Wood Science and Technology 30 (1996) 253–263 © Springer-Verlag 1996

Determination of Young's modulus for spruce, fir and isotropic materials by the resonance flexure method with comparisons to static flexure and other dynamic methods

D. W. Haines, J. M. Leban, C. Herbé

Summary Dynamic methods provide rapid and accurate means to determine Young's modulus, i.e. the modulus of elasticity, of wood. For dry, clear specimens of épicéa commun (Norway spruce, *picea excelsa*) and sapin pictiné (silver fir, *abies amabilis*) we present a comparison of results from tests by a resonance flexure method with results obtained from four-point static flexure tests. For a wide range of specimen size the resonance flexure method provides a simpler, more rapidly performed alternative to the classical static flexure method, giving Young's modulus values which are for the spruce and fir specimens of this study, nearly identical to those calculated from the static flexure tests. Results are also presented which show that a resonance longitudinal method yields higher values of Young's modulus and an ultrasonic method yields still higher values. We provide also a comparison of the four test methods applied to isotropic materials.

Introduction

Dynamic test methods have developed in parallel with static test methods for many types of solid materials including wood. Static methods are generally more familiar and more widely used. However, dynamic tests can often be performed more rapidly, particularly with the availability of modern electronic test equipment. The availability of digital storage oscilloscopes with fast Fourier transform capability greatly simplifies dynamic test analysis, particularly for resonance methods.

Haines (1979, 1980) used resonance flexure in steady state excitation for the measurement of Young's modulus for the characterization of wood for musical instruments. Other applications of resonance flexure were reported for the study of

---

Received 28 July 1995

Daniel W. Haines
Mechanical Engineering Department
Manhattan College, Riverdale, NY 10471, USA

Jean-Michel Leban
Equipe de Recherches sur la Qualité des Bois
Institut National de la Recherche Agronomique
54280 Champenoux, France

Christian Herbé
Equipe de Recherches sur la Qualité des Bois
Institut National de la Recherche Agronomique
54280 Champenoux, France

The authors wish to thank Pierre Michel and André Perrin for preparing the test specimens and components of the test apparatus for this study

plywood and laminated lumber (Sobue 1983; Sobue and Iwasaki 1981a, b) and for the study of the mechanical properties of epoxy-poplar composite materials (Moore et. al. 1983).

In this paper we present results obtained from vibrational resonance created by impact of the wood specimen in states of free support conditions. The free support conditions provide the most accurate support conditions achievable. Modern instrumentation enables this method to be applied with ease. Impact induced free-free resonance flexure vibrations were employed for the determination of Young's modulus of small clear specimens by Sobue (1986b) and Bordonne (1989), on samples of commercial size logs (Arima et al. 1991), structural lumber (Sobue 1988) and clear and knotty specimens by Chui (1991).

Several authors have reported on comparisons of dynamic and static tests of wood. Comparison of cantilever resonance, static three-point bending and ultrasonic tests for the determination of Young's modulus for Douglas-fir was reported by Sinclair and Farshad (1987). Comparison of Young's modulus calculated from free-free resonance flexure tests and static three and four-point bending tests were performed for 16 types of wood by Curie (1989).

Hearmon (1948) reported on the determination of Young's modulus of beech and pine by means of flexural and longitudinal tests. He found values calculated from longitudinal tests to be 5% to 8% greater than those calculated from flexural tests. Results of Brenndorfer (1972) and Radu and Brenndorfer (1976) also noted differences between the Young's modulus deduced from longitudinal and flexural modes of vibration.

Longitudinal vibrations in wood, produced by induced stress waves on which the velocity was the measured parameter have been reported by Dunlop (1978 and 1980) and Gerhards (1982). By tapping the specimen with a hammer and by receiving the signal with a wireless microphone, Sobue (1986a) determined the Young's modulus of various sizes of wood specimens. The resonances were determined from a frequency analysis of the signal, and a microcomputer permitted treatment of the data.

Ultrasonic techniques have been applied to wood by Hearmon (1965) and Bucur (1985) to determine the elastic constants of wood.

In this paper we present a discussion of the three dynamic tests and a comparison of these results with tests of the same specimens by the four-point static flexure method. The specimens tested are nominally 360 mm long with square cross section measuring 20 by 20 mm. This size is the French standard size for the four-point static flexure test; it is also taken as the standard size for the dynamic tests. Throughout this report we note nominal dimensions, however, all measurements were taken with accuracy of at least three significant figures. All references to Young's modulus pertain to the measure of stiffness in the fiber direction. The methods discuss in this paper may be applied to determine the stiffness in directions other than the fiber direction, however this report is restricted to discussion of the Young's modulus in the fiber direction.

It is shown that for the dry, standard specimens of both épicéa commun (Norway spruce, *picea excelsa*) and sapin pictiné (silver fir, *abies amabilis*) the mean value of Young's modulus as calculated from resonance flexure tests results nearly matches that calculated from the static flexure results. The mean value from the resonance longitudinal results is above the resonance flexure and static results, and the mean value of the ultrasonic longitudinal results is above those of all other results. All tests of the same set of specimens were performed within a seven hour period in order to minimize ambient effects. The moisture content at the time of the tests was approximately 12%.

In addition, full size specimens of construction lumber were tested by resonance flexure and static flexure. The two sets of tests were performed several years apart.

These specimens were of rectangular cross section and measured nominally three meters long. The results for Young's modulus for these large specimens show the resonance flexure values to be very close to those of the static flexure method. The small difference between the results of the two types of tests is believed to be due to different environmental conditions in existence during the two tests.

Finally, we present a discussion and presentation of the results of resonance flexure tests of small specimens less than 1 mm thick. These specimens represented early and late wood of spruce and had been tested in static tension several years prior to the resonance flexure tests. Very close correlation is found between the results of the two types of tests.

Dynamic methods

The two resonance methods employ excitation of the specimen by a sharp blow with a hammer at a known antinode of vibration. The weight of the hammer must be great enough to deliver enough energy to excite the vibrations and light enough not to deliver so much momentum to carry the specimen with it as a rigid body. For many situations these requirements lead to a hammer with a weight in the same range as the specimen. A microphone is placed in close proximity to a second known antinode to receive the radiated sound from the specimen due to the blow of the hammer. The electrical signal generated in the microphone is sent to a digital storage oscilloscope with fast Fourier transform processing capability. For our purposes the *LeCroy* 9314M oscilloscope served well. In a properly posed experiment, the frequency of the strongest resonance displayed on the screen of the oscilloscope is the resonance frequency from which Young's modulus may be calculated.

The through-transmission ultrasonic test requires a pair of piezoelectric transducers, a signal generator to produce the ultrasonic pulse signal and a device to measure the time of travel of the ultrasonic pulse through the specimen. Often the signal generator and the time measurement instrument are contained in one unit. For the tests reported herein the instrument was the *Sattec* 1Mhz unit.

The four-point static flexure tests were performed in accordance with the French standard NF B 51-016 "Determination of modulus of elasticity in static bending using small clear specimens."

Resonance flexure vibration mode

The resonance flexure vibration test utilizes the fundamental mode of vibration of a free-free beam, i.e. a beam with no constraints. Figure 1 shows the test set-up. Light support may be provided at two points located 22% of the length from each end (Timoshenko, 1955). This distance is not critical, provided the support does not unnecessarily constrain the specimen. The points of support are the nodes of the fundamental mode of vibration. With the standard 20 × 20 × 360 mm specimens this dimension is 79 mm. Support by light threads held in tension by elastic bands serves well for this purpose. The microphone is placed above and close to the specimen near one end. With appropriate settings of the oscilloscope it is easy to capture the frequency of the fundamental mode of flexural vibration when the center of the specimen is struck from above and perpendicular to its length. The entire process requires no more than a few seconds. With this frequency (700 Hz–900 Hz for the standard specimens of spruce and fir) Young's modulus is calculated from the equation

$$E = 0.946\,\rho f^2 L^4/h^2 \tag{1}$$

where E is Young's modulus, ρ is the density of the wood specimen (mass per unit volume), f is the measured frequency, L is the length of the specimen (nominally 360 mm) and h is the height, i.e. vertical thickness of the specimen (nominally 20 mm).

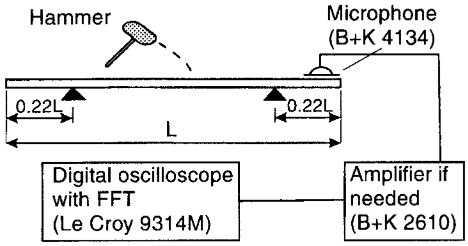

Fig. 1. Resonance flexure test

For all wood specimens in this study, the density, ρ, represents the mean density of the entire specimen. The derivations of Eq. 1 and Eqs. 2 and 3 of following sections may be found in Timoshenko (1955) and Morse (1948).

Resonance longitudinal vibration mode

The longitudinal mode of vibration is produced by striking one end of the specimen directly and parallel to the long dimension of the specimen while the specimen is held in the hand close to its center. Figure 2 shows the test set-up. When properly executed this action establishes the fundamental longitudinal vibration of the specimen acting as a bar. The technique to excite this resonance requires practice, for example, the specimen must be struck centrally at the end and directly in line with the specimen length. Flexural vibrations are also easily excited by this method, therefore, it is important to identify the correct resonance. For the standard specimens of spruce and fir, the fundamental resonance longitudinal occurs between 5000 Hz and 9000 Hz. The antinodes, i.e. the points of maximum motion are the two ends of the bar. The end opposite to the struck end is available for placement of the microphone nearby to receive the sound radiating from the free end. The signal is passed to the oscilloscope as with the flexure test and the resonance frequency is recorded. As for the resonance flexure tests the required time to perform the test is only a few seconds. Young's modulus is calculated from the equation

$$E = 4\rho f^2 L^2, \tag{2}$$

where f is the frequency of longitudinal vibration and again E is the Young's modulus, ρ is the density and L is the total length of the bar (nominally 360 mm).

Ultrasonic longitudinal through-transmission mode

The ultrasonic method requires the placement of two piezoelectric transducers in contact with the ends, one serves to emit a longitudinal stress pulse of very short duration which travels with the speed of sound in the wood to the transducer at the other end. Figure 3 shows the set-up of this experiment. The time of pulse travel for the standard specimen of spruce, and fir falls in the range between 60 and 85 microseconds. With this time of travel and the known distance separating the transducers, i.e. the length of the specimen (nominally 360 mm), the velocity of the stress pulse is calculated. Young's modulus is then calculated from the equation

$$E = \rho v^2, \tag{3}$$

where v is the calculated velocity and again ρ is the density of the wood specimen.

Static flexure test

The static tests were performed in accordance with the French standard NF B 51-061 which is the classical four-point bending test. Deflection is measured in the central

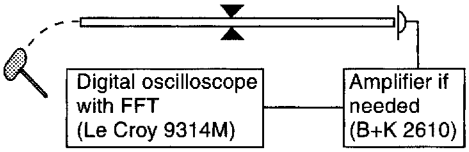

Fig. 2. Resonance longitudinal test

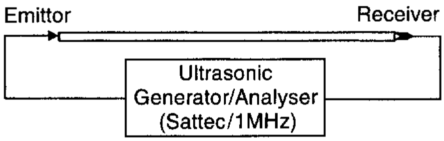

Fig. 3. Ultrasonic longitudinal test

region, a zone of pure bending in the absence of shear deformation. Figure 4a shows the support points separated by 320 mm, the load application points separated by 160 mm and the contact points of the deflection measurement gauge separated by 120 mm. Figure 4b shows the specimen deformed due to the application of load. Young's modulus is given by the equation

$$E = 3P(L_s - a)\,m^2/(8bh^3u), \qquad (4)$$

where P is the total force applied to the specimen, L_s is the supported length of the specimen (320 mm), a is the distance separating the points of load application (160 mm), m is the distance separating the deflection measurement gauge contact points (120 mm), b is the width of the specimen (nominally 20 mm), h is the height of the specimen (nominally 20 mm) and u is the deflection measured at the midpoint between the gauge contact points.

Test results of the standard wood specimens (20×20×360 mm)

Figures 5 and 6 show the results of the static flexure tests performed on the 25 specimens of spruce and 80 specimens of fir, respectively. The results show the anticipated rise of Young's modulus with increase in density.

Figure 7 shows the plot of the data points for the spruce specimens, indicating the Young's modulus as found by static flexure and the percentage differences given by the three dynamic tests.

Figure 8 is a similar plot for the data for the fir specimens. It is evident in Fig. 8 that the spread widens considerably between the results of the four types of tests for Young's modulus below 9000 MPa. The cause of this interesting observation is under study.

Table 1 shows the comparison of the mean values of Young's modulus and density for spruce and fir as obtained by each of the four methods. In this study the results for the resonance flexure method nearly match the results for the static flexure method; the mean Young's modulus by resonance flexure for spruce and fir fall above those for static flexure by 0.4% and 2.3%, respectively. The mean value of Young's modulus from the longitudinal resonance method is greater than the mean value obtained from the flexure methods by 6% and 11% for spruce and fir, respectively. These results are similar to those obtained by Hearmon (1948). The mean value of Young's modulus from the longitudinal ultrasound method exceed the mean value obtained from the flexure methods by 17% and 22% for spruce and fir, respectively. The results reported herein are also consistent with the findings of Sinclair and Farshad (1987).

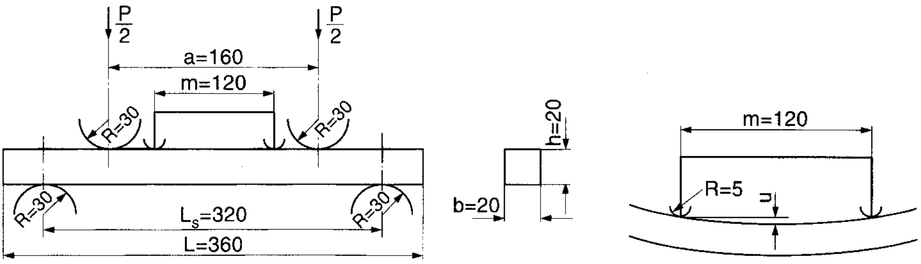

Fig. 4. a Static flexure test, b Static flexure deflection measurement

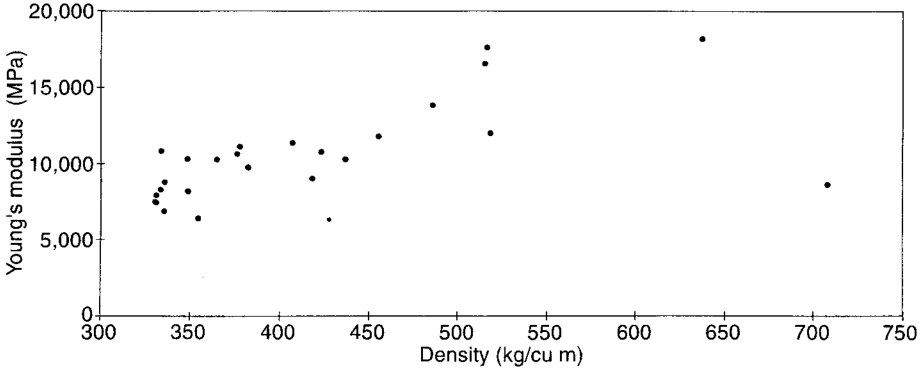

Fig. 5. Young's modulus v. density for 25 specimens of spruce

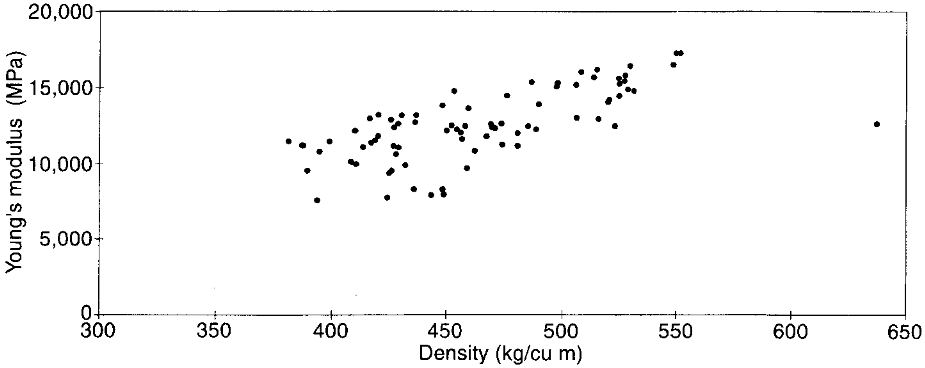

Fig. 6. Young's modulus v. density for 80 specimens of fir

One may know that for high ratios of height to length, shear deformation is significant and reduces the resonance frequency of beams vibrating in flexure. However, the dimensions of the standard specimen are such that this effect is negligible for the fundamental mode of vibration, and Eq. 1 is valid. This fact was verified by experiment in this study.

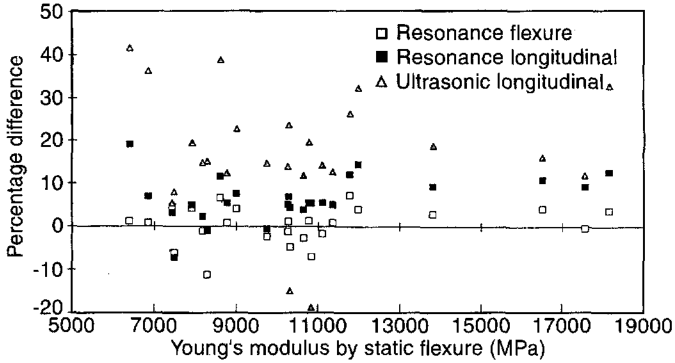

Fig. 7. Comparison of Young's modulus found from dynamic methods with those obtained from the static flexure method for the 25 specimens of spruce

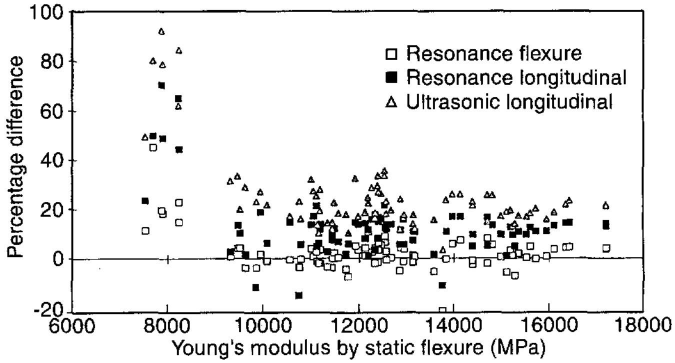

Fig. 8. Comparison of Young's modulus found from dynamic methods with those obtained from the static flexure method for the 80 specimens of fir

Tests of 20×20×360 mm homogeneous, isotropic specimens

Equations 1, 2, 3 and 4 were derived for ideal elastic materials which are also homogeneous, i.e., their material properties do not are constant from point to point, and isotropic, i.e., their properties vary with direction around a given point. No material fully achieves this ideal; compared to most metals and plastics, wood is highly non-homogeneous and anisotropic because of its grain structure.

Whenever test results are compared of material as complex as wood it is imperative to verify the accuracy of the test methods. One way to achieve this verification is to use the same method to test more ideal materials and compare the results. A close comparison indicates that the test methods are accurate. To so verify the test methods, each of the four methods were applied to two specimens each of duralumin, a material of greater stiffness than wood and two types of nylon, a material of less stiffness than wood. All specimens were nominally 20 × 20 × 360 mm, as for the principal series of wood in this study. These materials are nearly homogeneous and isotropic. The results given in Table 2 show that with the exception of the high ultrasonic longitudinal results for nylon, the methods give results which agree quite closely. These results provide further confidence in the test methods, particularly the resonance flexure method.

Table 1. Mean values of Young's modulus and density for wood

	Young's modulus				
	Density kg/cu m	Static flexure MPa	Flexural resonance MPa	Longitudinal resonance MPa	Longitudinal ultrasound MPa
Spruce 25 specimens	416	10600	10600	11300	12400
Fir. 80 specimens	465	12500	12700	14000	15400

Table 2. Young's modulus and density for isotropic materials. Means of tests on two specimens each

	Young's modulus				
	Density kg/cu m	Static flexure MPa	Flexural resonance MPa	Longitudinal resonance MPa	Longitudinal ultrasound MPa
Duralumin	2800	73600	71600	73400	73300
Nylon A	1410	3570	3600	3700	4450
Nylon B	1140	3300	3400	3500	4280

Tests of wood specimens of other sizes

The ease of application and accuracy of the resonance flexure method led to experimentation with specimens of larger and smaller size than the standard size.

Large specimens of construction lumber

A total of 20 specimens of construction lumber spruce were available for dynamic tests for which Young's modulus had been determined previously by static flexure. The weights of each specimen were nearly identical to those determined when the static tests were conducted; the mean densities of the two sample sets were nearly identical (412 kg/cu m for the static flexure tests and 411 kg/cu m for the resonance flexure tests). Experience has shown that Young's modulus remains constant if the mass (closely related to moisture content) is constant. Therefore, tests by the resonance flexure method could be readily compared with the earlier static flexure tests. The lumber dimensions were a standard European construction size, nominally three meters long with 50 by 150 mm rectangular cross section. Tested in flexure about the strong axis the resonance flexure frequencies fell between 65 and 80 Hz.

Figure 9 shows the results of these tests. The Young's modulus values for the resonance flexure tests fall slightly below the values calculated from the static tests; the mean values are 10300 MPa by the resonance flexure method and 10900 MPa by the earlier static flexure method, a difference of less than 6%.

Small specimens

Following the application of the resonance flexure method to a number of specimens of varying size, we pondered the question, "How small may the sample be to apply this method?" The smallest specimens available were those of a study by Chantre (1989) of early and late wood within the tree ring of spruce. Chantre prepared specimens for

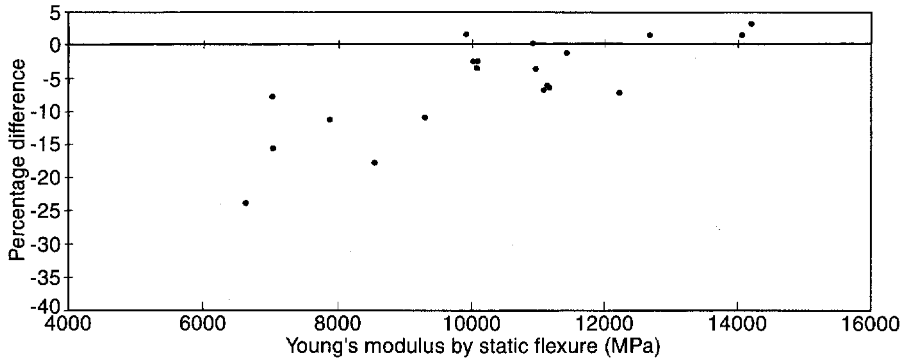

Fig. 9. Comparison of Young's modulus found from the resonance flexure method with those obtained from the static flexure method for 20 specimens of spruce construction lumber

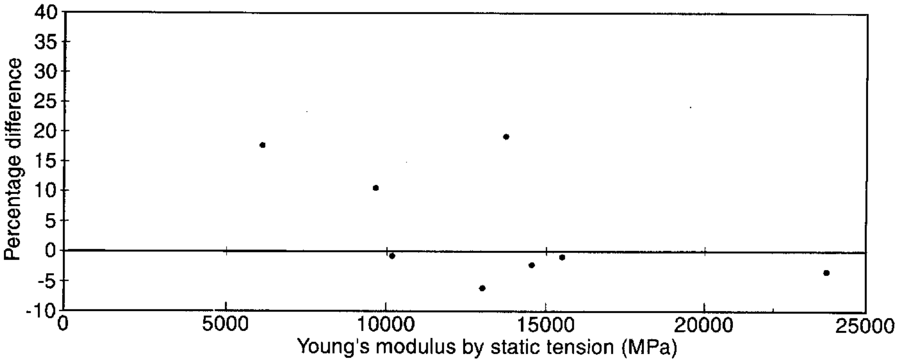

Fig. 10. Comparison of Young's modulus found from the resonance flexure method with those obtained from the static tension method for 8 spruce specimens less than 1 mm in thickness

tensile tests which were less than 1 mm thick and nominally 5 mm wide and 30 mm long. To test these specimens in flexural vibration it was necessary to hold the specimen between a two pairs of tightly stretched threads spaced such that each was positioned at or near a node (22% of the length from each end). We found it impossible to strike a specimen of this size effectively, however it was found that the desired fundamental mode of vibration could be excited by deflecting the center of the specimen downward with an instrument having a pointed tip such as a pencil until resistance is felt, then releasing the specimen as one would pluck a string. The frequency range of the resonance flexure of these small specimens fell between 2500 Hz and 3500 Hz.

Figure 10 shows the results of these tests in comparison with the earlier static tensile tests. Because of the time span of several years between the two sets of tests, the ambient conditions were different, and therefore the density of each specimen had changed. Therefore these results only yield a qualitative result. Nonetheless, the mean values are 13300 MPa for resonance flexure and 13600 MPa for static tension, a difference of less than 3%.

Conclusions

The resonance flexure method provides a useful tool to determine Young's modulus of wood. With the availability of modern instrumentation, this dynamic test can be performed rapidly and Young's modulus can be calculated from a simple formula. The standard specimen size adopted for static flexure tests is ideal also for the resonance flexure test. For clear coniferous wood specimens, resonance flexure test results provide a close match to the Young's modulus expected from the static flexure results, and it is more easily performed than the static flexure method.

Comparison of resonance flexure results with those from static tests for sizes other than the standard size show less than a 6% difference for the large construction lumber and less than a 3% difference for the small specimens of less than 1 mm thickness.

The differences between the Young's moduli obtained from the flexure methods and those from the longitudinal dynamic methods require discussion. First, note that the longitudinal methods contain high frequencies; the frequencies of the ultrasonic method are the higher. Both longitudinal dynamic methods yield high values for Young's modulus; the ultrasonic values are the higher. Equations 1, 2 and 3 were derived on the basis of the material being ideal on the macroscopic level, i.e. isotropic, homogeneous and elastic. The definition of Young's modulus likewise assumes these ideal properties. No material fully meets the ideal, but many are nearly so, e.g., most metals. Wood in this sense is less ideal than metals. On the macroscopic level, wood is clearly nonhomogeneous and anisotropic, due primarily to the pattern of early and late wood and the difference in their physical properties. The known capability for the damping of vibrations (Haines 1979) establishes wood as more viscoelastic than metals, i.e. less elastic.

Viscoelasticity is the most likely source of the differences noted. Both longitudinal methods employ a pulse; that of the ultrasonic method is shorter in time. Shorter pulses are related to waves of higher frequency content. Ferry (1961) reported that bulk longitudinal waves travel with greater velocity at higher frequencies in rubber and relates this phenomenon to the attenuation (viscoelastic property) of rubber. Kolsky (1963) provided analysis of a complex viscoelastic model which predicts higher velocities of longitudinal waves at higher frequencies. Since Young's modulus in the longitudinal modes is proportional to the square of the longitudinal velocity, these results of Ferry and Kolsky predict the higher value of calculated Young's modulus for wood as we obtained from longitudinal resonance and the even higher values as we obtained from longitudinal ultrasound.

Nonhomogeneity may play a role also. The higher density and stiffer late wood passes longitudinal stress waves with greater velocity. The longitudinal wave may follow the path through the late wood. Further study is required to quantify the differences noted. The main point of this paper is that the flexural resonance method to determine Young's modulus for wood is accurate and easily performed.

References

Arima, T.; Nakamura, N.; Maruyawa, N.; Hayamura, S. 1991: Classification of logs based on sound analysis and its application in product processing. Proc. Int. Timber Eng. Conf., London 2: 266-2,2.273

Bordonne, P. A. 1989: Module dynamique et frottement intérieur dans le bois mesures sur poutres flottantes en vibrations naturelles. Doctoral thesis, l'Institut National Polytechnique de Lorraine, Nancy

Brenndorfer, D. 1972: Nondestructive testing of wood with acoustic vibrations. Industria Lemn. 6: 234–240

Bucur, V. 1985: Ultrasonic velocity, stiffness matrix and elastic constants of wood. CAS Journal 44: 23–28

Chantre, G. 1989: Liaison entre rigidité et densité du bois a l'intérieur du cerne. DEA Etude (Masters Thesis) INRA, Nancy

Chui, Y. H. 1991: Simultaneous evaluation of bending and shear moduli of wood and the influence of knots on these parameters. Wood Sci. Technol 25: 125–134

Curie, P. 1989: Essais comparatif pour la mesure du module d'élasticité. Provisional report of the Centre Technique Forestier Tropical, Nogent-sur Marne

Dunlop, J. I. 1980: Testing of particleboard by acoustic techniques. Wood Sci. Technol 14: 69–78

Dunlop, J. I. 1978: Damping loss in wood at midkilohertz frequencies. Wood Sci. Technol. 12: 49–62

Ferry, J. D., Viscoelastic Properties of Polymers, John Wiley & Sons, New York, 1961, p. 420

Gerhards, C. 1982: Longitudinal stress waves for lumber stress grading: factors affecting applications. States of art. Forest Prod. J. 32(2): 20–35

Haines, D. W. 1979: On musical instrument wood, CAS Newsletter 31: 23–32

Haines, D. W. 1980: On musical instrument wood – Part II, surface finishes, plywood, light and water exposure. CAS Newsletter 3: 19–23

Hearmon, R. F. S. 1948: Elasticity of wood and plywood Forest Prod. Res. Spec. Rep. No. 7. His Majesty's Stationery Office, London, 87 pp

Hearmon, R. F. S. 1965: The assessment of wood properties by vibrations and high frequency acoustic waves. Proc. 2nd Symp. on the NDT of wood, Washington State University, 49–65

Kolsky, H; Stress Waves in Solids, Dover, New York, 1963, p. 121

Moore, G. R.; Kline, D. E.; Blankerhorn, P. R. 1983: Dynamic mechanical properties of epoxy poplar composite materials. Wood Fiber Sci. 15(4): 358–375

Morse, P.; Vibration and Sound, 2nd edition, McGraw-Hill, New York, 1948

Radu, A.; Brenndorfer, D. 1976: Zur zerstorungsfreien prufung des solzed durch schwingungsversuche. Holz als Rohund-Werkstoff 34(6): 219–222

Sinclair, A. N.; Farshad, M. 1987: A comparison of three methods for determining elastic constants of wood. Journal of Testing and Evaluation JTEVA 15(2): 77–86

Sobue, N.; Iwasaki, Y. 1981a: Effect of veneer construction on anisotropy of dynamic viscoelasticity of plywood. Mokuzai Gakkaishi 27(6): 457–462

Sobue, N.; Iwasaki, Y. 1981b: Effect of adhesive on dynamic viscoelasticity of laminated lumber of two-ply sawn laminae. Mokuzai Gakkaishi 27(7): 597–601

Sobue, N. 1983: Prediction of dynamic Young's modulus and loss tangent of plywood in bending. Mokuzai Gakkaishi 29(1): 14–19

Sobue, N. 1986a: Measurement of Young's modulus by the transient longitudinal vibration of wooden beams using a FFT spectrum analyser. Mokuzai Gakkaishi 32(9): 744–747

Sobue, N. 1986b: Instantaneous measurement of elastic constants by analysis of the tap tone of wood. Application to flexural vibration of beams. Mokuzai Gakkaishi 32(4): 274–279

Sobue, N. 1988: Simultaneous determination of Young's modulus and shear modulus of structural lumber by complex vibrations of bending and twisting. Mokuzai Gakkaishi 34(8): 652–657

Timoshenko, S., Vibration Problems in Engineering, 3rd edition, D. Van Nostrand, New York, 1955