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PROLOGUE-FUNCTORS

1. Davidson's analysis of "saying that"

According to Donald Davidson[1] the sentence

Galileo said that the earth moves.

can be analyzed in the following way:

Galileo said that.
The earth moves.

One might therefore think that it is of the logical form

aRb.
p.

But it is unsatisfactory to say that in the above case the expressions 'Galileo said that' and 'the earth moves' are two independent sentences. From

Galileo said that the earth moves

one may *not* infer

The earth moves.

This shows that in this case 'the earth moves' is not an independent sentence, but that the two expressions 'Galileo said that' and 'the earth moves' are rather two parts of a complex construction.[2]

Davidson himself points out the nature of the connection involved: the two expressions are connected like introducer and introduced or, as I shall say in this paper, like prologue and play. Therefore they should not be separated by a full stop, but it would be better to link them always by means of the symbol ':'. Replacing at the same time 'that' by the more explicit expression 'that which follows' we thus obtain[3]

Galileo said that which follows: the earth moves.

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One might now think that the construction is of the form

$$aRb : p.$$

But some reflection indicates that the expression 'that which follows' is not a name in the same logical sense in which 'Galileo' is a name. For, while the existential generalization

$$(\exists x)\ (x \text{ said that which follows: the earth moves})$$

i.e.

Somebody said that which follows: the earth moves

is easily understood, the meaning of the following is less clear:

$$(\exists x)\ (\text{Galileo said } x : \text{the earth moves}).$$

One might claim that this means

Galileo said something, namely: the earth moves.

But in this case 'namely' must be short for 'namely he said that which follows'. This shows that through this existential generalization we have actually amplified the original construction and that the variable 'x' has not really replaced the expression 'that which follows'. It would seem, therefore, that the expression 'that which follows' and the symbol ':' form an intimate unity which cannot be broken up into a name in the ordinary logical sense and a connective. The reason is not that the expression 'that which follows' is a demonstrative expression. For instance, from

Lightning struck. God caused that.

we *can* infer

$$(\exists x)\ (\text{Lightning struck} \wedge \text{God caused } x).$$

It is true that we loose thereby the factual information that the x in question is the event of lightning, but the resulting existential generalization is meaningful without any amplification. In the case of 'Galileo said that the earth moves', on the other hand, such an existential generalization without amplification is not admissible, because it would not merely cause a loss of factual (historical) information concerning Galileo's utterances, but it would also remove the necessary *linguistic* information how the *present* expression 'the earth moves' is to be understood, namely as a saying not of the present speaker but of Galileo.

It is therefore advisable to return to the pre-Davidsonian practice of articulating the logical form of the entire construction as

$$S(a, p).$$

This does not mean, of course, that Davidson's analysis has contributed nothing to our understanding of such constructions. On the contrary, although it seems wrong to claim that the expression 'that' in '*a* says that *p*' is a name in the ordinary logical sense, it remains nevertheless very illuminating to explicate

a says that *p*

as

a says that which follows: *p*.

This explication *does* clarify the peculiar nature of the functor '*S*', since it shows that '*a* says that which follows:' is related to '*p*' like a prologue is related to the play which follows it. It shows that the functor '*S*' is *not* the expression of a two-place relation, i.e. of a relation holding between pairs of *objects*. Rather, '*S*' is what might be called a two-place one-play prologue-functor, i.e. a two-place functor whose second argument is not the name of an object but a sentence, namely a sentence functioning as a play.

In the remainder of this paper I will point out some of the important consequences which follow from the peculiar nature of prologue-functors.

2. Prologue-Functors vs. Metalinguistic Functors

It is an important feature of the expression

Galileo said that which follows: the earth moves

that it seemingly talks *about* a sentence (or *about* the corresponding proposition), namely about the sentence (or the proposition) 'the earth moves', while as a matter of fact *no name* of that sentence (or of that proposition) is used. What is printed after the prologue is not the *name* of a sentence but simply that sentence *itself*. Since no names referring to expressions are used, therefore we are in fact still in the object-language and have not moved up into the metalanguage.

The prologue-functors are thus a useful device for avoiding an ascent into metalanguage while at the same time obtaining benefits usually associated with such an ascent.

The truth-functional connectives provide a simple illustration of this. For instance, by adding the metalinguistic predicate 'is false' to the name of the sentence 'The earth moves' we obtain the new sentence

> 'The earth moves' is false

which compared to the original sentence has the opposite truth-value. But it is well-known that we can obtain a sentence with the opposite truth-value without ascent into metalanguage, simply by prefixing the expression of a truth-functional negation to the original sentence (not to the *name* of the original sentence, but to the original sentence itself!):

> It is not the case that the earth moves.

A truth-functional negation can be regarded as a prologue-function, as the following reformulation shows:

> What that which follows asserts is herewith denied: the earth moves.

Or more briefly

> The following is herewith denied: the earth moves.

As a matter of fact all truth-functional connectives can be read as prologue-functors. Thus '$p \wedge q$' can be read

> What the following two items assert is herewith asserted: p, q.

And '$p \supset q$' can be read

> Of the following two items either what the second asserts is herewith asserted, or what the first asserts is herewith denied: p, q.

Etc.[4]

3. PROLOGUE-FUNCTORS VS. SET-THEORETICAL FUNCTORS

Prologue-functors can also be used to avoid another kind of ascent, namely an ascent into a set-theoretical language which contains *names of sets*, i.e. which turns *extensions* of expressions into namable objects.

For instance, instead of saying set-theoretically

> The set of men is identical with the set of
> featherless bipeds

we can say without using names for sets (and without ascending into metalanguage either!):

> The following two items are extensionally identical
> (i.e. have-the-same-extension):
> man, featherless biped.

Or in symbols

> man ○ featherless biped.

According to this interpretation the functor '○' is not an expression for a relation holding between pairs of objects but a two-place two-play prologue-functor whose arguments are not individual names of sets but general names functioning as plays. (General names *have* extensions, but they *do not name* their extensions; just as sentences might be said to *have* propositions as their meanings, but certainly *do not name* propositions.)

Leśniewski's Ontology can be viewed as the perfect example of an exact formal theory which takes full advantage of all the possibilities of using prologue-functors instead of set-theoretical functors. The teaching of Ontology has long been hampered by the fact that no paraphrase of its formulas was available which was both accurate and capable of making the analogy with set-theoretical formulas perspicuous.[5] With the help of prologue-play constructions it is now for the first time possible to systematically provide not merely an account of the truth-conditions but to give also a perspicuous intuitive paraphrase for all these formulas of Ontology.

4. Prologue-Quantifiers vs. Referential or Substitutional Quantifiers

Many philosophically astute logicians have hesitated to quantify over propositional variables because they thought that they were faced with the dilemma of either having to use referential quantification and thereby be committed to the existence of a new kind of objects, namely of propositions, or else having to use substitutional quantification and then having to worry about a possible lack of inscriptions. (An analogous dilemma seemed to exist with respect to the quantification over predicate expressions and general names.)

As a matter of fact, however, this dilemma can be avoided by using prologue-quantification, i.e. a new kind of quantification involving a prologue-play construction.

For instance

$$(p)\,(q)\,(r)\,[((p \supset q) \wedge (q \supset r)) \supset (p \supset r)]$$

can be read [6]

> Whatever the following three items are taken to say:
> $p, q, r,$
> the following is herewith asserted:
> $((p \supset q) \wedge (q \supset r)) \supset (p \supset r)$.

Notice that no *names of propositions* [7] and no *names of sentences* are used. Instead of being named, the sentences themselves are introduced by means of prologues.

By contrast, according to a referential reading of the quantifiers, the above mentioned formula says

> For any three propositional objects p, q, and r,
> $((p \supset q) \wedge (q \supset r)) \supset (p \supset r)$.

Here the variables 'p', 'q', and 'r' are considered as ranging over certain abstract objects in the universe of discourse. Furthermore, in order to avoid category mistakes, all truth-functors must be regarded as name-forming functors, i.e. in our example the formula after the quantifier should be read

> The material implication between, on the one hand the conjunction of the material implication of p and q and the material implication of q and r, and on the other hand the material implication of p and r, does hold.

The referential reading contains no names of expressions and is therefore not metalinguistic. It is rather like a set-theoretical reading in that it reifies meanings into abstract objects.

According to the substitutional reading of the quantifiers the above mentioned formula says that

> Every result obtained by substituting in the expression '$((p \supset q) \wedge (q \supset r)) \supset (p \supset r)$' in place of the variables 'p', 'q', and 'r' tokens of three arbitrary sentences is a true sentence.

This is clearly a metalinguistic statement since it contains names of expressions and refers to entities in the metalinguistic universe of discourse.

Leśniewski's logical systems are perfect examples of exact formal theories involving all possible types of prologue-quantifiers. Again, I believe that thanks to our insight into the functioning of prologue-play constructions we are now for the first time able to give a satisfactory intuitive paraphrase of Leśniewskian quantifiers.[8]

A revealing example of how prologue quantification is used in Leśniewski's Ontology is the following theorem:

$$\sim (x)\, ex\, (x).$$

Which can be read

> The following is herewith denied:
> whatever the following item is taken to name (i.e. whatever extension the following item is taken to have): x
> the following is herewith asserted: x exists.

For instance, if 'x' is taken to name the same as 'Pegasus', then 'x' is in fact an empty name which names nothing, i.e. which has an empty extension; and in this case the sentence 'x exists' is not true and should not be asserted.

5. A NEW EXPLICATUM OF THE PREDICATE 'IS TRUE'

The great significance of the method of using prologue-functors is demonstrated by the formal properties of the following semantical system (based on the extended propositional calculus) which has been developed by Charles C. Davis:[9]

Axiom 1. $(p)\,[N(Q(p), p)]$.
Axiom 2. $(A)\,(p)\,(q)\,[(N(A, p) \wedge N(A, q)) \supset (p \equiv q)]$.
Definition. $(A)\,[(\exists p)\,(N(A, p) \wedge p) \equiv Tr(A)]$.

The following could serve as an alternative definition of 'Tr':

$$(A)\ [(p)\ (N(A, p) \supset p) \equiv Tr(A)].$$

It is suggested that the definition of '*Tr*' is an explicit definition of the predicate 'is true', and it can be proven that the system is consistent.

This is a very striking result which obviously could not be obtained if the predicate 'is true' were understood in Tarski's sense. But if '*Tr*' is not a predicate in Tarski's sense, then present-day logicians are at a loss to understand what the real meaning of '*Tr*' is. I will therefore try to explain how, with the help of prologue-play constructions, a clear intuitive understanding of '*Tr*' and of the other special constants of Davis' system can be reached.

I propose that the formula

$$N(Q(p), q)$$

can be taken to mean the same as

> Samesayings of the following item: *p*,
> say the same as the following item: *q*.

According to this interpretation the functor '*N*' is a two-place one-play prologue-functor which can take another prologue-play construction as its first argument. If the expression 'samesayings of the following item: *p*' belongs to the semantical category $n(s)$ and the expression q belongs to the semantical category s (i.e. *s*entence), then the functor '*N*' is of the semantical category

$$\frac{s}{n(s) \quad s},$$

i.e. it is a functor which takes an expression of the category $n(s)$ and a sentence as arguments, and forms together with them a sentence.[10]

According to this interpretation the expression

$$Q(p)$$

has to be read

> samesayings of the following item: *p*,

where the functor '*Q*' is taken to be a one-place one-play prologue-functor which takes a sentence as its argument and forms together with it an expression of the semantical category $n(s)$. Thus 'Q' is of the semantical category

$$\frac{n(s)}{s}.$$

The expression 'samesayings of the following item: p' is a certain general name of sayings, the extension of which consists of all sayings which say the same that 'p' is saying. One might think that names of sayings are simply names, i.e. that the semantical category $n(s)$ is simply identical with the semantical category n. But such an identification would immediately give rise to semantical antinomies.[11] The non-identity of the two semantical categories $n(s)$ and n is, admittedly, a complication,[12] but it seems to agree rather well with our intuitive feelings. For instance

Desks say the same as the following item: the earth moves

sounds nonsensical and not merely false. Of course, one might insist that while sayings are not substances like desks, they are events like lightnings. But

Lightnings say the same as the following item:
the earth moves

sounds still rather odd.

The interest of the prologue-functor 'Q' lies in the fact that prefixing 'Q' to a sentence is like putting this sentence into quotation-marks. But whereas traditionally quantification into quotation-marks has not been allowed, there is no difficulty to quantify over the argument of a prologue-functor. For instance the following is a necessary semantical truth:

$$(p)\ [N(Q(p), p)]$$

i.e.

Whatever the following item is taken to say: p,
samesayings of the following item: p
say the same as the following item: p.

Furthermore, whereas traditionally quotation-marks have been taken to form an *individual* name, I have preferred to interpret 'Q' as forming the analogue of a *general* name. For instance, whereas the expression "The earth moves'' is usually treated as the individual name of one saying or of one set of sayings, in my interpretation 'Q (The earth moves)' is treated as a general name which has a set of sayings as its extension, but does not name this set.[13]

Actually the measure for the sameness of sayings can be chosen in many different ways, ranging from material equivalence to the most detailed intensional isomorphism, and covering also various kinds of equiformity in pronunciation and/or writing. That is, with the help of different equivalence-relations one could define many different Q-functors and N-functors.[14]

As a matter of fact the functor 'N' is analogous to the relational expression '*names*'. But 'names' takes the name of a name of an expression and a name of an expression as arguments. We say, for instance,

'Gödel's theorem' *names* Gödel's theorem.

The functor 'N', on the other hand, has the advantage that it requires no such ascent into meta-metalanguage. We can, e.g., say

> N (Q(There is a true arithmetical statement that is not demonstrable in arithmetic), There is a true arithmetical statement that is not demonstrable in arithmetic).

This is of the logical form

$$N(A, p)$$

where 'A' is a variable of the semantical category $n(s)$ (i.e. a variable for names of sayings), and it is not of the logical form N('A', 'p') resp. N(''p'', 'p'). But while the functor 'S' of Section 1 belongs still to the object-language, the functor 'N' is metalinguistic since his first argument is a name of sayings.

After these preliminaries, the interpretation of the functor 'Tr' presents no more difficulties. The formula

$$Tr(Q(p))$$

can easily be read

> Samesayings of the following item: p
> are true.

And

$Tr(A)$

becomes

*A*s are true.

The advantage which the functor '*Tr*' has over Tarski's predicate 'is true' consists precisely in the fact that thanks to the prologue-functor '*Q*' it can have '*p*' as an argument of its argument. Thus instead of getting bogged down at Tarski's schematic condition *T*

$(a \text{ is true}) \equiv p$

where the difference between the variables '*a*' and '*p*' cannot be bridged, we can now go on and write the theorem

$(p)\,[Tr(Q(p)) \equiv p].$

The functor '*Tr*' is an exact analogue of the truth-functional assertion-functor '*As*' which, together with a sentence, forms a new sentence having the same truth-value. That is, the functor '*Tr*' is nothing else but a metalinguistic truth-functor; it is simply the non-prologue analogue in the metalanguage of a certain prologue-functor of the object-language. The difference between '*Tr*' and '*As*' is merely that '*Tr*' is of the semantical category $s/n(s)$, whereas '*As*' is of the semantical category s/s.

6. PROLOGUE-FUNCTORS AS ILLOCUTIONARY-FORCE-INDICATORS

In keeping with the paraphrases given in Section 2 the formula '$As(p)$' has to be read

What the following item asserts is herewith asserted: *p*.

It might be objected that this reading is unsatisfactory, because it renders the theorem

$(p)\,[Tr(Q(p)) \equiv As(p)]$

false, since something may be true without anybody asserting it, and something may be asserted even if it is false. But my paraphrases are not to be understood as metalinguistic *reports* of the historical fact that something is being asserted, rather they have to be understood as themselves *making* those assertions. If this is kept in mind, then the objection becomes groundless, since asserting that certain assertions are true is certainly materially equivalent with *making* those assertions.

However, my paraphrases *do* have the fault that they sound too much like *reports about* assertions. A better way to convey the intended meaning of the formula '$As(p)$' might therefore be to read it:

> What the following item asserts *is* the case: *p*

(with emphasis on the 'is'); or simply

> Yes: *p*.

The difference between

> What the following item asserts is herewith asserted: *p*

and

> Yes: *p*

is similar to the difference between

> I promise herewith to bring about what the following item asserts: *p*

and

> Promised: *p*.

This shows that the functor '*As*' can actually be identified with Searle's illocutionary-force-indicator '⊢'.[15] The functor '*As*' can be said to emphasize or make explicit the assertive force of '*p*'. As a matter of fact all the so-called illocutionary-force-indicators can be regarded as a special kind of prologue-functors, namely as those prologue-functors of the object-language whose function it is to transform assertions respectively into assertions, promises, requests, questions, mere considerations, warnings, etc.

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NOTES

[1] D. Davidson, 'On saying that', *Synthese* **19** (1968–69) 130–146.

[2] Cf. also R. J. Haack, 'On Davidson's paratactic theory of oblique contexts' *Nous* 5 (1971) 351–361.
[3] To be fully explicit we would have to replace the expression 'that' not merely by the phrase 'that which follows' but by the long phrase 'something of which that which follows is a samesaying'. Wilfrid Sellars has strongly insisted on the importance of the notion of "samesaying" or "playing-the-same-linguistic-role," and he has introduced the use of dot-quote-expressions as general names whose extension consists of all utterance-tokens (from all possible languages) which play the same role as that normally played by expression-tokens equiform to the expression-token occurring between the dots. Therefore his explication of our example would presumably read:

Galileo uttered something which is a
· the earth moves ·.

But this Sellarsian paraphrase involves a metalinguistic term, namely a dot-quote-expression. Our explication has, as we shall see, the advantage that it allows to postpone such an ascent into the metalanguage.

For a clear account of Sellars' views on this and related topics see Michael J. Loux 'The ontology of Wilfrid Sellars', to appear in a collective work on the philosophy of Sellars.
4 Cf. Section 6 for an objection against these paraphrases. Section 4 will point out a further advantage of using prologue-functors instead of metalinguistic functors, and Section 5 will show the usefulness of metalinguistic prologue-functors.
5 Cf. G. Küng and J. T. Canty, 'Substitutional quantification and Leśniewskian quantifiers' *Theoria* **36** (1970) 165–182, footnotes 4 and 6 on p. 166–167; G. Küng, *Ontology and the Logistic Analysis of Language*, Dordrecht: D. Reidel, 1967, footnote 73 on p. 123. D. P. Henry, *Medieval Logic and Metaphysics*, London: Hutchinson Library, 1972, p. 37 interprets '$a \bigcirc b$' as saying the same as 'Only all a is b'; this paraphrase is accurate, but it does not make the analogy with a set-theoretical identity statement perspicuous.
[6] One might mark the scope of prologues by putting double quotes around the expressions which are used as plays. But this would probably mislead the reader into thinking that when expressions are used as plays they become a kind of metalinguistic names; and it is precisely the aim of this paper to point out that expressions used as plays are *not* metalinguistic names. As a matter of fact such double quotes could only be useful in the paraphrases. In the symbolic formulations the scope of the prologues is clear and no special quotation-marks are needed.
[7] In the case of the *universal* quantifier it is especially easy to show that prologue-quantifiers involve no reference to meanings, i.e. no reifying and naming of meanings. The readings of the *particular* quantifier are in this respect less perspicuous, because here it seems not possible to avoid using some *substantive* such as the word 'meaning'. We might for instance read the formula '$(\exists p)\ (p)$' as follows

For some meaning of the following item: p,
the following is herewith asserted: p.

This difficulty may have been one of the reasons why Leśniewski did not want to use the particular quantifier in his logical systems. However, if it is clear that the universal quantifier involves no naming of meanings, then it can also be made clear that the particular quantifier involves no naming of meanings, since the particular quantifier can be explicitly defined in terms of the universal quantifier and the negation sign.

[8] In Küng and Canty (1970), we were able to clarify that the Leśniewskian quantifiers are ranging over extensions or meanings, and that the range of quantification has to be distinguished from the universe of discourse. But the paraphrases mentioned (cf. p. 180), which actually coincided with formulations given by Leśniewski himself, were still unsatisfactory, since they contained metalinguistic names of expressions. Cf. also Küng (1967), p. 117–118: "From the point of view of a Frege-Russellian system Leśniewski's quantified sentences hover curiously between object-language and metalanguage..."
[9] Charles C. Davis 'Some Semantically Closed Languages' to appear in this issue. The original version of Davis' paper has been written before my paper and I have profited a great deal from it.
[10] The notation for categories of functors is explained in K. Ajdukiewicz 'Die syntaktische Konnexität', *Studia Philosophica* 1 (1935) 1-27 (English translation in S. McCall (ed.), *Polish Logic 1920–1939*, Oxford, Clarendon Press 1967).
[11] Cf. Davis' paper, Sections 10 and 12.
[12] Actually one is led to distinguish not only two but infinitely many different semantical categories of "names", cf. note 14. But on the other hand it proves unnecessary to distinguish different semantical categories of sentences. Compare how in Leśniewski's Ontology "copulas" of higher semantical categories can be defined, and how assertions in terms of these higher order copulas still belong to the basic semantical category *s*.
[13] This is similar to what Wilfrid Sellars says about dot-quote-expressions, cf. note 3. But Sellars has no clear equivalent of my notion of prologue-functor.
[14] Cf. Davis' paper, Section 20. Quotation-marks can also be applied to other expressions than sentences, and thus analoguous functors '$Q_{n(n)/n}$' $Q_{n(s/n)/s/n}$', etc., (and corresponding *N*-functors,) can be introduced which take respectively names, sentence-forming functors of names, etc., as arguments. But, as the subscripts indicate, these *Q*-functors must all be said to form expressions of new and different semantical categories. Cf. Davis' paper, Section 18.
[15] J. R. Searle, *Speech Acts*, Cambridge University Press, 1969, p. 31.