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A THEORY OF VAGUENESS

0. INTRODUCTION

One generally makes a distinction between extensional and intensional vagueness. A predicate is extensionally vague iff there are borderline cases for it. A predicate is intensionally vague iff it is logically possible that it is extensionally vague. This paper puts forth a semantic theory about extensional vagueness – vagueness, for short – at the level of Langue. It describes vagueness of the logical deep structures of the English Langue, i.e., of English, considered as a theoretical system. The paper contains a theory about vagueness – about a domain of phenomena I assume the reader capable of delimiting. In Section 1, I will make a distinction between 'fuzziness' – a property of objects – and vagueness – a property of words. In Section 2, vagueness for predicates is defined. In Section 3, vagueness for non-predicates is defined. In Section 4, some consequences of my theory will be presented.

For the sake of simplicity, I study a fragment of language in which all truth value gaps will be 'due to' vagueness of fuzziness. In this fragment, no phrases are ambiguous, meaningless, have presuppositions or can give rise to category mistakes. I also exclude phrases containing positions which are not purely referential[1] such as "Caesar appears to be approaching".

1. FUZZY OBJECTS

Consider the following two sentences:

(1.1) The sun is a hot star.

(1.2) The sun has a diameter of exactly 1.39×10^9 metres.

The sun is a borderline case for the predicate "is a hot star" and (1.1) is therefore neither true nor false. (1.2) is neither true nor false, for the sun has no distinct surface – the sun is a fuzzy object. However, the predicate "Has a diameter of exactly 1.39×10^9 metres" is a precise predicate and so

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there should be no borderline cases for it. We must then distinguish between:

(1.3) A predication is neither true nor false 'due to' the vagueness of the predicate.

(1.4) A predication is neither true nor false 'due to' the fuzziness of the object of which the predicate is predicated.

As (1.1) and (1.2) are predications of the same object, the distinction between (1.3) and (1.4) cannot depend only on the object of the predication. I consider it a basic fact that there is a distinction between (1.3) and (1.4) and the aim of this section is to provide a theory which explains that distinction by elimating the metaphorical "due to" in (1.3) and (1.4). The theory will explain fuzziness for physical objects and such objects are the only objects I will consider in this paper. My notion of physical object owes much to Quine:

Physical objects, conceived thus four-dimensionally in space–time are not to be distinguished from events or, in the concrete sense of the term, processes. Each comprises simply the content, however heterogeneous, of some portion of space–time, however disconnected and gerrymandered. (Quine 1960, p. 171.)

(1.5) DEF A non-empty subset of four-dimensional space–time is said to be an object base. An object is to be a pair set of object bases, $x = \{y, z\}$ where $y \subseteq z$. We put $\text{Max}(x) = z$ and $\text{Min}(x) = y$. x is said to be fuzzy iff $\text{Min}(x)$ is a proper subset of $\text{Max}(x)$. The doubtful region of x is the set $\{\text{Max}(x) - \text{Min}(x)\}$. (Note that the doubtful region of x is an object if $\text{Max}(x) - \text{Min}(x)$ is nonempty).

The intuitive motivation of these notions is this: if x is a fuzzy object, there will be space–time points which definitely belong to x and these will all be in $\text{Min}(x)$. Some points are definitely not in x and they will be in the complement of $\text{Max}(x)$. $\text{Max}(x) - \text{Min}(x)$ contains those space–time points which are neither definitely inside x, nor definitely outside x. And if x is non-fuzzy, $\text{Max}(x)$ and $\text{Min}(x)$ will coincide. We now define:

(1.6) DEF The object x is a part of the object y iff $\text{Max}(x) \subseteq \text{Min}(y)$.

(1.7) DEF The fusion of the objects x and y is the object z such that $\text{Max}(z) = \text{Max}(x) \cup \text{Max}(y)$ and $\text{Min}(z) = \text{Min}(x) \cup \text{Min}(y)$.

(1.8) DEF The object x is a borderline object for the object y iff x is a part of the doubtful region of y.

(1.9) DEF A reasonable delimination of an object y is any object, x, such that x is a part of $\{\text{Max}(y)\}$ and $\{\text{Min}(y)\}$ is a part of x.

(1.10) THM x is a reasonable delimitation of y iff one of the following is true:

(1.10.1) $\text{Min}(x) = \text{Min}(y)$.

(1.10.2) There is a z which is a borderline object for y such that x is the fusion of z and $\{\text{Min}(y)\}$.

(1.11) THM The following sentences are equivalent:

(1.11.1) x is fuzzy.

(1.11.2) There exists a borderline object for x.

(1.11.3) There are several different reasonable deliminations of x.

Thus, x is a part of y iff every space–time point which either definitely is inside x or inside the doubtful region of x is definitely inside y. z is a reasonable delimitation of y iff $\text{Min}(y) \subseteq \text{Min}(z) \subseteq \text{Max}(z) \subseteq \text{Max}(y)$. Some caution is needed here. My notion of reasonable delimitation is not sufficient to capture everything we may demand of an object in order to accept it as a reasonable substitute for another. For instance, connectedness may be essential to some objects – my body, for instance – while disconnectedness may be essential to others – the solar system, for instance. A complete theory of reasonable substitutes would have to divide objects into those for which connectedness is essential, those for which disconnectedness is essential and the others. Similarly for other topological notions.

The semantic notion of independence of fuzziness will now be introduced and used to explain the difference between (1.3) and (1.4).

(1.12) DEF The predicate P is said to be independent of the fuzziness of x iff there are no two reasonable delimitations, y and z, of x such that P is true of y and P is false of x.

Consider now the two predicates:

(1.13) is a hot star,

(1.14) has a diameter of exactly 1.39×10^9 metres.

"The sun" names an object, u, which is fuzzy. All reasonable delimitations have roughly the same temperature and so (1.13) is independent of the fuzziness of *u*. But (1.4) is not independent of the fuzziness of *u*. For there are two reasonable delimitations of *u*, one which has a diameter of exactly 1.39×10^9 metres and one reasonable delimitations which definitely does not. We can replace the unanalysed (1.3) and (1.4) with:

(1.3.a) A predication is neither true nor false and the predicate is independent of the fuzziness of the object of which the predicate is predicated.

(1.4.a) A predication is neither true nor false and the predicate is *not* independent of the object of which the predicate is predicated.

The theory of fuzziness explains not only the difference between (1.3) and (1.4) but also the truth value gap of (1.2). (1.2) lacks a truth value because there are several reasonable delimitations of the object, *u*, which are named by "the sun" and the predicate in (1.2) will be true of some reasonable delimitations of *u* and false of others. If someone were to assert (1.2) and we had all the facts about the sun at hand, we would neither agree, nor disagree with him. Instead, we would say: "That depends on what you take the sun to be". The truth or falsity of (1.2) depends on how the sun is delimited and as long as no particular delimitation is chosen, (1.2) is neither true nor false. This explanation of the truth value gap of (1.2) is similar to the explanation of truth value gaps provided by supervaluation theory.

Finally, something ought to be said about the status of fuzziness. Is fuzziness at bottom a property of our conceptual system or is it a property of objects in the world? Two questions ought to be distinguished here:

(1.15) Is it possible to define "*x* is fuzzy" by a second order sentence which, except for logical constants, contains only names of predicates and predicates of predicates?

(1.16) Do we actually justify our knowledge of the truth of "*x* is fuzzy" by a deduction from a second order sentence?

I have no answer to the first question. Fuzziness is defined by spatio-temporal concepts and if the Leibnizian theory that all such concepts are definable by non-spatio-temporal concepts is true, the answer to (1.15) is yes. There can be little doubt, however, that we do know the truth of some sentences asserting spatio-temporal relations without deducing these sentences from any other sentences. We perceive directly some spatial relations among objects. Fuzziness is also directly perceived by us, at least in some cases. The ability to discriminate very fuzzy objects from relatively non-fuzzy objects is an ability we share with animals like rats and pigeons. The discriminations these animals make are not based on deduction but on perception. The answer to the second question is therefore no – at least in some cases we do perceive fuzziness directly and do not base our knowledge of the truth of "*x* is fuzzy" on deduction. Quine (1960, p. 126) and Alston (1964, p. 92) employ a notion of vagueness of individuation which is a property of predicates and is very similar to my notion of fuzziness which I ascribe to objects. My discussion of (1.15) and (1.16) should make it clear that my theory need not logically contradict theirs. But I consider their theory to be defective for epistemological reasons.

2. VAGUENESS FOR PREDICATES

With the distinction between fuzziness and vagueness at hand, it is easy to define vagueness for predicates within our fragment:

(2.1) DEF x is a borderline for the predicate P iff all of the following conditions are true:

(2.1.1) x exists.

(2.1.2) P is neither true nor false of x.

(2.1.3) P is independent of the fuzziness of x.

(2.2) DEF The predicate P is extensionally vague iff some x is a borderline case for P.

(2.3) DEF The predicate P is intensionally vague iff it is logically possible that P is extensionally vague.

(2.4) THM If $x = y$, then x is a borderline case for P iff y is a borderline case for P.

The notion of borderline case is thus extensional. An object can be given, presented, to persons in different ways and they may fail to know whether two descriptions actually pick out the same object. Knowledge, belief, mental states and dispositions in general depend on how an object is given. The question whether a certain object is a borderline case for a certain predicate does not depend on how the object is given. My theory of vagueness is incompatible with those theories which make vagueness depend on how objects are given.

3. VAGUENESS FOR NON-PREDICATES

In our fragment of English, the vagueness of a sentence consists in its lack of a truth value. A predicate is vague if it has borderline cases. Using sentences and predicates as basic elements, I will define vagueness for arbitrary phrases by a recursive definition. The leading idea I will here work out is that a non-predicate is vague iff it sometimes 'causes' vagueness of a sentence of which it is a part – more precisely: if there is a vague sentence which, except possibly for the phrase under consideration, contains only precise phrases.

I assume that the logical deep structures of English are given by a categorial grammar as in Lewis (1970). There are two basic categorial indices, S and N. Whenever $c, c_1 \dots c_n$ are categorial indices, then so is $(c/c_1 \dots c_n)$. We pick out a finite number of phrases to serve as simple phrases and assign to each of them a categorial index. Complex phrases are formed by the rule:

(3.1) Whenever P has index $(c/c_1 \dots c_n)$ and P_1 has index c_1 and P_2 has index c_2 and ... and P_n has index c_n, then the sequence $PP_1P_2 \dots P_n$ has index c and is well formed.

Below an example of assignment of categorial indices:

Kind of expression	*Examples*	*Categorial index*
Predicates	isn't a man, run, snore move, hit John	(S/N)
Relationship expressions	hit, break, push	(S/NN)
Quantifier phrases	everything, something many men, every man	$(S/(S/N))$
Quantifiers	every, some, many	$((S/(S/N))/(S/N))$
Proper names	John, Mary, the sun	$(S/(S/N))$
Verb phrase modifiers	rapidly, with a hammer	$((S/N)/(S/N))$
Noun phrase modifiers	heavy, hard, long	$((S/N)/(S/N))$

In each category there will be variables and I assume lambda abstraction for binding these. In what follows, my metavariables for phrases will not range over phrases which are variables or contain free variables. I assume that each such phrase has a categorial index which is composed entirely out of the indices S and (S/N) and (S/NN) and (S/NNN) etc. No phrases among the deep-structures of English will have a categorial index like, N (N/N),

$(N/(S/N))$, etc. This assumption is needed in order to guarantee that the definition (3.3) will define vagueness for all phrases in the English deep structures. Most authors in the Montague tradition claim this assumption to be true.

Instead of making the underlying model theory explicit, I will use the notion of a possible phrase. The model I take for granted assigns a meaning to each phrase of the English deep structure. For instance, each n-place relationship expression is assigned a set of n-tuples built up from elements of the domain of objects. There will, however, be such sets of n-tuples which are not assigned to any actual relational expressions. I will say that such sets of n-tuples are assigned to possible relational expressions – there could have been an actual relational expression to which this set was assigned. Similarly for phrases which are not relational expressions. My variables below range over possible expressions – otherwise the vagueness of some phrases would depend on which phrases happened to be actual in English. We define:

(3.2) DEF A phrase in category S i.e. a sentence, is vague iff it is neither true nor false.

(3.2.1) A phrase in any of the categories (S/N), (S/NN), (S/NNN), etc. is vague iff there are borderline cases for it.

(3.2.2) A phrase P category $(c/c_1 \ldots c_n)$ where $(c/c_1 \ldots c_n)$ is not $(S/N \ldots N)$ is vague iff there are possible phrases $P_1 \ldots P_n$ in categories $c_1 \ldots c_n$ respectively such that:

(3.2.2.a) Each P_i is non-vague.

(3.2.2.b) The phrase $PP_1 \ldots P_n$ is vague.

(3.3) DEF A phrase, P, is precise iff P is not vague.

(3.4) DEF A phrase, P, is intensionally vague iff it is logically possible that P is vague.

(3.5) THM If all constituent phrases in the complex phrases P are precise, then P is precise.

(3.6) THM If P is a vague complex expression and all constituents in P, except perhaps P_1, are precise, then P_1 is vague.

The theorem (3.6) gives a method of showing a phrase P vague – find a vague expression Q containing P which except for P, contains only precise phrases. Consider, for instance, the vague sentence "Many photons passed through the slide S between t_1 and t_2". Suppose there are no borderline cases for the predicate "passed through the slide S between t_1 and t_2". Then this predicate is precise and it follows that the quantifier phrase "many photons" is vague. Suppose furthermore that there are no borderline cases for the predicate "is a photon". It follows that the quantifier "many" is vague. By this method, modifiers can be shown vague. My theory of vagueness can be used to show that the logical constants are precise, for instance:

(3.7) THM On its normal interpretation, "there is" is precise.

The proof uses (2.1) and (1.12) and the following normal interpretation of "there is":

(3.8.1) "there is a P" is true iff there is an object x such that P is true of x.

(3.8.2) "there is a P" is false iff P is false of every existing object.

Is there any connection between fuzziness and vagueness? In his seminal paper on vagueness, Russell (1923, pp. 86–87) argues from the fuzziness of Ebenezer Wilkes Smith to the vagueness of "Ebenezer Wilkes Smith". The following theorem shows his argument to be justified:

(3.9) DEF A phrase P with categorial index $(S/(S/N))$ is a proper name of x iff x exists and for every possible predicate, Q, PQ is true (false) iff P is true (false) of x.

(3.10) THM Let N be a proper name of x. Then x is fuzzy iff N is vague.

The proof employs the fact that if x is fuzzy, there will be some (possible) precise predicate which talks about spatio-temporal extension but which fails to be true of x.

4. THE INFECTION THEORY OF VAGUENESS

In this section, I will use my theory of vagueness to discuss how the vagueness of complex phrases depends on the vagueness of their constituent phrases. First, we observe that the following is false:

(4.1) If P is a syntactic argument for Q and if P is vague, then so is Q.

The falsity of (4.1) follows from the fact that the existential quantifier is precise in spite of the fact that it takes vague arguments. It has been argued that "true" is vague because it takes vague arguments,[2] but this argument rests on a false premiss. Another assumption often made in the literature on vagueness is:

(4.2) If all the constituent phrases of the complex phrase *P* are vague, then *P* is vague.[2]

I will call (4.2) "the infection theory of vagueness" and show it false. The following two complex phrases are counterexamples to (4.2):

(4.3) Caesar attained much power.

(4.4) is a well known British philosopher, born around 1870, dead around 1970, whose original work in the foundations of mathematics and logic deeply inspired the Vienna Circle, whose pacifist views outraged his fellow countrymen during the First World War.[3]

(4.3) is true and thus precise in spite of the fact that both "Caesar" and "attained much power" are vague. (4.4) shows that we can combine a large collection of vague predicates and obtain a precise predicate, true of a unique individual and false of all others. The falsity of the infection theory has the following interesting consequences. Suppose that all observational predicates of a language are vague. It is then logically possible to define complex expressions which are perfectly precise from these vague observational predices. I will now consider a related theory, called "the infection theory of intensional vagueness". This theory says:

(4.5) If all constituent phrases of the complex phrase *P* are intensionally vague, then *P* is intensionally vague.

The following sentence is a counterexample to (4.5):

(4.6) This square is round.

(4.6) is logically false and thus not intensionally vague. However, "this square" is vague if it names a physical object and "is round" is vague when applied to physical objects. Having falsified two infection theories, we now proceed to consider the spread of the infection and the cure of it. We observe:

A. Sometimes the vagueness of a sentence is cured by replacing an expression in the sentence with a vaguer expression. Consider the following sentences:

(4.7) John ran fast during the latter part of the race.

(4.8) John ran at an average speed exceeding 24.382 km/h during the latter part of the race.

The predicates of these sentences are respectively "ran fast" and "ran at an average speed exceeding 24.382 km/h". Say that (4.8) is neither true nor false, i.e., vague, for its truth value depends on how we delimit the latter part of the race. And say that (4.7) is true, for John ran fast, no matter how we delimit the latter part of the race within reasonable limits. Thus, substitution of a vague expression – "ran fast" – for a preciser expression in (4.8) yields a sentence which is true and thus precise.

B. Sometimes a sentence gets infected with vagueness if a vague expression in it is replaced with a preciser expression. This is equivalent to A, but the substitution goes in the other direction.

Thus, it is quite conceivable that some philosopher should 'explicate' the parts of a sentence in a way which will make the whole sentence vaguer. Some of the explicated expressions may fit so badly with the remaining vague parts that a vague sentence results. Furthermore, one must be careful in stating the supervaluation theory of vagueness. Consider the following:

> ... an expression is made more precise through making its simple terms more precise ... an expression is made more precise *only* through making its simple terms more precise ... (Fine 1975 pp. 274–275).

In the general case, this is false in both directions, as is seen in A and B. It might be true for regimented languages of the kind Fine considers, however.

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NOTES

* My view on language owes much to my teachers Sören Halldén and Bengt Hansson, both of Lund. Criticism from them, from Ingemar Nordin, of Lund, and from anonymous referees helped improve earlier versions of this paper.

1 The position held by the phrase P in the sentence ... P ... is said to be purely referential iff, whenever P and P_1 normally have the same extension, the sentence ... P ... is true iff the sentence ... P_1 ... is true.

[2] Russell (1923) makes these assumptions – see Rolf (1980).
[3] (4.4) can be considered to consist of one predicate and a large number of modifiers – I need not assume that any conjunction occurs in (4.4).

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