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RESEARCH ARTICLE

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Stability properties of human reaching movements

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Abstract Through an experimental study of the stability properties of the human neuromuscular system while it performs simple point-to-point arm movements, this paper evaluates the concepts of equilibrium and virtual trajectories as a means of executing movement of the arm. Human subjects grasped the instrumented handle of a two-link robot manipulandum and performed specified point-to-point planar arm trajectories. Computer-controlled brakes were used to subtly change the movements by constraining the trajectory along an arc of radius equal to the length of one link of the manipulandum. Target points were arranged to lie along the arc so that the subject could complete the movement even when constrained. Three situations were tested: (1) unconstrained throughout the movement, (2) constrained through the entire movement, and (3) initially constrained and then released during movement. Experimental results showed that the constraint evoked significant forces strongly oriented so as to restore the hand to the unconstrained hand path. In addition, when released from the constraint, these forces caused a strong tendency to return the hand to the unconstrained path before the end of the movement was reached. Such strong positional stability properties of the arm reinforce the notion that a moving attractor point dominates the dynamics of the arm during movement. Additionally, bounds on the shape of the virtual trajectory were found which indicate that the equilibrium point remains close to the actual movement produced. These results, showing that a controlled equilibrium point may be used for planning and coordinating multijoint movements, are consistent with an equilibrium point hypothesis.

Key words Multijoint movements · Reaching movements · Equilibrium point · Motor control · Interaction control · Human

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Introduction

The central goal of this paper is to determine whether muscle-generated equilibrium points have sufficient influence to generate two-joint arm movements. Interestingly, reasonable doubts have been raised about whether this fundamental assumption of the so-called equilibrium-point hypotheses could ever be disproved. These doubts result mainly from the fact that a force-length relation is intrinsic to the structure of muscles and their reflex loops, and therefore "equilibrium" or "attractor points"[1] will always exist as a natural description of the muscle activation. However, the key idea of the virtual trajectory hypothesis is that the attractor point is a variable that the central nervous system can use to simplify the computational burden of generating movement and controlling contact. Consequently, this hypothesis can be disproved if the attractor point is too weak. In fact, several recent studies have shown that, although an attractor point may exist during movement, its overall influence over movement may be relatively small compared with the dynamics of movement (Bennett et al. 1992; Gomi et al. 1992; Kawato et al. 1992; Lackner and DiZio 1992). Given these recent results, it is the goal of this paper to clarify the role of attractor points in the dynamics of human arm movements.

Attractor or virtual trajectories allow movements to be encoded in terms of kinematic variables. In 1987, Flash took experimental kinematic data from various planar, point-to-point two-joint arm movements and showed that a single virtual trajectory might be the foundation for the planning and generation of these movements. Us-

[1] During motion, the limb does not have to achieve its equilibrium position, and in fact the equilibrium point during movement is not constrained to remain within the accessible workspace of the limb. Furthermore, the term equilibrium does not necessarily indicate system behavior, i.e. an equilibrium point can be stable, unstable, or both stable and unstable simultaneously. The property discussed in this paper is a stronger statement than simply equilibrium. It is a statement that all state trajectories show a tendency to approach this point. As the term "equilibrium" is ambiguous, the terms "attractor" or "attractor point" will be used

ing a single, measured movement and various assumptions of the size and shape of the stiffness and damping ellipses, Flash calculated that the virtual trajectory generating this movement was straighter than the actual hand path. Furthermore, using this derived virtual trajectory as the driving input to a computer simulation, Flash showed that a motor program consisting of an estimated straight-line trajectory of attractor points was competent in reproducing the kinematics of the actual experimental data.

Conversely, several recent studies have cast doubt upon the ability of a controlled attractor to simplify the planning and execution of coordinated multijoint movements. In particular, the magnitude of the joint stiffness used by Flash during the simulated movements was called into question. The stiffness data used by Flash were taken from postural measurements and multiplied by constant scaling factors, ranging from 1 to 3 times postural stiffness. Yet Bennett et al. (1992) and Gomi et al. (1992) found that the mechanical stiffness of the elbow during single- and two-joint movements, respectively, drops to well below the levels used by Flash.

Sufficiently low stiffness during movements would compromise the ability of virtual trajectories to simplify movement planning by requiring large displacements in order to generate the forces needed to induce motion. Using these low-stiffness estimates, Kawato et al. (1992) computed virtual trajectories under the assumption that the human desired perfectly straight point-to-point hand trajectories. Since large displacements were required to generate the necessary joint torques, the virtual trajectories of Kawato et al. were substantially more complex than the ones derived by Flash in her work, thereby removing the postulated advantage of simplicity.

The question of the relative magnitude of stiffness during movement seems at first glance to be important when considering the validity of any equilibrium point hypothesis. Therefore it seems obvious that a direct measurement of stiffness would be required to make such an evaluation. This was the technique used by Bennett et al. (1992) and Gomi et al. (1992) in their evaluations of the virtual trajectory hypothesis. Yet such a method is vulnerable not only to systematic experimental uncertainties (due to the fact that a full impedance measurement of a complex system is difficult by its very nature) but also to assumptions in formulating a model of the arm's impedance (order, linearity, time-invariance, structure), making it difficult to assess the accuracy of the findings. Furthermore, estimating stiffness parameters may be a needless detail. What is truly important when considering the positional stability of the arm during movement is whether the stiffness dominates the inertial dynamics of the human arm.

Compounding the impedance identification problem is that the dynamic behavior of a general two-link articulated manipulator is strongly nonlinear and nonintuitive. The dynamics of the complete human arm moving in the horizontal space can be written in the following general form:

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = F_{\mathrm{m}}(\bullet) \quad (1)$$

where $\boldsymbol{q}$ is the vector of generalized coordinates, $\boldsymbol{H}(\boldsymbol{q})$ is the manipulator inertia matrix, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\ \dot{\boldsymbol{q}}$ is a vector of velocity-dependent acceleration, and $F_{\mathrm{m}}(\bullet)$ is the force produced by the neuromuscular system. As stated earlier, finding a specific form for $F_{\mathrm{m}}(\bullet)$ is a difficult proposition. This study focused on showing that the position-dependent component of the neuromuscular force is significant during movement production. Therefore, although it is reasonable to assume that the neuromuscular force is created by some combination of intrinsic muscle properties as well as reflex action, no assumptions need to be made about the neuromuscular force except that it is an unknown function of the kinematics as well as central command and reflex action. It is sufficient to show that position dependence is important relative to movement, regardless of linearity or assumptions of the underlying force-producing mechanisms. (The term "stiffness" is often used in this context. Strictly speaking, it refers to a tangent linear approximation to a nonlinear force-length function. Because we need make no assumption about linearity, we avoid use of this term.)

The experimental paradigm used for this study was designed to assess the influence of attractor points without requiring any specific model of the neuromuscular system, i.e., $F_{\mathrm{m}}(\bullet)$ in Eq. 1. This method has the advantage that all conclusions may be based on observable data and not on an attempt to parameterize and model the human neuromuscular system of the arm. In addition, this method also removes another major objection of the virtual trajectory hypothesis. Smith and Humphrey (1991) argued that the large number of uncertain parameters in the human neuromuscular system makes any claims made through simulation suspect. In contrast to the work of Flash or Kawato et al., the conclusions in this paper will be made directly from the data and not through model-based simulations.

This paper will demonstrate that the attractor properties of muscles cannot be ignored when studying movement control. Specifically, it will be shown that a stable attractor point generated by the neuromuscular system does exist and that this attractor point is significant in terms of its ability to influence the dynamic behavior of the arm segments moving in space. In addition, we will also show that the location of this attractor at any point during the movement is bounded to lie close to the actual trajectory performed by the arm. These results are significant because they support the hypothesis that movements are generated through the motion of a neurally defined attractor point.

Materials and methods

Five right-handed male subjects made point-to-point reaching movements between two targets located along a horizontal plane. All subjects were between 21 and 30 years of age and had no history of neuromuscular disease. They completed all of the trajectories, except subject E, who did not complete the third trajectory set. Each subject gave their informed consent before participating in these experiments. This study was approved by the Committee on the Use of Humans as Experimental Subjects, which is the governing ethics body over such experiments at MIT.

The experimental apparatus

Subjects were seated in front of a computer-controlled, parallel four-bar two-joint robot. At the end-effector, the robot had a handle containing a force transducer (Lord LTS-210F), which the subject grasped at shoulder level. The arm of the subject was suspended with a sling so that the upper arm and forearm remained in plane with the arms of the robot. In addition, the subject wore a wrist cuff, which effectively immobilized the wrist. A pair of light-emitting diodes (LEDs) were suspended over the workspace of the robot to serve as targets for the point-to-point movements to be performed. These target lights were approximately 18 cm apart from each other.

The robot itself was equipped with two electromagnetic brakes (Dodge FB30-10–713), placed on each motor shaft. These brakes were capable of independently locking the shoulder or elbow joints of the robot with an impedance sufficiently large that the human would be incapable of causing a displacement along that degree of freedom. Robot joint angles were measured by two conductive film potentiometers (NEI F78SC502 5 k).

Procedures

Three different trajectories were tested, as shown in Fig. 1A. These three trajectories were performed using the same two target lights. To generate the different configurations, the chair of the subject was translated and rotated relative to the targets. As a result, the start and end points remained the same in the reference frame of the robot while the human perceived a different trajectory to be performed, thereby ensuring that the inertia tensor of the robot at each position along the path would be the same for all three trajectories. Therefore any observed differences in the trajectories will be due to a change on the human side rather than on the robot side.

For each of the trajectories, the subject started with the handle under the illuminated target light. The subject was prompted to make a movement when the LED at the current position was extinguished and the other LED at the target position was illuminated. The subject was instructed to "move to the target as if reaching for some object at that point, ignoring any abnormalities during the movement." Once at the target, the subject was instructed to stop and wait until the other LED had been illuminated. As a result, through the course of a single trial, the subject made several movements back and forth between the two points. The speed of the movement was not specified in the instructions.

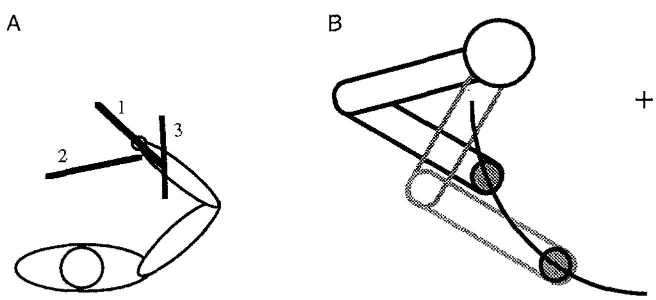

Fig. 1 **A** The three trajectories shown relative to the shoulder of the subject. **B** The kinematics of the robot when the elbow joint is locked. The outer link is locked relative to the ground reference frame, not to the upper arm. Therefore, the handle of the robot is constrained to lie along a circle whose center does not coincide with the shoulder joint of the robot

At intervals, perturbations were applied to the subject through one of the magnetic brakes. These perturbations took two forms. The first type locked the brake associated with the elbow joint of the robot from the start of the movement to the end. This removed a degree of freedom from the robot constraining the path of the handle to a 23.2-cm-radius circle, as shown in Fig. 1B. The constraint was applied in such a way that both the start and target positions lay upon the circular path. As a result, the movement task could still be completed even in the presence of the constraint.

The second type of constraint locked the elbow joint at the beginning of the movement. However, during movement, the constraint was released without warning, and the arm was free to move in both degrees of freedom. The release was triggered by position and was consistently released when the hand had moved 5 cm away from the start, measured along the straight line between the start and target.

During an experimental trial, these two types of constraints were applied only after at least three unperturbed movements had been performed by the subject. The number of unperturbed movements before the next applied constraint was randomly varied from a minimum of three to a maximum of ten movements. These variations were intended to suppress any anticipation by the subject of a perturbed trial.

For a single configuration, 20 constrained cases and 20 constrained and released cases were tested. Ten of each case were applied when the subject moved from target A to target B and the other ten, when the subject moved in the opposite direction from target B to target A. In addition to the constraint cases, 24 (12 in each direction) of the unconstrained point-to-point movements were sampled and recorded by the computer. This same pattern of applied constraints and unperturbed movements was presented to the subject for the three different trajectories (chair positions). Due to the number of movements, it is unlikely that the subject could use past knowledge of a previous trial to anticipate constraints in the new trajectory. Subjects could rest between trajectories to minimize fatigue.

Analysis

The data from the movement experiments consist of the time history of the robot joint angles and the applied force at the handle during each movement. Since the robot was equipped solely with potentiometers, numerical differentiation of the position data was necessary to estimate joint velocities. Previous work has shown that optimal smoothing algorithms are superior in performance to Butterworth filtering and numerically differentiating data (Murphy 1990). The algorithm used for this work is based on the work by Dohrmann et al., using a state space formulation of the smoothing and differentiating problem with cubic splines (Dohrmann et al. 1988). Murphy (1990) demonstrated the ability of this technique to accurately smooth and estimate the first and second derivatives of noisy kinematic data.

Incomplete data for the trajectories was filtered out of the main group before processing. Such data resulted from premature termination of sampling by the computers. Therefore, any movement which was sampled for less than 250 ms was removed from the final group of data to be processed. Only 13 of more than 700 movements recorded were deleted in this way. The remaining data were then assumed to contain complete trajectory information and were thereupon sent through the processing procedure.

As stated previously, the processing of the experimental data was designed to condense these large data sets into meaningful numbers without using a priori models of the underlying motor strategy or estimates of physical parameters of the system. Therefore, the accuracy of these purely quantitative results will not depend on any assumptions within the analysis stage. The results hinge only on the major assumptions of the experimental paradigm: that the subject makes a "normal" repeatable reaching movement and that the motor strategy for the unconstrained, constrained, and constrained/released cases are identical. The former assumption will be shown to be reasonable based on the data of the unconstrained movements to be presented later. The latter assumption will be discussed in the following subsection.

Natural movement data

Most problematic of psychophysical experiments is their reliance on the assumption that the subject completes the specified task in the perturbed trial in the same manner as the unperturbed trial. Perturbations are applied to make the measurements; however, the subjects may respond to the disturbance by changing their behavior, causing the measurement of a system with a different neural state. To ensure proper measurements, the applied perturbations either have to be sufficiently subtle that they go unnoticed by the subject or measurements must be taken before the human can implement a response.

For these movement experiments, the constraint causes a relatively small maximum deflection of approximately 3 cm from the unconstrained trajectory. In addition, care was taken to make the application of the constraint appear to be unpredictable to the subject. However, one cannot be sure that the subject completes the movement in total ignorance of a change in the system. Therefore, a series of experiments were performed to quantify the ability of the human subject to detect and react to the constraint.

The subject was placed in the configuration for trajectory 1 of Fig. 1A. After making a series of unconstrained movements, the constraint was applied for the entire duration of a movement. Instead of being asked to ignore changes to the system, the subject was instructed to stop the movement and reverse the direction of the force he was applying to the handle when the constraint was detected. Force was chosen as the state to be changed because it has the most relevance for this type of constrained motion (Mansfield 1992). A sample of the results from this test is shown in Fig. 2.

Consistently subjects were unable to make measurable voluntary changes within the first 500 ms of the movements. For these reasons, only data up to 500 ms into a movement were used to estimate the original intent of the motor program. This time is much longer than the 100 ms found by other researchers (Jeannerod 1991). However, instead of clear tactile cues, we are measuring the time required to react to a subtle perturbation. In fact, it is possible that the subject cannot discern the perturbation until the displacements grow to significant magnitudes. Moreover, an entire class of prior psychophysical experiments have been based on the notion that the human subject can suppress their voluntary reactions by times much greater (i.e., 800 ms, Asatryan and Feldman 1965; 500 ms, Mussa-Ivaldi et al. 1985; >750 ms, Bennett et al. 1992; 700 ms, Gomi et al. 1992; 8 s, Shadmehr et al. 1993). Latash (1993) found that voluntary response times during movements under large force perturbations to be within the 300- to 500-ms range. He also found that subjects under the "do not intervene" instruction were able to suppress a voluntary response for periods larger than 1 s.

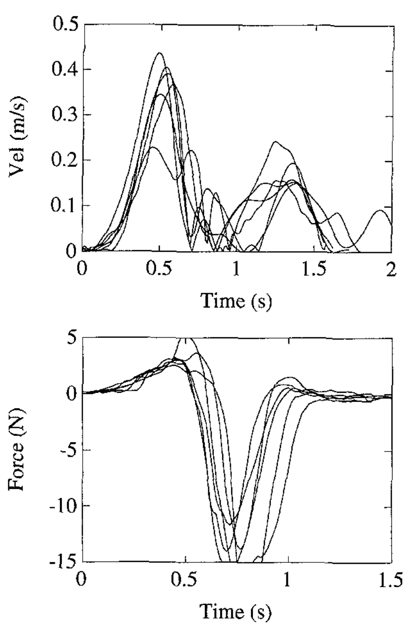

Fig. 2 Velocity profiles and force components in the x-direction (parallel to subject's torso) for the "do not intervene" tests for subject A. Voluntary response to perturbation occurs when the velocity decreases and the force direction changes

Results

For each of the three trajectories, there are three constraint conditions applied to both forward and reverse movements for a total of six cases per trajectory. The movement can be unconstrained throughout the motion, constrained throughout the movement, and constrained then released during the movement. Since the important qualitative features were consistent in both movement directions, movement data for only a single direction will be shown for each of the three movements. However, data from all of the movement directions will be included in the data analysis.

Unconstrained reaching movements

The point-to-point planar arm movements can be considered to be the most "conventional" motor control experiment of the three – conventional because there is a large body of research with which to compare these results. This provides a means of calibrating the experiment. Since this set of experiments is based on the assumption that the subject is using a standard and repeatable motor program, the unconstrained movements allow a comparison with experiments by other researchers in order to check that the subject is making a "normal" movement.

Figure 3 displays typical movement data for three of the subjects. Data for the other two subjects as well as for trajectories in the opposite direction are similar. To accentuate the consistency between movements, the velocity profiles of the different trials shown in these plots were shifted so that their peaks were aligned. This data not only agrees with previous research (Morasso 1981) but, in addition, the excellent internal consistency of these trials indicates that the subjects were capable of performing repeatable point-to-point movements, thereby verifying a major experimental assumption.

Constrained reaching movements

The constraint applied by the magnetic brake forced the nominally straight hand path along a circular constraint. Figure 4 shows typical data for this type of trajectory. The hand path is now circular in all cases, as expected, with smooth velocity profiles that have only been slight-

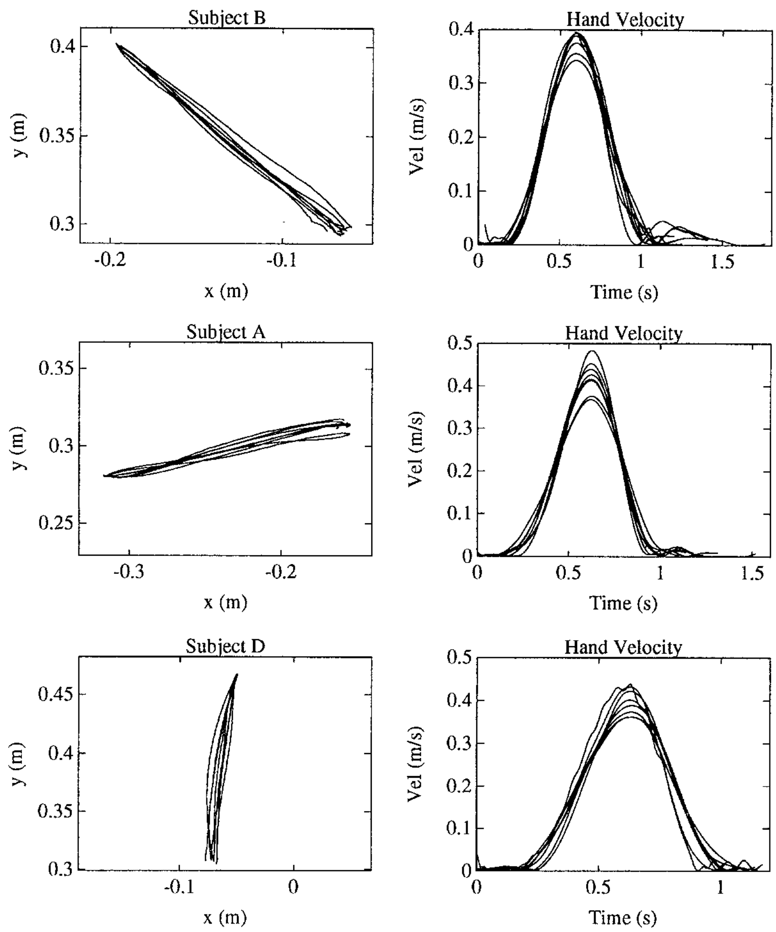

Fig. 3 Typical unconstrained movement data for the three sets of start and end points. The path of the hand is given on the plots to the *left*, while corresponding tangential hand velocity profiles are given to the *right*. Velocity profiles were shifted so that their velocity peaks would be aligned

ly distorted from the typical shape of the unperturbed movements. The vectors plotted along the trajectory paths are the measured forces applied by the hand onto the handle. The forces are plotted at 50-ms intervals.

Figure 5 demonstrates the internal consistency of the result by providing time history plots in which the magnitude and direction of the force vectors have been plotted as a function of time for a single subject performing a single trial. The force vectors have been plotted in a reference frame defined relative to the constraint circle. The horizontal direction represents not only the time axis but also the component of the force tangent to the circular path with a positive (rightward) component indicating a vector oriented with the velocity vector. The vertical direction represents the radial direction where upward strokes represent force directed radially inward onto the constraint.

In all of the data shown in Fig. 5, the circular constraint evokes forces from the arm to the handle that are strongly oriented inward toward the center of the circular constraint. In fact, the vast majority of the data follows this trend. An exact quantitative summary of this result as well as of the consistency of the data will be presented in the following sections.

Force magnitude

The applied constraint evoked forces from the human arm which increased as the difference between the constrained path and the unconstrained path increased. Figure 6 presents the mean maximum force magnitude measured during the first 500 ms of movement. As argued previously, the data of the first 500 ms of movement were assumed to be free of any intervention by the subject. The maximum force magnitude for each constrained trajectory was found and then averaged with the maximum magnitudes found in the other trials of the same trajectory and the same subject. The maximum force magnitude is similar across all three trajectories and all five subjects. The overall maximum force ranges from 3–5 N. These magnitudes indicate that a small displacement (<3 cm.) can cause large increases in force (>300–500%).

Force direction

The evoked forces have strong directional characteristics. To show this, the dot product between the measured force vector and the radial vector (the displacement vector between the present position and the center of the circular constraint) was calculated at each time step, normalized

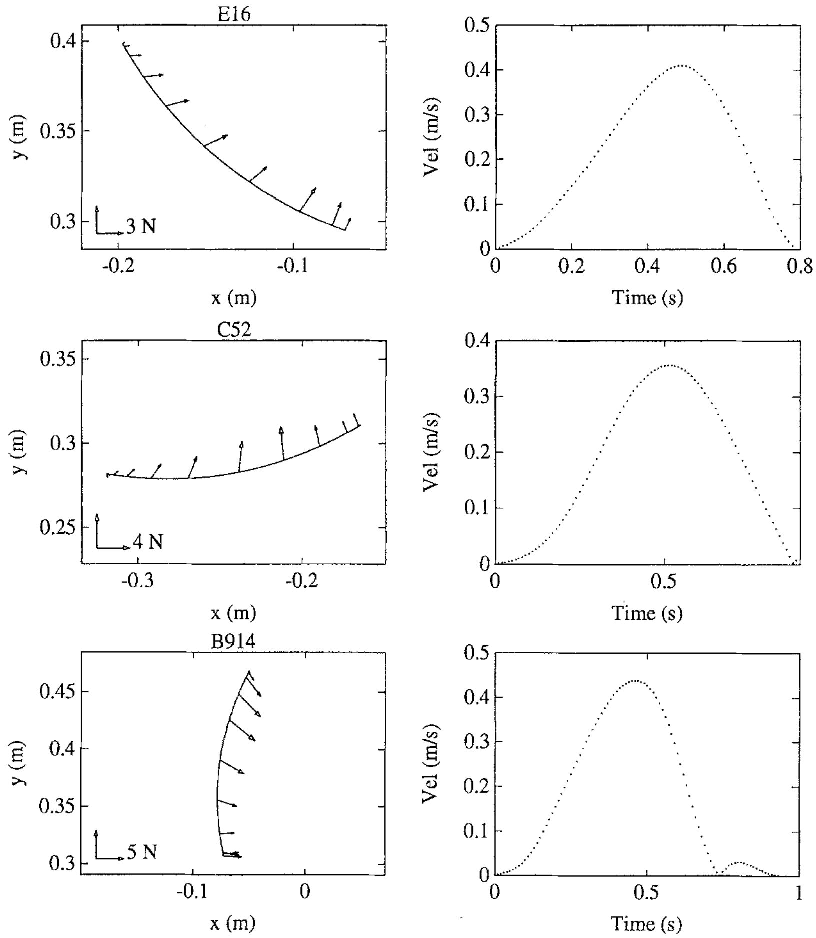

Fig. 4 Fully constrained trajectories for the three sets of start and end points. The path of the hand is given on the plots to the *left*, while corresponding tangential hand velocities profiles are on the *right*. Vectors drawn along the hand path represent the force that the subject applies to the handle of the robot. Plot titles contain letter designation of subjects

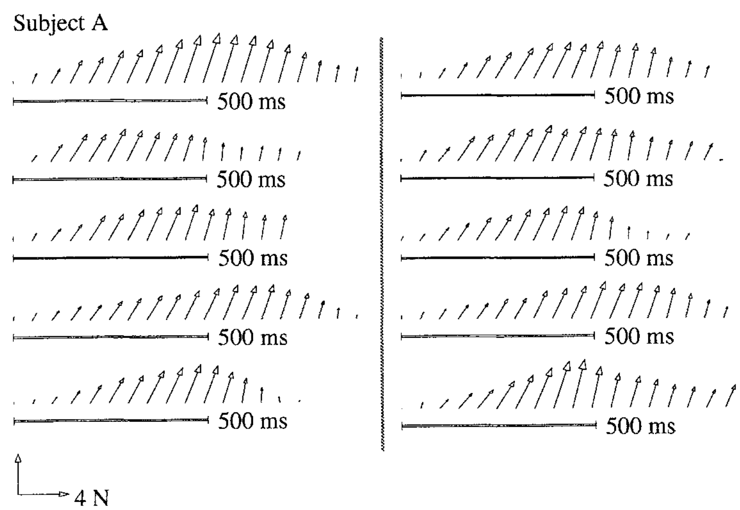

Fig. 5 Force vector histories for the trajectories of one subject performing the same trajectory. The upward component of the forces is the component along the radial direction of the constraint. *Upward* implies radially inward. The horizontal component is the component along the tangent of the constraint circle. *Right* implies pointing along the velocity vector. The 500-ms *bar* indicates the time span where the subject is unlikely to intervene. The vector legend at the *bottom* translates vector lengths to force magnitude

by the lengths of the two vectors, and then averaged over the first 500 ms of the movement. As this product is simply the cosine of the angle between the two vectors, a value near 1 implies that the forces point radially inward; zero implies forces point tangentially along the constrained path; values less than zero imply forces which point outward, away from the center of the constraint.

Figure 7 displays this quantitative measure of the direction of the forces averaged over the first 500 ms of movement. This figure shows a pronounced inward orientation for the direction of the force vectors. The total pooled projection mean is 0.822, which translates into an angle of 28° relative to the radial direction. Indeed, since all the measures are strongly positive, we see that the evoked forces measured during the trajectories consistently point inward.

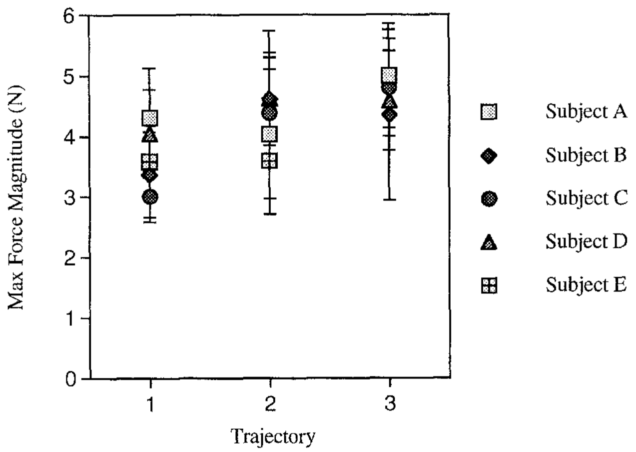

Fig. 6 The magnitudes of the forces evoked by the constraint for the five different subjects moving between the three sets of start and end points shown in Fig. 1. Standard deviations are also shown. Note that magnitudes are between 3 and 5 N

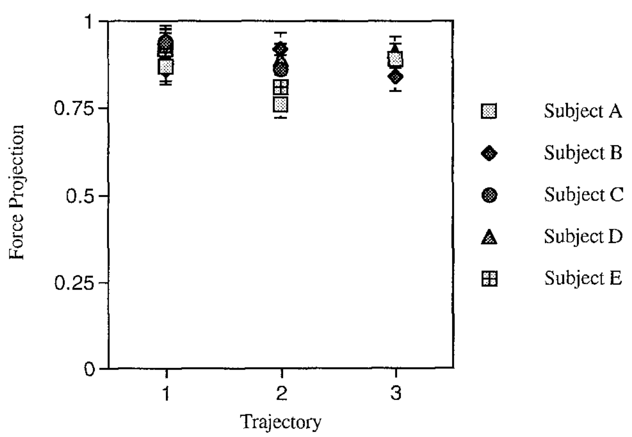

Fig. 7 The mean component of the evoked force along the radial direction for the five subjects and the three sets of start and end points. Standard deviations are also shown. Note that this measure is not only positive but significantly different from zero. This supports the finding that the forces have a consistent inward orientation during movement with the constraint

Constrained and released reaching movements

It remains to be shown that the forces evoked by the constraint deflection can significantly affect the motion of the system. Figure 8 presents representative data for the constrained and released trials. Almost all of the data show a marked inward return of the hand after release from the constraint. This overall trend can be seen in these plots, where all the trajectories recorded for each subject show some degree of inward return.

In order to quantify this tendency to inward return, we defined two measures, the positive and negative areas. The positive area represents the area residing between the hand path and the circle defined by the constraint passing through the start and end paths. The area between the two curves is counted as a positive area only if the hand path is inside the circle. Otherwise the area is a negative area. The units of these areas are unimportant, since these measures of positive and negative areas will be compared with one another and not used as an absolute measure.

Figure 9 summarizes the shape of the responses after release for all the data in terms of positive and negative area. The data presented in Fig. 9 show a very strong tendency for the hand path to move inward after release. The positive area for a given trajectory is at least 80 times larger than the corresponding negative area. The negative areas are in general so small that it is difficult to separate them from inaccuracies of the area-calculating algorithm. The data show that the forces evoked by the constraint are strong enough to cause the hand to make a significant return toward the unconstrained path.

Discussion

The data collected from the experiment showed that forces were evoked as a result of perturbing the natural trajectory along a circular constraint. Furthermore, these evoked forces had sufficient magnitude and the appropriate orientation to generate a definite inward return of the arm after release from the constraint. The question remains whether these forces truly result from the spring-like nature of the neuromuscular system.

Significance of the evoked forces

One might argue that the inward-pointing forces are simply the result of accelerating the hand away from the unperturbed trajectory. Yet this argument cannot account for the consistency of the inward direction of the forces throughout the entire 500-ms analysis period. Figure 10 displays a force-displacement plot which will help to make this clear. In this figure, the vector difference of measured forces between a typical constrained and unconstrained trajectory is plotted with its corresponding displacement. Note that when the hand reaches the point of furthest lateral displacement, the force vector is not only large but inward-pointing as well. (The lateral direction is defined as the direction perpendicular to the straight-line path between the start and end points. Therefore, the hand is at its largest lateral displacement in the middle of the movement at the top of the circular arc defined by the constraint. Since the slope of the circle is zero relative to the straight path at this point, the lateral velocity is therefore zero.) At this point, lateral velocity is zero and tangential velocity (indicated by the vector **V** in Fig. 10) has reached a maximum. In addition, since the hand moves through this maximum velocity, its tangential acceleration is small. Because the hand path is circular, a centripetal (inward-directed) acceleration occurs. The corresponding measured force would be an outward-pointing (centrifugal) force (shown as the vector **F** in Fig. 10). Yet the evoked forces are at their largest and consistently point inward throughout the re-

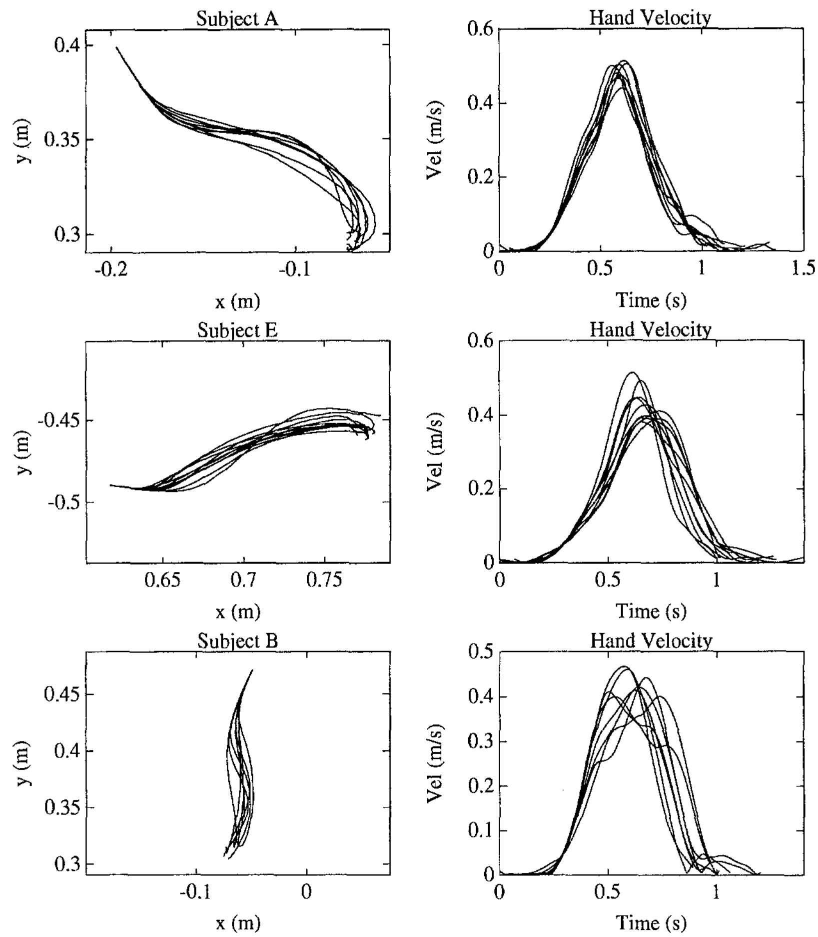

Fig. 8 Constrained and release trajectories for the three sets of start and end points. The path of the hand is given on the plots to the *left*, while corresponding tangential hand velocity profiles are on the *right*

gion. Therefore, under these assumptions, the inward-directed evoked forces can only be due to neuromuscular effects.

Because the human arm is a kinematically coupled mechanism, there are more than simple point mass forces involved; one might argue that the dynamic coupling effects could have sufficient influence to account for the measured inward-pointing forces. Again, this argument can be discounted. Instead of the isotropic mass tensor of the point mass, the full multijoint arm has a nonisotropic apparent endpoint mass. However, since this mass tensor is required by physical principles to be symmetric positive definite, the rotation it induces relative to the point mass force is bounded by 90° in either direction. In fact, since mass tensors of the human arm tend to be very nonsingular in the center of the workspace, the possible angles of rotation will actually be much less than 90°. Therefore forces due to acceleration can never cause inward-pointing forces along a circular path.

Discounting the acceleration-dependent forces, only the coupling effects due to velocity-dependent accelerations remain. Given that the arm was constrained to move along a plane, the forces due to these centripetal accelerations (velocity-squared terms) can be approximated by the following equation:

$$\bar{F}_{\mathrm{cent}} = J^{-T}(\theta)\, C(\theta) \begin{bmatrix} \dot{\theta}_S^2 \\ \dot{\theta}_E^2 \end{bmatrix} \qquad (2)$$

where $J(\theta)$ is the manipulator Jacobian and $C(\theta)$ is the centripetal acceleration matrix given by the following:

$$C(\theta) = m_{\mathrm{f}} l_{\mathrm{a}} c_{\mathrm{f}} \sin(\theta_S - \theta_E) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad (3)$$

where m_{f}, l_{a}, and c_{f} are the mass of the forearm, the length of the upper arm, and the distance from the elbow to the centroid of the forearm, respectively.

Using the submultiplicative property of operator norms[2], the magnitude of the centrifugal force can be bounded by the following equation:

$$\|\bar{F}_{\mathrm{cent}}\|_2 \leq \|J^{-T}(\theta)\|_{i2} \|C(\theta)\|_{i2} \|\dot{\theta}^2\|_2 \qquad (4)$$

Given Eq. 2, it is easy to see that this bound becomes:

$$\|\bar{F}_{\mathrm{cent}}\|_2 \leq |m_{\mathrm{f}} l_{\mathrm{a}} c_{\mathrm{f}}| \|J^{-T}(\theta)\|_{i2} |\sin(\theta_S - \theta_E)| \|\dot{\theta}^2\|_2 \qquad (5)$$

Taking the radially inward and outward trajectories (trajectory 3) as a simple example, we can now calculate an upper bound on the magnitude of the centrifugal force. Again, taking the point of largest lateral deflection, the velocity of the hand is solely in the y-direction and its magnitude is bounded by 0.5 m/s, which was the largest tangential velocity measured in the experiments. Trans-

[2] The induced 2-norm of a matrix A is simply the largest singular value of the matrix:

$$\|A\|_{i2} = \sigma_{\max}(A) = [\lambda_{\max}(A^T A)]^{1/2}$$

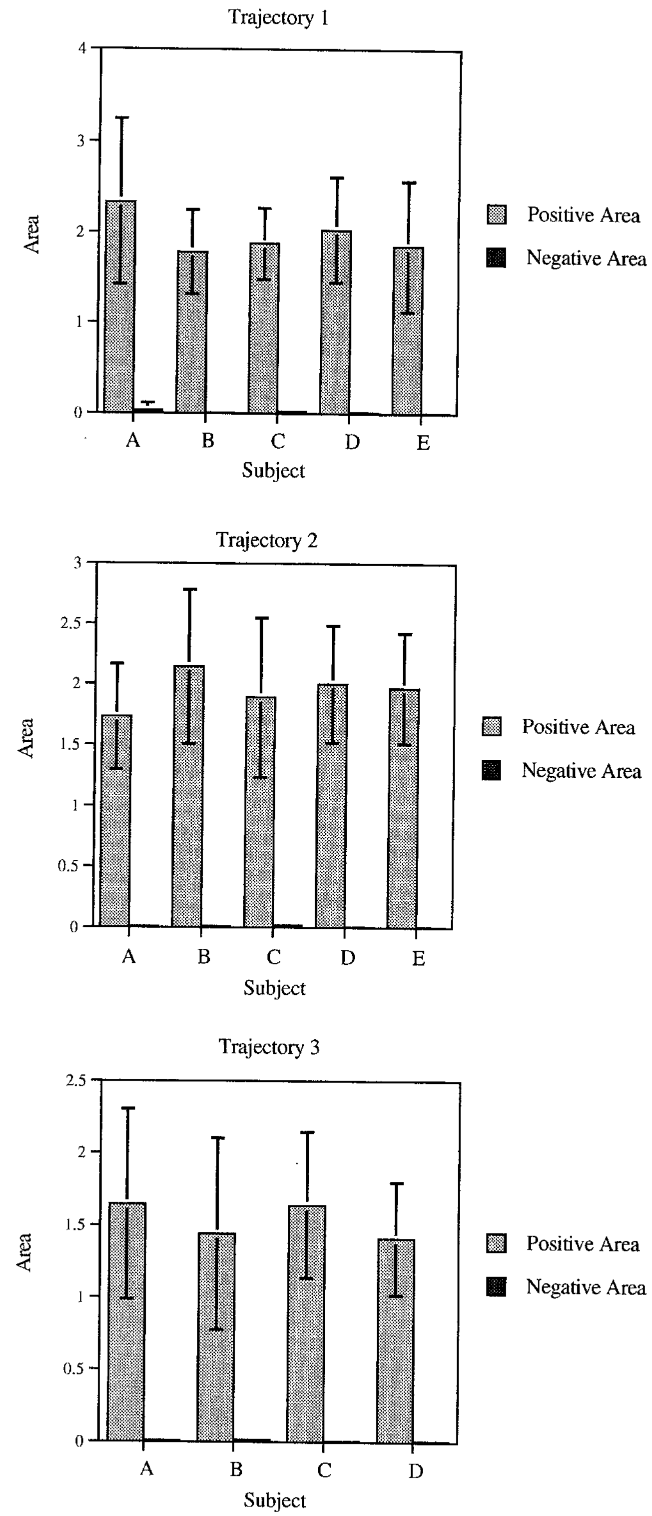

Fig. 9 The positive and negative areas for the three trajectories. Standard deviations are also shown. Note that the positive area measure is significantly larger than the negative area. This demonstrates the consistent tendency of the arm to move against the constraint after release

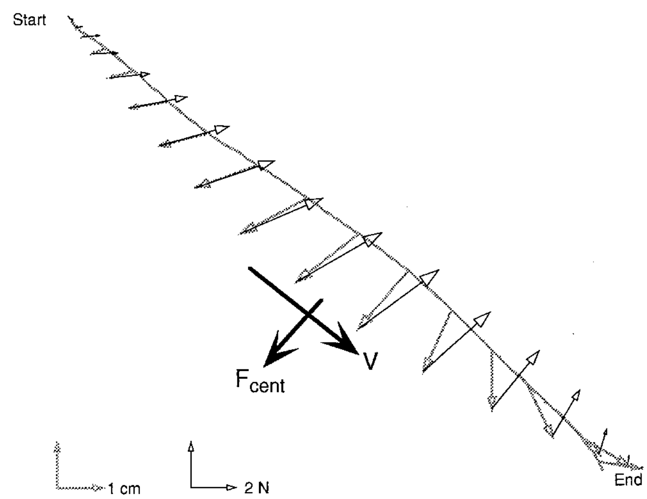

Fig. 10 A sample force-displacement profile comparing the unconstrained and fully constrained trajectories. The *gray trajectory* is the unconstrained trajectory, with the *gray vectors* representing the displacement of a similar point in time on the constrained trajectory. The *black vectors* represent the vector difference of the forces between the two trajectories

forming this endpoint velocity through the Jacobian, squaring, and taking the norm, we have the value of the largest possible joint velocity squared at that point. Assuming link lengths as well as mass parameters in order to maximize each induced norm, we find the following bound on the centrifugal force at this point along the trajectory:

$$\left\|\vec{F}_{cent}\right\|_2 \leq 0.47\,N \tag{6}$$

This bound shows that the 5-N inward force measured experimentally is approximately an order of magnitude too large to be explained by forces generated from velocity-dependent acceleration. Therefore the action of the arm inertias cannot account for the large inward forces we observed. Note that this bound is conservative in two respects. It bounds the total magnitude of the centrifugal force vector, but the radially directed component of the force may be much smaller than this bound because of the triangle inequality. Secondly, each of the norms of $J^{-T}(\theta)$, $\mathbf{C}(\theta)$, and $\dot{\theta}^2$ were maximized even though these maxima cannot be attained simultaneously. As a result, the number given in Eq. 6 represents a worst-case estimate of the force due to centripetal force and is insensitive to the assumptions made about the underlying mechanics such as the planar mechanism.

The last possible cause of nondisplacement-dependent forces is the velocity dependence of the muscles. Yet, when the hand is at its furthest lateral displacement, both the magnitude and the direction of the velocity are essentially identical for the perturbed and unperturbed cases. Therefore, velocity-related forces (i.e., viscous effects) are insufficient to account for the observed differences in the measured forces. Given that velocity- or acceleration-dependent effects are insufficient to account for the data, we are left with the conclusion that the evoked endpoint forces are primarily due to the apparent spring-like properties of the neuromuscular system.

Shape of the virtual trajectory

By nature of this experimental paradigm, an exact measurement of the location of the attractor point during movement could not be performed. However, enough information exists from the experimental results to place accurate bounds on the possible locations. As shown in Fig. 7, the arm when constrained consistently generated an inward force against the constraint. We have just rigorously shown that inertia and viscosity are incapable of generating these forces. Therefore, given that these inward forces must be a result of an attractor point acting on the system, then the only possible location for an attractor or virtual trajectory must be within the circle defined by the constraint.

Additionally, the available experimental evidence indicates that the general behavior of the forces evoked by displacing the arm is fairly regular in all directions (Mussa-Ivaldi et al. 1985; Shadmehr et al. 1993). Displacing the arm from posture in any direction evokes forces directed against the displacement. (Note that the presence of forces opposing displacement from posture does not necessarily imply a conservative force field; see Hogan 1985.) Therefore, if an experiment were performed where the subject was perturbed along an arc on the other side of the free trajectory, then a similar result, i.e., evoked forces pointing inward against the new constraint, would be expected. Given this regularity argument, we infer another circular bound where the attractor point may lie. The intersection of these two circles bounds the possible locations of the attractor point.

Although a specific measurement of a virtual trajectory was not performed, the inferred bounds on its possible locations define a very small area surrounding the unperturbed trajectory. Since the circular arc has a maximum lateral displacement of less than 3 cm. from the unperturbed trajectory, the maximum width of the bounded area defined by the two arcs is then less than 6 cm. By this argument, the attractor point never deviates more than 3 cm laterally away from the unperturbed trajectory.

This result is also supported by the unconstrained and constrained-and-released experiments. The unconstrained movements show very small forces which, when constrained, grow to be very large. The constrained-and-released experiments show a system which generate large forces upon forced deviation from a path. After release in response to these forces, the system moves to return to its unperturbed path and on occasion settles to that path before completion of the movement.

The results presented here bound the shape of the virtual trajectory. First of all, the virtual trajectory does not deviate very far from the actual trajectory performed, which supports the simulations of Flash rather that Kawato et al. In addition, the tight bounds given as well as the behavior of the system after release from the constraint add additional support to the idea that the virtual trajectory resembles the actual unperturbed trajectory performed.

Contact stability

Another interesting finding of this experiment was the ability of the human subjects to handle the unexpected imposition of a hard constraint onto the trajectory. As shown in the data, the constraint caused no trouble in terms of stability for the subjects; in fact, as a result of the interaction, relatively large forces were applied to the interface without any noticeable degradation in the performance of the movement. This is not a trivial observation. The task was designed to force the subject into an interaction task, while executing the motor plan for a free movement that a robot with similar kinematics and inertial parameters would more than likely exhibit when performing a similar task. In fact, Whitney (1977) considered the stabilization of contact as one of the central problems of robotic control.

If control of contact force can be controlled through the use of an attractor point, simply moving the attractor point within a object will cause the limb to exert a force on that object. This in itself does not guarantee contact stability for the human arm. However, it has been shown that an actively controlled manipulator (mechanical or biological) can maintain stable contact with an arbitrary passive object if and only if the mechanical impedance of the manipulator is equivalent to a passive object (i.e., if the amount of energy that can be drawn from it is not greater than the amount stored within it. See Colgate and Hogan 1988). In other words, the dynamic behavior *about* an attractor can in itself guarantee contact stability with any passive environment.

Mussa-Ivaldi et al. (1985) were able to quantify one component of this behavior in humans by measuring the static behavior of the arm around its set attractor point. A major feature of these static measurements was that the human arm near equilibrium exhibited conservative or spring-like behavior which satisfies the passivity criterion. Therefore in at least the static case, this result implies that contact force can be stably controlled through the attractor point. Yet this result is by no means trivial. Recent experiments by Lacquaniti et al. (1993) have shown significant curl in the static force field about equilibrium for the elbow-wrist system; this is not consistent with the passivity criterion and therefore cannot guarantee contact stability with all passive systems.

However, the experiments in this paper demonstrate that interaction may simply be an extension of movement. A free trajectory was performed, but due to an imposed constraint the actual trajectory and the desired trajectory did not coincide, which resulted in the generation of significant forces at the point of contact. Turning the problem around, to interact stably with a circular wall such that a radially directed inward force is always applied to the wall, all that is required is to execute a straight free movement from one end of the wall to the other. The forces will automatically result from the spring-like behavior. It has been stated that control of force applied to an environment cannot be derived from position information alone, due to the incompatibility between the torques needed to direct a force and the motion of the joints to generate the movement (Van Ingen Schenau et al. 1992). However, this experiment clearly shows that force can be controlled by position information alone, even though the motion of the joints are not compatible with the torques required. By executing a virtual trajectory which does not coincide with the actual trajectory, this incompatibility can and does occur. Indeed, this is the central mechanism for applying the force.

The virtual trajectory hypothesis

A common complaint about the virtual trajectory hypothesis is that "it is difficult to test adequately or, more specifically, to disprove" (Smith and Humphrey 1991). This difficulty results from the possibility of creating several different hypotheses which could be classified as "equilibrium point hypotheses". However, all versions of this hypothesis are challenged by the apparent observation of low stiffness (insufficient to influence movement during its production).

Several researchers besides Bennett et al. (1992) and Gomi et al. (1992) have suggested that stiffness is indeed low during movement. In particular, the work by Lackner and DiZio (1992) shows results which are difficult to explain within an equilibrium-point framework. They reported errors in movement trajectories and endpoints in response to Coriolis force perturbations applied by rotating the subject. Significant endpoint errors should not occur if an attractor point exists strong enough to control movement. Therefore it would seem that a virtual trajectory is not controlling these reaching movements; however, it is unclear how the rotating reference frame influences the perception of the subject and their ability to generate an accurate virtual trajectory.

Another criticism of the virtual trajectory hypothesis is that, since the theory is not tied to any biological substrate, it is easy to recreate any movement by tuning the parameters of the system. "Given an observed movement and certain assumptions about the elastic, stiffness, and viscous parameters of muscles and joints, one can seemingly always construct a sequence of equilibrium-point positions such that observed movement is 'predicted' by the hypothesis (Smith and Humphrey. 1991)". This is indeed true if any set of parameters can be used. However, most simulation studies have constrained themselves to use stiffness, damping, and inertial tensors as well as virtual trajectories of a particular shape, thereby shrinking the space of achievable solutions into a proper subset of all possible movements. Furthermore, all reaching-movement simulations to date have attempted to recreate free movements only. This paper has shown that there is also an entire realm of stable behavior when the movement has been constrained in some way to involve contact. Recent studies have shown that different control methods can display the same kinematic behavior, but examination of the dynamics at the point of contact reveals clear differences (Mansfield 1992; Won 1995). Since the pattern of the contact forces measured in this paper have such a clear signature, simulations of contact tasks would provide a more stringent test of the motor program used to generate free movements.

Conclusions

Significant position dependence is a fundamental stability property underlying all versions of the equilibrium idea. Therefore, in order to pursue the hypothesis, the foundational properties must be shown to hold. In our experiments, it was rigorously shown that the large forces evoked from the small displacements could only result from spring-like actuators. By then observing that these forces were sufficient to modify the dynamic behavior of the arm, the work presented in this paper provided clear evidence that stiffness is significant. This evidence was attained in the simplest and most direct manner without introducing linear mechanical models such as those used by Bennett et al. (1992) and Gomi et al. (1992) or confounding perturbations such as the noninertial reference frame used by Lackner and DiZio (1992).

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