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THE TRUE MODAL LOGIC

For Arthur Prior, the construction of a logic was a supremely philosophical task. As a logician, he could of course appreciate a finely crafted formal system for its own sake, independent of its "meaning" or philosophical significance. But a genuine *logic* – a good one, at least – lays bare the nature of those concepts that it purports to be a logic *of*; and this comes only by way of deep reflection and insightful philosophical analysis.

This sort of reflection and analysis is no more evident than in Prior's own search for the "true" modal logic.[1] In this paper, I want to trace the course of Prior's own struggles with the concepts and phenomena of modality, and the reasoning that led him to his own rather peculiar modal logic **Q**. I find myself in almost complete agreement with Prior's intuitions and the arguments that rest upon them. However, I will argue that those intuitions do not of themselves lead to **Q**, but that one must also accept a certain picture of what it is for a proposition to be possible. That picture, though, is not inevitable. Rather, implicit in Prior's own account is an alternative picture that has already appeared in various guises, most prominently in the work of Adams [1], Fine [5], and more recently, Deutsch [4][2] and Almog [2]. I, too, will opt for this alternative, though I will spell it out rather differently than these philosophers. I will then show that, starting with the alternative picture, Prior's intuitions can lead instead to a much happier and more standard quantified modal logic than **Q**. The last section of the paper is devoted to the formal development of the logic and its metatheory.

By way of preliminaries, then, let **T** be the basic propositional modal logic obtained by adding to the propositional calculus the axiom schemas $\Box\varphi \supset \varphi$ and $\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$, and the inference rule of necessitation $\vdash\varphi \Rightarrow \vdash \Box\varphi$. **S5** is the modal logic **T** plus the axiom schemas $\Diamond\varphi \supset \Box\Diamond\varphi$ and $\Diamond\varphi \equiv \neg\Box\neg\varphi$. Let **QT**

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and **QS5** be the result of adding classical quantification theory to **T** and **S5**, respectively. By *classical* modal logic we shall mean any logic that includes **T**, and by *classical quantified* modal logic any logic that includes **QT**. Finally, 'QML' will abbreviate the expression 'quantified modal logic'. I will be using formalized English throughout the paper, and so will be both using and mentioning formulas of predicate logic, as well as metalinguistic variables over formulas, terms, etc., in a rather free and undisciplined way. Occasionally, where necessary, I will use quotes of the appropriate sort, though most often, where no confusion will result, I will use expressions metalinguistically as names for themselves.

1. MODAL LOGIC AND THE NECESSITARIAN MYTH

1.1. *The Rejection of Possibilism*

Classical QML, Prior points out, is "haunted by the myth that whatever exists exists necessarily" ([22], p. 48). That is, at first sight anyway, it appears to be incompatible with even the mere possibility of contingency. Since it is fundamental to our conception of modality that there are contingent beings, classical QML, its formal elegance and simplicity notwithstanding, cannot be the true modal logic.

Prior's earliest hint that something was amiss was his realization that **QS5** entails the Barcan formula[3]

BF $\Diamond \exists x \varphi \supset \exists x \Diamond \varphi$.

This formula, though, as Prior notes,[4] seems false. For intuitively, the consequent asserts that some actually existing thing could φ, whereas apparently "the antecedent doesn't commit us to that much." For example, most all of us believe that John Paul II could have had a grandchild, or a bit more formally, that

JP It could have been the case that something is John Paul II's grandchild,

even though, intuitively (assuming the Pope's lifelong chastity), no existing thing could have had this property.[5] More generally, **BF** is incompatible with the thesis that there could have been objects other than those that there are in fact. For Prior, the only sensible justification for this claim is the assumption that all the objects there could be already exist necessarily.[6] And thus the specter of necessitarianism makes its first appearance.

Prior, however, in typically judicious fashion, was not so quick to dismiss the view. For there is a somewhat weaker and more plausible version of necessitarianism to explore, viz., *possibilism*. Consider, for instance, that in discussing the counterexample to **BF** above we might say that whoever could have been JP II's grandchild would have had to have been *someone who does not actually exist, but might have*. This in turn might be taken to imply that, in some sense, *there is* something that does not exist in fact, but only might have, something which, though nonexistent, "nevertheless can . . . be talked about, or at all events can be a value for the variables bound by our quantifiers" ([23], p. 30).

The proposal, then, is that, in addition to all the things that exist, there are also mere *possibilia*, objects which, though not actual, nonetheless possess a certain sort of being. In addition, it is assumed that the pool of objects generally, existing and otherwise, necessarily remains the same (though of course those fortunate enough to exist could vary in countless ways). If we then let our quantifiers range over this pool of necessary beings,[7] **BF** comes out true, as we can easily see: for if it is possible that some one of our necessary beings a φ, then of course some such being, indeed the very same one a, *could* φ.[8] In terms of our example above, since it is possible that there be a grandchild of JP II, there is some nonexistent *possibile* that is possibly JP II's grandchild.

Now, at first sight, possibilism fails Prior's desideratum as well, since for the possibilist all things are in a sense necessary. But not so, for armed with the distinction between being and existence he can still reconstruct a robust notion of contingency: just introduce a primitive predicate $E!$, true of all and only *existing* things; a contingent being is thus a possible object that doesn't *exist* necessarily, i.e., $CONTINGENT(x) \equiv \neg\Box E!x$. The possibilist can thus have his modal cake and eat it too. He can accept the truth of the Barcan formula in a way that is compatible with the existence of contingent beings.

Prior thus cannot reject possibilism on grounds that it fails to get the facts right about contingency. But he rejects it nonetheless. The problem is that, on the possibilist's account, there are, as Prior puts it, *facts about* nonexistent possibles: the possibilist truth conditions for **JP**, for instance, entail that there facts of the form *a is distinct from every existing thing* and *a is possibly JP II's grandchild* for possibilia *a*. But what then does it mean that these entities don't exist? For

[w]here 'x' stands for a proper name, it seems to me that the form 'x exists' must be logically equivalent to, and definable as, 'There are facts about x', $\exists FFx$. If there *are* facts about x, I cannot see what *further* fact about x would consist in its existing ([23], p. 31).

Prior's objection here, and the presuppositions behind it, are especially revealing, so we will consider them in some detail. The first thing we want to get clear about is the idea of a fact being *about* an object. This notion, and the notion of a *proposition* being about an object, are pervasive in Prior's work.[9] Since facts just are true propositions for Prior (see, e.g., [29], p. 5), we can concentrate on the latter notion.

For Prior, any sentence involving a "logical proper name" of an individual x, i.e., a name that can "in principle be replaced by a demonstrative" ([23], p. 33), expresses a proposition *about* x, or in Russellian terms, a *singular proposition* about x (call x the *subject* of the proposition). This is the case irrespective of the sentence's logical form, i.e., irrespective of whether its logical form is atomic or complex – negated, quantified, modalized, etc. Thus, for instance, of himself Prior says that, not only is it a fact about him that he is a logician, it is also a fact about him that he is not a logician without being a logician, and indeed it is a fact about him that there are facts about him. (See [23], pp. 34–5.) More formally, then, let ψ be a formula (in an interpreted language) in which the variable y occurs free, and t any term that is free for y in ψ, and suppose that a_t is the denotation of t. Then for Prior, so long as t is being used as a logical proper name for a_t, the formula ψ^y_t expresses a singular proposition about a_t.[10]

The first premise of Prior's objection, then, which we will formulate as a reductio, is that, according to the possibilist, there are such facts as the one expressed by

(1) $\forall y(E!y \supset x \neq y)$.

Since (1) involves a name for some (merely possible) object x,[11] it expresses a singular fact about x. It follows in general that there are facts about x. Prior cashes this claim – that there are facts about x – by means of the second-order formula $\exists FFx$, i.e., in effect, the claim that x exemplifies some *property*. In doing so, Prior exploits the intimate connection between singular facts and exemplification encoded in the second-order principle

λ-Con $[\lambda x_1 \ldots x_n \varphi]y_1 \ldots y_n \equiv \varphi^{x_1 \ldots x_n}_{y_1 \ldots y_n}$,[12]

where $\ulcorner[\lambda x_1 \ldots x_n \varphi]\urcorner$ is a second-order term for the n-place relation expressed by the formula φ. So if in particular ψ^y_x expresses a fact about x, it follows that x exemplifies the property $[\lambda y\ \psi]$ of being an y such that ψ. By second-order existential generalization, it follows from **λ-Con** that if ψ^y_x expresses a fact about x, then x exemplifies some property or other,

Facts $\varphi \supset \exists FFt$, where t occurs free in φ,

and hence $\exists FFx$ seems to be a reasonable rendering of 'There are facts about x'.

As an instance of **Facts** we have

(2) $\forall y(E!y \supset x \neq y) \supset \exists FFx$,

and so from (1) and (2) it follows that

(3) $\exists FFx$.

Now, Prior argues that existence ought to be defined in terms of singular facts; or more precisely, in light of the connection between singular facts and exemplification, in terms of predication:

(4) $E!t =_{df} \exists FFt$.

Given this, it follows from (3) that

(5) $E!x$.

But (1) entails

(6) $\neg E!x$,

contradiction, and so (1) must be false. An analogous argument can of course be constructed for any alleged singular fact that could only be about possibilia, and hence possibilism must be rejected.

The key premise here is the proposed definition (4) of existence in terms of singular facts. The premise as it stands weakens Prior's objection in two ways. First, one needn't be a possibilist to balk at the idea that existence is nothing more than the exemplification of properties; one might well want to consider it a full-fledged property in its own right, with all the attendant benefits and privileges. Second, and more important, the definition obscures the real issue. Given the context, it is clear that what is uppermost in Prior's mind is not the *definability* of existence, but rather, as Prior first indicates in the quote, the *logical connection* between existence and singular facts, or equivalently in light of **Facts**, between existence and exemplification. The possibilist has torn the two logically asunder, and *that* is what Prior finds so puzzling. Thus, whether we heal this metaphysical wound by definition – to exist just *is* to be the subject of some singular fact – or by a weaker logical connection is immaterial. And in fact the argument above goes through just as well with definitional identify $=_{df}$ replaced by equivalence $\equiv$; indeed, all we need instead of (4) is

(4′) $\exists FFt \supset E!t$.

Since the converse of (4′) is an instance of **Facts**, the equivalence of the two notions becomes a theorem.

However, now note that, with the weakening of (4) to (4′), the second-order apparatus in the argument is extraneous. Prior's strategy in his objection is to highlight the connection between singular facts and the existence of the subjects of those facts. But this can be expressed straightaway by the first-order schema

PSA $\varphi \supset E!t$, where t occurs free in φ,

which follows from **Facts** and (4′). (2), (3), and (4′) above can then simply be replaced by the following instance of **PSA**:

(2′) $\forall y(E!y \supset x \neq y) \supset E!x$.

and the contradiction follows just as before.

PSA then is a key Priorean component of the true modal logic. It is easy to show that **PSA** is equivalent simply to $E!t$. And that is just as we should expect since if there are no singular propositions about nonexistents, it should follow in particular that anything nameable exists. We want to highlight **PSA**, though, rather than the principle $E!t$, since what is of particular interest is how we got here via **λ-Con, Facts,** and Prior's definition of existence. Moreover a cousin of **PSA** will be of considerable significance below.

The name '**PSA**' indicates that, on Prior's view of things, the principle can be thought of as expressing a form of *serious actualism*,[13] i.e., a form of the view that exemplification entails existence. For given Prior's thesis that sentences involving names express singular propositions, and given the connection between singular facts and exemplification expressed in **Facts**, a sentence involving a name as much expresses a predication as a singular proposition. Hence, one reading of **PSA** is that if the denotation a_t of t exemplifies the property $[\lambda y\ \varphi(y)]$ expressed by the open formula $\ulcorner\varphi(y)\urcorner$ (where y replaces one or more free occurrences of t in φ), then a_t exists. It is easy to show that possibilism's denial, or *actualism* – the thesis that everything exists, $\forall yE!y$ – follows directly from **PSA**,[14] and hence in general that possibilism is not an option for Prior. So while possibilism both preserves classical QML and avoids the necessitarian myth, it does so at the cost of serious actualism. And for Prior that cost is just too high. A solution to the problem of finding the true modal logic must be found elsewhere.

1.2. *Isolating the Culprits*

Prior's approach to a solution to the problem proceeds by way of isolating the logical culprits behind the failures of classical QML, in particular, the necessitarian myth and, in the case of **QS5**, the Barcan formula.

1.2.1. *The Interdefinability of* $\Box$ *and* $\Diamond$. The first of these culprits is the interdefinability of $\Box$ and $\Diamond$. He argues as follows ([23], p. 48):

If we so interpret 'It could be that *p*' that it *is* true if and only if *p could be* true, then my non-existence is something that could not be, since 'I do not exist' . . . is not a thing that could be true. But in the ordinary systems [of model logic] 'It could not be that I do not exist' entails 'It is bound to be that I do exist', which makes gods of us all.

The premise that catches one's eye here – to say the least – is Prior's claim that his nonexistence is not possible. At first blush, it might appear that Prior is giving a very bad argument for this premise, viz., that his nonexistence is not possible because (trivially) it is not possible that the *sentence* 'I do not exist' could be used truly by him. It is only Prior's somewhat injudicious formulation of his actual argument that suggests this imposter.[15] To the contrary, Prior's focus is on what the sentence *says* in his mouth, on the *proposition* expressed. And his claim is that that proposition, i.e., the proposition *Prior does not exist*, could not possibly be true.

What then is Prior's argument here? To make matters clearer, for a given sentence φ, let $[\varphi]$ be the proposition that φ expresses. Given this, the claim in the first clause seems to be that a sentence of the form $\ulcorner \Diamond\psi \urcorner$ is true just in case the proposition $[\psi]$ could be true. This claims runs together two distinct Priorean theses that it is useful to separate. The first is a general statement of the intuitive truth conditions for *sentences*:

(7) A sentence φ is true if and only if the proposition $[\varphi]$ that it expresses is true,

while the second, the central premise of the quote, is a statement of the intuitive truth conditions for *propositions* of the form $[\Diamond\psi]$:

(8) The proposition $[\Diamond\psi]$ is true if and only it could be the case that $[\psi]$ is true.

Now, as an instance of (8), where '*a*' is a logical proper name for Prior, we have,

(9) $[\Diamond\neg E!a]$ is true if and only if it could be the case that $[\neg E!a]$ is true.

However, Prior claims,

(10) It could not be the case that $[\neg E!a]$ is true,

and hence,

(11) $[\Diamond\neg E!a]$ is not true,

so by (7)

(12) '$\Diamond\neg E!a$' is not true,

that is, by the usual rule for $\neg$,

(13) '$\neg\Diamond\neg E!a$' is true.

So by Tarski

(14) $\neg\Diamond\neg E!a$, it could not be the case that Prior fail to exist.

Given (7) and (8), which seem reasonable enough, the crux of the reasoning behind (14) is (10). Why does Prior think that it is true?

The answer once again of course is the logical connection between singular facts and existence. Since the connection between the two is a *logical* one, Prior had no intention that it should be limited to what happens in fact to be the case, as in **PSA**, though that was all that was needed for the argument against possibilism. Rather, not only does it just *happen* to be the case that no subject of a singular fact fails to exist, no proposition *could* have been a fact without its subjects existing, i.e., for any singular proposition $[\varphi]$ about a_t,

(∗) It is not possible both that $[\varphi]$ be true and a_t fail to exist.

Since, then, $[\neg E!a]$ is a singular proposition about Prior (since '$\neg E!a$' is a formula involving a logical proper name for Prior), it couldn't be true without Prior existing. But, of course, by its very nature, it couldn't be true without Prior *failing* to exist either. So it couldn't possibly be true at all, just as (10) says.

More formally, then, for Prior, the schema

PSA* $\neg\Diamond(\varphi \wedge \neg E!t)$, where t occurs free in φ,

being the first-order representation of (∗), should be a theorem of the true modal logic. If so, then where φ is '$\neg E!a$', then – assuming $\neg\Diamond(\neg E!a \wedge \neg E!a) \equiv \neg\Diamond\neg E!a$ will also be a theorem – **PSA*** entails (14) straightaway.

Now of course we have the problem Prior raises in the quote: in classical modal logic, (14) is equivalent to the thesis that Prior's existence is *necessary*, $\Box E!a$. And that, if we know anything about necessity at all, is just not so. The world simply might have lacked his presence; there might have been no information, no singular propositions, about him at all. So in the true modal logic, to avoid necessitarianism and get the modal facts right, $\Box$ and $\Diamond$ must not be interdefinable in the usual way.

1.2.2. *Necessitation.* The second culprit Prior places behind the failures of classical modal logic is the rule of necessitation: $\vdash \varphi \Rightarrow \vdash \Box\varphi$. In any reasonable modal logic, it should turn out to be a theorem that if Prior is a logician, then he is a logician, $La \supset La$, for its truth is guaranteed by its logical form. By necessitation, it follows that this conditional is necessary, $\Box(La \supset La)$. But, Prior objects, "if it is necessary that if I am a logician then I am a logician, it is necessary that I am" ([23], pp. 48–9).

(*) is again at the heart of Prior's reasoning. More specifically, suppose that 'Necessarily, if Prior is a logician, then he is a logician', '$\Box(La \supset La)$', is true. By (7) and (8′), this is so only if it is necessarily the case that the proposition $[La \supset La]$ expressed by 'If Prior is a logician, then he is a logican' is true. But once again, since $[La \supset La]$ is a singular proposition about Prior, by (*) $[La \supset La]$ can be necessarily true only if it is necessarily the case that Prior exist. So if it is necessary that Prior is a logician only if he is a logician, it is necessary that he exist, as he had claimed. But of course the antecedent *is* necessary in classical modal logic; so therefore, is the consequent.

More formally once again, $\Box E!a$ follows straightforwardly from propositional logic ($\vdash La \supset La$), **PSA** (where φ is $La \supset La$), necessitation, and $\Box$-distribution (which Prior finds obviously true but of "limited applicability" ([21], pp. 151–2)). So yet again, necessitarianism rears its head.

2. PRIOR'S Q

Prior's philosophical analysis thus reveals two standard received modal principles to be at the heart of the failures of classical QML.

Both lead to the necessitarian myth, and in the context of full quantified **S5**, they jointly entail the Barcan formula, the theoremhood of which, recall, was Prior's initial clue that not all was well with classical modal logic.[16] With the culprits behind the failings of classical QML unmasked, and with a distinctive conception of modality as his guide, Prior turned to the task of reconstructing the true modal logic, which he called **Q**.

2.1. *Statability*

The logical connection between singular facts and existence is the central intuition underlying Prior's conception of modality. But Prior often puts the thesis more strongly than this, for this statement of the matter does not in and of itself rule out the possibility of there being singular *falsehoods* about an object x that doesn't exist, e.g., the proposition $[E!x]$ that it *does* exist. Of course, if there were such a proposition, then it is most reasonable to assume that there would be its negation as well, and hence facts about x after all, but this is nonetheless an additional assumption. Thus, Prior often puts his thesis more broadly that there would be no *propositions* about x if it were not to exist; there would be no *information* about x at all, good or bad.

To accomodate this thesis more effectively, Prior introduces the notion of *necessary statability* and a corresponding sentential operator S. Intuitively, stability is propositional existence;[17] a proposition is statable just in case all the individuals it is about – all of its *subjects* – exist. Thus, a proposition $[\varphi]$ is *necessarily* statable, $S\varphi$, just in case $[\varphi]$ is either wholly general, so that it is not about any particular object, or it is a singular proposition whose only subjects are necessary beings.

2.2. *The System*

With the picture in mind, then, we turn to the actual axiomatics for **Q**. Most of these axioms and rules are drawn directly out of Prior's paper "Tense Logic for Non-Permanent Existents" in [22].[18] The system takes the operators $\Diamond$ and S as primitive. A proposition can then be defined to be necessary in the usual sense just in case it is *both* necessarily statable and not possibly false; formally,

PDef□: $\Box\varphi =_{df} S\varphi \wedge \neg\Diamond\neg\varphi$.

In addition to some standard complete set of nonmodal schemas, the only propositional axiom schema for the system is the $\Diamond$ version of the characteristic axiom of the basic modal system **T**:

T◇: $\varphi \supset \Diamond\varphi$

For quantifier axioms Prior uses two rules of inference that are equivalent to the following, more familiar three axiom schemas and the rule of generalization:

Qu1: $\forall x(\varphi \supset \psi) \supset (\forall x\varphi \supset \forall x\psi)$

Qu2: $\varphi \supset \forall x\varphi$, x not free in φ

UI: $\forall x\varphi \supset \varphi^x_t$, t free for x in φ

Gen: $\vdash\varphi \Rightarrow \vdash\forall x\varphi$.

Though he makes use the identity predicate '=', he doesn't explicitly introduce any axioms for it in [22], but it is clear that he assumes the standard schemas:[19]

Id: $t = t$, for any term t

Ind: $t = t' \supset (\varphi \supset \varphi')$, where φ is atomic, and φ' is just like φ except that t' replaces 0 or more (free) occurrences of t in φ.

The purpose of the following two rules is to establish the connection between the statability of a complex proposition and the existence of its subjects. If all one needed to worry about were terms, the necessary existence of a propositional subject a_t could be expressed simply as $\Box\exists x(x = t)$. But the language here contains n-place predicates for possibly every n, and these might express properties or relations like *being an admirer of Prior* that do not exist, are not "there", necessarily. Hence, a way is needed of applying a notion of necessary statability to them as well. Prior achieves the desired effect uniformly for both predicates and terms by affixing the operator to simple formulas that explicitly contain the predicates and free terms occurring in a complex formula. Thus, for a given formula φ, the proposition $[\varphi]$ is necessarily statable only if the propositions $[\forall x_1 \ldots x_n P x_1 \ldots x_n]$ $(=[P]$, if P is a 0-place predicate) and $[t = t]$ are necessarily statable, for all n-place predicates P and free terms t occurring in φ. $\ulcorner \mathsf{S}t = t \urcorner$ thus in effect expresses that a_t exists necessarily, and $\ulcorner \mathsf{S}\forall x_1 \ldots x_n P x_1 \ldots x_n \urcorner$ that the property or relation (or proposition) $\boldsymbol{r}$ expressed by P exists necessarily.

To make things a little neater, for any n-place predicate P $(n > 0)$, let $\ulcorner \mathsf{S}P \urcorner$ abbreviate the formula $\ulcorner \mathsf{S}\forall x_1 \ldots x_n P x_1 \ldots x_n \urcorner$, for any term t, let $\ulcorner \mathsf{S}t \urcorner$ abbreviate the formula $\ulcorner \mathsf{S}t = t \urcorner$, and let $\ulcorner \mathsf{S}\{P_1 \ldots P_n, t_1 \ldots t_m\} \urcorner$ abbreviate the formula $\ulcorner \mathsf{S}P_1 \wedge \ldots \wedge \mathsf{S}P_n \wedge \mathsf{S}t_1 \wedge \ldots \wedge \mathsf{S}t_m \urcorner$. Then we have:

RS1a: $\vdash \mathsf{S}\varphi \supset \mathsf{S}P$, where P occurs in φ

RS1b: $\vdash \mathsf{S}\varphi \supset \mathsf{S}t$, where t occurs free in φ.

A related rule gives us a sort of converse to **RS1a** and **RS1b**. Intuitively, on the picture that is guiding us, not only is it the case that a proposition exists only if all its subjects do, it is also true that the existence of the subjects is sufficient for the existence of the proposition. Thus, if it can be shown that the subjects of a given proposition $\boldsymbol{p}$ all exist necessarily – by showing that certain other propositions of which they are subjects are necessarily statable – then it follows that $\boldsymbol{p}$ is necessarily statable. Formally,

RS2: $\vdash \mathsf{S}\{P_1 \ldots P_n, t_1 \ldots t_m\} \supset \mathsf{S}\varphi$, where $P_1, \ldots, P_n$ are all the predicates in φ, and $t_1, \ldots, t_m$ are all the terms occurring free in φ.

The following powerful principle serves as the central modal schema of the logic:

RS◇ $\vdash \varphi \supset \psi \Rightarrow \vdash \mathsf{S}\{P_1 \ldots P_n, t_1 \ldots t_m\} \supset (\varphi \supset \neg\Diamond\neg\psi)$, where all predicates and terms (bound or free) occurring in φ are within the scope of a $\Diamond$ or an S, and where $P_1, \ldots, P_n$ are all the predicates that occur in φ but not in ψ, and $t_1, \ldots, t_m$ are all the terms occurring free in φ but not in ψ.

RS◇ is equivalent in **Q** to a schema relating S and ◇ directly

$$\text{S}\Diamond: \qquad \Diamond \text{S}\varphi \supset \text{S}\varphi$$

and the following Priorean version of □-distribution (where $\text{S}\{P_1 \ldots P_n, t_1 \ldots t_m\}$ is as in **RS◇**), the characteristic **S5** axiom schema, and necessitation:

$$\textbf{PK:} \qquad \neg\Diamond\neg(\varphi \supset \psi) \supset (\text{S}\{P_1 \ldots P_n, t_1 \ldots t_m\} \supset \neg\Diamond\neg\varphi \supset \neg\Diamond\neg\psi)$$

$$\textbf{PS5:} \qquad \Diamond\neg\Diamond\neg\varphi \supset \neg\Diamond\neg\varphi,$$

$$\textbf{DR}\Diamond\textbf{:} \qquad \vdash\varphi \Rightarrow \vdash \neg\Diamond\neg\varphi.$$

As anticipated, the standard rule of necessitation fails in **Q**; rather we get only the expected weakening

$$\textbf{DR}\Box\text{:} \qquad \vdash\varphi \Rightarrow \vdash \text{S}\varphi \supset \Box\varphi,$$

i.e., if a theorem is *necessarily statable* then its necessitation is a theorem. So in particular such propositions as that Prior is a logician only if he is a logician, while still logical theorems, are not necessary truths, as Prior had argued. As he puts it in [23] (p. 50), where "formulable" = "statable",

> To be logically necessary a statement must not only be incapable of being false, but necessarily formulable. But a statement exemplifies a logical law so long as its *truth* is secured by its logical form alone, even if its *formulability* depends on what is usually a contingent matter, the existence of such objects as are directly referred to in it. This distinction is unavoidable, it seems to me, in a modal logic for contingent beings. . .

Just as Prior finds it necessary to restrict his ◇-introduction rule **RS◇** to take into account the contingency (i.e., non-necessary stability) of some propositions, similar considerations about contingent individuals drive him to introduce a restricted version of modus ponens:

PMP: $\vdash\varphi, \vdash\varphi \supset \psi \Rightarrow \vdash \text{S}\{P_1 \ldots P_n, t_1 \ldots t_m\} \supset \psi$, where $P_1, \ldots, P_n, t_1, \ldots, t_m$ are all the predicates and terms that occur free in φ but not in ψ.

Prior's chief reason for the restriction is that, since **Q** is to be compatible with the existence of contingent beings, he wants it to be compatible with the possibility that *all* individuals are contingent and hence also with the more general possibility that no individuals exist. But with unrestricted *modus ponens*, $\neg\Diamond\neg\exists x(x = x)$ follows straight away from **Id** ($\vdash x = x$), **UI** ($\vdash x = x \supset \exists x(x = x)$), **DR**$\Diamond$, and some propositional logic. For all we know, of course, $\neg\Diamond\neg\exists x(x = x)$ might well be true. In particular, it might well be that there are necessary beings – theists routinely assert as much, and Prior himself frequently makes use of the "Christian hypothesis" in his examples. But his point, I take it, is that the question of necessary beings, and more generally the question of whether there might have been nothing at all, is not something that should be decided by *logic*, and hence that $\neg\Diamond\neg\exists x(x = x)$ is not an appropriate theorem for **Q**.

The system thus far seems to be what Prior pretty much explicitly had in mind by quantified **Q**. But there is a further thesis implicit in nearly all of Prior's informal discussions that doesn't receive full expression in the logic as it stands, viz., the assumption that logical proper names are "rigid", that they retain their actual denotation in modal contexts. This thesis is typically expressed in modal logic by means of axioms that also express the related thesis of the necessity of identity and difference.[20] In **S5**'ish logics like Prior's, one needs only the necessity of identity – or in clumsier, but in our context more accurate, Priorean terms, the impossibility of the nonidentity of identicals:

PNI: $\quad t = t' \supset \neg\Diamond t \neq t'.$

Though he explored other approaches to names – description theories for names of objects that no longer exist, for example (see [23], pp. 32ff.) – and though he did register some mild reservations about the necessity of identity and difference,[21] this view of names seems to exhibit the best fit with Prior's overall views, and in particular the view of names as they function in **Q**. So we will make them explicit in the system.

2.3. *Q Triumphant?*

Thus **Q**. The one crucial difference that separates **Q** from its classical counterpart is that **Q** is able to distinguish between propositions that might not exist and those that exist, or are statable, necessarily; indeed, all it takes to collapse **Q** into **QS5** (plus **PNI**) is the schema

Q-collapse: $\vdash \mathrm{S}\varphi$

which of course encodes the thesis that all propositions are necessarily statable ([21], p. 155). But with this one difference, **Q**, unlike classical QMLs, is cleansed of the necessitarian myth, as Prior had intended; that is where $E!$ is $\exists x(x = t)$ (x a variable distinct from t), the formula $\forall y \Box E!y$ is not a thesis of **Q**. This can be shown rigorously by providing an appropriate model theory for **Q** and then an interpretation in which the necessitarian formula is false. This would take us beyond the scope of this paper, though, and so for purposes here we will content ourselves with noting how neither the necessitarian thesis nor the Barcan formula is provable in **Q** in the usual way.

Thus, following the usual argument, we do have $E!x$, by **Id**, **UI**, **PMP** and the definition of $E!$. But by **RS**$\Diamond$ and **Gen**, all we are permitted to infer from $E!x$ is the impossibility of every object's nonexistence, $\forall x \neg \Diamond \neg E!x$, a claim we've already seen Prior defend. In order to prove $\forall x \Box E!x$ we should have first to prove $\mathrm{S}E!x$ in order to exploit **PDef**$\Box$; but of course Prior has made no provision for that since in the metaphysics behind **Q** it is just another expression of the necessitarian myth.

Similarly, the Barcan formula is not provable in the usual way either, since as we've seen its proof depends on both the usual interdefinability of $\Box$ and $\Diamond$ and the unrestricted rule of necessitation, neither of which holds in **Q**.

PSA is provable in **Q** given $E!t$ and some propositional logic, so its modalized counterpart **PSA*** follows immediately from by **DR**$\Diamond$, as required. So Prior has indeed met his desiderata and appears to have fashioned a genuine actualist quantified modal logic for contingent beings.

2.4. *Problems for Q*

Alas, **Q** is not without its difficulties. Least among these is the loss of the simplicity and elegance of a logic in which $\Box$ and $\Diamond$ are interdefinable. But of course, if this interdefinability cannot be incorporated in a logic that, on analysis, gets the modal facts right, we have no choice but to do without it. All the same, we should prefer to have it back if the problems **Q** addresses could be avoided some other way.

There are deeper problems, though. First, though he slides rather quickly over the issue, the loss of $\exists x(x = x)$ as a theorem is no small matter. For since $t = t$ is a theorem for any t, then if $\exists$ is to have its intended meaning, $\exists x(x = x)$ is going to turn out to be a logical truth; it will have to come out true in any model of **Q**, for any reasonable **Q**'ish model theory, since the base world can never be empty. But then **Q** is incomplete.

The only way out here, it seems, aside from abandoning classical quantification theory (see below), is to accept full modus ponens, thus the thesis $\exists x(x = x)$, and hence the thesis $\neg\Diamond\neg\exists x(x = x)$. But this seriously undetermines Prior's views about logic – though logic is hardly independent of metaphysics, it shouldn't decide issues on which one has taken no stand, and it most certaily shouldn't force one to accept theses that one doesn't believe. Yet this would seem to be precisely the position Prior finds himself in.[22]

There is a further problem, deeper still.[23] Prior is not insensitive to the apparent untoward consequences of his view. In particular, he agrees, that, intuitively, the proposition

P Prior might not have existed

is true. As noted, the most natural logical form of **P** – $\Diamond\neg E!a$ – is incompatible with **Q**; its negation is a theorem. Nonetheless, Prior claims ([21], p. 150),

> . . . there *is* a sense of '[x] might not have existed' in which what is says could be the case (and generally is), i.e., the sense: 'It is not the case that (it is necessary that (x exists))' '$\neg\Box E!x$'.

The sort of possibility expressed by $\Diamond$, that is to say, is not the sense relevant to **P**. Rather, the reading on which **P** is true involves a weaker notion of possibility $\Diamond_w$ such that $\Diamond_w =_{df} \neg\Box\neg$, so that a proposition is weakly possible just in case its negation is not necessary. The sense of **P** corresponding to our intuition, then, is the reading $\Diamond_w\neg E!a$, which is true because $[E!a]$ is not necessary. So by making the distinction between the two notions of possibility, Prior is able to have his cake and eat it too: he can both argue that metaphysical and semantical considerations force us to admit that **P** is false, that his nonexistence is not possible, and nonetheless claim that he has satisfied the intuition that it is true.

But this will not do. Consider, for example,

P′ Prior might have been both a logician and not a logician (simultaneously).

As with **P**, the most natural reading of **P′**, $\Diamond(La \wedge \neg La)$, is false; it is inconsistent with **Q**, as we should hope. But also like **P**, if we replace the ordinary notion of possibility here with its weaker counterpart, $\Diamond_w(La \wedge \neg La)$, i.e., $\neg\Box\neg(La \wedge \neg La)$, **P′** turns out to be true as well. For it is not necessarily the case that the proposition $[\neg(La \wedge \neg La)]$ is a fact, since it wouldn't have been statable if Prior hadn't existed. Thus, in this weaker sense, both $[\neg E!a]$ and $[La \wedge \neg La]$ are equally possible.

More generally, it is provable in **Q** that $\neg S\varphi \supset \Diamond_w\varphi$, i.e., that *any* proposition that is not necessarily statable is weakly possible,[24] and hence that any logically contradictory proposition is weakly possible so long as some contingent being is among its subjects. Surely, however, $[\neg E!a]$ is possible in some sense in which obvious impossibilities like $[La \wedge \neg La]$ are not; surely the former in some way characterizes the world as it might have been, and the latter does not. Weak possibility cannot distinguish between the two, however, and so cannot be the sense of possibility in which **P** is true.

The irony of this point cannot easily be overstated. **Q**'s sole purpose, its *raison d'etre*, was to serve as the true modal logic for contingent beings, to be compatible with the existence of things that might have failed to be. But the most natural way of expressing contingency is inconsistent with **Q**, and the only remaining alternative cannot distinguish the expression of contingency from the possibility of manifiest repugnancies. Contingency still seeks its champion.

3. A KINDER, GENTLER SOLUTION

3.1. *Haecceitism*

What is to be done? As we saw, the central premise of Prior's argument for the impossibility of his nonexistence is (10), and the crux of the argument for (10) is (∗), the thesis that it is not possible both that a singular proposition be true and its subject fail to exist. Is there any reason for thinking this latter thesis to be false that is compatible with actualism? Plantinga's work suggests a solution that involves the idea that every object has a *haecceity* or thisness – the property of *being that very object*. It is currently fashionable to think of propositions as in some sense "built up" logically from other entities. This has caused some controversy in regard to the nature of singular propositions on such a view. Some philosophers, most notably Russell [25] (p. 47), have it that the subject of a singular proposition $\boldsymbol{p}$ is itself a logical component of $\boldsymbol{p}$. Others, most notably Plantinga himself ([15], pp. 8–9), have found this puzzling, something on the order of a category mistake; in particular, how can an abstract entity have concrete parts? Suppose then we replace the role of concrete objects in Russellian propositions with their haecceities, and say that a name expresses the haecceity of its referent. The singular proposition $[\varphi]$ expressed by a given sentence φ thus contains just the properties expressed by the predicates in φ and the haecceities expressed by the names in φ among its logical constituents, and a singular proposition is *about* an object x just in case it involves x's haecceity. Singular propostions thus remain abstract through and through.

Now like all properties, this view has it, haecceities exist necessarily, and hence Prior's haecceity would have existed even if he hadn't. Thus, singular propositions about Prior would have existed even if he hadn't as well, and in particular the proposition $[\neg E!a]$ that Prior doesn't exist. In such a circumstance, that proposition would of course have been true, and hence, contrary to (∗), there could have been singular facts about Prior – i.e., facts involving his haecceity – even if he hadn't existed. Nonetheless, since the view is committed only to the existence of actually existing objects, properties, and propositions, it comports fully with actualist strictures against possibilia.

There are a number of problems with the haecceitist approach that have been discussed elsewhere (e.g., [1], [13], and [14]). But the chief difficulty from our current standpoint is that haecceitism is wholly un-Priorean. Prior has no difficulty with the notion of a haecceity;

there's no doubt he thinks there are such things, in whatever sense he thought there were properties at all.[25] Where he would balk, of course, is at the idea that there could be any such property in the absence of the object x that exemplifies it any more than there could be a proposition about x. Granted, there would have been all the *general* properties that x exemplifies in fact, and it would have been possible that there be *such* an object as x, i.e., an object with all those same general properties ([22], pp. 70–1). But on Prior's view, if it hadn't existed, there would have been nothing that singled out *that* object, nothing such that, necessarily, it is associated unqiuely with x if and only if x exists. There would have been, so to say, no telltale trace of the object whatever, whether concrete *possibile*, or abstract haecceity. As Peirce put it, whom Prior quotes approvingly,

> [t]he possible is necessarily general and no amount of general specification can reduce a general class of possibilities to an individual case. It is only actuality, the force of existence, which burst the fluidity of the general and produces a discrete unit.[26]

So haecceitism is not an option, at least not if we want to remain faithful to Prior's intuitions. The question is whether there is a way to remain faithful without affirming **Q**. I think that there is.

3.2. *Two Conceptions of Propositional Possibility*

My charge is that Prior's argument is ambiguous between two conceptions of what it is for a proposition to be possible, or possibly true. On the one conception, Prior's argument is sound and **Q** is inevitable. On the other, both (10) and the underlying principle (∗) are false and the door is open for an appropriate alternative logic.

The ambiguity in question might be seen as arising out of two intuitive conceptions of "the world" (both of which can be found in Prior's writings), viz., the world as the *totality of facts*, and the world as a maximal *configuration of objects*. It is the former that yields (10). If the world is best thought of as the totality of all true propositions, then to say that a proposition p is *possible* is to say it could have been one among the totality of facts, in which case there would have to have *been* such a fact as p. On this reading, (∗) amounts to the claim that a singular proposition cannot be one of the existing facts without its subjects existing, which seems most reasonable. But then it follows that the proposition $[\neg E!a]$ that Prior does not exist is not possible, it could not have been among the totality of facts.

Things work out rather differently on the second conception. To get at the idea, we need first the notion of a *possible state of affairs*. We must be careful though. Lest we be accused of begging any questions, the sense of possibility here must be one acceptable to Prior; it will not do, for example, to explicate our alternative conception of possibility in terms of "merely possible" states of affairs, or by assuming that there could be true propositions that don't exist. We do this as follows. A *state of affairs*, we shall say, is a structured configuration of objects, i.e., a collection of objects existing together exemplifying certain properties and standing in certain relations. We can thus think of a state of affairs as a triple $\langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$, where $\mathbf{O}$ is a set of individual objects, $\mathbf{R}$ a set of n-place relations ($n \geqslant 1$), and $\mathbf{ext}$ a function that specifies how the properties and relations in $\mathbf{R}$ are actually exemplified within $\mathbf{O}$. The world, then, is a *maximal* configuration $\mathscr{A}' = \langle \mathbf{O}', \mathbf{R}', \mathbf{ext}' \rangle$ in the sense that it is not subsumed by any "larger" state of affairs, i.e., there is no state of affairs $\mathscr{A}'' = \langle \mathbf{O}'', \mathbf{R}'', \mathbf{ext}'' \rangle$ such that $\mathscr{A}'' \neq \mathscr{A}'$ and $\mathbf{O}' \subseteq \mathbf{O}''$, $\mathbf{R}' \subseteq \mathbf{R}''$, and $\mathbf{ext}' \subseteq \mathbf{ext}''$. I'll assume henceforth that the world in this sense exists.[27]

States of affairs are all actual as I'm defining them. So, to get at our intended notion, let $\mathscr{A} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$, where $\mathbf{O}$ is any set of individual objects, $\mathbf{R}$ a set of n-place relations, and $\mathbf{ext}$ any function from the members of $\mathbf{R}$ to sets of n-tuples of members of $\mathbf{O}$. Then we say that $\mathscr{A}$ is a *possible state of affairs* just in case it is possible that it be a state of affairs, i.e., just in case the members of $\mathbf{R}$ could have precisely the extensions in $\mathbf{O}$ assigned by $\mathbf{ext}$. Note that a possible state of affairs is possible *in Prior's sense of the term*. For if $\mathscr{A}$ were an (actual) state of affairs, then it would exist – since, by serious actualism, only existing things are in the extensions of properties and relations – and hence the proposition that $\mathscr{A}$ is a state of affairs would be both statable and true. Furthermore, all possible states of affairs *exist*; their components are sets of existing objects, properties, and relations, and functions from the latter to sets of (n-tuples of) the former. We are thus not begging any important metaphysical questions by introducing possible states of affairs.

Say now that $\mathscr{A} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$ is *possibly maximal*, or a *possible world*, just in case it is possible that $\mathscr{A}$ be a maximal state of affairs, i.e., that it be an actual state of affairs that is not subsumed by any larger state of affairs.[28] (That there are such possible states of affairs in addition to the world is unproblematic – or at least no more problematic than the world itself – since it seems clear that only the members of some proper subset of the individuals there are could have existed, or that the same objects could have existed with rather different properties and relations.) Henceforth, I will use 'state of affairs' to mean both actual and possible states of affairs. A proposition $\boldsymbol{p}$ can be said to *exist in*, or be *statable in*, a state of affairs $\mathscr{A} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$ just in case all its subjects are members of $\mathbf{O}$ and all its component relations are members of $\mathbf{R}$.

On this conception of things, unlike the totality of facts conception, propositions are not first-class ontological citizens; they are not considered on the same footing with objects, properties and relations. Indeed, as on Prior's own view when all is said and done, perhaps they needn't be considered ultimate elements of our ontology at all. However that works out, the important point here is that there is a clear sense in which certain propositions can be said to *characterize* possible states of affairs in which they don't exist. Prior himself suggests the idea. In the course of his argument against $\Box/\Diamond$ interdefinability in [21] (pp. 150–1), he writes that

> [t]here are, then, no possible states of affairs in which it is the case that $\neg E!x$, and yet not all possible states of affairs are ones in which $E!x$. For there are possible states of affairs in which there are no facts about x at all; and I don't mean ones in which *it is the case that there are not* facts about x, but ones such that *it isn't the case in them that there are* facts about x.

The idea, then, is that while such propositions as that there are no facts about x, $[\neg\exists FFx]$, and that x does not exist, $[\neg E!x]$, could not be among the facts that exist in any state of affairs $\mathscr{S}$ in which x does not exist, they nonetheless still say something true about $\mathscr{S}$, i.e., characterize it in a certain way. Adams, Deutsch, and Fine have expressed similar ideas vividly in terms of perspective: though "within" $\mathscr{S}$ such propositions are not true, since they don't exist in $\mathscr{S}$, nonetheless, from our "vantage point" in the actual world ([1], p. 22), from our "point of view" ([4], p. 176), they still truly describe it. As Adams puts it, while they are not true *in* $\mathscr{S}$, they are nontheless true *at* $\mathscr{S}$ (pp. 20ff.); or in Fine's jargon ([6], p. 163), while not true in the *inner* sense at $\mathscr{S}$, they are nonetheless true in the *outer* sense.[29]

This same basic idea is captured in our current framework as follows. For the moment, we'll restrict our attention to nonmodal propositions. Let $\mathscr{S} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$ be a state of affairs:

C1 An atomic proposition $[Ra_1 \ldots a_n]$ characterizes $\mathscr{S}$ iff $\langle a_1, \ldots, a_n \rangle \in \mathbf{ext}(R)$;

C2 $[\neg \psi]$ characterizes $\mathscr{S}$ iff it is not the case the $[\psi]$ does;

C3 $[\psi \wedge \theta]$ characterizes $\mathscr{S}$ iff both $[\psi]$ and $[\theta]$ do;

C4 $[\exists x \psi]$ characterizes $\mathscr{S}$ iff $[\psi]$ does for some $a \in \mathbf{O}$ when 'x' means a.

Say that a proposition $[\varphi]$ is *true in* $\mathscr{S}$ just in case it both characterizes $\mathscr{S}$ and exists in $\mathscr{S}$. It is clear that for an atomic proposition – $[Rab]$, say – to characterize a state of affairs $\mathscr{S} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$ it must be true in $\mathscr{S}$; all of its "components" a, b and R must exist in $\mathscr{S}$ in order for $\langle a, b \rangle$ to be in the extension of R in $\mathscr{S}$. Not so however when we turn to negated propositions like $[\neg Rab]$. For all that is required for $[\neg Rab]$ to characterize $\mathscr{S}$ is that it $[Rab]$ fail to do so, i.e., that $\langle a, b \rangle \notin \mathbf{ext}(R)$. And that can happen irrespective of whether or not $[\neg Rab]$ exists in $\mathscr{S}$. That is, it can fail to be the case that $\langle a, b \rangle \in \mathbf{ext}(R)$ either because (*i*) a, b and R all exist in $\mathscr{S}$ but a just doesn't happen to bear R to b in $\mathscr{S}$; or (*ii*) because a, b, or R simply fails to exist in $\mathscr{S}$; if a, say, is not among the objects in $\mathscr{S}$, then *ipso facto* it doesn't bear R to b in $\mathscr{S}$. In case (*i*), then, as we might put it, following the lead of Adams and Fine, $[\neg Rab]$ characterizes $\mathscr{S}$ *internally*, since it exists in $\mathscr{S}$, whereas in (*ii*), since it does not exist in $\mathscr{S}$, it characterizes it *externally*. In either case, though, there is a clear intuitive sense in which $[\neg Rab]$ is true of $\mathscr{S}$, a clear sense in which the state of affairs *is* as the proposition *says*.

The relevance of all this to the issue at hand, of course, is that the notion of characterization yields a precise sense in which the proposition $[\neg E!a]$ that Prior does not exist is possible. Specifically, say that a proposition is *possible*, or *possibly true*, if there is some possibly maximal state of affairs, some possible world, that it characterizes.[30] Suppose in particular that $\mathscr{S} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$ is maximal and that Prior is not among the individuals in **O**. Then where a is Prior, since Prior $\notin$ **ext(existence)**, $[E!a]$ fails to characterize $\mathscr{S}$, and so $[\neg E!a]$[31] characterizes $\mathscr{S}$. Thus, there is a possibly maximal state of affairs that $[\neg E!a]$ characterizes, i.e., in the above sense, $[\neg E!a]$ is possible, contrary to (10).

On this way of carving things up, Prior's totality of facts conception of possibility amounts to the more stringent combination of characterization plus statability: for him, a proposition is possible if it is *true in* some possible world $\mathscr{S}$, i.e., just in case it both characterizes and exists in $\mathscr{S}$. Since it does not appear that our notion of possibility can be explicated in terms of Prior's notion, this indicates that our notion of possibility based on characterization is in fact a more basic notion than Prior's.

Not only does it turn out to be false that Prior's nonexistence is not possible on our new conception, more generally (*) turns out to be false as well. In terms of its formal counterpart **PSA***, if we read $\Diamond$ in accord with the totality of facts conception, then as we've seen **PSA*** says that the conjunction of any singular proposition with the proposition that one of its subjects does not exist could not have been one among the totality of facts. This thesis is no longer expressed by **PSA*** if we interpret $\Diamond$ to express our alternative conception of possibility. Rather, on that reading, it says that no singular proposition can characterize a (possibly maximal) state of affairs in which any of its subjects fail to exist. And that, as we've just taken such pains to argue, is false. Prior's argument for (14) – that his nonexistence is impossible – thus appears to fail. Unlike the haecceitist move, however, we have not obviously violated any of Prior's central intuitions. So there remains hope that we can find a reasonable Priorean alternative to **Q**.

3.3. *A Natural Alternative*

The question then is where Priorean intuitions lead once we adopt our alternative notion of possibility. The natural course is to see how our alternative conception of possibility affects **Q** and make the appropriate changes. This is what we will do. However, note first that to proceed in a fully rigorous and general way we should have to extend our notion of possibility so that it applies both to propositions like *JP II has a grandchild* that do not characterize any existing state of affairs, and to modal propositions generally. Those of the former sort requires us to consider "merely" possible states of affairs, i.e., ones that we can't construct only out of existing objects (since, in this case, no existing thing could be JP II's grandchild), and those of the latter sort in general require us to consider the entire class of all possible states of affairs, merely possible and otherwise. For the actualist, of course, there are no such things as merely possible states of affairs, and this might appear to be a fatal limitation of our alternative conception of possibility. However, as I have argued at length elsewhere, we can achieve the effect of such possibilia by using necessarily existing surrogates – sets, say – that, as far as our modal semantical needs go, play the same role.[32]

Thus, henceforth when I speak of (possible) states of affairs, where necessary I should be taken to be referring to surrogate states of affairs constructed out of appropriate surrogate possibilia.

The obvious place to begin is with Prior's definition **PDef□** of □ in terms of S and ◇. As we've seen, Prior's argument for (14) fails, and (14) is the linchpin of his argument against the standard definition:

Def□: $\Box\varphi =_{df} \neg\Diamond\neg\varphi$.

There is thus no apparent reason to abandon the standard definition. More than this, though, from our new perspective, **Def□** is intuitively correct. To see this, we extend the notion of characterization in the obvious way:[33]

C5 $[\Diamond\varphi]$ characterizes $\mathscr{S}$ iff $[\varphi]$ characterizes some possible world;

C6 $[\Box\varphi]$ characterizes $\mathscr{S}$ iff $[\varphi]$ characterizes all possible worlds.

If we then say that a proposition is *true* just in case it characterizes the actual world, we have that $[\neg\Diamond\neg\varphi]$ is true iff $[\Diamond\neg\varphi]$ is not true iff it is not the case that $[\Diamond\neg\varphi]$ characterizes the actual world iff it is not the case that $[\neg\varphi]$ characterizes any possible world iff $[\varphi]$ characterizes every possible world iff $[\Box\varphi]$ is true. Our first move, then, is to replace **PDef□** with **Def□**.

The upshot of this is of course dramatic, since the logical separation of $\Box$ from $\neg\Diamond\neg$ drives Prior's entire system. First and foremost, full necessitation is now provable. Indeed, more generally, we can replace the Priorean versions of □-distribution (**PK**), the characteristic **S5** axiom schema (**PS5**), and necessitation (**DR◇**) with their full-blooded counterparts:

K: $\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$,

S5: $\Diamond\varphi \supset \Box\Diamond\varphi$

Nec: $\vdash\varphi \Rightarrow \vdash\Box\varphi$.

At first blush this might seem to be what we'd expect, since Prior's argument against necessitation now fails. That argument, recall, depended on the claim that the proposition $[La \supset La]$ that Prior is a logician only if he is a logician is necessary only if it is necssarily the case that Prior exists. But this of course depends on the notion of necessity appropriate to the world as the totality of facts. On our chosen picture, we should say rather that a proposition is necessary just in case it characterizes every possible state of affairs. And for that to be the case, it needn't exist in those states of affairs. $[La \supset La]$ is such a proposition. For in any possible state of affairs $\mathscr{S}$ either Prior is in the extension of the property *being a logician* or he is not, whether in virtue of being in the extension of *being a nonlogician* in $\mathscr{S}$ or simply in virtue of being absent from $\mathscr{S}$. Either way, $[La \supset La]$ characterizes $\mathscr{S}$, and hence on this way of looking at things is necessary.

Despite the apparent soundness of this reasoning, however, with full necessitation back in the picture we are in necessitarian hot water all over again. For by **Id**, **UI** and **PMP**, we get $\vdash E!t$ (i.e., $\exists x(x = t)$) for any term t, and hence $\vdash\Box E!t$. Or again, from a different direction, by **UI** and some simple propositional modal logic we have $\vdash\Diamond\neg E!t \supset \Diamond\exists x\neg E!x$, and hence since $\vdash\forall x(x = x)$, we have $\vdash\Box E!t$ once again. Our reasoning thus far has led us essentially right back to **QS5** where Prior started.

A common solution to this problem (see e.g., [3] and [7]) is to abandon classical quantification theory in favor of *free logic*, and in particular to replace **UI** with the schema

FUI: $\forall x\varphi \supset (E!t \supset \varphi^x_t)$, t free for x in φ.

Thus, from **Id**, **FUI**, and Necessitation we can prove only the innocuous $(E!t \supset E!t)$, and from **FUI** and modal propositional logic we have no more than the equally unobjectionable $\Diamond\neg E!t \supset \Diamond(E!t \supset \exists x\neg E!x)$.

Free logic in and of itself is nearly as un-Priorean in spirit as haecceitism. Prior is loathe to tamper with classical quantification theory generally in dealing with problems involving names[34] (his abandonment of $\exists x(x = x)$ as a theorem notwithstanding), and free logic collides with this inclination notably in two ways. First, Prior's intention is that the terms of **Q** be logical proper names. That is, though he is well aware of other functions for names than as purely denotative tags, it is *that* role to which he confines his attention in the context of **Q**. The problem with free logic is that, while it provides resources for dealing effectively with the logic of names in modal contexts, it also throws in unwanted resources for dealing with non-denoting names with the bargain as well. Second, as we stressed throughout, Prior's search is for a modal logic of contingent *beings*, i.e., things that actually, but not necessarily, *exist*, and that is most naturally suited by a logic in which all names denote. The loss of $E!t$ as a theorem in free logic militates against both these central Priorean intentions.

However, in the context of our broader search for a revised Priorean modal logic, it runs out that we can exploit the modal benefits of free logic without violating these Priorean intentions. A central, arguably *the* central, Priorean intuition is still evident in our notion of characterization, viz., serious actualism, the view that exemplification entails existence. However, its manifestation in the logic takes a rather weaker form. Recall that for Prior a sentence involving a name as much expresses a predication as it does a singular proposition – and that, importantly, no less for sentences embedded in modal contexts than for those that are not. Thus, for Prior, the proposition $[\varphi^x_a]$ is possible if and only if it is posible that a

have the property $[\lambda x\ \varphi]$. This is appropriate in the context of the totality of facts conception, since the proposition $[\varphi^x_a]$ could not have been among the facts without a exemplifying $[\lambda x\ \varphi]$, and conversely. Given serious actualism, it follows that $[\varphi^x_a]$ could not have been among the facts without a existing – just the principle **PSA*** was intended to express.

However, the symmetry between facts and exemplification breaks down in modal contexts when possibility is thought of in terms of characterization. Most notably, once again, $[\neg E!a]$ can characterize a state of affairs, and hence be possible, without it being the case that a has the property $[\lambda x \neg E!x]$ of nonexistence in that state of affairs.[35] Thus, contrary to Prior, sentences involving names don't in general express predications in modal contexts. But a vestige of Prior's account remains in the case of atomic sentences: in general, $[Ra_1 \ldots a_n]$ characterizes a state of affairs $\mathscr{S}$ if and only if $a_1, \ldots, a_n$ stand in the relation R in that state of affairs, and hence (by serious actualism) only if they exist in $\mathscr{S}$. Thus, though **PSA*** is false in general, we still have the weaker principle

SA*: $\neg\Diamond(Pt_1 \ldots t \ldots t_{n-1} \wedge \neg E!t)$, for any any n-place predicate P.

Now, given **SA***, or indeed given only the nonmodal

SA: $Pt_1 \ldots t \ldots t_{n-1} \supset E!t$, for any n-place predicate P,[36]

when we take **FUI** as our universal instantiation axiom, $E!t$ is still provable directly from **Id** and **PMP** (Prior's version of modus ponens). Furthermore, **UI** itself is provable from **SA**, **Id**, **PMP**, and **FUI**. Thus, every term denotes and the quantification theory remains classical, as Prior desires.

Of course, with $E!t$ back once again as a theorem we've gained nothing so long as we also have full necessitation. But the serious actualist considerations just noted suggest needed Priorean restrictions. As we saw above, Prior's serious actualism compels him to distinguish between logical truths and necessary truths – a distinction that he claims is "unavoidable in a modal logic for contingent beings." The distinction, though less pervasive due to our milder implementation of serious actualism, is no less valid here. In particular, the logic of identity requires that we count all instances of **Id** as logical truths: in any model, the denotation a_t of a term t will be identical with itself. However, they do not in general express necessary truths: by serious actualism, since no contingent being a stands in the identity (or any other) relation with itself in possible states of affairs in which it doesn't exist, $[a = a]$ does not characterize any such state of affairs. We thus restrict necessitation accordingly:

Nec′: $\vdash\varphi \Rightarrow \vdash\Box\varphi$, so long as φ is provable without any instance of **Id**.[37]

Note now that the use of **Id** is essential to proving both $E!t$ and **UI**, and hence neither of the necessitarian proofs above go through with **Nec′** as our rule of necessitation. So **FUI**, **SA**, and **Nec′** together let us satisfy Prior's desire for a classical quantification theory without running afoul of the necessitarian myth.

A couple of last tweaks and our system will be complete. First, the restriction on **Nec′** also makes Prior's restriction on modus ponens in his rule **PMP** otiose, since the only way to prove $\neg\Diamond\neg\exists x(x = x)$ is to apply necessitation to $\exists x(x = x)$, which can only be proved by using **Id**. So we can safely assume full modus ponens.

Second with no axiom or rule any longer relating S and the modal operators, S becomes a fifth wheel. Necessary statability is relevant only if it is required that a proposition exist in a state of affairs in order for it to be evaluated with respect to it. As we've seen, however, this is not in general a requirement when evaluation is based on characterization. Where statability *is* relevant, viz., the atomic case, **SA** alone suffices. Hence, S is otiose; it plays no essential role in the logic of modality on this way of thinking about things. Accordingly, we drop the operator S from our language and so also the axioms **RS1a**, **RS1b**, **RS2**, and S$\Diamond$ from our system.

Third, though $\Box t = t$ is not in general true, $\Box\forall x(x = x)$ is. However, as things stand $\forall x(x = x)$ is provable only from **Id** and **Gen**, and hence its necessitation is not provable from **Nec′**. We thus add $\forall x(x = x)$ explicitly to the system as an axiom.

Finally, we no longer want Prior's version of the necessity of identity

PNI: $t = t' \supset \neg\Diamond t \neq t'$, for otherwise we should fall prey to necessitarianism once again (by **SA** and **K**). Rather, what we want in light of our reflections on identity above is the weaker

NI: $t = t' \supset \Box(E!t \supset t = t')$.

This in particular avoids the counterexample to the stronger principle that would arise with respect to worlds in which the denotation of t doesn't exist.

I will call the full system '**A**'. But for only a few minor changes, the very Priorean **A** is almost exactly the very un-Priorean **QS5** that spawned Prior's original search. But with these simple changes, driven by the notion of characterization, **A** retains Prior's key intuitions – serious actualism and classical quantification theory, in particular – without the high logical cost of **Q**. All in all, I think, a much seemlier candidate for Prior's true modal logic.

4. THE TRUE MODAL LOGIC FORMALLY CONSIDERED

Like any first-order logic worth its salt, **A** deserves a sound and complete metatheory. In this section we provide it.

4.1. *Syntax*

I will use a standard first-order modal language that includes an infinite set of variables v_i, and a possibly infinite set of constants c_i and n-place predicates $P_i^n (0 < n < \omega)$[38] that contains at least the predicate P_0^2. $\exists$, $\wedge$, $\neg$, and $\Diamond$ are the basic logical operators, and the usual definitions of the other standard operaters hold; in particular, $\Box =_{df} \neg\Diamond\neg$. Formulas are constructed in the usual way. We will usually use $=$ for P_0^2 along with the usual infix notation. For a language $\mathscr{L}$, TRM ($\mathscr{L}$) is the set of terms (i.e., constants and variables) of $\mathscr{L}$, PRED ($\mathscr{L}$) is the set of predicates, and FLA($\mathscr{L}$) the set of formulas.

4.2. *Model Theory*

As we'll see shortly, it is most convenient for defining **A**'s model theory and for proving completeness to define **A** as one of *two* logics, the other being roughly a free sublogic of **A**. We therefore have two corresponding notions of an interpretation.

Thus, a *general interpretation* (*G-interpretation*) **I** for a language $\mathscr{L}$ is a 5-tuple $\langle W, @, D, d\ V \rangle$, where W and D are disjoint sets, $@ \in W$, $d: W \to \mathscr{P}(D)$, $D = \bigcup range(d)$, and V is a valuation function on TRM($\mathscr{L}$) $\cup$ (PRED($\mathscr{L}$) $\times$ W) satisfying the following conditions:

(M1) For all $t \in$ TRM($\mathscr{L}$), $V(t) \in D$.

(M2) For all $P^n \in$ PRED($\mathscr{L}$), $w \in W$, $V(P^n, w) \subseteq d(w)^n$; in particular, $V(P_0^2, w) = \{\langle a, a \rangle : a \in d(w)\}$.

If instead of (M1) we have

(M1′) For all $t \in$ TRM($\mathscr{L}$), $V(t) \in d(@)$,

then we say that **I** is an *actualist* interpretation, or simply, an *interpretation*, for $\mathscr{L}$.[39]

Note that constants and variables are not distinguished semantically; both receive rigid denotations in a given model by the valuation function, so there is no need for separate variable assignments. This seems to me to be more consonant with Prior's own use of variables in his work.

Truth for a formula at an index, or "world," w is defined recursively relative to an interpretation in more or less the familiar way in terms of a total extension $\bar{V}$ of V from the formulas of $\mathscr{L}$ into the set of "truth values" $\{\top, \bot\}$. The clauses are all standard. Specifically, for the atomic, quantificational, and modal cases:

(A) $\bar{V}(Pt_1 \ldots t_n, w) = \top$ iff $\langle V(t_1), \ldots, V(t_n) \rangle \in V(P, w)$.

(Q) $\bar{V}(\exists x\psi, w) = \top$ iff for some $a \in d(w)$, $\overline{V_a^x}(\psi, \omega) = \top$, where $\overline{V_a^x}$ is $V - \{\langle x, V(x) \rangle\} \cup \{\langle x, a \rangle\}$.

(M) $\bar{V}(\Diamond\psi, w) = \top$ iff for some $w' \in W$, $\bar{V}(\psi, w') = \top$.

We note for future reference that the definition satisfies the usual principle of substitution:

LEMMA 1. *For any formula φ of $\mathscr{L}$, if t is free for x in φ, then for any $w \in W$, $\bar{V}(\varphi^x_t, w) = \top$ iff $\overline{V^x_{V(t)}}(\varphi, w) = \top$.*

A formula φ of $\mathscr{L}$ is said to be *true* in **I** iff $\bar{V}(\varphi, @) = \top$, and φ is said to be *valid* ($\models \varphi$) iff it is true in all interpretations, and *G-valid* ($\models_{\mathrm{G}} \varphi$) iff it is true in all G-interpretations. An interpretation (G-interpretation) in which all the formulas of a set Γ of formulas of $\mathscr{L}$ is true is said to be a *model* (*G-model*) of Γ.

4.3. *Logic*

Let $E!t$ abbreviate $\exists x(x = t)$ where x is the first variable distinct from t. Consider the following system **S** of axioms.

PC:	Propositional Tautologies
K:	$\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$
T:	$\Box\varphi \supset \varphi$
S5:	$\Diamond\varphi \supset \Box\Diamond\varphi$
Qu1:	$\forall x(\varphi \supset \psi) \supset (\forall x\varphi \supset \forall x\psi)$
Qu2:	$\varphi \supset \forall x\varphi$, x not free in φ
FU1:	$\forall x\varphi \supset (E!t \supset \varphi^x_t)$
SA:	$Px_1 \ldots t \ldots x_{n-1} \supset E!t$, P any n-place predicate
NI:	$t = t' \supset \Box(E!t \supset t = t')$
Ind:	$t = t' \supset (\varphi \supset \varphi')$, where φ is atomic, and φ' is just like φ except that t' replaces one or more (free) occurrences of t in φ.

The usual restrictions on substitution apply in **FUI** and **Ind**.

By **G** we will mean the system that results from adding to **S** the following axiom schemas and rules of inference:

◇E!:	$\Diamond E!t$
GId:	$\forall x(x = x)$

GMP: $\vdash_G \varphi, \vdash_G \varphi \supset \psi \Rightarrow \vdash_G \psi$

GGen: $\vdash_G \varphi \Rightarrow \vdash_G \forall x\varphi$

G□I: $\vdash_G \varphi \Rightarrow \vdash_G \Box\varphi$

where as usual $\vdash_G \varphi$ iff there is a finite sequence $\varphi_1, \ldots, \varphi_n = \varphi$ such that for $1 \leqslant i \leqslant n$, either φ_i is an axiom of **G** or φ_i follows from previous formulas in the sequence by one of the rules of inference of **G**. If Γ is a set of formulas of $\mathscr{L}$, the $\Gamma \vdash_G \varphi$ iff there is a finite $\Gamma' \subseteq \Gamma$ such that $\vdash_G \wedge \Gamma' \supset \varphi$, where $\wedge \Gamma'$ is any conjunction of the formulas in Γ'. I will often write $\Gamma, \varphi \vdash_G \psi$ for $\Gamma \cup \{\varphi\} \vdash_G \psi$.

A will be the system that results when to **S** the following axiom schema and rules of inference are added:

Id: $t = t$, for any term t

MP: $\vdash_A \varphi, \vdash_A \varphi \supset \psi \Rightarrow \vdash_A \psi$

Gen: $\vdash_A \varphi \Rightarrow \vdash_A \forall x\varphi$

A□I: $\vdash_G \varphi \Rightarrow \vdash_A \Box\varphi$.

Several remarks are in order here. Note first **A□I**: the necessitation of any formula provable *in **G*** (not **A**) can be inferred in **A**. Since the only difference between **A** and **G** is the presence of **Id**, **A□I** is essentially the same rule as the rule **Nec′** above in our original development of **A**, and hence these are just equivalent formulations of the same system.

It is easy to see that **G** is a subtheory of **A**, since ◇**E**! follows straightaway from **Id**, **SA**, and **T**, and **GId** is just the generalization of **Id**. The reverse is not true, however, To see this, note first that since there are G-interpretations in which the domain of @ is empty, and since predicates are interpreted locally, it follows that no atomic formula is G-valid. By the Soundness Theorem below for **G**, it follows that no atomic formula is provable in **G**. But $t = t$ provable in **A** for every t. Thus, it is not a subtheory of **G**. Note also that all the theorems of classical **S5** modal propositional logic are theorems of **G**, and hence of **A**.

Henceforth we will drop the subscripted 'A' on the turnstile, so that $\vdash\varphi$ just means $\vdash_A \varphi$. As usual, we will say that a set Γ of formulas of

$\mathcal{L}$ is *consistent* (*G-consistent*) iff there is no formula φ such that $\Gamma \vdash \varphi$ and $\Gamma \vdash \neg\varphi$ ($\Gamma \vdash_{G} \varphi$ and $\Gamma \vdash_{G} \neg\varphi$).

4.4. *Soundness and Completeness*

THEOREM 1 (*Soundness*). *If* $\vdash\varphi$, *then* $\vDash\varphi$; *similarly for* $\vdash_{G}$ *and* $\vDash_{G}$.

Proof. The soundness of all the axioms and rules follow directly from the definition of an interpretation. The only one whose proof is slightly less than routine is A□I. Thus, suppose $\vDash_{G} \varphi$ but that $\nvDash \Box\varphi$, i.,c., that there is an (actualist) interpretation $\mathbf{I} = \langle W, @, D, d, V\rangle$ such that $\bar{V}(\Box\varphi, @) = \bot$. Then for some index $w \in W$ we have $\bar{V}(\varphi, w) = \bot$. It is easy to see, however, that the structure $\mathbf{I}' = \langle W, w, D, d, V\rangle$ just like $\mathbf{I}$ except that it has w as its "actual world" is a G-interpretation (though not necessarily an actualist interpretation). But since the falsity of φ at w in $\mathbf{I}$ doesn't depend on which world is actual,[40] we still have $\bar{V}(\varphi, w) = \bot$ in $\mathbf{I}'$, contrary to our assumption that φ is G-valid.

We turn now the Completeness Theorem. For our construction we will draw upon methods developed by Gallin [7]. Similar techniques (apparently discovered independently) are also found in Fine [5]. We begin with some preliminary definitions; let Γ be any set of formulas of a language $\mathcal{L}$:

DEFINITION 1. Γ is *maximal* iff for any formula φ of $\mathcal{L}$, either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$.

DEFINITION 2. Γ is *G-∃-complete* iff for any formula φ of $\mathcal{L}$, if $\exists x\varphi \in \Gamma$, then for some (substitutable) $t \in \mathrm{TRM}(\mathcal{L})$, $E!t$, $\varphi^x_t \in \Gamma$.

DEFINITION 3. An ω-sequence $W = \langle W_0, W_1, \ldots\rangle$ of sets of formulas of a language $\mathcal{L}$ is G-consistent just in case, whenever Γ_i is a finite subset of W_i for all $i < \omega$, the set $\{\Diamond \bigwedge \Gamma_i : i < \omega\}$ is G-consistent.

Let $W[i,\{\varphi_1, \ldots, \varphi_n\}]$ be the result of replacing W_i in W with $W_i \cup \{\varphi_1, \ldots, \varphi_n\}$. Where $n = 1$, I will write $W[i, \varphi]$ instead of $W[i, \{\varphi\}]$.

DEFINITION 4. Let $j < w$ and $\varphi \in \text{FLA}(\mathscr{L})$. Then we say that φ is *G-consistent$_j$ with W* just in case $W[j, \varphi]$ is G-consistent.

Now, let Γ be a consistent (i.e., A-consistent) set of sentences in a language $\mathscr{L}$. (We will assume for simplicity that $\mathscr{L}$ is countable, but our proof is easily generalized to uncountable languages.) Let **Var** be a countable set of new variables, and let $\mathscr{L}'$ be $\mathscr{L}$ + **Var**. Let $\mathscr{E} = \{\langle j, \varphi\rangle_i\}_{i<\omega}$ be an enumeration of all pairs $\langle j, \varphi\rangle$ where $j < \omega$ and $\varphi \in \mathbf{FLA}(\mathscr{L}')$. Let ord_i be the ordinal number (i.e., the first element) in the ith pair in $\mathscr{E}$, and let φ_i be the ith formula.

We define the following sequence of sequences W^i of sets W^i_j, i, $j < \omega$. Let $W^0_0 = \Gamma \cup \{E!t : t \in \text{TRM}(\mathscr{L})\}$, and $W^0_j = \varnothing$, for $j > 0$. (Note that W^0_0 is consistent (hence G-consistent) if Γ is since $\vdash E!t$ for every $t \in \text{TRM}(\mathscr{L})$.) For W^{i+1}, if φ_i is not G-consistent$_{ord_i}$ with W^i, then $W^{i+1} = W^i$. Otherwise,

(i) If φ_i is of the form $\exists x\psi$, then $W^{i+1} = W^i[ord_i, \{\varphi_i, \psi^x_y, E!y\}]$, where y is a new variable out of V Var.
(ii) if φ_i is of the form $\Diamond\psi$, then $W^{i+1} = W^i[\text{ord}_i, \varphi_i][n, \psi]$, where n is the least ordinal > 0 such that $W^i_n = 0$.
(iii) if φ_i is of neither of the above forms, then let $w^{i+1} = W^i[ord_i, \varphi_i]$.

First we need three lemmas.

LEMMA 2. $\vdash_G \forall x E!x$.
Proof. By **SA** ($x = x \supset E!x$), **GGen**, **Qu1**, **GId**, and **GMP**.

LEMMA 3. $\Gamma \vdash_G \varphi \Rightarrow \Gamma \vdash_G \forall y\varphi$, *if y is not free in* Γ.
Proof. By **GGen**, **Qu1**, and **Qu2**.

LEMMA 4. *If* $\Sigma = \{\Diamond \wedge \Gamma_k : k < w\}$, *where each* Γ_k *is a finite set of formulas of* $\mathscr{L}'$, *then if* $\Sigma \cup \{\theta\}$ *is G-inconsistent, so is* $\Sigma \cup \{\Diamond\theta\}$.
Proof. Suppose $\Sigma \cup \{\theta\}$ is G-inconsistent. The for some finite $\Sigma' \subseteq \Sigma$, $\Sigma' \cup \{\theta\}$ is G-inconsistent. Hence, where $\Sigma' = \{\Diamond \wedge \Gamma_{k_1}, \ldots, \Diamond \wedge \Gamma_{k_n}\}$, $\vdash_G \wedge\Sigma' \supset \neg\theta$. Thus, by **G□I**, $\vdash_G \Box(\wedge\Sigma' \supset \neg\theta)$, and by **K**, $\vdash_G \Box\wedge\Sigma' \supset \Box\neg\theta$. By the propositional thesis $(\Box p \wedge \Box q) \supset \Box(p \wedge q)$ it follows that

$\vdash_G (\Box\Diamond\bigwedge\Gamma_{k_1} \wedge \ldots \wedge \Box\Diamond\bigwedge\Gamma_{k_m}) \supset \Box\neg\theta$, and so by **S5**, $\vdash_G (\Diamond\bigwedge\Gamma_{k_1} \wedge \ldots \wedge \Diamond\bigwedge\Gamma_{k_n}) \supset \Box\neg\theta$, i.e., $\vdash_G \bigwedge\Sigma' \supset \neg\Diamond\theta$. So $\Sigma' \cup \{\Diamond\theta\}$ is G-inconsistent. Hence, so is $\Sigma \cup \{\Diamond\theta\}$.

Now for the more important lemma that uses these.

LEMMA 5. *Each W^i is G-consistent, for $i < \omega$.*

Proof. By induction on i. If $W^0 = \langle W_0^0, \emptyset, \emptyset, \ldots \rangle$ were not G-consistent, then there would be some finite $\Gamma' \subseteq W_0^0$ such that $\{\Diamond\bigwedge\Gamma'\}$ is not G-consistent. But this could not be. For $W_0^0 = \Gamma \cup \{E!t : t \in \mathrm{TRM}(\mathscr{L})\}$ is consistent, by assumption. And for any finite $\Gamma' \subseteq W_0^0$, $W_0^0 \vdash_G \bigwedge\Gamma'$, by **PC**, and $\vdash_G \bigwedge\Gamma' \supset \Diamond\bigwedge\Gamma'$, by **T**, and so $W_0^0 \vdash_G \Diamond\bigwedge\Gamma'$. Hence, $\{\Diamond\bigwedge\Gamma'\}$ is G-consistent.

Suppose then W^i is G-consistent, and let $\langle j, \varphi\rangle$ be the ith pair in $\mathscr{E}$. We show that W^{i+1} is G-consistent as well.

(i) If φ is not G-consistent$_j$ with W^i, then $W^{i+1} = W^i$, and so is G-consistent, by our induction hypothesis.

(ii) If φ is G-consistent$_j$ with W^i, then:

Case 1: φ is $\exists x\psi$. Then $W^{i+1} = W^i[j, \{\varphi, \psi_y^x, E!y\}]$, where y is a new variable. Suppose W^{i+1} is not G-consistent. Then there are finite sets $\Gamma_k \subseteq W_k^{i+1}$ such that $\{\Diamond\bigwedge\Gamma_k : k < \omega\}$ is not G-consistent. Let $\Delta = \{\Diamond\bigwedge\Gamma_k : k < w, k \neq j\}$, and let $\Gamma_j' = \Gamma_j - \{\varphi, \psi_y^x, E!y\}$. Then $\Delta \cup \{\Diamond(\bigwedge\Gamma_j' \wedge \varphi \wedge \psi_y^x \wedge E!y)\}$ is G-consistent. Thus,

$$\Delta \vdash_G \neg\Diamond(\bigwedge\Gamma_j' \wedge \varphi \wedge \psi_y^x \wedge E!y)$$

$$\Delta \vdash_G \Box\neg(\bigwedge\Gamma_j' \wedge \varphi \wedge \psi_y^x \wedge E!y)$$

$$\Delta \vdash_G \Box(\neg\bigwedge\Gamma_j' \vee \neg\varphi \vee \neg\psi_y^x \vee \neg E!y)$$

$$\Delta \vdash_G (\neg\bigwedge\Gamma_j' \vee \neg\varphi \vee \neg\psi_y^x \vee \neg E!y), \text{ by } \mathbf{T}$$

$$\Delta \vdash_G (\bigwedge\Gamma_j' \wedge \varphi) \supset (\neg\psi_y^x \vee \neg E!y)$$

$$\Delta, \bigwedge\Gamma_j' \wedge \varphi \vdash_G \neg\psi_y^x \vee \neg E!y$$

$$\Delta, \bigwedge\Gamma_j' \wedge \varphi \vdash_G E!y \supset \neg\psi_y^x$$

$$\Delta, \bigwedge\Gamma_j' \wedge \varphi \vdash_G \forall y(E!y \supset \neg\psi_y^x), \text{ by Lemma 3}$$

$$\Delta, \bigwedge\Gamma_j' \wedge \varphi \vdash_G \forall yE!y \supset \forall y\neg\psi_y^x, \text{ by } \mathbf{Q1}$$

$$\Delta, \bigwedge\Gamma_j' \wedge \varphi \vdash_G \forall y\neg\psi_y^x, \text{ by Lemma 2.}$$

But $\vdash_G \forall y \neg \psi^x_y \equiv \neg\varphi$, in which case $\Delta \cup \{\bigwedge\Gamma'_j \wedge \varphi\}$ is G-inconsistent. By Lemma 4, $\Delta \cup \{\Diamond(\bigwedge\Gamma'_j \wedge \varphi)\}$ is G-inconsistent, contradicting the G-consistency$_j$ of φ with W^i.

Case 2: φ is $\Diamond\psi$. Then $W^{i+1} = W_i[j, \varphi][n, \psi]$, where n is the least ordinal > 0 such that $W^i_n = \emptyset$. Suppose W^{i+1} is not G-consistent. Then again there are finite sets $\Gamma_k \subseteq W^{i+1}_k$ such that $\{\Diamond\bigwedge\Gamma_k : k < \omega\}$ is not G-consistent. Or again slicing things up a bit differently, where $\Delta = \{\Diamond\bigwedge\Gamma_k : k < \omega, k \neq j, k \neq n\}$, $\Delta \cup \{\Diamond\bigwedge\Gamma_j\} \cup \{\Diamond\psi\}$ is not G-consistent. (By the G-consistency$_j$ of φ with W^i, Γ_n must be $\{\psi\}$, so that $\Diamond\bigwedge\Gamma_n = \Diamond\psi$.) Thus, $\Delta, \Diamond\bigwedge\Gamma_j \vdash_G \neg\Diamond\psi$, and hence by the theorem $\Diamond(p \wedge q) \supset \Diamond p$, we have that $\Delta, \Diamond(\bigwedge\Gamma_j \wedge \varphi) \vdash_G \neg\Diamond\psi$. (Note that, for all we known, φ needn't be a member of Γ_j.) However, $\vdash_G \Diamond(\bigwedge\Gamma_j \wedge \varphi) \supset \Diamond\varphi$, by the above theorem once again, and $\vdash_G \Diamond\varphi \supset \varphi$, i.e., $\vdash_G \Diamond\Diamond\psi \supset \Diamond\psi$, by **T** and **S5**. Thus, $\Diamond(\bigwedge\Gamma_j \wedge \varphi) \vdash_G \Diamond\psi$, so $\Delta, \Diamond(\bigwedge\Gamma_j \wedge \varphi) \vdash_G \Diamond\psi$, hence $\Delta \cup \{\Diamond\bigwedge(\Gamma_j \cup \{\varphi\})\}$ is not G-consistent, contradicting the G-consistency$_j$ of φ with W^i.

Case 3: If φ is of neither of the above two forms, then since φ is G-consistent$_j$ with W^i, $W^{i+1} = W^i[j, \varphi]$ is G-consistent. So our lemma is proved.

Now, for all $j < \omega$, let $W_j = \bigcup_{k<\omega} W^k_j$, and let $W = \langle W_0, W_1, \ldots \rangle$.

LEMMA 6. *W is G-consistent.*

Proof. Suppose not. Then, because of the finiteness of proofs, there would be some finite number n of finite Γ_{j_i}, where $\Gamma_{j_i} \subseteq W_{j_i}$, such that $\{\Diamond\bigwedge\Gamma_{j_i} : 1 \leqslant i \leqslant n\}$ is G-inconsistent. Since each Γ_{j_i} is finite and there are only finitely many of them, there is a least $k < \omega$ such that $\Gamma_{j_i} \subseteq W^k_{j_i}$, for $1 \leqslant i \leqslant n$. But then W^k is G-inconsistent, contrary to Lemma 5.

LEMMA 7. *If φ is G-consistent$_j$ with W, then $\varphi \in W_j$.*

Proof. If φ is G-consistent$_j$ with W, then φ is G-consistent$_j$ with W^i, for all $i < \omega$. (If not, the same things that would make φ G-inconsistent$_j$ with any W^i would also make it G-inconsistent$_j$ with W, by construction of W.) In particular, then, where $\langle j, \varphi\rangle$ is the nth element of the enumeration $\mathscr{E}$, φ is G-consistent$_j$ with W^n. By our construction, then, $\varphi \in W_j^{n+1}$, and so $\varphi \in \bigcup_{k<\omega} W_j^k = W_j$.

LEMMA 8. *Each W_j is maximal, for all $j < \omega$.*

Proof. Let φ be any formula. We want to show that either $\varphi \in W_j$ or $\neg\varphi \in W_j$. Either φ is G-consistent$_j$ with W or it isn't. If it is, then, $\varphi \in W_j$, by Lemma 7. If it isn't, then there are finite $\Gamma_k \subseteq W[j, \varphi]_k$ such that $\{\Diamond \bigwedge \Gamma_k : k < \omega\}$ is not G-consistent. Since W is G-consistent by Lemma 6, φ must be an element of Γ_j, and so where $\Delta = \{\Diamond \bigwedge \Gamma_k : k < \omega, k \neq j\}$ and Γ_j^- is $\Gamma_j - \{\varphi\}$, it follows that $\Delta \vdash_G \neg\Diamond(\bigwedge \Gamma_j^- \wedge \varphi)$, and so $\Delta \vdash_G \Box(\bigwedge \Gamma_j^- \supset \neg\varphi)$.

We now claim that, where $\langle j, \neg\varphi\rangle$ is the nth element of $\mathscr{E}$, $\neg\varphi \in W_j^{n+1}$. For suppose not. Then $\neg\varphi$ is not G-consistent$_j$ with W^n, and hence, there are finite $\Gamma_k' \subseteq W_k^n$ such that $\{\Diamond \bigwedge \Gamma_k' : k < \omega\}$ is not G-consistent. Parallel to the above, let Δ' be $\{\Diamond \bigwedge \Gamma_k' : k < \omega, k \neq j\}$ and let $\Gamma_j'^-$ be $\Gamma_j' - \{\neg\varphi\}$; then $\Delta' \cup \{\Diamond \bigwedge (\Gamma_j'^- \wedge \neg\varphi)\}$ is not G-consistent. As above, it follows that $\Delta' \vdash_G \Box(\bigwedge \Gamma_j'^- \supset \varphi)$. Thus, we have both that $\Delta \cup \Delta' \vdash_G \Box(\bigwedge \Gamma_j^- \supset \neg\varphi)$ and that $\Delta \cup \Delta' \vdash_G \Box(\bigwedge \Gamma_j'^- \supset \varphi)$. By basic modal logic it follows that $\Delta \cup \Delta' \vdash_G \neg\Diamond \bigwedge (\Gamma_j^- \cup \Gamma_j'^-)\}$ and hence that $\Delta \cup \Delta' \cup \{\Diamond \bigwedge (\Gamma_j^- \cup \Gamma_j'^-)\}$ is not G-consistent. Using the theorem $\Diamond(p \wedge q) \supset (\Diamond p \wedge \Diamond q)$, it is easy to show that every element of $\Delta \cup \Delta'$ is derivable in **G** from $\{\Diamond \bigwedge (\Gamma_k \cup \Gamma_k') : k < \omega, k \neq j\}$, and hence $\{\Diamond \bigwedge (\Gamma_k \cup \Gamma_k') : k < \omega, k \neq j\} \cup \{\Diamond \bigwedge (\Gamma_j^- \cup \Gamma_j'^-)\}$ is not G-consistent. But $\Gamma_j^- \cup \Gamma_j'^-$ is a finite subset of W_j and each $\Gamma_k \cup \Gamma_k'$ is a finite subset of W_k, contradicting the G-consistency of W.

So where $\langle j, \neg\varphi\rangle$ is the nth element of $\mathscr{E}$, $\neg\varphi \in W_j^{n+1}$. Thus, since $W_j = \bigcup_{k<\omega} W_j^k$, $\neg\varphi \in W_j$.

LEMMA 9. *Each W_j is G-$\exists$-complete.*

Proof. Straightforward from our construction.

LEMMA 10. *For all $j < \omega$, $\Diamond\psi \in W_j$ iff $\psi \in W_n$, for some $n < \omega$.*

Proof. Left to right follows directly from construction. For the other direction, suppose not, i.e., that $\psi \in W_n$, for some n, but $\Diamond\psi \notin W_j$. Then $\Box\neg\psi \in W_j$, by Lemma 8. Let $\Gamma_n = \{\psi\}$, $\Gamma_j = \{\Box\neg\psi\}$, and let $\Gamma_k = \emptyset$, for all other $k < \omega$. Then

$\Gamma = \{\Diamond \wedge \Gamma_k : k < \omega\} = \{\Diamond\psi, \Diamond\Box\neg\psi\}$ is G-consistent, by the G consistency of W. But this can't be, since $\Diamond\Box\neg\psi \equiv \Box\neg\psi \equiv \neg\Diamond\psi$, by **S5**.

Now, for each term t of our language $\mathscr{L}'$ let $[t] = \{s : \text{for some } j < \omega, t = s \in W_j\}$, and let $V(t) = [t]$. For n-place predicates P of $\mathscr{L}'$, let $V(P, j) = \{\langle [t]_1, \ldots, [t]_n \rangle : Pt_1 \ldots t_n \in W_j\}$. Let $D = \{[t] : t \in \text{TRM}(\mathscr{L}')\}$, and let $d(j) = \{[t] : E!t \in W_j\}$. Finally, let $\mathbf{M} = \langle \omega, 0, D, d, V \rangle$.

LEMMA 11.*M is a G-interpretation for $\mathscr{L}'$.*

Proof. This is easily verified by comparing the definition of **M** with the definition of a G-interpretation. That **M** satisfies (M2) in particular follows from **SA**.

LEMMA 12. *For all $\varphi \in$ FLA($\mathscr{L}'$), for all $j < \omega$, $\varphi \in W_j$ iff $\bar{V}(\varphi, j) = \top$.*

Proof. By induction on φ. The atomic case and the connective cases are straightforward. The quantifier case follows in the usual way from Lemma 1, Lemma 9, and **FUI**. The modal case follows directly from Lemma 10.

Let $\mathbf{M}' = \langle \omega, 0, D, d, V' \rangle$ where V' is the restriction of V to $\text{TRM}(\mathscr{L}) \cup (\text{PRED}(\mathscr{L}) \times \omega)$.

LEMMA 13. **M**′ *is an actualist interpretation for $\mathscr{L}$.*

Proof. Again, straightforward, given that by construction $E!t \in W_0$, for all $t \in \text{TRM}(\mathscr{L})$.

LEMMA 14. *For all $\varphi \in FLA(\mathscr{L})$, $\varphi \in W_0$ iff $\bar{V}'(\varphi, 0) = \top$.*

Proof. Immediate from Lemma 12.

COROLLARY. *For all $\varphi \in \Gamma$, $\bar{V}'(\varphi, 0) = \top$.*

Proof. $\Gamma \subseteq W_0$.

Thus, **M**′ is a model of **Γ**. Since $\mathscr{L}$ and Γ were chosen arbitrarily, we have:

LEMMA 15. *Every consistent set of formulas of a (countable) language has a model.*

Hence,

THEOREM 2 *(Completeness). For any formula φ of a language $\mathscr{L}$, $\models\varphi$ only if $\vdash\varphi$.*

Proof. As usual, given Lemma 15.

Note that it is the use of the weaker notion of G-consistency in the construction that allows us to get away without verifying the Barcan formula **BF**.[41] For since the quantification theory of **G** is free, W_0 can contain formulas of $\mathscr{L}'$ of the form $\neg E!y$ whose negations $E!y$ will occur in other W_j (by construction, since $\vdash_G \Diamond E!y$ for all y). Thus, the model can contain "individuals" that exist at other worlds and that falsify **BF**. However, since we added $E!t$ to Γ for every term in the *original* language $\mathscr{L}$ at the initial stage W^0 of the construction we get an actualist interpretation when we restrict V down to $\mathscr{L}$, and hence an interpretation that preserves classical quantification theory.[42]

NOTES

[1] The expression comes from [24], p. 104.

[2] While I was writing this paper Deutsch and I discovered that we were independently working out quite similar approaches to the problems of contingency in modal logic. Deutsch invokes a double index semantics inspired by Kaplan's work on indexicals that is in certain respects more general than mine, though this added generality brings about some important divergence in regard to validity (see his [3]).

[3] See [23], p. 26. Prior's proof of the implication is found in [19], pp. 60–62.

[4] See [23], p. 29. Prior makes the point with regard to tense, but as with most of his remarks about time and ontology, it held for him no less in the modal case.

[5] I assume here, in current fashion, that no one could have had different parents, and hence in particular that no one could have had JP II as a parent (hence grandparent).

[6] [23], p. 29. The context again is temporality.

[7] Ordinary quantification can then be capture by introducing a special predicate $E!$ for existence and restricting the quantifiers only to possible objects that happen to exist. Thus, $\forall x\varphi$ becomes $\forall x(E!x \supset \varphi)$ and $\exists x\varphi$ becomes $\exists x(E!x \wedge \varphi)$. Compare [12], pp. 5ff.

[8] For such a view, see, e.g., [29], ch. 3, exp. 68ff; also [26].

[9] Though he did not wish to commit himself *ultimately* to the actual existence of such things. His view was that propositions were "logical constructions," i.e., that talk which

apparently commits one to propositions can always be paraphrased away into talk about ordinary objects (see, e.g., [20], Chs. 1 and 2, esp. p. 12) and hence that such talk is innocuous. It's not at all clear to me that Prior's own propositional talk can be reinterpreted in this way. However, I don't think this has any direct bearing on the issues here, and I will simply be taking Prior's remarks at face value.

[10] Where ψ^y_t is the result of replacing every free occurrence of y in ψ with an occurence of t.

[11] It needn't be the case for the possibilist, of course, that we could actually establish a referential connection between such a name and a mere possibile. The point is rather just that there are facts of that form, in the same way that if Fermat's last theorem is false, we know there are facts of the form $x^n + y^n = z^n$, even though we don't know what the subjects of any such facts might be.

[12] Prior discusses λ-abstraction, and endorses λ-conversion, in [20], pp. 43–4.

[13] See, e.g., [15], pp. 11–15, [17], pp. 126–9, and [16], pp. 316–324.

[14] Let φ be $\neg E!y$.

[15] Prior himself gives stern warning against identifying propositions with sentences in [20], pp. 5ff.

[16] As Prior later discovered, **BF** is also provable from the two principles and **QT** plus the weaker Brouwersche axiom schema $\varphi \supset \Box\Diamond\varphi$. See [21], p. 146.

[17] See, e.g., [22], p. 149. The use of the term 'statability' arises apparently from Prior's desire to avoid any ultimate ontological commitment to propositions, as of course 'propositional existence' strongly suggests (to say the least). As he himself points out, the locution 'statable' "has the disadvantage of suggesting that the difficulty here is simply with our mechanisms of reference," ([22], p. 147) i.e., that the issue is just linguistic or epistemological.

[18] See esp. pp. 157–8.

[19] See, for instance, the footnote on p. 205 of Prior's [18], where he approves both of these axioms in the context of **S5**. That they still hold in **Q** reasonably follows from the fact that none of his arguments against **S5** tell in any way against either schema.

[20] See, e.g., [8], p. 255.

[21] [21], p. 148; see also his discussion of the doctrine in [18], pp. 205–207.

[22] Prior indicates some discomfort with his solution in [22] (p. 159), where he suggests as an alternative revising the system so that his version of necessitation **DR**$\Diamond$ as it stands is not a derivable rule. The system **A** below can be seen as an implementation of this strategy.

[23] The substance of the following objection is derived from Plantinga [15]. Fine notes further difficulties for **Q** in [24], pp. 148ff.

[24] From the definition of $\Box$ and the rules **RS1a**, **RS1b**, and **RS2** it is provable that $\Diamond_w\varphi \equiv \Diamond\varphi \vee \neg S\varphi$, and hence by propositional logic $\neg S\varphi \supset \Diamond_w\varphi$.

[25] See, for example, [22], p. 68.

[26] [9], p. 147 (4.172); quoted in [23], p. 114; see also [22], pp. 71–2.

[27] Certain restrictions on membership in **O**′ and **R**′ are of course required if we are to make this assumption feasible – we can't for instance include all the sets among the individuals. Perhaps one way to work this out is to define states of affairs that are maximal *relative* to a given property **r** that carves out a domain of individuals and a corresponding set **R*** of n-place relations "appropriate" for things that have **r**. So, for example, **r** might be the property of being of a concrete object, and **R*** a set of appropriate relations. A state of affairs ⟨**O**, **R**, **ext**⟩ is maximal relative to **r** and **R*** then just in case everything in **O** has **r**, **R** = **R*** and there is no state of affairs ⟨**O**″, **R**, **ext**″⟩

such that $\mathbf{O} \neq \mathbf{O}''$, everything in $\mathbf{O}''$ has $\mathbf{r}$, and $\mathbf{O} \subseteq \mathbf{O}''$. We could then make all the same moves without worrying about imminent paradox.

[28] In accord with the previous note, we could define possible maximality for $\mathscr{A}$ relative to some property $\mathbf{r}$ and a corresponding set $\mathbf{R}^*$.

[29] Fine's distinction, though, rests on an unanalyzed conception of possible worlds that I find difficult to understand in an actualist framework.

[30] Note this is not a definition but only a sufficient condition; for a full equivalence we need to extend charcterization to propositions of any form. This is discussed in more detail below.

[31] Or if you please, $[\neg\exists x(x = a)]$. Using the clause for the quantifier things work out essentially the same way.

[32] See [14]. I don't think this is terribly controversial, and it strikes me that most philosophers who claim to be actualists but still use possible world semantics think along these lines. Although it is not obvious how to carry out this idea in detail, at bottom it is quite straightforward, especially for the simpler cases. For example, there is no possible state of affairs that the proposition $[\exists xGxj]$ that JP II has a grandchild characterizes since any such state of affairs would have to involve an object that doesn't exist. Nonetheless, we can let some abstract necessary being – some purely qualitative relation $\mathbf{r}^*$, say – go proxy as it were for JP II's "possible grandchild". That is, suppose the set of individuals were just like the actual set except that it included a grandchild of JP II (orphaned, then, I suppose); then the resulting possibly maximal state of affairs $\mathscr{S} = \langle \mathbf{O}, \mathbf{R}, \mathbf{ext} \rangle$ that would exist under those circumstances would be isomorphic to a structure $\mathscr{S}' = \langle \mathbf{O}', \mathbf{R}', \mathbf{ext}' \rangle$ just like $\mathscr{S}$ except that $\mathbf{O}'$ would contain $\mathbf{r}^*$ instead of JP II's grandchild, and $\mathbf{ext}'$ would be altered accordingly. Unlike $\mathscr{S}$, though, $\mathscr{S}'$ actually exists in the here and now, and hence can itself go proxy for that "merely possible" state of affairs. The idea, then, is to extend the definition of possibly maximal states of affairs, i.e., possible worlds, so that they include such surrogates as $\mathscr{S}'$. This cannot be done casually, for one must make certain that one's surrogate possibilia play consistent roles from one surrogate possible world to another in one's collection of *all* possible worlds. Much of [14] is devoted to carrying this out in detail. But it can done, and so this move provides us with a way of talking about all possible worlds without committing us to genuine possibilia.

[33] Here is where Adams and I emphatically part company. Adams takes modal statements of the form $\ulcorner\Box\varphi(t)\urcorner$ and $\ulcorner\Diamond\varphi(t)\urcorner$ in effect to express atomic propositions in which the modal properties $[\lambda x\, \Box\varphi'_x]$ and $[\lambda x\, \Diamond\varphi'_x]$ are predicated of a_t ([1], pp. 28ff). But, first, this is at odds with the treatment of negation above (why not take '$\neg E!a$' to express the predication of the property $[\lambda x\, \neg E!x]$ of a, hence take it to be false at worlds in which a doesn't exist), and, second, leads to a rather awkward (and Q'ish) modal logic.

[34] See, e.g., his discussion of 'Bucephalus' in [23], pp. 32ff.

[35] One way of putting the matter then is that the derived rule $\vdash\varphi \supset \psi \Rightarrow \vdash\Diamond\varphi \supset \Diamond\psi$ will break down in general in a logic containing λ-conversion that is based on characterization. (It breaks down for Prior as well, but in different cases; the instance with $\varphi = \neg E!t$ and $\psi = [\lambda x\, \neg E!x]t$ in particular would valid in $\mathbf{Q}$ with λ-conversion.) For related difficulties with λ-conversion in modal contexts, see [27].

[36] Compare [24], p. 132.

[37] This rule achieves roughly the effect of Adams' suggested weakening of Necessitation in [1] (p. 27) to the theorems of the free fragment of his system.

[38] I omit 0-place predicates only for convenience, as they often require separate, though straightforward, treatment.

[39] In terms of Hodes' convenient categorization scheme in [10] (pp. 375–6), G-interpretations here are *extensionwise actualistic*, and actualist interpretations are both extensionwise actualistic and *denotationwise actualistic*.

[40] Unlike, e.g., logics that contain an actuality operator; see e.g., [11].

[41] Garson suggests in his excellent survey article [8] (p. 277) that completeness proofs for quantified **S5** systems using Gallin's methods required the Barcan formula **BF**. Theorem 2 shows that this is not true in general.

[42] An **S4** version of **A** and its metatheory were developed when I was a fellow at the Center for Philosophy of Religion at Notre Dame in the spring of 1988. My thanks go out to the Center and to the following faculty/fellows/visitors who were there at the time: Toomas Karmo, George Mavrodes, Barry Miller, Tom Morris, Al Plantinga, Ken Sayre, Howard Wettstein, and especially Michael Kremer. A long phone call with Jim Garson during this time was a big help, as were comments from Tom Jager and Ken Konyndyk on a rather inchoate, disorganized and skeptically received talk at Calvin College. Subsets of the paper were presented at the ASL meetings at Berkeley in January 1990, and at the Prior Memorial Conference at the University of Canterbury in Christchurch, New Zealand in August 1989, where I received valuable comments from Harry Deutsch, Philip Hugly, and Mark Richard. I have benefited especially from discussion with Deutsch since first encountering his work. I also want to thank the referee for useful comments, and Ed Zalta and my colleagues at Texas A & M for discussion, criticism, and encouragement.

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