at the Irvine Summer Institute on the Philosophy of Language in 1971. It was expanded in 1973. The initial discursive material was written on April 20, 1973 as part of a research proposal. This paper was intended as a companion piece to and progress report on the material in 'Dthat'. A more

extensive presentation occurs in my manuscript *Demonstratives.* This work was supported by the National Science Foundation.

[2] Or possibly, just in case ϕ was true at every index *which differed from the given index only in possible world coordinate.*

BIBLIOGRAPHY

Kaplan, D., 'Dthat', in Peter Cole (ed.), *Syntax and Semantics*, Vol. 9; *Pragmatics*, Academic Press, New York, 1978, pp. 221–243. Also reprinted in *Contemporary Perspectives in the Philosophy of Language*, ed. by Wettstein *et al.*, The University of Minnesota, Morris, forthcoming.

Kaplan, D., *Demonstratives* (Draft # 2) mimeographed, UCLA Philosophy Department, 1977.

Kripke, S., 'Naming and Necessity', in Donald Davidson and Gilbert Harman (eds.), *Semantics of Natural Language*, Reidel Publishing Co., Dordrecht-Holland, 1972, pp. 253–355; Addenda, pp. 763–69.

Strawson, P., *Introduction to Logical Theory*, John Wiley & Sons, New York, 1952.