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SQUARED PAPER IN THE NINETEENTH CENTURY: INSTRUMENT OF SCIENCE AND ENGINEERING, AND SYMBOL OF REFORM IN MATHEMATICAL EDUCATION

Abstract. The paper traces the gradual adoption of squared paper from its exclusive use as a research tool in the early 19th century to its universal use for a variety of purposes in mathematical education by the end of the first decade of this century. Three underlying causal factors are explored – the growth of new educational philosophies; the development of science teaching and the associated need for mathematics correlation; and the growing demands of engineering and technical education. Whilst the focus on squared paper is a narrow one, it is argued that its adoption in education generally was symptomatic of a much wider transformation of mathematical curricula in response to various demands which, significantly, arose *outside* the academic mathematical community.

Squared paper, a marvellous instruction which ought to be in the hands of every one who works in mathematics from the kindergarten to the university. (Laisant, 1904, p. 23.)

The graphical presentation of experimental data has a number of advantages for the physical sciences, as Dr. Laura Tilling (1975) indicates in her useful paper on early experimental graphs. She elaborates upon this 'graphical method' as follows:

Its greatest strength lies in the clarity and succinctness with which it displays the information contained in tabulated results: for the experimenter a graph provides a rough and immediate check on the accuracy and suitability of the methods he is using, and for the reader of a scientific report it may convey in a few seconds information that could only be gleaned from a table of measurements by hours of close study. (p. 193.)

A graph's power to summarize data can also be utilized in economic, social and other quantitative fields but, as Tilling's paper makes clear, the graph – something that is nowadays part of the visual furniture of every newspaper and *Punch* cartoonist – was something only rarely used by eighteenth- and early nineteenth-century experimentalists. Two examples may be cited. A paper published by the physician and chemist, William Prout (Prout, 1813, p. 332) in 1813 on human carbon dioxide output over a 24 hour period uses squared paper, and the French chemist and physicist, Gay Lussac (1819), published a graph of solubilities in 1819. However, as Tilling notes (Tilling, 1975, p. 210), John Herschel (1833) explicitly recommends the usefulness of squared paper in his 1833 paper on revolving stars as if few scientists had adopted the technique.

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. . . let a sheet of paper be procured, covered with two sets of equidistant lines, crossing each other at right angles, and having every tenth line of each set darker than the rest. By these, the whole surface of the paper will be divided into large squares by the dark lines, and each of these sub-divided into 100 smaller ones by the faint ones. Such charts may be obtained, neatly engraved; and are so very useful for a great variety of purposes, that every person engaged in astronomical computations, or indeed, in physico-mathematical inquiries of any description, will find his account in keeping a stock of them always at hand. (p. 178.)

It is clear that until the 1820s and 1830s the graphical method and, in particular, squared paper were *rarely* used. Moreover, until the 1870s, as we will show, such methods remained in relatively uncommon use in Great Britain and were not employed by students as part of their elementary scientific and mathematical education.[1] One obvious inhibiting factor was the cost of squared paper. Before 1876 the price of a single sheet of paper engraved in the manner described by Herschel was as much as (old) 8d. Yet by 1887 it could be obtained for a little over ¼d a sheet or 8d. a quire (a quire being a printer's term for 24 sheets of paper); by 1899, 7½d a quire; by 1903, 7d a quire; and by 1913, 6d a quire (exactly ¼d a sheet) (Ayrton, 1887, Perry, 1899). By 1903 a number of firms were marketing various forms of squared paper in both single sheets, for as little as 2¾d a quire with ¼ inch rulings, and variously mixed with normally ruled papers in the form of exercise books. It might also be added that polar graph paper, published in books of 40 sheets at 1s a book, first appeared in England in 1902 (Salmon, 1903). Needless to say, this dramatic fall in price and increasing availability coincides with the adoption of various forms of squared paper for a variety of purposes in both science and mathematics within technical, elementary and secondary education. The publication of textbooks like Richard Wormell's *Plotting or Graphic Mathematics* (1888) provides an early illustration of the trend in mathematics, though Charles Godfrey (1912), in his report on a survey of teaching methods in English secondary schools, prepared for the International Commission on the Teaching of Mathematics (ICTM), claimed:

The use of graphical methods in elementary algebra teachings is universal and entirely a 20th-century development. Other aspects of the same movement are the adoption of descriptive geometry by the mathematicians, the use of handy 4-figure tables, and of graphical methods in statics, and, though, in these cases, the victory is less complete than that of the 'graph', it is remarkable and equally modern.

Godfrey also found that the use of squared paper in connection with the topic of area was a standard part of the curriculum for lower and middle forms by this time. In 1912 also, the Board of Education's (1912) *Suggestions for the Teaching of Arithmetic* noted that squared paper was to be found in any well-equipped elementary school, reflecting the tendency towards 'concrete methods'. However, the Board went on to issue the following warning concerning its uses:

Any risk of injury to eyesight by the excessive use of squared paper should be avoided. Its use for practical geometry, etc., will only be occasional and little danger to health is likely if no paper with rulings less than one-tenth of an inch apart is used. The use of paper ruled in squares for the working of arithmetical examples has no real educational advantages and might well be discontinued. (p. 8).

What caused the change from a fairly unusual and always exclusively research use of squared paper to it becoming a versatile, familiar and elementary tool of the mathematical tyro? We should like to suggest three underlying factors:

(1) The break-up of the narrow 'instrumentary' tradition in elementary education, which Selleck (1968) has described, and the parallel erosion of the dominant classical tradition in secondary education, within which mathematics was regarded as an 'art', which disciplined the mind, rather than a 'science'.

(2) The rise and development of science teaching in schools, and the associated problem of correlating mathematics and science teaching (and, indeed, finding mathematicians who could teach mathematics for scientists).

(3) The development of engineering in its various forms and the associated movement for technical education which brought about the resurgence of a broader conception of mathematics, 'practical mathematics', to replace the dominant, narrow and inward-looking 'academic mathematics'.

Now there may well be other factors that others would emphasize, such as the decline of British industrial supremacy and attempts to alleviate it, or the availability in the stationer's printing shop of cheaper dividing engines for producing squared paper from copper engravings or by machine rule, but these seem to us the most important. Moreover, for once the historian finds a simplifying thread linking these factors, namely the careers of just three men and their pioneering work in just two geographically distant institutions. These are Henry Armstrong, William Ayrton and John Perry, and the Royal Engineering College, Tokyo and Finsbury Technical College in London (Brock, 1979).

1. NEW EDUCATIONAL PHILOSOPHIES

The various features of the 'new education' in the period 1870–1914 have been disentangled and ably discussed by Selleck (1968). The new thinking influenced teaching methods from the kindergarten through to the secondary stage and, for our purposes, some of the important arguments and influences are those which emanated from the 'practical educationists'. Armstrong, Perry and many other reformers pressed for more practical approaches in education, which implied a more active role for the learner, whose education should be based upon 'concrete' experiences and related to the circumstances of life. The training of hand and eye through learning by observation and 'doing' also came to be valued as an end in itself. Drawing on squared paper was one valuable activity for such purposes, even at the kindergarten stage, where it might go along with other exercises such as paper folding, tearing and laying, bead laying, string measurement and division, clay modelling and colour sorting (Selleck, 1968, p. 111). By 1902, the educational reformer J.J. Findlay (Findlay, 1902), was able to single out 'self-activity' as one of the principles governing the reform movement in mathematical education generally. The principle could be pursued:

> by 'using' squared paper, by measuring, by plotting curves, etc. (p. 185.)

In 1908, Benchara Branford (1908), a London County Council Inspector, observed:

> Attention is now beginning to be paid in rapidly increasing measure to mathematics on its experimental and graphical side, and is exemplified by the use of drawing-boards, improved mathematical instruments, squared paper and spherical blackboards.

Branford went on to mention one further and more dramatic sign of a changed outlook, namely the provision of 'mathematical laboratories':

> . . . well stocked with clay, cardboard, wire, wooden, metal, and other models and material, and apparatus for the investigation of form, mensuration and movement . . . .

The gradual adoption of 'practical geometry' within a liberal secondary education and the overthrow of the strictly Euclidean treatment of geometry have been considered by Brock (1975). The gradual transformation of other branches of mathematics was more subtle and cannot be adequately discussed here. It is, however, significant that the International Commission on the Teaching of Mathematics chose as a major focus for international comparisons, at Cambridge in 1912, 'methods of intuition and experiment in secondary schools'. Such methods embraced the graphical study of statistics, functions, vectors and statics, as well as the use of slide rules and mathematical tables. It is clear that a more practical approach had been adopted universally by this time and as American David Eugene Smith (Smith, 1913) noted:

> Of the value of squared millimetre paper there is no question anywhere. (p. 614.)

One important argument for the use of squared paper, and indeed for practical tendencies generally, has yet to be considered.

In the nineteenth century the common justification for practices within both the instrumentary and classical traditions in mathematical education was in terms of the disciplinary value for both the mind and the character. The mind was modelled as a set of discrete faculties (like mental muscles) which could be exercised, and thereby developed, by working upon suitable subject matter. Thus the formal literary study of Euclid could be justified as a training for general faculties such as those of reasoning and memory, taking transfer of training as axiomatic. Algebra as an elaborate symbolic game divorced from both arithmetic and reality, and apparently leading nowhere, could be similarly justified. No consideration was given to the learner's perceptions and interests. Like physical training, the educational process might be very uncomfortable but it was considered to be good faculty training for the learner. However, this extreme doctrine of mental and moral discipline was gradually undermined towards the latter part of the nineteenth century. For younger pupils, alternative rationales emphasizing the idea of natural growth from within, rather than the process of forming the mind from without, were derived from the writings of Pestalozzi, Froebel, and subsequently Montessori in particular. More generally, the writings of Herbart provided further inspiration for the reformers, and shifted the focus to questions of motive and interest in education and to the growth of ideas in the individual (Selleck, 1968, pp. 179–272). In particular, mathematical educators such as Godfrey, Branford and Percy Nunn reflected the influence of Herbart and all developed philosophies before the First World War, which emphasized the role of motives in the historical development of mathematics and in the learner, and stressed the value of the outward-looking aspects of mathematics in particular (Branford, 1908; Price, 1976). In this shift away from a crude and simplistic rationale of mental discipline squared paper provided one means for implementing the doctrine of interest in mathematics. In 1902, Godfrey (1902) made a number of detailed suggestions for preparatory schools on the uses of squared paper, and a square ruled blackboard, for work in fractions, decimals, areas and algebra. He boldly claimed:

> GRAPHS have found their way into elementary work, and are now recognised as quite the most valuable instrument in our possession for awakening interest. (p. 289.)

2. MATHEMATICS AND SCIENCE TEACHING

The development of 'practical' or 'experimental' mathematics as an approach to teaching the subject involving the use of apparatus of various kinds, already mentioned, and perhaps a special room – a mathematical laboratory such as that devised by Godfrey *et al.* (1905) and Bell (1912) at Winchester – has obvious parallels with the somewhat earlier emergence of practical science teaching. John Perry (1850–1920) taught physics at Clifton College between 1870 and 1873 and claims to have established the first workshop within a public school in England, where boys could make apparatus to repeat experiments for themselves (Perry, 1900, pp. 73, 94). At the same school, from 1875, A. M. Worthington pioneered a course of practical work in elementary mensuration and hydrostatics which subsequently became generally adopted as the first stage in practical physics – 'elementary physical measurements' in public schools, and science classes and schools under the Department of Science and Arts (Fawdry, 1912; Jenkins, 1979). The growth of both physics and chemistry teaching, as well as demanding the use of various materials and apparatus, was accompanied by the rise of a new approach to teaching, 'heurism', or the method of discovery, which was so strongly advocated by H. E. Armstrong (1848–1937). The search for functional relationships between quantities, and hence the use of squared paper, was an important component of the heuristic method in the laboratory (Salmon, 1903; Ostwald, 1894). However, apart from practical plane and solid geometry and practical mensuration, which were supported by examinations such as those of the Department of Science and Art (DSA) and the City and Guilds of London Institute, the elementary mathematics curriculum in nineteenth century institutes was predominantly 'academic' in character, limited in scope, and out of step with the increasing practical uses of mathematics in science (and engineering, as we will show). The situation was well captured by Professor G. H. Bryan in his response to Perry's agitation and the British Association debate of 1901, and Bryan (1902) deserves quoting in some detail at this point:

> Formerly our schools and colleges were given over mainly to the study of classical and literary subjects, and mathematics was looked upon as a portion of an arts course. The rigid deductive system was undoubtedly admirably suited to the object then held in view. With the development of experimental science new teachers have been appointed all over the country for physics, chemistry and biology, *but next to nothing has been done to meet the greatly increased demand for mathematical teaching thus produced.* The same teachers who provided efficiently for the teaching of mathematics on the classical side have now thrust upon them an influx of new pupils having quite different requirements. (p. 90) (Bryan's stress.)

As well as the new demands of science, there also arose the more tangible demands of the Civil Service Commission's examinations, particularly for entrance to the army, which stimulated the growth of modern sides and army classes in British public schools, and virtually forced a more practical mathematics curriculum upon certain classes, including laboratory work (Montgomery, 1965; Boyt, 1906). By 1902, Findlay (1902) was able to distinguish the issue of 'correlation' as one of the factors behind the general reform of mathematical education:

> This whole movement [correlation] is the outcome of the desire to rank mathematics in its place as the handmaid of science. Not, be it observed, as a tool for engineers, but in close correlation with the needs of the science syllabus right through the school. And, in another aspect, it emphasizes the necessity of bringing the various parts of elementary mathematics in correlation with each other. (p. 185.)

The issue of correlation was pursued further by mathematics and science teachers over the first decade of the twentieth century and the debate involved the Mathematical Association and the Association of Public School Science Masters (APSSM), but we need not consider the detail here. What is more important for our argument is the earlier shift from a state of *no* correlation to one which demanded at least *some* correlation, and a new outlook from teachers of mathematics in particular. Finding teachers who were able to adapt to the new demands was, however, problematic, given the dominance of the Cambridge tradition in mathematics. Thus, as Perry (1902) scathingly remarked in 1902:

> We want mathematical masters from Cambridge, but we must do without them, and indeed severely reject them, as candidates for posts, if they will not give boys such teaching as befits the twentieth century. (p. 203.)

It could be argued that Perry subsequently pushed his case too far by crudely opposing the 'mere mathematician' and the practical man with wider scientific interests. In 1908, he remarked (Perry, 1908):

> ... the marriage of mathematics and science seems to me like that of December and May – the marriage of a man of seventy with old bachelor habits to a bright young virgin of seventeen. (p. 461.)

However, it is significant that pioneers in mathematical education like Wormell, Findlay, Nunn and W. D. Eggar of Eton College (a founder member of the APSSM in 1900), were trained in both mathematics and science, and could teach both subjects heuristically (Findlay, 1902; Nunn, 1903; Wormell, 1900). Findlay and Nunn both devised correlated curricula for their schools, before they entered the field of teacher training, and Eggar (1903) in the preface to his *Practical Exercises in Geometry*, an early innovatory textbook for general educational use, emphasized:

> This book is an attempt to adapt the experimental method to the teaching of Geometry in schools. The main object of this method, sometimes called 'heuristic', is to make the student think for himself, to give him something to do with his hands for which the brain must be called in as a fellow-worker. The plan has been tried with success in the laboratory, and it seems to be equally well-suited to the Mathematical classroom.

The book demanded the use of squared paper for work on areas and included a separate chapter on graphs. Graphs subsequently became subsumed under algebra, and indeed the rise of a new subject called 'graphic(al) algebra' was a source of some confusion for schools generally, The board of Education (1909) attempted to clear the air with *Circulars* on the subject in 1909 and 1914. The Board acknowledged that one of the roots of graphical work lay in science teaching:

> Even where graphs are neglected in the mathematical classroom they inevitably make their appearance in laboratory work. Generations of boys had indeed been familiarised with graphs in connection with physical observations before ever graphic work was recognised as an essential element in the teaching of algebra.

However, there were other roots of graphical work, and indeed of the reform of mathematical education in general. As a perceptive member of the College of Preceptors (1903) commented in the *Educational Times* in 1903:

> . . . the change was inevitable . . . When teachers began to teach science – as well as other subjects – scientifically and with more due regard to the pupils' share in education, when the demands of the laboratory and the workshop, of technical schools and colleges, made themselves felt, reform was bound to come in the *teaching* of mathematics, as of logic and languages and other subjects. It is part of a general movement. (p. 465.)

The demands of engineering and technical education remain to be considered.

3. ENGINEERING, TECHNICAL EDUCATION, AND INTRODUCTION OF SQUARED PAPER

In the older engineering colleges only civil engineering was taught and the mathematical demands were strictly limited. However, the requirements increased somewhat with the development of shipbuilding and mechanical engineering, and increased greatly with the advent of high-speed machinery and the development of electrical engineering (Perry, 1912). What is particularly interesting is that the teaching of graphical methods, within a reorganized and broader conception of mathematics, 'practical mathematics', first arose in the later nineteenth century in response to engineering's various demands and that the subsequent general pattern of adoption was a common one internationally. The American D. E. Smith (1913) was able to claim at Cambridge in 1912 that:

> Graphic methods of one form or another are now found in the courses in mathematics [in secondary schools] . . . in all countries, having gradually made their way from engineering, through thermodynamics and general physics, to pure mathematics. (p. 622.)

We need not dwell upon the causes of the movement for the promotion of technical education in Great Britain during the 1870s. Suffice to say that it is bound up with the reports of successive Royal Commissions, the question of maintaining British industrial supremacy or preventing its decline, and the professionalization of science itself by way of providing jobs for science and mathematics teachers. In addition to the older DSA, two of the principal bodies involved were the City of London Corporation and the ancient Guilds of London. These two latter bodies came together in 1878 as the City and Guilds of London Institute, followed a year later by their sponsorship of evening lectures in chemistry and electricity for artisans in what was to become Finsbury Technical College. Armstrong and W. E. Ayrton (1847–1908) were the first teachers and they were joined by Perry in 1882. Armstrong (1920) subsequently referred to the pioneering trio as the 'Finsbury Mohicans'. It was at Finsbury that Perry developed a syllabus of practical mathematics for engineers which, in 1899, was adopted by the DSA (soon to become the Board of Education) as an addition to the various stages of pure mathematics, and practical plane and solid geometry. The new elementary syllabus which crystallized out at Finsbury included an emphasis on decimals with approximation in arithmetic and the use of logarithmic tables, and the slide rule. In algebra the use of formulae was stressed, and the study of functions and graphs using squared paper, with an early introduction to the ideas of the calculus. Practical methods, including the use of squared paper, were encouraged in mensuration and geometry. Numerical trigonometry, also involving the use of tables, simple work in three dimensions and vectors were also included. All these elements were to be variously mixed, all the time keeping close links with science and laboratory work (Perry, 1901; pp. 44–56). As Perry (1912) later somewhat acidly remarked, this conception was at the time:

> . . . exceedingly different from what used to be the study of the mere mathematician on the same subjects. (p. 35.)

Unfortunately, Perry is ambiguous as to when he first used squared paper for teaching purposes. It may possibly have been at Clifton College between 1870 and 1873,[2] following which he joined William Thomson in Glasgow before going on to Japan as Professor of Mechanical Engineering at Yedo (Tokyo) in 1875. In 1871, soon after arriving at Clifton (one of the new middle-class proprietary schools which took science seriously), at, he claims, the headmaster's (John Percival) suggestion Perry established a physical laboratory and engineering workshop in which he put boys through a series of quantitative experiments in mechanics using a kind of standardized 'Meccano kit' apparatus which Robert Ball had devised for lecture demonstrations at the Royal College of Science in Dublin. (Perry had learned of the apparatus from his brother, who still lived in Ireland; Ball's apparatus was itself adapted from that devised by Robert Willis (1800–1875.) Perry (1926) may be quoted at this point:

There is a piece of apparatus which is generally employed to illustrate the laws of blocks and tackles by which a man's pulling force can lift a great weight. In this apparatus a small weight is shown to be able to balance a much larger weight. The student having been told about this in a lecture, he is taken to the laboratory, where he finds *printed instructions* on the piece of apparatus. A scale pan containing the larger weight is, say, A and that of the smaller is B. He puts a certain weight in A. He now adds to B until, on giving B a start downwards, a steady motion of the apparatus results. He now says that B balances A and friction. He takes another weight A and repeats, and so on until he has obtained a great number of observations. He now *plots the weights* B which balance weights A on squared paper, obtains the law for efficiency of the machine, and obtains the law connecting friction and load. It is impossible for a boy to perform this process without obtaining exact ideas of work, loss of energy by friction, co-ordinate geometry, the meaning of a physical law, and the algebraic expression of a physical law. (p. iv.)

Unfortunately, this is later Perry speaking – in 1887 – and we have to note that a few sentences later he qualifies this description by saying that:

. . . this *method* of instruction was not very completely carried out at Clifton College.

We have already referred to the prohibitive cost of squared paper while Perry was at Clifton and without further evidence we take Perry to mean, probably, that the graphical method was *not* developed there.

However, we do have Perry's (1926) explicit word, as well as corroborative evidence, that the use of squared paper was 'more fully elaborated and tested' in Japan. Again, in his lectures of 1899 for the DSA on practical mathematics for working men, Perry (1899) says that in 1876:

Professor Ayrton and I used [sheets of squared paper] extensively in Japan, and when we returned to London and introduced, at the Finsbury Technical College, our methods of teaching mechanical and electrical engineering and laboratory work . . . we saw that one essential thing was the manufacture of cheap squared paper.

William Ayrton (1887) implicitly confirms this in the first edition of his *Practical Electricity*, published in 1887. Here he remarks that before the foundation of Finsbury by the City and Guilds of London in 1879:

Squared paper was practically used in England only for the recording of results of original experiments. And as these results, rather than the *training* of the experimenter, were the most important part of the investigation, the paper was very accurately divided, and sold at a high price totally out of reach of students. It became, therefore, necessary to have squared paper specially made, cheap, and at the same time sufficiently accurately divided for students' purposes.

Final independent corroboration, for use in Japan, comes from an anonymous report on Ayrton's laboratory, in the *Japan Weekly Mail* (1878) in October 1878, soon after Ayrton (but not Perry) had returned to England. The writer goes upstairs and visits a drawing office:

... some of the students were making working drawings of instruments, but the majority were reducing observations and drawing curves on squared paper.

This drawing office was to be found duplicated on the second floor of the new Finsbury College building opened in 1884.

Given the very close collaboration and rapport that emerged between Ayrton and Perry in Japan – they published dozens of joint papers – it will probably be impossible to say which of them first developed the use of squared paper for technical students. All we can say with confidence is that the technique was introduced in Tokyo in 1876 or 1877 and was developing at Finsbury by 1882.

We have dwelt on what some may feel is a trivial matter because the introduction of squared paper is a vital symbol of a development which was of tremendous significance for both technical education and mathematical education in general. It is, we believe, in that Japanese experience of Ayrton and Perry, and its transculturation to the City of London, that we must look to find the early roots of the amazing transformation of mathematical curricula in Britain which had become inevitable by 1900, as well as one of the roots of technical education and, concomitantly, the tension which grew between advocates of the study of mathematics for its own sake and for its utility which came to a head following Perry's address at the Glasgow meeting of the British Association in 1901.

In the late nineteenth century, the engineering profession, and Perry in particular, was responsible for some of the earliest agitation concerning the general teaching of mathematics in this country and the content of journals such as *Nature, Electrician* and *Engineering* testifies to this. In as early as 1880, Perry outlined his conception of elementary mathematics in a paper read before the Society of Arts (Perry, 1900; MacGillivray, 1902). By the turn of the century agitation had spread to the columns of general educational periodicals such as the *Educational Times, Journal of Education* and *School World* (Findlay, 1902; Young, 1920, pp. 87–90).

It should be added that we are *not* here considering the early attempts to reform geometry teaching which can be traced to the 1860s. In this earlier period the movement came from *within* the mathematical community, it met powerful resistance from certain leading academics, and was not helped forward by the more general and potent forces to which we have drawn attention. In consequence, the efforts of the Association for the Improvement of Geometrical Teaching achieved comparatively little and as the Mathematical Association, from 1897, this group was initially conservative in outlook and was forced to react in the early years of the present century on a much changed general situation which was largely not of its own making, though we will not pursue the detail here (Brock, 1975). What is more important is Perry's campaign to extend the adoption of his 'practical mathematics' from within the education of engineers to the wider education of intending teachers in the colleges and pupils in the schools. As a 1903 'Correspondent' (1903) pointed out in *Engineering*:

> It is well known that Professor Perry holds the opinion that the method which advocates of teaching mathematics to engineers is the best method of teaching children, for whatever life intended. At the same time it will be easily realised that the new system would from its very nature prove less startling to those who are trained to work hand-in-hand with Nature, and would therefore be easier to introduce to the engineering world than to any other. Practical mathematics, in fact, is a system of teaching mathematics to all persons, of all kinds and all ages. (p. 803.)

Perry left Finsbury in 1896 to become Professor of Mathematics and Mechanics at the Royal College of Science, South Kensington, where it was understood that he would develop further his system of mathematical teaching. Significantly, in 1899, the DSA first offered 'elementary practical mathematics' within its scheme of examinations in science and art, with Perry appointed as an examiner, and in the same year the Department published Perry's (1899) six lectures for working men on the subject, which was soon to be followed by a crop of new textbooks on practical mathematics by writers such as Frank Castle (1901), who came under Perry's direct influence at South Kensington. The success of this new subject in the technical field was quite extraordinary over the first decade of the twentieth century. The Board of Education (1900) reported that in 1899, the first year, there were only 212 papers worked compared with around 12 500 for practical plane and solid geometry and 24 000 for mathematics (the traditional diet). By 1903 the Correspondent (1903, p. 803) in *Engineering* could refer to 'enormous' progress, and added:

> How else describe the effect at one college in these islands, at which preparation was made for ten or a dozen students in the new mathematics and to which several hundreds came?

Interest was spreading rapidly and in 1909 the new subject had stolen the lead, with 6500 candidates compared with 6000 for mathematics and 3500 for geometry. The numbers actually attending the evening courses would have been at least two or three times greater (Holmes, 1910, p. 300). P. Abbott (1912), in his paper of 1912 for the ICTM, stated:

> It is probably correct to say that nobody has done more to influence the teaching of Mathematics in this country during the last 15 or 20 years than Professor Perry; and in no branch of the work has he brought about greater changes than in the teaching of Mathematics to technical students . . . In 1910, in England alone, 6964 students presented themselves in Practical Mathematics while only 2841 presented themselves in the first three stages of Pure Mathematics.

By 1913, when he was forcibly retired, Perry (1899, 1913, pp. vii–viii) was able to claim that there were more students of science engaged in his subject than in any other and also that it was more popular with students than the traditional course, taking the level of attendance through the year as an indicator. In the same period up to the First World War Perry also contributed to the process of dissemination by giving short summer courses at South Kensington on the teaching of practical mathematics, as well as courses on mechanics, practical geometry and graphics (Board of Education, 1904). It should also be added that in 1903 it was the Institution of Civil Engineers which appointed a Committee to consider the general education of all classes of engineer. The Committee reported in 1906, following a survey, and supported a number of Perry's proposals for a more practical and broader approach in the schools, including trigonometry, logarithms, contracted methods and mental arithmetic in particular (Thompson, 1911, p. 287). What is remarkable is that the non-engineers were also carried forward on the wave of curriculum reform.

There is not space in this paper to pursue Perry's general campaign and the various responses of the British, Mathematical and Assistant Masters' Associations, the Headmasters' Conference, the Board of Education, the various important examining bodies, and the educational suppliers and publishers, all of whom became involved in what became known as the 'Perry movement' in Great Britain, 'Perryismus' on the Continent and the 'Laboratory Method' in America. Such was the range of Perry's influence (Young, 1920, pp. 87–121; Wolff, 1915; Perry, 1901; British Association, 1907). However, Perry's early hopes for the general adoption of his scheme in toto were to prove too sanguine. At the Belfast meeting of the British Association (1903), in 1902, he suggested:

It seems probable that at the end of another five years no average boy of fifteen years of age will have been compelled to attempt any abstract reasoning about things of which he knows nothing; he will be versed in experimental mathematics, which he may or may not call mensuration; he will use logarithms, and mere multiplication and division will be a joy to him; he will have a working power with algebra and sines and cosines; he will be able to tackle at once any curious new problems which can be solved by squared paper; and he will have no fear of the symbols of the infinitesimal calculus . . . Five years hence it will be called 'elementary mathematics'. Four years ago it was an unorthodox subject called 'practical mathematics' . . .

However, as Findlay (1902) pertinently remarked, in the same year:

I do not think Prof. Perry and his friends quite realize . . . what a great gulf separates their work with adults and artisans from that conducted by teachers of boys and girls in school classes. (p. 184.)

Certainly the mathematical requirements for intending army officers within the public schools, and at the Royal Naval Colleges of Osborne and Dartmouth, which admitted naval cadets at 13, moved quite rapidly at an early stage towards Perry's ideals and laboratory work involving the mathematical masters was therby stimulated (Boyt, 1906; Board of Education, 1912). As Godfrey (1912, p. 431) pointed out, in 1912, concerning the preliminary education of intending engineers, surveyors and army entrants:

> ... the teaching ... tends to be of a more intuitional and experimental character than that of other boys ... The Army requirements during the last 10 years have no doubt had a powerful reaction on the character of the teaching throughout English public schools.

However, in elementary and secondary schools generally 'practical mathematics' variously interpreted as both a new approach and a wider conception of mathematical possibilities, also gradually gained a hold, with some support from the important university examining bodies within the chaotic English examination system. The new requirements were accompanied by a flood of new textbooks and stimulated the supply of mathematical tables, instruments and aids of various kinds for the classroom and, of course, squared paper in a variety of forms. In all this there was clearly much enthusiasm, experimentation and also confusion in the schools. In particular, 'graphic(al) algebra' might mean different things to different teachers, and working through a chapter on 'graphs' in an arithmetic, algebra, or geometry textbook (writers generally 'cashed in' on the craze) could itself become as tedious and mechanical as the rote learning of Euclid's demonstrations[3]. However, squared paper had entered the mathematical classroom to serve a variety of purposes, and its place has not been undermined to this day. It was one of the engineer's tools by which it was thought the mathematical heritage might best be made available to all.

NOTES

[1] This is probably also true of the continent, though Cajori (Cajori, 1917, p. 299) found Strehlke (1842) recommending "the graphic representation of functions ... [as] an effective aid in instruction" as early as 1842.

[2] In his practical physics texts, Perry's successor at Clifton College, A. M. Worthington, still used ratios to test the proportionality between results. Thus in Worthington (1886, p. 131) the law of the simple pendulum is evaluated by recording observations of L/T^2 for different lengths and periods. Only at the end of the book and in only *one* experiment, on the calibration of the galvanometer, is 'a square foot of millimetre curve paper' used. (See Worthington 1886, pp. 294, 308.) We thank T. B. Akrill, Stone Science Librarian at Clifton College for this argument.

[3] These general remarks are partly based on a scrutiny of a complete run of the monthly *School World*, 1899–1918, which contains dozens of articles, reviews, reports, commentaries and letters on mathematical education. The *Journal of Education* and the

Educational Times are similarly illuminating for the period. The *Mathematical Gazette* on the other hand is not very helpful concerning general curricular tendencies until about 1907.

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