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# CHANGE IN MATHEMATICS EDUCATION SINCE THE LATE 1950's – IDEAS AND REALISATION U.S.A.

## *The United States Experience with "New Math"*

In 1959 the Commission on Mathematics of the College Entrance Examination Board (CEEB) published its *Program for College Preparatory Mathematics* in a report that provided strong motivation and specific guidelines for a succeeding decade of vigorous research, curriculum development, and school innovation designed to bring major change to the form and substance of United States school mathematics instruction. But, by 1973 concern about the character and effectiveness of resulting mathematics programs made Morris Kline's book, *Why Johnny Can't Add*, a best seller among books on American education. And in 1977 the Mathematical Association of America and the National Council of Teachers of Mathematics issued a joint statement on college preparatory mathematics which, while acknowledging positive changes during the 1960's, embodied an unmistakeable renunciation of the most daring reform ideas and an urging to emphasize more traditional instructional goals and methods. This prestigious professional statement and widespread public calls for "back to basics" education promise continued retreat from the program initiatives known collectively as "New Math – American Style."

It would be easy for innovators to conclude that the once promising revolution in United States school mathematics has been lost. But the apparent public and professional preference for conventional content and methods in school mathematics has not gone unchallenged. Professional meetings, journals, and the popular press have provided regular forums for lively discussion of fundamental questions such as:

> What mathematical knowledge and abilities are important preparation for students of varying aptitudes and interests?
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> What curriculum development and implementation strategies will most effectively put good programs into action?
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> How can individual student and program achievements be appraised accurately?
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> What major research and development efforts are needed to produce the programs called for by individual and societal needs?

There have been several recent attempts to base answers to these questions on accurate understanding of the status and dynamics of change in United

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States mathematics education (NACOME, 1975; Osborne and Suydam, 1977; Price, et. al., 1977; Graeber, et. al., 1977). At the outset one must admit serious difficulties in characterizing school mathematics programs across the United States. There is a large measure of local school control of education, with no official national or state syllabi or examinations and no standard method of teaching. Would-be reformers criticizing school mathematics in the 1950's could aim at curricular and instructional practices that had, through force of tradition, grown remarkably uniform. But the goals and the effects of the "New Math" efforts to change traditional program patterns have been extremely diverse.

The following report describes and analyzes an era in United States school mathematics that saw promises of striking improvement lead to elaborate development efforts, modest impact on schools, and persistent dismay with the content and teaching style of most elementary and secondary school programs. But the story should yield valuable insight to those who hope to change school mathematics in the future.

## WHAT CHANGES WERE SOUGHT?

The case for reform in United States school mathematics was made by the 1959 CEEB Commission on grounds similar to those stated around the world: The spectacular twentieth century growth of mathematics and its increasingly pervasive applications call for enhanced secondary school instruction bringing capable students more quickly to the frontiers of pure and applied mathematics. The Commission Report suggested that topics from logic, modern algebra, probability, and statistics should be included in the secondary school curriculum. But the major recommendations concerned new and efficient ways to reorganize traditional school topics. Plane and solid geometry were to be integrated in a single course; trigonometry was to merge with advanced algebra; inequalities were to be treated along with equations; and judicious use of deductive method, the process of pattern searching, and structural concepts like set, relation, and function were to provide unity to the entire curriculum.

The first reform proposals were generated and shaped largely in response to changes in the character of contemporary mathematics. But Jerome Bruner's 1960 publication of *The Process of Education* provided psychological bases for emphasis on unifying structures of the discipline and on the active, discovery learning thrust in mathematics teaching that also became a prime initiative of the major innovative curriculum projects.

The curriculum development work of the School Mathematics Study Group (SMSG) and the University of Illinois Committee on School Mathematics

(UICSM) soon produced model high school course materials embodying the CEEB proposals. To support high school mathematics programs organized around abstract structuring concepts and processes, developers proposed enriched elementary and junior high school programs as well. In materials developed by the University of Maryland Mathematics Project (UMMaP) and SMSG, the traditional junior high school review of arithmetic was augmented by informal geometry, probability, algebra, and study of structure in number and numeration systems. These experimental secondary school materials were in wide classroom trial use by the early 1960's and they had a pronounced impact on the commerical textbooks produced later in that decade.

Work on modernized elementary school curricula proceeded somewhat more slowly. But the early SMSG program for grades K-6 incorporated informal geometry and algebra as well as close attention to the algebraic properties of the number systems. The clear thrust of these experimental elementary programs was to replace the traditional rote instruction in arithmetic with more meaningful attention to a broader sample of topics from mathematics. After the first round of new elementary programs, many conceived in a naive optimism that young children could achieve far more than had ever been expected of them, curriculum developers influenced by Piaget's work on cognitive development began the laborious process of understanding and accounting for the influence of human development and readiness on curricular possibilities. Various interpretations of that work have led to extensive concern for use of physical materials in teaching elementary mathematics through laboratory-like investigations.

From a 1978 vantage point it is hard to imagine that so many ingredients of the "New Math" proposals were completely foreign to most mathematics programs and teachers in 1960. At professional meetings and in-service education institutes and workshops, thousands of teachers learned the algebra of sets and logic, the ordered field structure of the real number system, and the elements of probability. They saw demonstrations of discovery lessons and learned how to use manipulative materials in a laboratory learning environment that would encourage students to experience a more open-ended and creative style of developing mathematical ideas.

The innovative themes described thus far – enriched content, emphasis on understanding unifying concepts and structures, and discovery learning – were characteristics of the first round of "New Math" development. While these ideas found their way into a variety of early experimental programs, those first experiences quickly led to another round of proposals and programs. The Secondary School Mathematics Curriculum Improvement Study (SSMCIS), the Comprehensive School Mathematics Project (CSMP), and SMSG each produced programs that more thoroughly unified the school mathematics curriculum – breaking the long tradition of separate algebra and geometry courses.

Based on an odd mixture of Piagetian concern for developmental readiness and Gagne's behavioral task analysis ideas, several other projects began work on individualized systems of instruction. The best known program, Individually Prescribed Instruction (IPI), prepared a package of over 300 learning objectives and the self-instructional materials to support independent student progress through the learning hierarchy. While IPI and other individualized programs consciously chose to make no content innovations, their instructional management systems changed the roles of teachers and students. The teacher in an individualized system became a busy tutor and administrator – not an expositor; the student learned primarily from self-study of written material – not by participating in or listening to discussion with teachers or fellow students.

Many innovative programs of the 1960's were designed for mathematically capable students heading for further academic study of mathematics. Decisions about selection or presentation of the content were generally based on considerations of internal mathematical structure and current fashion, not coordination with other school subjects or preparation for post-school occupations. As a result, most "New Math" programs could be characterized fairly as emphasizing pure mathematics and neglecting the traditional diet of "life survival" applications that were prominent in the junior high school and senior high school programs they replaced. Reaction to these emphases in "New Math" programs during the late 1960's and early 1970's led to a call for more practical course materials that would better suit the needs and aptitudes of less able students. In fact, the interest in applied or applicable mathematical content has spread to all types of course, creating a trend toward more informal and intuitive style of curriculum and instruction at all levels with emphasis on physical models of mathematical ideas.

Another form of reaction against the ambitious, academic emphasis of "New Math" era programs is the widespread concern for specifying standards of minimal competence in mathematics to be attained by all students – either for progression within the normal school program or for graduation from secondary school. In policy discussions and development efforts at all levels the optimistic "sky is the limit" enthusiasm of the 1960's has been overtaken by a pessimistic focus on the least that can be expected. Both the substance and the mood of these activities reflect deep disillusion with earlier objectives and accomplishments, disillusion that represents a crisis of confidence for leaders in mathematics education over the past several decades.

## WHAT CHANGE WAS REALIZED

The "New Math" plans for new content, new curriculum structure, and new pedagogical styles constituted an ambitious agenda for those charged with materials development, teacher education, and program implementation. But the enterprise was fired by a tremendous sense of urgency and an optimistic belief that all things were possible. It now seems perfectly clear that nearly everyone involved in "New Math" innovation projects overestimated the likely benefits of new curricular structures and teaching styles and seriously underestimated the problems of developing appropriate new curricular materials, preparing teachers to use those materials, and convincing both the public and the mathematics education profession that changes were needed.

Estimates of the extent and value of changes in school mathematics stimulated by the "New Math" movement vary immensely, and data to support any opinion are hard to assemble. The most visible, and embarrassing, signs of change have been the reports of sharp decline in tested mathematical proficiency of students at all age levels. Mean scores on two widely used college entrance tests and on several general achievement test batteries used in grades 7–9 have declined steadily since the mid-1960's. Many mathematics educators point out that the mathematics test score decline is paralleled, if not exceeded, by declines in performance in nearly every other school subject area, and they question the validity of the tests as indicators of quality in contemporary mathematics programs. But the decline in scores, coupled with public concern about rising costs of education, has provoked furious debate over the merits of recent program changes.

Unfortunately, reasoned appraisal of the nature and causes of apparent declines in school mathematics achievement is stymied by an almost total lack of information that would indicate just how much of the "New Math" reform agenda has become common school practice. While the goals of "New Math" curriculum projects were often formulated in a language whose vocabulary featured catch words like "sets", "properties of the number systems", and "discovery", the transformation of these ideas into working school programs yielded an extremely diverse array of reform options. Furthermore, when the experimental programs were modified for mass consumption in the United States commercial textbook marketplace and filtered through the personal teaching style preferences of over a million different classroom teachers, the gap between rhetoric and reality was bound to be significant.

With no uniform national syllabus, text, or test, one is forced to look for indirect evidence of change in the school mathematics programs that students really experience. The picture is different at various grade levels in school and it is continually changing in response to changing public and educational values. The following observations are reasonably representative of the situation as measured by consensus of professional opinion and a few limited survey studies:

1. In the high school college-preparatory program most content changes recommended by the CEEB Commission on Mathematics were effected soon after 1959 and remain in place. Plane and solid geometry have been integrated (many feel to the detriment of solid geometry), trigonometry and advanced algebra have been integrated (though there are indications of a return to separate semester courses in trigonometry for less able students), and inequalities are now commonly treated along with equations throughout algebra. Only the recommendation concerning emphasis of probability and statistics remains largely unsatisfied.

The effort to make significant use of unifying concepts and structures was, in original experimental curricula, not particularly imaginative. As a result, one finds the linguistic trappings of set theory and algebraic structure properties, but very little of the spirit of unity. Algebra and geometry are still taught as separate and very distinct year courses, to be followed by another year of algebra, a course of elementary functions (with analytic geometry) and, for the best students, a year of calculus in many schools.

2. Current junior high school textbooks contain a rich menu of mathematical topics including informal introductions to geometry, probability, and algebra. But there is a clear trend to emphasize those topics that were main fare of the junior high school curriculum 20 years ago – arithmetic of common fractions and decimals and application of those skills to problems of "life survival", primarily consumer problems involving money and percents. For many students the program reflects very little "New Math" impact either in content, organization, or pedagogy.

3. Like the current junior high texts, the curriculum materials being used in most elementary schools today are strikingly different from the drab arithmetic program of the 1950's. However, data from several recent surveys of classroom practice indicate that the novel topics in those books receive very little attention. Most teachers apparently choose to spend a heavy majority of their class sessions on development of skill with arithmetic of whole numbers and fractions.

Though the preceding survey of "New Math" era curricular changes is unavoidably vague and supported by sketchy data, change in the approach to teaching mathematics is even harder to accomplish and to assess. For many "New Math" developers the essence of needed change was not specific content variation, but a major re-orientation in the way students acquired mathematical understanding. Evidence of change in classroom use of discovery teaching strategies is practically non-existent. The Price, et. al. (1977) survey suggests that elementary school mathematics is still taught primarily by a pattern of short teacher explanation and extended student practice. While there is evidence of using laboratory activities and manipulative aids to teaching mathematics at that level, the extent of use is certainly modest in relation to hopes and expectations. Despite widely expressed enthusiasm for principles of individualized instruction, on a national basis one suspects the extent of its use is also modest. At the junior and senior high school level the recent enthusiasm for instruction based on detailed behavioral objectives suggests dominance of expository or heavily guided discovery instruction and very little of the open-ended problem solving activity that so appealed to proponents of new curricula during the 1960's. Hard data on the patterns of teacher/student interaction in mathematics classes is extremely rare.

The discouraging progress of attempted changes in curricula during the recent era has suggested to many mathematics educators that "New Math" is really largely a phantom culprit in the current problems of mathematics teaching in the United States. The "New Math" couldn't be responsible for poorer mathematical performance in schools because it never became a part of the substance or style of instruction in any major share of United States classrooms. Others argue:

> The content innovations K-12, the emphasis on student understanding of mathematical methods, the judicious use of powerful unifying concepts and structures, and the increased precision of mathematical expression have made substantial improvment in the school mathematics program. Unfortunately, the innovations have not fulfilled the euphoric promise of the 1960 beginning, and current debate seems intent on locating blame for failures in real or imagined "new math" programs.
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> (NACOME, 1975)

But there is a substantial segment of professional and public opinion arguing that the "New Math" reforms produced poor results and/or failed to gain broad acceptance because they were urging inappropriate mathematical substance and pedagogical principles. These critics, led by Morris Kline, see the "New Math" movement characterized by excesses of abstraction, symbolism, and deductive instruction – all designed to teach mathematical topics (such as set theory, Boolean algebra, or topology) that are not suitable or important for elementary or secondary school students.

## WHAT HAS BEEN LEARNED

There is probably some wisdom and some error in each of the assessments of "New Math" promise and achievement. But the most disturbing feature of current activity in United States mathematics education is the growing evidence that we have learned very little about the appropriate process or product of mathematics curriculum change, despite 20 years of effort and experience gained by very capable people. It is not uncommon to hear mathematicians criticizing the schools in much the same way that their predecessors complained 20 years earlier; recommendations on the proper direction of educational practice and policy seem to reflect no awareness of, or curiosity about, the historical experience with similar problems or conjectured solutions. Furthermore, popular reports of current debates suggest an inevitable and bitter polarization of the mathematics community according to stands on various dichotomous issues:

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| old math | versus | new math |
| skill emphasis | versus | concept emphasis |
| intuition | versus | rigor and precision |
| applied math | versus | pure math |
| teachers | versus | mathematicians |

What are the lessons that can be learned from recent experience?

*The development process.* When the "New Math" era curriculum development efforts began in the late 1950's, one of the striking innovations was a new form of development enterprise. The most influential United States projects were supported generously by funds from the federal government and private foundations concerned with improvement of science education. The development projects were team efforts employing classroom teachers, supervisors, teacher educators, and research mathematicians. The most common working procedure was a systems approach including syllabus planning conferences, team writing sessions, classroom trials of draft curriculum materials, revision based on tryouts, and subsequent broad implementation. Each of these features of the curriculum development process represents a major departure from the then standard process in which recommendations of syllabus bodies were translated into classroom curricula by independent authors working for the commercial textbook market place. If anything, the team systems approach to curriculum development has become more elaborate in recent years, but this and several other process innovations have not been judged unexceptionable successes.

Federal government support for curriculum development has recently encountered substantial political resistance. Prompted by emergence of several highly controversial social studies programs, conservative congressmen have seen in government supported projects the ominous prospect of national curricula. As a result, it appears the only projects receiving government support in the future will be those that are widely agreed to meet a national need. Skeptics fear that such policies will bar support for the daring ideas most needing venture capital. Thus, while massive government support has been welcomed and used creatively in curriculum development, it has not been free of problems – encouraging unhealthy dependence on the "big project" model and holding prospect of further entrenchment of safe ideas.

The collaboration of teachers, teacher educators, and mathematicians was probably the most exciting and volatile aspect of "New Math" era projects. Much of the stimulus for curriculum development came from criticisms by the professional mathematics community, and many fine mathematicians devoted tremendous energy to production of new and better school programs. In many cases this activity led mathematicians and teachers to a new and healthy respect for each other's challenges and abilities. However, as the fruits of "New Math" began to accumulate mixed critical reviews, many mathematicians and public school teachers engaged in vigorous debate placing the blame for problems on each other. The mathematicians claimed that only the superficial aspects of their ideas were adopted in actual classrooms; the teachers claimed that mathematicians had sold them a high powered pure mathematics program that was not appropriate or teachable for real elementary and secondary students. The lessons from this experience are clear, but not easy to heed. Mathematicians must realize the complexity of transforming their ideas into realistic school programs through cooperative, not dogmatic approaches to change. On the other hand, classroom teachers must be careful to avoid the easy excuse, "You don't know what it's like in the schools," and to realize their natural inclination to conservatism that discourages needed innovation. The interplay of ideas and experiences shared by varied professional and lay groups interested in mathematics education has been shown to be very productive. But wresting full benefit from the collaboration requires a cooperative and sympathetic spirit that does not come easily in the contentious arena of educational policy-making.

Many of those involved in the large, university-based development projects of the 1960's feel that the systems development model proved its value convincingly. But critics, who point to subsequent problems with implementation of these programs, argue that development must involve more intimately the classroom settings and teachers who will be potential curriculm users. This position implies localized curriculum development initiative and energy, rather than centralized national efforts such as those characteristic of the 1960's.

There is probably real virtue in user involvement. However, many developers, who have carefully crafted a coherent program to reflect considered professional opinion on content and best available psychological knowledge to shape pedagogy, are deeply dismayed by the kind of chaotic curriculum that results when elementary teachers are given a smorgasbord of materials and urged to build their own mathematics programs one day at a time. Certainly optimum curriculm development process lies somewhere between the untested text produced during a summer of writing at some luxurious university conference and the helter-skelter result of day by day planning in individual classrooms. The clear message from experiences of the 1960's is that extensive field trials and subsequent revision must be part of any curriculum development effort that hopes for reasonable acceptance and success.

The big project model for development offers the promise of high quality product from shared talents of many people. But it also involves the risk of discouraging truly creative and daring innovations by its demand of broad acceptability for ideas. To remind us of the potential contribution from older, smaller scale curriculum development modes, we have seen several very good recent one-person products in applied algebra and transformation geometry. It seems vital that such avenues for curricular innovation be kept open through regular support of individual initiatives.

*The implementation process*. As mentioned earlier, apologists for the real or imagined failures of "New Math" programs frequently argue that the true intentions of the reform movement never really found their way into the actual curricular or instructional practices of most schools. While this might be a fair appraisal of the situation, it implies failing marks for the implementation strategies employed to sell the reform ideas.

For the most part, United States "New Math" programs were sold by a combination of persuasive, crisis atmosphere rhetoric and the weight of endorsement from prestigious sponsors. For a time in the early 1960's, "New Math" was a top-priority educational fad which every system or school had to be involved with. As a result, schools cheated on the essential background work of testing the programs, examining critically the curricular goals and materials, and preparing teachers to convey the substance and spirit of new programs.

At the secondary school level, government sponsored teacher institutes involved many thousands of teachers in retraining. But this effort still reached only a fraction of the teachers, often the best already, and the institute programs probably concentrated too exclusively on deep background mathematics, ignoring careful study of the actual classroom programs to be implemented. At the elementary level, where teachers normally have the most limited background understanding of mathematics, teacher preparation for "New Math" implementation was inexcusably most limited. Grade school teachers frequently returned to a new school year in September greeted with the news that they were to begin using a "New Math" program immediately. As a result, despite the best intentions of skillful and open-minded elementary teachers, the spirit and substance of new programs were probably most poorly realized in grades K-6. The reaction against frivolous use of set theory or non-decimal numeration or algebraic properties of number systems was probably inspired by horror stories from misguided elementary teaching.

In retrospect it is almost comical that National Science Foundation teacher education programs were specifically prohibited from serving elementary teachers. Now nearly every would-be curriculum reformer acknowledges the crucial role of classroom teachers in implementation of new programs. A variety of new innovation models are now in use, each making a stronger effort to focus on classroom teachers as effective change agents. There is a realization that teacher education must be continual activity involving – in a cooperative way – university, supervisory, and classroom teachers. Very recently militant teacher organizations have begun making control over curricular change a part of their negotiated teaching contracts. This movement promises to institutionalize and further complicate the curriculum implementation process in United States schools.

Developers of the "New Math" also learned the critical role, in the United States, of the commercial textbook industry as an agent of change. In the early 1960's the label "New Math" was fashionable, and commercial texts used by most classes proudly advertised inclusion of the best known ideas. However, to discerning eyes it was clear that much of this innovation was superficial or cosmetic change laid over a substantially unchanged curriculum. Chapter 1 of each text covered the language and operations of sets, but subsequent chapters made no use of the unifying concepts and language thus available. Furthermore, when criticism of "New Math" began to build during the 1970's, the commercial publishers, guided as much by market research as by author professional judgement, quickly jumped on the "back to basics" bandwagon. Thus the competitive marketplace, instead of offering variety for schools and opportunity for novel ideas, seems to enforce a generally conservative curriculum.

Those who have sought school system support for curricular innovation have also experienced the influence of another powerful factor in determining school programs – the expectations, real or imagined, of colleges or universities that admit secondary school graduates. Those "New Math" era projects that tried altering the traditional pre-college curriculum of algebra/geometry/algebra faced an extremely skeptical school marketplace, where administrators' first questions always focused on how the proposed new programs would satisfy college entrance requirements. In most cases this anxiety about acceptance of innovative programs was far greater than justified by reality; but it is just another instance of the fundamental conservatism in American education – a reluctance to adopt new ideas that contrasts sharply with the stereotype of American fascination in face of change and progress. It is not clear that anyone has learned how to circumvent the variety of conservative factors in order to change school programs. However, the naive optimism of 1960 has been replaced by a healthy respect for the necessity and complexity of considering numerous forces that shape potential innovation in school mathematics.

*The evaluation process*. The very first "New Math" programs were adopted in schools on the basis of professional opinion supporting the intentions of those programs. But developers and school users both soon sought more objective evidence that the new programs really worked. The first such evaluations naively sought answers from administration of available standardized mathematics tests. But as soon as this testing showed little advantage to the new programs, innovators sought data from tests that assessed newer emphases fairly. This raised the crucial paradox of trying to compare effectiveness of programs with substantially different goals, a problem that is clearly at the heart of much current controversy over school mathematics.

Attempts to provide adequate evaluations of the recent curricular and instructional innovations have stimulated extensive and striking improvements in the measurement of mathematical abilities and attitudes. The most influential such effort, the National Longitudinal Study of Mathematical Abilities (NLSMA), began with the hypothesis that mathematical ability and achievement consist of many components rather than a single unitary trait. Then NLSMA sought understanding of the relationship between curriculm characteristics and a broad collection of mathematical ability, achievement, attitude, and psychological variables. The NLSMA approach to comparing curricula placed less emphasis on determining which was best and more emphasis on determining the achievement profile associated with each program. This point of view, together with the numerous and varied measurement instruments developed by NLSMA, has produced a remarkably more sophisticated technology and theory of evaluation that that available in 1960.

Unfortunately, one of the facts of life in the curriculum development business is the inevitable impatience of those who support or seek to utilize the new programs. In retrospect, nearly all of the "New Math" curriculum and instructional ideas appear to have been rushed too quickly from planning to production, field test, and evaluation. Just as the second round of development efforts was beginning to produce some clever and balanced approaches to difficult problems, like unified structure in curriculum, the fundamental ideas were being rejected on the basis of hasty early efforts.

While evaluation of curricular initiatives should ideally follow a deliberate pattern of extensive testing, it seems inevitable that the battle for acceptance must involve a certain amount of public contention among advocates of competing programs. Too often the participants in these debates over "New Math" have aimed their critical barbs at caricatures of each other's programs. Such comparisons are further complicated by the failure of any national body to monitor regularly the substance and achievements of mathematics curricula and instruction in United States schools. As a result, critics claim things are "worse than they used to be"; but no one has the longitudinal data that could confirm or deny the charge. Recently many groups in mathematics education have called for the useful regular status surveys. The United States National Assessment of Educational Progress and periodic IEA international studies should provide useful benchmarks for rational evaluation of change.

The goals of most "New Math" projects in the 1960's were so ambitious that evaluation was bound to show that the efforts fell short. Experience with evaluation of those programs has probably led to permanently more modest professional expectations and more insightful ways of assessing program quality. But it is also likely that American education will continue grasping at innovative straws to serve as final solutions for difficult problems.

*The influence of educational and social context.* When, in the early 1960's, new school mathematics programs were presented to an expectant and hopeful American public, developers calmly assured school officials and parents that they shouldn't be alarmed by changes in traditional language or vocabulary of mathematics programs. Though it became difficult for parents to understand new program goals or to assist their children with homework, they were told that one of the key program objectives was to make students better independent workers who would explore and discover mathematical ideas on their own.

As public sympathy for "New Math" has declined during the 1970's, there are curriculum developers who claim that failure to educate the public on reform aims and substance was a crucial mistake. There were some efforts – through popular media and school meetings for parents – to inform the public of the new program intentions. But very often these exhibitions of "New Math" were forced to emphasize a few simple and most unusual aspects of innovation, at the expense of adequate understanding of the fundamental rationale and substance in changes.

Even allowing for difficulty attributable to inadequate public education about "New Math", the rising tide of disenchantment in the 1970's seems to reflect quite different sources of public concern about education. The criticism of "New Math" is no greater than that directed at 1960's innovations in teaching of language and social studies. Furthermore, the criticism coincides with a period of straitened economic conditions in the schools and for the public at large. Much of the challenge to make school programs more practical and accountable for their effectiveness seems to reflect anxiety about personnal economic pressures much more than philosophical disagreement about educational policy.

This rather pessimistic view concerning influence of broad school and societal conditions implies that the narrow discipline orientation of curriculum reform during the 1960's was probably hopelessly unrealistic. Schools and societal expectations of schools appear to change very slowly. Further, the receptivity to reform and the effectiveness of new programs are probably much more strongly influenced by factors not directly controlled by mathematics educators than by any content or pedagogical policies that are restricted to specific classrooms. Especially in the United States, education seems easily swept by superficial fads which consume enormous energy of innovators but burn out quickly when they fail to yield quick and permanent solutions to deep and longstanding problems. Furthermore, innovators are seldom given a second chance, to learn from the successes and errors of their first efforts.

*Knowledge about students, teachers, and curricula.* The specific school programs commonly labelled as "New Math" might have had more limited impact on mathematics teaching than their developers wished. However, the broad movement toward improved programs was a dramatic stimulus to the mathematics education profession in the United States. The enthusiastic search for new and better school mathematics attracted thousands of very capable people to careers in mathematics teaching, teacher education, research, and development. The varied investigations produced knowledge about learning, teaching, and curriculum that greatly enriches our understanding of mathematics education. The "New Math" has produced few simple answers to perennial problems of teaching, but it sparked research and development efforts that will, for many years to come, yield a rich harvest of educational ideas and experiences.

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