## MAGNETOPAUSE STABILITY THRESHOLD FOR PATCHY RECONNECTION

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Abstract. This review is devoted to the problem of the internal fine structure of the Earth's magnetopause. A number of theoretical and experimental papers dealing with this subject is discussed from a unified viewpoint. The Vlasov kinetic approach is used to study the stability of magnetopause magnetic surfaces that can be destructed by the growth and overlapping of magnetic islands. The stochastic wandering of magnetic field lines between the destructed surfaces can result in magnetic percolation, i.e. the appearance of a topological connection of interplanetary and geomagnetic field lines. Such a process may be considered as a mechanism of the macroscopic (but spatially localized) reconnection. We discuss this in relation with the phenomena of spontaneous 'patchy' reconnection, recently observed at ISEE satellites and now known as flux transfer events.

Drift tearing mode, which is responsible for the growth of magnetic islands can be stabilized due to its coupling with ion sound waves, and the process of percolation will be interrupted if even a thin region with smooth stable magnetic surfaces exists within the magnetopause. Accordingly, we obtain a magnetopause stability threshold for localized reconnection. It is represented in the form of dependence of marginal dimensionless thickness of the magnetopause on the angle of magnetic field rotation within it.

Further, we discuss the possible role of lower hybrid turbulence permanently observed within the magnetopause and speeding up the process of reconnection. Nonlinear calculations supporting the developed model are given in the appendices. We consider briefly the motion of reconnecting flux tubes and evaluate the time necessary for the accomplishment of percolation. The calculations show that the appearance of reconnection 'patchies' at the dayside magnetopause cannot occur too far from the stagnation region. The latter agrees with experimental indications on the most probable site of the formation of flux transfer events. In the concluding part of the review we discuss the necessary limitations on the theory, possible lines of its future advance and comparison with the experimental data.

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#### 1. Introduction

The study of the magnetopause – the external boundary of the Earth's magnetosphere – remains a most challenging and intriguing problem that is equally important for general plasma physics and for an understanding of solar-terrestrial relations. Although an experimental study of the magnetospheric boundary started in the early 1960s, it suffered from the intrinsic insufficiency of single satellite measurements, related with the spatial mobility of the boundary. Significant progress has only recently been achieved due to the dual ISEE 1 and 2 satellite mission. Even more promising for the study of the structure and dynamics of the magnetopause should be the multi-satellite measurements that are now being planned for the next decade.

A theoretical analysis of the magnetopause processes also meets a number of difficulties. The conditions of the applicability of the MHD approach (often used for these problems) are not exactly clear. Some problems are also related to the choice of the initial equilibrium self-consistent plasma configurations properly describing the magnetopause (Kan, 1972; Harris, 1962; Lee and Kan, 1982). In our paper a most general kinetic description (a self-consistent system of the Vlasov-Maxwell equation) is used for the analysis of the processes within the magnetopause. To model the initial magnetopause structure we have chosen the 'generalized' (i.e., with the guiding magnetic field superimposed) Harris's equilibrium as was previously done by Galeev and Zeleny (1977). To our mind this model gives an adequate qualitative description of the principal properties of the magnetopause as a transitional layer where the magnetic field vector rotates by an angle that is always less than  $180^{\circ}$  from its magnetosheath direction  $\mathbf{B}_{\mathcal{M}}$ to the direction in the magnetosphere  $\mathbf{B}_{G}$  (Berchem and Russell, 1982b, Swift and Lee, 1983). Despite the different topologies of the magnetic fields on both sides of the magnetopause, a great number of convincing experimental facts that these fields are (at least sometimes) topologically connected have been accumulated up to the present time. It is quite natural that the magnetic reconnection process responsible for the establishment of such a topological connection had become the subject of numerous theoretical and experimental efforts, especially after the famous paper by Dungey (1961). Now reconnection is thought to be one of the main processes responsible for the energy supply from the solar wind to the magnetosphere and for the mass exchange between magnetosphere and magnetosheath. Although one can imagine other ways the solar wind plasma may break through the magnetopause (for example, impulsive penetration, as discussed by Lemaire, 1977, and Schindler, 1979) dayside magnetic reconnection from the viewpoint of global magnetospheric dynamics remains the paramount and determining process (see, for example, the review paper by Sonnerup, 1984). The next important step is related to the experimental search for the spatial-temporal evidence of the dayside reconnection. The main theoretical models developed since the end of the 1950s (Parker-Sweet-Petschek-Sonnerup models, thoroughly discussed and compared in review by Vasyliunas, 1975) treat the reconnection as a 'forced' quasi-stationary process. All 'forced' reconnection models, despite some internal differences between them, predict the existence of plasma flows, accelerated in the course of reconnection along the magnetopause (see, for example, Levy et al., 1964). Although ISEE 1 and 2 dayside measurements showed a number of cases that can be interpreted as experimental evidence of quasi-stationary magnetic reconnection (Sonnerup et al., 1981), it seems that such a situation cannot be considered as more or less typical for the magnetopause (Eastman and Frank, 1982). Moreover, the important question concerning the origin of the neutral line (sheet, point) where, in fact, tearing of the magnetic field line occurs, remains unclear in the 'forced' reconnection models. The situations in which the dayside reconnection occurs as a localized spontaneous process (so-called (*patchy* reconnection) happen to be much more general and typical. Plasma and magnetic field measurements at the ISEE 1 and 2 satellites, initially presented in the paper by Elphic and Russell (1979), have shown that the localized reconnection manifests in the so-called *flux transfer events* (FTEs). It is interesting to mention that some indication of the 'patchy' nature of the dayside reconnection had been discussed in the literature even earlier (Cambou and Galperin, 1974). Rather abundant experimental material concerning the properties of the FTEs, their structure and their relation to interplanetary conditions have been published during the last few years (Paschmann et al., 1982; Berchem and Russell, 1984; Daly et al., 1984; Rijnbeek et al., 1984; Saunders et al., 1984). The typical diameter of the reconnected FTE flux tubes is of the order of one Earth radius  $(1-2 R_F)$ ; that is comparable with the typical spatial scale of the magnetosheath magnetohydrodynamic turbulence. The localized spontaneous nature of the phenomenon already drives one to the nonstationary and spatially limited theoretical models of spontaneous reconnection or, in other words, to the study of the tearing-mode instability.

The tearing-mode theory was initially developed for the problems of plasma confinement by a magnetic field in a thermonuclear device (Laval et al., 1966). Such a dangerous phenomenon as the disruptive instability in tokamak discharge is now discussed in terms of the growth of a tearing mode (Kadomtsev, 1975). The important aspects of magnetotail dynamics have been related with the tearing-mode instability in the pioneering paper by Coppi et al. (1966). Later this approach has enabled us to explain the principal qualitative feature of the magnetospheric substorm – the spontaneous and fast release of stored magnetic energy (Schindler, 1974; Galeev and Zeleny, 1975). A little later, similar spontaneous processes resulting in the growth of magnetic islands began to be discussed for the case of a sheared magnetic field in relation with magnetopause reconnection (Galeev and Zeleny, 1977). The formation of chains of magnetic islands within the magnetopause for the southward direction of the interplanetary magnetic field has been demonstrated in laboratory simulation experiments by Podgorny and colleagues (1980). Later Greenly and Sonnerup (1981) sought experimental evidence for the growth of a tearing mode within the magnetopause on the basis of measurements made by the OGO-5 satellite. Recently, an interesting model of the multiple X-line reconnection at the Earth's magnetopause due to the growth of a tearing mode was proposed by Lee and Fu (1985).

Decisive arguments logically justifying the applicability of the spontaneous tearing models to the magnetopause have been given, however, by the experimental observations of the FTE events mentioned above. The main problem one meets in studying the reconnection processes is to understand the bursty, sporadic (sometimes explosion-like) character of these phenomena. It seems as if the system possesses metastable properties with respect to the reconnection process.

If one studies these phenomena with the help of plasma instability formalism it is clear that some linear or nonlinear thresholds should exist for the growth of tearing-mode perturbations. It is interesting that the importance of an investigation of the threshold conditions for the onset of magnetopause reconnection was appreciated in the review paper by Sonnerup (1984). Analysing theoretical and observational studies of FTEs, he pointed out 'that these thresholds must be associated with local conditions at the reconnection site rather than with global boundary conditions'.

The present paper is devoted to a thorough investigation of the stability of the magnetopause boundary layer (MBL). The generalized equilibrium Harris-type model used in the paper may, of course, be refined in future. Nevertheless, it contains the main shear property of the magnetopause – the rotation of a magnetic field vector within the MBL. Due to the plasma depletion from the central part of the flux tubes (Zwan and Wolf, 1976), the pressure balance on the dayside magnetopause is maintained by magnetic field pressures, so, in the first approximation, one can assume  $|\mathbf{B}_G| \approx |\mathbf{B}_M|$  and treat the magnetopause as a rotational discontinuity (Sonnerup and Ledley, 1974). The slab equilibrium model, which is under discussion here, have plane undestructed magnetic surfaces, so one can imagine the appearance of reconnection 'patchies' (or, in other words, the formation of reconnected flux tubes) as a result of the magnetic field line diffusion across the MBL in some spatially limited region of it. The motion of such a reconnected flux tube (FTE) is determined by its convection with the solar wind plasma flow in the magnetosheath to the high-latitude regions or to the flanks of the magnetosphere. Flux tubes can experience twisting in the course of this convection, which results in the generation of rather large field-aligned currents within them (Saunders et al., 1984). However, we shall focus our attention here only on the nontrivial subject of the initial formation of the reconnection 'patchy' on the magnetopause.

The diffusion of the magnetic field within the MBL is related to the stochastic wandering of the magnetic field line between the destructed magnetic surfaces (Galeev and Zeleny, 1978; Galeev, 1984). The destruction of magnetic surfaces occurs due to the overlap of magnetic islands growing on the neighbouring magnetic surfaces (Rosenbluth *et al.*, 1966). The attempt to visualize this process schematically is shown in Figure 1: originally smooth magnetic surfaces become 'shaggy' due to the growth of magnetic islands and the field lines experience random Brownian walks between them. Finally, the magnetic field line can diffuse in this manner from one side of the MBL to another (the magnetic field line diffusion coefficient,  $D_F$ , is evaluated in Appendix 3). In a macroscopical sense this indicates the accomplishment of the reconnection of interplanetary and geomagnetic field lines. This growth process of the magnetic islands and the subsequent overlapping and stochastic diffusion of magnetic field lines through the destructed magnetic surfaces can be imagined as a *magnetic field percolation* through





Fig. 1. Diffusion of magnetic field lines through the MBL. (a) All magnetic surfaces within the MBL are destructed. The stochastic wandering of magnetic field lines through the layer results in magnetic percolation. Bold solid curve – the magnetic field line penetrating from the magnetosphere to the magnetosheath. (b) A region with stable magnetic surfaces exists within the MBL. The stochastic wandering of magnetic field lines does not result in percolation if the width of this region is larger than  $\delta_{\varphi}$ . The topological connection of magnetic field lines on both sides of the MBL is absent.

the entire MBL. Taking into account the stochastic nature of the process, this terminology seems to be even more appropriate for the phenomena under discussion than the sometimes misleading use of such words as 'merging' or 'reconnection'. As will be shown in Appendix 2 (see also Biscamp, 1977; Swartz and Hazeltine, 1984; Galeev et al., 1985) the growth of magnetic islands saturates rather quickly at a finite island width  $W^*$ . So, if even a very narrow region (but wider than  $W^*$ ) with stable magnetic surfaces exists within the plasma layer it cannot be overlapped by nearby growing magnetic islands and thus it appears to be impenetrable for the diffusing field lines. One can come to the conclusion that the necessary condition for magnetic percolation to occur through the MBL will be the destruction of almost all magnetic surfaces within it. Figure 1(a) schematically shows the stochastic diffusion of magnetic field lines through the destructed MBL. The process of magnetic field stochastic wandering will be interrupted if a region (even a thin region) with the smooth, well-defined magnetic surfaces exists within the MBL. This case is shown in Figure 1(b) where there is no topological connection of magnetic field lines on both sides of MBL, and so reconnection (in a global or macroscopic sense) is absent (despite the possible observations of developed magnetic turbulence within the magnetopause). Thus, the conditions for magnetic percolation through the MBL (that, in our terms, means the formation of a reconnected flux tube with diameter  $\lambda$ ) should be defined as the conditions for the growth of tearing modes with wavelengths  $\leq \lambda$  at all magnetic surfaces within the layer. The possible appearance of undestructed domains within the MBL can be attributed to the effect of the tearing-mode stabilization in high-temperature plasma, recently discussed by a number of authors in collisionless and weakly collisional limits (Bussac et al., 1978; Coppi et al., 1979; Galeev and Zeleny, 1982; Drake et al., 1983). In many practically significant situations tearing perturbations can be made stable due to their coupling with the field-aligned propagation of the ion sound waves in a nonuniform plasma. This effect appears to be of principal importance in thermonuclear devices, but the results obtained in papers by Bussac et al. (1978) and Coppi et al. (1979) (where the assumption about the existence of a strong toroidal magnetic field has been made) are poorly applied to the magnetopause case where the 'azimuthal' magnetic field component (produced by field-aligned current) may be comparable to, or even larger than, the 'toroidal' field. Nevertheless, the approach developed below permits us to study this important aspect which we shall refer to as the 'integral' case.

The general magnetopause stability threshold is therefore obtained in our paper by a comparison of the stability thresholds for all magnetic surfaces within the MBL and by the selection of the destabilization condition for some domain within the MBL with magnetic surfaces that are the most stable against reconnection. Moreover, the finiteness of the time of the flux tube convection imposes an additional restriction on the parameters of the magnetopause - the magnetic islands must have enough time to grow to an overlapping dimension in order to destruct the magnetic surfaces. Briefly speaking, the time of percolation should be less than the characteristic time of the flux tube convection. The importance of similar conditions has been emphasized recently by Quest and Coroniti (1981a) and by Coroniti and Quest (1984). We shall also discuss limitations of this kind in the concluding part of the paper and show that although they are not too stringent in comparison with the stabilization thresholds, their study can give some clue to the most probable site of reconnection at the dayside magnetopause. In the last section we discuss the theoretical results obtained (the dependencies of the threshold MBL thickness on the angle of magnetic field rotation within the MBL) in relation with the experimental results concerning the magnetopause structure and FTE properties obtained recently by ISEE 1 and 2 satellites. Appendices 1 and 2 are devoted to the details of the mathematical calculations in the nonlinear regime and the 'integral' case. Appendix 3 is devoted to the magnetic field line diffusion across the MBL.

## 2. The Model of the Magnetopause Boundary Layer. The Set of Equations for Electromagnetic Perturbations

Let us consider the transitional layer with a characteristic thickness L(-L < X < L), where the magnetic field rotates by an arbitrary angle  $\theta_0$  from a value  $\mathbf{B}_M$  at one side to a value  $\mathbf{B}_G$  at the other side of the layer. Here  $\mathbf{B}_M$  and  $\mathbf{B}_G$  are asymptotic values of the magnetic field  $(|\mathbf{B}_{M}| = |\mathbf{B}_{G}|)$  at far edges of the layer. Such a layer can be modelled by the well-known Harris distribution generalized for the case where the plasma in this layer is magnetized by the current-aligned magnetic field

$$\mathbf{B} = B_0 \operatorname{th} (X/L) \mathbf{e}_z + B_y \mathbf{e}_y, \qquad n = n_0 \operatorname{ch}^{-2} (X/L),$$
$$\mathbf{B}_{MG} = B_y \mathbf{e}_y \pm B_0 \mathbf{e}_z. \tag{1}$$

To clarify this geometry we can address ourselves to Figure 10(d), which is given for convenience in Section 8. Below we shall use configuration (1) as a model for the magnetopause boundary layer (MBL). The self-consistent distribution functions  $f_{0j}$ , corresponding to configuration (1) are in fact simple Maxwellian functions shifted by the diamagnetic drift velocity  $u_j = 2cT_j/e_jB_0L$ , j = e, *i*. The parameter  $b_y = B_y/B_0$  may be considered as a measure of the magnetic field rotation through the layer  $\theta_0 = 2 \arctan b_y^{-1}$ . The parameter

$$\varepsilon_j = u_j / v_{Tj} = \frac{\rho}{L} \sqrt{(1 + b_y^2)} \frac{m_j T_j}{m_i T_i} = v_{Tj} m_j c / (eB_0 L)$$

(where  $v_{Tj}$  is the thermal velocity of the *j*-species,  $\rho = v_{Ti}m_ic/e\sqrt{B_0^2 + B_y^2}$  is the ion gyroradius in an asymptotic magnetic field) characterizes both the plasma anisotropy and the relative thickness of the layer.

We can consider the stability of plasma (configuration (1)) with respect to the excitation of low-frequency electromagnetic perturbations in terms of the perturbations of vector and scalar potentials

$$\begin{array}{l} \mathbf{A} = \mathbf{A}(X) \\ \varphi = \varphi(X) \end{array} \exp(-i\omega t + ik_z z + ik_y y).$$

The gauge condition in the low-frequency limit gives  $\mathbf{kA} = 0$ . The subscript '||' will be used for the projections of vector quantities on the local direction of the magnetic field. The magnetic field-aligned electric field consists of two parts: inductive and potential

$$E_{\parallel} = \frac{i\omega}{c} A_{\parallel} - ik_{\parallel} \varphi.$$
<sup>(2)</sup>

For any magnetic surface within the layer  $X = X_s$ , the wave vector  $\mathbf{k} = k_y \mathbf{e}_y + k_z \mathbf{e}_z$  can be selected such that

$$k_{\parallel}(X_s) = \mathbf{k}\mathbf{B}(X_s)/|\mathbf{B}| = 0.$$
(3)

Figure 10(d) illustrates the geometry appropriate for the excitation of the tearing mode within configuration (1). The inductive and potential parts of  $E_{\parallel}$  cannot compensate for each other in the  $\delta_{\varphi}$ -vicinity of the  $X = X_S$  surface, so the parallel electric field  $E_{\parallel}(x)$ has a finite value in this region. This results in a breaking of a frozen-in condition and in a strong nonadiabatic interaction of electromagnetic perturbations with particles. The plane  $X = X_S$  is usually called the singular surface for the k-mode. The region

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 $|x| = |X - X_S| < \delta_{\varphi}$  around  $X = X_S$  having a thickness  $\delta_{\varphi}$ , where  $E_{\parallel} \neq 0$ , will be called the *interaction region*. In order not to restrict ourselves to the small ion Larmor radius limit ( $\rho_i < \delta_{\varphi}$ ) we shall write the starting equations in a general operator form. Following the method discussed by Galeev (1963) for the drift modes, one can represent the *x*-dependence in a quasi-classical form  $\sim \exp(i \int k_x(x) dx)$  and introduce the differential operator

$$\hat{k}_x = -i\frac{\partial}{\partial x}.$$
(4)

The main idea of the following derivation of the equations describing the dynamics of electromagnetic perturbations is that one can operate with terms containing operator (4) as though (4) were an ordinary algebraical quantity, and return to its differential form only at the last moment. It is convenient to introduce the scalar quantity  $A_{\parallel}(x) = \mathbf{A}(x)$  $\mathbf{B}(X_S)/|\mathbf{B}(X_S)|$  and to write the linearized system of Maxwell–Vlasov equations for  $A_{\parallel}(x)$  and  $\varphi(x)$ 

$$\{-\hat{k}_x^2 - k^2 - V_o(x)\}A_{\parallel} = \frac{\omega}{k_{\parallel}c}\sum_j \frac{1}{d_j^2}\frac{\omega - \omega_j}{\omega}Z_{1j}\hat{f}_j\left(\varphi - \frac{\omega A_{\parallel}}{k_{\parallel}c}\right) = \hat{V}A_{\parallel}, \quad (5)$$

$$\{-\hat{k}_{x}^{2}-k^{2}+d_{e}^{-2}+d_{i}^{-2}\}\varphi=\sum_{j}\frac{1}{d_{j}^{2}}\frac{\omega-\omega_{j}}{k_{\parallel}v_{\tau j}}\hat{f}_{j}\left(\frac{v_{\tau j}}{c}A_{\parallel}Z_{1j}-\varphi Z_{0j}\right),\qquad(6)$$

where

$$\hat{k}^2 = k_y^2 + k_z^2, \qquad \hat{k}_x^2 = -\frac{\partial^2}{\partial x^2}, \qquad x = X - X_s,$$

$$v_{Tj}, \qquad \omega_{Bj} = |e_j|B/m_jc, \qquad \omega_{Pj} = \sqrt{4\pi n e^2/m_j}, \qquad \omega_j = -\frac{c\mathbf{k}[T_j \nabla n \mathbf{B}]}{e_j B^2 n(x)},$$

 $d_j = v_{Tj}/\sqrt{2} \omega_{Pj}$ ,  $\rho_j = v_{Tj}/\omega_{Bj}$  are, respectively, the thermal velocity; cyclotron, plasma and drift frequencies; Debye and Larmor radii of *j*-species of particles (electrons (*j* = e), ions (*j* = *i*)).

The term with  $V_0(x) = -2L^{-2}\cos^2\theta \operatorname{ch}^{-2}((X_s + x)/L)$  on the left-hand side of Equation (5) is responsible for the adiabatic interaction of plasma with electromagnetic perturbations,  $\theta$  is the angle between the wave vector **k** and the Z-axis, tg  $\theta = k_y/k_z = -\operatorname{th}(X_s/L)b_y^{-1}$  (see Equation (3) and Figure 10(d)).  $Z_{nj}$  is the plasma dispersion function

$$Z_{nj} = Z_n \left( \zeta_j = \frac{\omega}{k_{\parallel} v_{Tj}} \right) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{t^n \exp(-t^2) dt}{t - \zeta_j - i\varepsilon \operatorname{sign} \zeta_j}, \quad \varepsilon \to 0.$$
(7)

Operator  $\hat{\Gamma}_i$  takes the form

$$\hat{I}_{j} = \exp\left(-\frac{1}{2}\hat{k}_{x}^{2}\rho_{j}^{2}\right)I_{0}\left(\frac{1}{2}\hat{k}_{x}^{2}\rho_{j}^{2}\right),\tag{8}$$

where  $I_0$  is a modified Bessel function. This comes from averaging the electromagnetic field over the particle motion along the Larmor orbit. Using the well-known asymptotic expansion of the modified Bessel functions for  $|\hat{k}_x \rho_i| \ge 1$  and the Taylor expansion series for  $|\hat{k}_x \rho_i| \ll 1$  we obtain

$$\hat{T}_{i} \simeq \begin{cases} 1 - \frac{1}{2} \hat{k}_{x}^{2} \rho_{i}^{2}, & |\hat{k}_{x} \rho_{i}| \ll 1, \\ 1/\sqrt{\pi \hat{k}^{2} \rho_{i}^{2}}, & |\hat{k}_{x} \rho_{i}| \gg 1. \end{cases}$$
(9a)  
(9b)

Since the Larmor radius of electrons is always small one can onsider  $\hat{T}_e = 1$ .



$$\frac{\delta_{\varphi}}{|\hat{k}_{x}\rho_{i}| \ll 1} |\hat{k}_{x}\rho_{i}| \gg 1 \qquad |\hat{k}_{x}\rho_{i}| \ll 1$$

$$-\rho_{i} - \delta_{i} - \delta_{e} 0 \delta_{e} \qquad \delta_{i} \quad \rho_{i} \qquad x$$

Fig. 2. (a) The spatial structure of the interaction of tearing perturbations with plasma particles near the singular surface  $k_{\parallel}(X_S) = 0$ . I – electron resonant region; II – region of interaction of electromagnetic perturbation with ions  $(E_{\parallel} \neq 0)$ ; III – adiabatic region where  $E_{\parallel} \rightarrow 0$ . Thick arrows show the direction of convective loss of the wave energy carried away from the interaction region by ion sound waves. Relative scaling of characteristic spatial parameters  $\delta_e$ ,  $\delta_{\varphi}$ ,  $\rho_i$ ,  $\delta_i$ ,  $1/|\hat{k}_x|$ ; (b) in the differential case  $(\rho_i < \delta_i)$ ; (c) in the integral case  $(\rho_i > \delta_i)$ .

The right-hand side of Equation (5) can be reduced to the form  $\hat{V}A_{\parallel}$  after inserting  $\varphi(x)$  expressed with the help of Equation (6) through the  $A_{\parallel}(x)$ . The operator  $\hat{V}A_{\parallel}(x)$  corresponds to the nonadiabatic response of particles to the perturbations of the electromagnetic field. The most effective nonadiabatic interaction with the given particle species takes place within the electron (j = e) and ion (j = i) resonant regions

$$|x| \lesssim \delta_j,\tag{10}$$

where the Doppler shift  $k_{\parallel}(x)v_{\parallel}$  of the wave frequency  $\omega$  seen by the particles is small. Accordingly we have

$$\delta_{j} = \frac{|\omega|}{k_{\parallel}' v_{\tau j}}, \qquad k_{\parallel}' = \frac{\partial k_{\parallel}(x)}{\partial x} \bigg|_{x=0}.$$
(11)

The nonadiabatic term  $\hat{V}A_{\parallel}$  is strongly peaked in the vicinity of  $X_s: (\hat{V}A_{\parallel}) \ge (V_0A_{\parallel})$ , but vanishes far from the singular surface  $|x| \ge \delta_{\varphi}$ , where the parallel electric field  $E_{\parallel}$ becomes negligibly small. The spatial structure of the resonant wave-particle interaction near the singular surface  $k_{\parallel}(X_s) = 0$  is shown in Figure 2(a). For the condition of the magnetopause two scalings of the  $\delta_i$ ,  $\delta_{\varphi}$  parameters are appropriate:

(a)  $\delta_e < \rho_i < \delta_{\varphi} < \delta_i$ , which is shown in Figures 2(a) and (b) (we shall refer to this scaling as the 'differential' case);

(b)  $\delta_e < \delta_i < \delta_{\varphi} \simeq \rho_i$ , which is shown in Figure 2(c) (we shall refer to this scaling as the 'integral' case).

However, we shall assume everywhere that the mode under consideration remains in a kinetic regime  $\delta_{\varphi} > \delta_e$ . The transition to the hydrodynamic case ( $\delta_{\varphi} < \delta_e$ ) requires too large a values of the parameter:  $b_y > \sqrt{m_i/m_e}$  (see Galeev and Zeleny, 1978). In the adiabatic region the solution for  $A_{\parallel}(x)$  is then given by the left-hand side of Equation (5) and in fact determines the general favourability of the growth of perturbations. For brevity, below we shall omit the subscript '||' for  $A_{\parallel}(x)$ .

## 3. Dispersion Equation in the Vicinity of the Drift-Tearing Mode Stability Threshold ( $\omega \simeq \omega_e + i\gamma, \gamma \rightarrow 0$ )

The dispersion equation for the perturbations A(x) of an arbitrary structure, and relevant expressions for the characteristic growth rates  $\gamma$ , have recently been obtained in the paper by Kuznetsova and Zeleny (1985). It has been shown, however, that near the stabilization threshold, use of the so-called 'constant A' approximation is well justified and the form of the perturbation A(x) may be treated as quasi-symmetrical with respect to  $X_s$ . In this limit the procedure for obtaining the dispersion equation is well known and straightforward (Laval *et al.*, 1966; Galeev and Zeleny, 1978; Quest and Coroniti, 1981b). It is necessary to match the solutions of Equation (5) in adiabatic and interaction regions, or, in other words, to equate the energy dissipated due to the resonant wave-particle interaction with the free energy of the mode gained in the adiabatic region. The general dispersion relation in this case acquires the form

$$\int_{0}^{\infty} \hat{V} \,\mathrm{d}x = 1/\xi,\tag{12}$$

where  $\xi$  ( $\mu$ ,  $\theta_0$ ,  $X_S$ ) is the inverse value of the free energy of the mode with the dimensionless wave number  $\mu = kL$  growing in the vicinity of the singular surface located at a distance  $X_S$  from the symmetry plane of configuration (1). The parameter  $\xi$  is determined by the solution of Equation (5) in the adiabatic region and can be expressed through the associated Legendre functions:

$$\xi(X_S) = \frac{P_v^{-\mu} \left( th \frac{X_S}{L} \right) P_v^{-\mu} \left( -th \frac{X_S}{L} \right)}{[P_v^{-\mu}(0)]^2} \frac{1}{\Delta'},$$
(13)

where  $\Delta'$  is the standard parameter commonly used in the theory of the tearing-mode instability (Laval *et al.*, 1966). The value  $1/\xi(X_S)$  is, in fact, the generalization of  $\Delta'$  to the more general case of the oblique perturbations, propagating under large angle  $\theta$ between the wave vector and z-axis. The latter is appropriate for large  $X_S$  which should be also considered as we are looking through all the magnetic surfaces of configuration (1). The quantity  $\Delta'$  has been found for the slab sheared magnetic field (1) by Galeev and Zeleny (1977):

$$\Delta' = -(\mu + \nu) \frac{\Gamma\left(\frac{\mu + \nu}{2}\right)\Gamma\left(\frac{1 + \mu - \nu}{2}\right)}{\Gamma\left(\frac{\mu - \nu}{2}\right)\Gamma\left(\frac{1 + \mu + \nu}{2}\right)} = \frac{L}{P_{\nu}^{-\mu}(0)} \left[\frac{\mathrm{d}}{\mathrm{d}X}P_{\nu}^{-\mu}\left(\mathrm{th}\frac{X}{L}\right)\right]_{X = 0},$$
(14)

 $v = \frac{1}{2}(\sqrt{1 + 8\cos^2 \theta} - 1), \mu = kL, \Gamma(t)$  is the gamma function. The characteristic spatial scale of the quasi-symmetrical function A(x) is much greater than the width of the interaction region  $(L \ge \delta_{\varphi})$ . In this case one can regard A(x) as a constant value along the Larmor orbit and assume that the operator  $\hat{\Gamma}_i$  affects only the function  $\varphi(x)$  which varies in space much faster. At the stability threshold ( $\omega \simeq \omega_e, \gamma \to 0$ ), Equation (12) takes a form that does not depend on the specific expression for the operator  $\hat{\Gamma}_i$ 

$$\frac{1}{\xi} = -\left(1 + \frac{T_e}{T_i}\right) \frac{2\omega_{pi}^2}{c^2} \int_0^\infty \left(1 - \frac{k_{\parallel}c}{\omega} \frac{\hat{f}_i \varphi}{A}\right) \zeta_i^2 Z_{1i}(\zeta_i) \,\mathrm{d}x =$$

$$= -\left(1 + \frac{T_e}{T_i}\right) \frac{2\omega_{pi}^2}{c^2} \int_0^\infty \left(1 - \frac{k_{\parallel}c}{\omega} \frac{\varphi}{A}\right) \frac{\zeta_i^2 Z_{1i}(\zeta_i)}{1 - Z_{1i}(\zeta_i)} \,\mathrm{d}x.$$
(15)

## 4. General Criterion of the Stabilization of Tearing Perturbations Due to Their Coupling with Ion-Sound Oscillations

The field-aligned propagation of ion-sound oscillations carrying away the wave-energy from the interaction region lead to slowing down of the growth of the tearing-mode and, finally, to the total stabilization of instability. At the stability threshold the energy carried away from the interaction region by the ion-sound waves (the right-hand side of Equation (15)) is exactly equal to the free energy of the instability (left-hand side of Equation (15)). Expressions for  $\hat{\Gamma}_i \varphi$  could be found separately in each limit (9a) and (9b). The commonly used approximation (9a) – which is well justified for transitional layers in laboratory devices of the tokamak type where  $b_y \ge 1$  – has been investigated in a number of papers (see, for example, Bussac *et al.*, 1978; Coppi *et al.*, 1979). Expressions for  $\varphi(x)$ ,  $\hat{\Gamma}_i \varphi$ , and  $\delta_{\varphi}$  take in this limit the forms

$$\hat{T}_{i}\varphi \simeq \varphi = \frac{\omega A}{k_{\parallel}c} \left( \frac{ix^{2}}{2\delta_{\varphi}^{2}} \int_{0}^{\pi/2} \exp\left( -\frac{ix^{2}}{2\delta_{\varphi}^{2}} \cos t \right) \sqrt{\sin t} \, \mathrm{d}t \right), \tag{16}$$

$$\delta_{\varphi} = \sqrt{\delta_i \rho_i} \sim \frac{1}{|\hat{k}_x|}.$$
(17)

From Equations (17) and (9a) one can see that expression (16) for  $\varphi(x)$  is valid only if  $\delta_i > \rho_i$ . The opposite case,  $\delta_i < \rho_i$ , is important for the magnetopause case where the parameter  $b_y$  can have an arbitrary value. Ion in its movement along the Larmor orbit with large radius ( $\rho_i \ge \delta_i$ ) feels the average scalar potantial

$$\hat{T}_{i}\varphi(x) = \begin{cases} 0, & x < \rho_{i}, \\ \frac{\omega A}{k_{\parallel}c}, & x > \rho_{i}. \end{cases}$$
(18)

Note that

$$|k_x \rho_i| \ge 1$$
 in the region where  $x < \rho_i$ ,  
 $|k_x \rho_i| \ll 1$  in the region where  $x > \rho_i$ ,  
(18a)

(see Appendix 1 for a detailed study of this question). It is seen from Equation (18) that the characteristic effective width of the interaction region is

$$\delta_{\omega} \simeq \rho_i$$
. (19)

Relative scaling of characteristic spatial parameters  $\delta_e$ ,  $\delta_{\varphi}$ ,  $\rho_i$ ,  $\delta_i$ ,  $|1/k_x|$  are shown in Figure 2(b) and (c) in two limiting cases ( $\rho_i < \delta_i$ , the differential case;  $\rho_i > \delta_i$ , the integral case).

Substituting Equations (16)–(19) into Equation (15) and matching approximately the results in both limits at the point where  $\rho_i = \delta_i$  we shall obtain the general criterion of

the stability

$$\xi(X_{S}, \theta_{0}, \mu_{cr}) = \xi_{cr},$$
  
$$\xi_{cr} = \frac{\sqrt{2}\Gamma(1/4)}{\pi\Gamma(3/4)} \operatorname{ch}^{2}\left(\frac{X_{S}}{L}\right) \sqrt{\frac{2b_{y}T_{e}}{\operatorname{sh}(2X_{S}/L)T_{i}(1+b_{y}^{2})}} \frac{\rho}{L\beta}D, \qquad (20)$$

where

$$D = \max\left\{1, \left(\frac{2b_{y}T_{i}}{\sinh(2X_{S}/L)T_{e}(b_{y}^{2} + \hbar^{2}(X_{S}/L))}\right)^{3/2}\right\},$$
  
$$\beta = \frac{8\pi n_{0}(T_{e} + T_{i})}{B_{0}^{2} + B_{y}^{2}}.$$
 (21)

The first value of D = 1 is obtained in the differential approximation when  $\rho_i < \delta_i$ ; the second value of D refers to the integral approximation in the case  $\rho_i > \delta_i$ .

# 5. Marginal Magnetopause Thickness and Spatial Scale of the Reconnection Patchies. Dependence on the Angle of Magnetic Field Rotation $\theta_0$

We shall try in this section to represent a series of the quantitative results resulting from the theory developed above. Expression (20) establishes, in principle, the relation between the magnetopause parameters (thickness L, angle of magnetic field rotation  $\theta_0$ ) and characteristics of the dynamic processes occurring at the magnetopause (spatial scale of reconnection patchies  $\lambda$ ). For applications and discussion it is convenient to represent the results in two cases: (1) assuming that L has a fixed value; (2) assuming that  $\lambda$  has a fixed value.

Figure 3 shows the results of the numerical solution of Equation (20). The figure represents the dependence of the wavenumber  $\mu_{cr}$  on  $X_S$  for different values of  $\theta_0$ . The thickness of the magnetopause layer L is kept constant and is equal to  $10\rho/\beta$ ;  $\beta$  and  $\rho$  are assumed to be independent of  $\theta_0$  (the possibility of the  $\beta(\theta_0)$  dependence will be considered later). Instability development on the magnetic surface  $X_S$  is possible only if its wavelength is larger than  $2\pi L/\mu_{cr}$ . Perturbations with wavelengths smaller than the critical one  $2\pi L/\mu_{cr}$  are completely stabilized. For small angles  $\theta_0(\theta_0 \leq 60^\circ)$ , the value of  $\mu_{cr}(X_S)$  reaches its minimum at magnetic surfaces in the central parts of the layer near the symmetry plane, i.e., the central region of the layer appears to be the most stable. On the other hand, for large angles ( $\theta_0 > 60^\circ$ ),  $\mu_{cr}$  reaches its minimum at the periferical surfaces, so the central region of the layer becomes the most unstable. The solid parts of curves in Figure 3 refer to the differential case ( $\rho_i < \delta_i$ ) and the dashed parts of curves correspond to the integral case ( $\rho_i > \delta_i$ ). One can see that in a rather wide range of angles  $\theta_0$  the calculations of ( $\mu_{cr}$ )<sub>min</sub> should be performed in the integral approximation.

It is reasonable to assume that the excitation of perturbations is possible in the layer only if its characteristic length  $\lambda$  is larger than, or at least of the order of, the wavelength



Fig. 3. The dependence of the dimensionless wavenumber  $\mu_{cr}$  marginal for the stability on the position of magnetic surface  $X_S/L$  within the layer for different values of  $\theta_0$ .  $L = 10\rho/\beta$ ,  $T_i = 4T_e$ . Solid parts of curves refers to the differential case ( $\rho_i < \delta_i$ ), dashed parts to the integral case ( $\rho_i > \delta_i$ ).

of the perturbations. If the characteristic length of the layer  $\lambda$  is smaller than marginal  $\lambda_m = 2\pi L/(\mu_{cr})_{min}$  then a region with smooth well-defined magnetic surfaces exists within the layer. If only its thickness is larger than the very small quantity  $\delta_{\varphi}$ , the presence of this region within the layer breaks the topological connection of magnetic field on both sides of it. When  $\lambda > \lambda_m$  all magnetic surfaces within the magnetopause are destructed. Though the instability saturates at a relatively low nonlinear level (as will be shown in Appendix 2, the maximum halfwidth of the magnetic island  $W^* \sim \delta_{\varphi}$ ) the overlapping of magnetic islands results in the effective stochastic wandering of magnetic field lines through the layer and finally in macroscopic reconnection of both sides of the magnetopause as the percolation of the magnetic field through it. In the framework of the developed model  $\lambda$  may be treated as a characteristic diameter of the reconnected flux tubes in the FTE events (spatial scale of reconnection patchies).

Figure 4 shows the dependencies of the dimensionless marginal size of the reconnection patchies (characteristic length of the layer)  $\lambda_m/\rho = 2\pi L/\rho(\mu_{\rm cr})_{\rm min}$  on the angle of magnetic field rotation  $\theta_0$  for different values of the dimensionless magnetopause thickness. To make the comparison with experimental data more convenient all quantities in Figures 4 and 5 are multiplied by the numerical factor of the order of unity  $(4T_e/T_i)\beta$ . For thin magnetopause layers the curve  $\lambda_m(\theta_0)$  has a flat shape and in a rather wide range of angles weakly depends on  $\theta_0$ . In the case of 'thick' layers the stability curves have pronounced minima, indicating that the conditions for percolation are most favourable (it can occur for a rather wide range of  $\lambda$ ) when the angle  $\theta_0$  is of the order of 60°.



Fig. 4. The dependencies of the dimensionless marginal spatial scale of the reconnection patchies  $\overline{\lambda}_m = \lambda_m 4\beta T_e/\rho T_i$  on the magnetic field rotation angle  $\theta_0$  for different values of dimensionless magnetopause thickness  $\overline{L} = L(4\beta T_e/\rho T_i)$ . Percolation occurs above the given curves.

Let us now consider another representation of the solutions of Equation (20). In Figure 5 (solid curves) the thresholds of the macroscopic magnetopause stability against reconnection are given in the form of the dependencies of dimensionless marginal magnetopause thickness  $L/\rho$  on the angle of magnetic field rotation  $\theta_0$ . The spatial scales of the reconnection patchies  $\lambda$  are assumed to be fixed. 'Constant  $\lambda$ ' representation appears to be more adequate – one can assume that  $\lambda$  may be determined by the external conditions, particularly by the typical spatial scale of the MHD-turbulence in the magnetosheath plasma flow. Moreover, such a form of representation appears to be more convenient for comparison with the available experimental data concerning the dependencies of magnetopause thickness on the interplanetary and magnetosheath conditions (the experimental statistics of the FTE properties is not, unfortunately, too representative thus far). In the last section of the paper we shall discuss the experimental data concerning the magnetopause thickness (see Figure 6(a) and (b)).

For the given value of  $\overline{\lambda} = 4T_e\beta\lambda/T_i\rho$  (see Figure 5) the transitional layers with thicknesses less than the marginal will be subjected to the percolation of magnetic field



Fig. 5. The dependencies of the dimensionless magnetopause thickness  $\overline{L} = L(4\beta T_e/\rho T_i)$  on magnetic field rotation angle  $\theta_0$  for different values of dimensionless spatial scale of the reconnection patchies  $\overline{\lambda} = \lambda(4\beta T_e/\rho T_i)$  (solid curves). Dashed lines satisfy the requirement of sufficiently fast instability growth ~  $\gamma_{SC}$  and show the critical magnetopause thicknesses obtained from condition  $\gamma_{SC} \tau_{conv} \simeq 1$  for the different characteristic times of magnetic flux tube convection  $\tau_{conv}(T_i = 4T_e, \beta = 1, B = 70 \text{ nT})$ . Large  $\tau_{conv}$ are appropriate for the convection near the nose of the magnetosphere. The region below the shown curves are subjected to percolation.

lines. All curves reach their maximum at  $\theta_0 \simeq 60^\circ$ ; therefore, according to our model, the angle  $\theta_0 \simeq 60^\circ$  seems to be the most favourable angle for percolation (i.e., at these angles percolation occurs even through the respectively 'thick' layers). The numerical values of  $\overline{\lambda}$  chosen for calculations in Figure 5 correspond to the ISEE data on the typical spatial scale of FT events:  $\lambda \sim 1-2 R_E$  (Berchem and Russell, 1984; Saunders *et al.*, 1984).

## 6. The Influence of the Convective Draping of Magnetic Flux Tubes Along the Magnetopause on the Reconnection Dynamics

The process of magnetic percolation through the magnetopause takes some time,  $\tau$ , needed for the growth of magnetic islands, their overlapping and corresponding



Fig. 6. The magnetopause thicknesses L versus the angle of magnetic field rotation within the magnetopause according to the ISEE 1 and 2 experimental observations. (a) Data from the paper by Elphic and Russell (1979). (b) Results given later in the paper by Berchem and Russell (1982a) (see discussion in the text). Theoretical thresholds of magnetopause stability (see Figure 5) are given for comparison by dashed lines. (1)  $\rho = 90$  km,  $\lambda = 2.5 R_{\rm E}$ ,  $\overline{\lambda} = 200$ ; (2)  $\rho = 70$  km,  $\lambda = 2 R_{\rm E}$ ,  $\overline{\lambda} = 200$ ; (3)  $\rho = 70$  km,  $\lambda = 1.5 R_{\rm E}$ ,  $\overline{\lambda} = 150$ ; (4)  $\rho = 50$  km,  $\lambda = 1 R_{\rm E}$ ,  $\overline{\lambda} = 150$ .

magnetic field line diffusion. The successful accomplishment of this process, which results in the formation of the reconnected flux tube, can be achieved, however, only if the value of  $\tau$  is less than  $\tau_{conv}$ , the characteristic time of convection of this magnetic flux tube with the magnetosheath plasma flow. Similar conditions have been discussed recently for the magnetopause tearing mode by Quest and Coroniti (1981a, b). These authors have assumed that the perturbations should have grown appreciably at the convection distance of 6  $R_E$  that gives  $\tau_{conv} \simeq 200-400$  s. The characteristic time  $\tau$  was estimated by Quest and Coroniti (1981a) as the growth time of the collisonless tearing mode  $\gamma_t^{-1}$  in a sheared magnetic field configuration

$$\tau \sim \gamma_t^{-1}, \qquad \gamma_t = \frac{1}{\sqrt{\pi}} \frac{T_e + T_i}{T_i b_y} \sqrt{\frac{m_e T_e}{m_i T_i} \left(\frac{\omega_{Bi}}{\beta}\right) \left(\frac{\rho}{L}\right)^3}.$$
 (22)

From the condition  $\gamma_t \tau_{conv} \simeq 1$ , authors have found the critical value of the magnetopause thickness  $L_{cr}$  which, for the wide range of  $\tau_{conv} = 100-1500$  s, does not exceed  $1-2 \rho$ . In the framework of the theory discussed here it is quite reasonable to estimate  $\tau$  (which we treat as the time for establiching the topological connection between the magnetic field on both sides of the magnetopause) as the inverse linear growth rate of the corresponding tearing mode. As was shown by Galeev and Zeleny (1978), the diffusion of a magnetic field is a rather quick process by itself (the diffusion coefficient  $D_F$  is evaluated in Appendix 3) and the duration of the linear stage of the growth of magnetic islands gives the dominant contribution to the value of  $\tau$ . However, to our mind, the value of  $\tau$  is significantly underestimated by Quest and Coroniti (1981a), who have considered the idealized case of quiet collisionless plasma. The presence of different modes of microturbulence can appreciably alter the reconnection characteristic times. This subject was also discussed recently by Lee and Fu (1985).

The question about the influence of the fast time scale plasma turbulence on the reconnection process has recently been studied by a number of authors. This problem is especially important for the reconnection theories in solar flares. Dendy and Ter Haar (1984) have shown that under the typical conditions of a solar corona the Langmuir turbulence can significantly increase the collisionless tearing-mode growth rate. Magnetosonic turbulence can also modify and enhance the reconnection processes, and corresponding mechanisms which trigger solar flares have been proposed by Sakai and Washimi (1982) and Sakai *et al.* (1984).

It is obvious that the magnetopause plasma should also be subjected to the influence of similar effects.

A permanent feature of the magnetopause crossings observed by Prognoz-8 (Vaisberg et al., 1983), Imp-6 (Gary and Eastman, 1979), and ISEE 1 and 2 (Anderson et al., 1982) satellites is the presence of small-scale, low-frequency (1–100 Hz) oscillations of the electric field. A comparison of the experimental data with theory shows that the lower hybrid drift instability is the most probable source of these oscillations. The amplitude of the electric field measured by the Prognoz-8 satellite in the low-hybrid range of frequencies is of the order of  $(1-5) \times 10^{-4} \text{ V/m}^{-1}$  (Vaisberg et al., 1983). The theoretical estimates of the instability saturation level in the limit  $2\rho/L > T_e/T_i$  has been given independently from different nonlinear models by Sotnikov et al. (1981), and Gary and Eastman (1979) (see also Huba and Papadopoulos, 1978). In the limit of a thick magnetopause  $(2\rho/L < T_e/T_i)$  one should use a slightly different expression (Galeev, 1982).

$$\frac{(\delta E)^2}{8\pi n_0 T_i} \simeq \frac{m_e}{m_i} \left(\frac{\omega_{Be} \times 2\rho}{\omega_{pe}L}\right)^2 \frac{T_i}{T_e(1+\beta)} \min\left\{1, \frac{2\rho T_i}{LT_e}\right\}.$$
(23)

The numerical value of the electric field amplitude obtained from Equation (23) on the basis of Prognoz-8 plasma and magnetic field measurements  $(n_0 \simeq 16 \text{ cm}^{-3}, T_i \simeq 100 \text{ eV}, T_e \simeq 25 \text{ eV}, B \simeq 30 \text{ nT})$  may be evaluated as  $(\delta E)^2/8\pi n_0 T_i \simeq 8 \times 10^{-8}$ , which satisfactorily agrees with experimental data.

The interaction of electrons with low-hybrid turbulence results in an anomalous loss of electron momentum. This corresponds with the appearance of some anomalous collision frequency with a value evaluated by Huba *et al.* (1977) (see also Sotnikov *et al.*, 1981) as

$$v_{ef} \simeq \sqrt{10\pi} \frac{m_i}{m_e} \frac{(\delta E)^2}{8\pi n_0 T_i} \left(\frac{\omega_{pe}}{\omega_{Be}}\right)^2 \frac{\omega_{\rm LH}}{\sqrt{1+\beta}},\tag{24}$$

where  $\omega_{LH} = \sqrt{\omega_{Be} \omega_{Bi}}$  is the frequency of the lower hybrid resonance. The presence of the lower hybrid turbulence within the mangetopause may alter the conditions of the tearing-mode growth. When  $v_{ef} \simeq 50 \text{ Hz} \gg \gamma_i \simeq 0.03 \text{ Hz}$  the mode passes into the so-called semi-collisional regime described earlier by Drake and Lee (1977a). The value of the semi-collisional growth rate  $\gamma_{SC}$  increases correspondingly in comparison with (22)

$$\gamma_{\rm SC} = \gamma_t^{2/3} \, v_{ef}^{1/3} \,. \tag{25}$$

It is very important for the present study that all calculations of the tearing-mode stabilization due to its coupling with ion sound waves remains absolutely unchanged even in the presence of weak collisions. This was shown, for example, in the paper by Bussac *et al.* (1978), where the unified treatment of collisionless and semi-collisional tearing-mode stabilization has been given. The fact is rather obvious because the stabilization condition (20) does not contain any singular electron contributions and may therefore, be used, up to a very high degree of collisionality when the mode passes into the hydrodynamic regime ( $\delta_{\varphi} < \delta_e$ ). It seems, therefore, that it will be more adequate to estimate  $\tau$  through Equation (25) rather than (22). The critical magnetopause thicknesses obtained from this relaxed condition  $\gamma_{SC}\tau_{conv} \simeq 1$  are shown in Figure 5 by dashed lines for the different characteristic times of magnetic flux tube convection  $\tau_{conv}$ . The presence of the lower hybrid turbulence accelerates the tearing-mode growth and appreciably restrains the Quest-Coroniti limitations on the magnetopause thickness.

We should also refine the estimate of  $\tau_{conv}$  for our model of reconnection with limited spatial scale  $\lambda$ . In a very qualitative manner one can have

$$\tau_{\rm conv} \simeq \int\limits_{\lambda} \frac{{\rm d}s}{v_0(s)},$$

where s and  $v_0(s)$  correspond to the coordinate (counting from the subsolar point) and magnetosheath flow velocity along the magnetopause.

One can see that the flux tubes passing far from the stagnation region  $s = s_0 > \lambda$  with  $\tau_{\text{conv}} \simeq \lambda/v_0(s_0) \lesssim 100-200$  s are not very promising as possible sites of reconnection. So, our estimates indicate the special importance of the hydrodynamic pattern of flow in the vicinity of the stagnation point. This question has been discussed in a number of papers by Spreiter and his colleagues (see, for example, Luhmann *et al.*, 1984). Very recently the problems of the magnetosheath flow in the vicinity of the stagnation point (or line) and, in particular, influence of the interplanetary magnetic field and aberration

of the position of the stagnation point were discussed by Crooker *et al.* (1984) on the basis of the ISEE experimental material.

To make the rough estimate  $\tau_{conv}$  for the flux tubes passing near the nose of the magnetosphere, one can use the expansion of the flow velocity near the stagnation point at the magnetopause surface discussed by Zwan and Wolf (1976):

$$\tau_{\rm conv} \sim \frac{R_M}{v_0^*} \ln \frac{R_M}{\lambda}.$$
(26)

Here  $R_M$  is the distance from the magnetopause to the Earth in the equatorial plane,  $v_0^*$  is the characteristic velocity of the magnetosheath flow far from the stagnation point, and  $\lambda$  is the spatial scale of reconnection region at the magnetopause. Equation (26) describes the simple fact that the field lines convected along streamlines passing very close to the stagnation point spend a long time in the vicinity of the magnetospheric nose. The logarithmic factor in Equation (26) describing this effect may, in principle, be arbitrarily large for the infinitely small diameter of magnetic flux tube  $\lambda$ , but acquires the limited value of order 2–3 due to the finite value of  $\lambda$ . So one can estimate the characteristic value of  $\tau_{\rm conv}$  for flux tubes with  $\lambda \simeq 1-2 R_{\rm E}$  passing near the nose of the magnetosphere:  $\tau_{conv} \simeq 400-800$  s. Returning again to Figure 5, one can see that for typical values of  $\lambda \simeq 1-2 R_{\rm E}$ ,  $\rho \simeq 50-100 \, {\rm km}$  ( $\overline{\lambda} \simeq 100-200$ ) the limitations on the magnetopause thickness L, related to the convection of the magnetosheath plasma (dashed curves), appear to be appreciably less stringent than the destabilization thresholds (20) (solid curves). Although the interplanetary magnetic field can influence the quantities entering Equation (26), it is clear that this effect cannot be very strong and in a first approximation one can neglect the dependence of  $\tau_{conv}$  on  $\theta_0$ . Nevertheless, the future progress of the theory requires a careful description of the draping of the reconnected flux tubes and their extension to the magnetotail (in particular, a recently published paper by Sibeck and Siscoe, 1984, is devoted to this subject).

Anaway, despite all the serious simplifications made above one can see that the percolation process may have enough time to be entirely accomplished only for the flux tubes passing through the rather close vicinity of the stagnation point. So, in accordance with these estimates, the possible site of the FTE formation lies not far from the stagnation region. This agrees fairly well with the experimental indications discussed recently by a number of authors (Berchem and Russell, 1984; Daly *et al.*, 1984; Russell *et al.*, 1984) that the most probable source of the FTE is in the equatorial plane.

In the papers by Quest and Coroniti (1981a) and Coroniti and Quest (1984) another effect relevant to the real magnetopause conditions – i.e., finite time  $\tau_F$  of resonant electron interaction with the tearing mode – was also taken into account. The value of  $\tau_F$  can be estimated as  $\tau_F \simeq \lambda/v_{Te} \simeq 1-10$  s, where  $\lambda$  is the length of the reconnection region along the magnetic field, which is approximately equal to the spatial scale of the FTE. In the limit  $\tau_F \gamma_t < 1$  the loss of electrons can be considered, in principle, as an additional mechanism of dissipation because the outgoing electrons carry away the energy gained in the course of their acceleration by the inductive electric field of the growing tearing mode. However,  $\tau_F \gtrsim 1$  s appears to be much larger than  $v_{ef}^{-1} \simeq 0.02$  s, which is the time of free electron motion between two effective collisions. Thus it can be shown that the finiteness of  $\tau_F$  influences the linear growth of the tearing mode much less than the effective collisions., As for the nonlinear stage, both effects ( $v_{ef}$  and  $\tau_F$ ) lead to the slow algebraic growth of the magnetic islands (Drake and Lee, 1977b; Coroniti and Quest, 1984). In principle, this nonlinear growth, which can be evaluated with the help of semi-collisional results given by Drake and Lee (1977b)

$$\frac{W(t)}{L} \simeq \frac{5m_i}{8\pi m_e} \frac{\lambda \rho^2}{L^3} v_{ef} t , \qquad (27)$$

might lead to the smearing of the linear stabilization thresholds obtained above. However, the numerical estimate of W(t) for  $t = \tau_{conv}$  shows that for the real magnetopause conditions this growth remains too slow ( $W(t = \tau_{conv}) \ll L$ ) to influence significantly the destruction of the magnetopause magnetic surfaces. To a greater extent this conclusion holds for the weaker nonlinear growth related with  $\tau_F$  discussed by Coroniti and Quest (1984).

Thus our discussion of the dissipation mechanisms operating at the magnetopause shows that even the strongest of them, the scattering of electrons at the lower hybrid oscillations, appears to be sufficiently slow. On the one hand, it can provide effective percolation only for the flux tubes passing near the stagnation zone and, on the other, it cannot result in the significant destruction of linearly stable magnetic surfaces at a nonlinear regime.

## 7. The Limitations of the Theory in Cases of Small ( $\theta_0 \simeq 0$ ) and Large ( $\theta_0 \simeq 180^\circ$ ) Magnetopause Rotation Angles

The theory of magnetopause stability developed in previous sections has some natural restrictions related to the limited applicability of the Harris' equilibrium model (1) for small rotation angles  $\theta_0$  and breaking of the drift approximation (i.e., violation of the strong 'guiding field' limit) for the large ( $\theta_0 \rightarrow 180^\circ$ ) angles of magnetic field rotation. We shall discuss these difficulties and the ways to solve them in this section.

For small values of magnetic field shear (large values of  $b_y \ge 1$  and small values of  $\theta_0$ ) the pressure balance condition in the form derived for the generalized Harris equilibrium model

$$\beta_0 = \frac{8\pi n_0 (T_e + T_i)}{B_0^2} = 1$$
(28)

could easily be violated, because the plasma may now be confined not only by the 'azimuthal' field  $B_0(x)$ , but also by a weak gradient of the field  $B_y$ . The less the angle  $\theta_0$ , the more pronounced this effect may be. To avoid this difficulty we have assumed  $\beta(\theta_0) = \text{const.}$  in the calculations represented in Figures 3-5. Now, to make our point clearer, it is reasonable to discuss some possible modifications of the adopted

equilibrium model (1). In particular, we modify the pressure balance (28) to the form

$$(T_e + T_i)n_0 = \frac{B_0^2 + \alpha B_y^2}{8\pi}.$$
 (29)

The parameter  $\alpha$  determines here the respective role of a current-aligned magnetic field in the magnetopause plasma confinement, i.e. the deviation of real equilibrium from the Harris model.

It is not very difficult to generalize the theory discussed above to the case of finite but not large  $(0 < \alpha < 1)$  values of  $\alpha$ . In this case the main effect of  $\alpha$  is to restrict the value of  $\beta$  when  $\theta_0 \rightarrow 0$ . The influence of  $\alpha \neq 0$  on expression (13) for  $\xi$  can be neglected.

Figure 7 shows the dependence of a critical magnetopause thickness  $L/\rho$  on  $\theta_0$  for different given values of  $\alpha$ . The case  $\alpha = 0$  corresponds to the original Harris' equilibrium with  $\beta_0 = 1$ . The case  $\alpha = 1$  corresponds to the  $\beta(\theta_0) = \text{const.}$  assumption and was considered above (see Figures 3–5). For arbitrary  $\alpha \in (0, 1)$  the value  $L(\theta_0)/\rho$  lies within the shaded region bounded by the two limiting curves  $\alpha = 0$  and  $\alpha = 1$  in Figure 7. With the  $\theta_0$  increase, the shaded region shrinks and becomes very narrow for  $\theta_0 > 60-70^\circ$ . Although this consideration is qualitative, it indicates some uncertainty of the stability



Fig. 7. Modification of results shown in Figure 5 with the account of the possible  $\beta$  dependence on  $\theta_0$ :  $\beta(\theta_0) = \sin^2(\theta_0/2) + \alpha \cos^2(\theta_0/2)$ . Solid curves show the dependencies of critical magnetopause thicknesses  $L/\rho$  on  $\theta_0$  for the different given values of  $\alpha(T_i = 4T_e)$ . Shaded regions represent the limits of possible scatter of instability thresholds for small  $\theta_0$  angles due to the weak definition of the value of  $\alpha$ .

threshold (20) for small  $\theta_0$  angles. This uncertainty arising especially for small  $\alpha$ -values is, however, not too crucial for the theory of stabilization, because the requirements of a sufficiently large growth rate of instability  $\gamma_{\text{instab}} \tau_{\text{conv}} > 1$ , discussed in the previous section, can be more stringent in the range of small angles  $\theta_0 < 30-40^\circ$ . Nevertheless, it would be interesting to reconsider the problem using the self-consistent equilibria having  $B_y(x) \neq \text{const.}$  A few numerical examples of such plasma equilibria have been given by Kan (1972).

Other difficulties are encountered when one is dealing with transitional layers with very large rotation angles. As was shown in the paper by Galeev and Zeleny (1977), the thermal ions become unmagnetized when  $b_{\nu} < \sqrt{\varepsilon_i}$ . Drift approximation, therefore, is not already appropriate for such small  $b_{y}$ . So the approach used above breaks down for all central magnetic surfaces in the  $\sqrt{\varepsilon_i}$  vicinity of the singular surface  $X_S = 0$ . The macroscopic reconnection of magnetic fields in such layers may be discussed in terms of nonlinear tearing modes for the case of  $b_{\nu} = 0$ , which was recently considered in relation to the Earth's magnetotail reconnection (Galeev et al., 1978). The estimates show that the nonlinear stabilization of magnetic islands occurs for the amplitude of magnetic perturbation of the order of  $B_1^* \sim (kL)^2 B_0$ . This value strongly exceeds the corresponding limiting values for  $b_{y} \gg \sqrt{\varepsilon_{i}}$  (see Appendix 2). This case needs a special treatment which is beyond the scope of the present paper (see details in Galeev, 1982). The matching of the nonlinear results for the  $b_{\nu} \rightarrow 0$  case ('pure' neutral sheet) to the results obtained in the drift approximation for plasma layers with a magnetic field shear (given in the Appendix 2) is rather poor at present, so we shall limit ourselves here only to the range of  $b_{\nu}$  in which the drift approximation may be valid for both sort of particles. For typical magnetopause conditions this is the range of  $\theta_0 < 110-120^\circ$ .

To summarize our results prior to comparing them with the existing experimental data – which will be done in the next section – it is convenient to introduce three different ranges of angles  $\theta_0$  according to differences in the theoretical consideration:

(1)  $0 < \theta_0 < 30-40^\circ$ . In this interval of angles the assumptions made about the structure of the equilibrium solution can influence the magnitude of  $L/\rho$  (see Figure 7) and make the theoretical value of the threshold magnetopause thickness rather uncertain.

(2)  $30-40^{\circ} < \theta_0 < 110-120^{\circ}$ . The main results obtained in this paper, and shown in Figures 4 and 5, refer to this angular range. The threshold magnetopause thickness is determined by coupling the drift tearing mode to the ion sound waves and is given by the general equation (20) for the marginal stability of tearing modes.

(3)  $110-120^{\circ} < \theta_0 < 180^{\circ}$ . This is the 'neutral sheet' limit where the drift theory breaks down and gives very reduced values of critical magnetopause thickness. We shall postpone the study of this special case.

Historically, from the pioneering paper by Dungey (1961), the attention of researchers in reconnection studies has been concentrated primarily on two limiting cases: strictly parallel ( $\theta_0 \rightarrow 0$ ) and strictly antiparallel ( $\theta_0 \rightarrow 180^\circ$ ) magnetic fields on both sides of the magnetopause. Instead of this, the consideration in our paper appears to be focused on the intermediate case of the large y-component (in GSM coordinates) of an interplanetary magnetic field (not to be mixed with the  $B_y$  field introduced in the coordinates of Equation (1)). This seems to be more typical for real magnetopause conditions. Therefore, the results given above emphasize the strong influence of the y-component of an interplanetary magnetic field on the magnetopause reconnection dynamics (in addition to the well-studied influence of the z-component).

#### 8. Macroscopic Pictures of Spontaneous Magnetopause Reconnection

We have dealt above mainly with the internal microscopic dynamics of the magnetopause reconnection. Now it is reasonable to discuss, at least briefly, the possible global pictures of the unsteady magnetopause reconnection.

From the end of the 1970s the importance of the tearing-mode processes for the dynamics of the sheared magnetic field configuration of the dayside magnetopause have been recognized in a series of papers by Galeev and Zeleny (1977, 1978, 1982). Figure 8



Fig. 8. The growth of two tearing modes at different locations  $X_{s1}$  and  $X_{s2}$  within the sheared magnetopause configuration (1). Magnetic islands are shown at the moment before they overlap (Galeev and Zeleny, 1977).

shows the structure of the perturbation of magnetic surfaces within the magnetopause boundary layer (Galeev and Zeleny, 1977). For simplicity the figure shows only two growing modes at the moment before they overlap.

The topology of a magnetic field near the magnetospheric boundary has been investigated in detail by Podgorny and colleagues (1980) in their experiments on a laboratory simulation of Earth's magnetosphere carried out in the Moscow Space Research Institue. Figure 9 shows the structure of the magnetic field at the dayside magnetopause according to the paper by Dubinin *et al.* (1980). The large-scale magnetic



Fig. 9. The geometry of magnetic field at the dayside magnetopause observed in laboratory simulation experiments by Dubinin *et al.* (1980).

curl forming in front of the magnetopause in the simulation experiment, with the southward direction of the frozen magnetic field, is torn on the chain of smaller scale magnetic islands elongated in the north-south direction. The formation of this chain is related by the authors to the growth of the tearing-mode instability. A little later Greenly and Sonnerup (1981) found some experimental evidence that such magnetic island chains had been observed during the dayside magnetopause crossings by the OGO-5 satellite.

A new impulse to the study of these problems has been given by the experimental findings related to flux transfer events. The discovery of FTEs have emphasized the importance of the localized impulsive forms of reconnection and, starting with the original paper by Russell and Elphic (1979), the unsteady 'patchy' reconnection models have begun to be intensively explored (Quest and Coroniti, 1981a; Galeev and Zeleny, 1982; Cowley, 1982; Sonnerup, 1984; Lee and Fu, 1985).

A number of more or less successful attempts exist to imagine the complicated three-dimensional magnetic geometry of FTEs following the initial picture by Russell and Elphic (1979). We have also tried to visualize the process of FTE formation and their subsequent convection in Figures 10 and 11. Here and in our previous publications we develop the theory attributing the mechanism of FTE formation to the spontaneous localized magnetopause reconnection due to the growth of tearing-modes within the MBL.

Figure 10(b) shows the IMF flux tube with a diameter  $\lambda$  approaching the appropriate geomagnetic flux tube somewhere at the nose of the magnetosphere. All the complicated and dramatic processes of the magnetic field percolation resulting in the formation of reconnected flux tubes (Figure 10(c)) occur in a rather thin ( $L \ll \lambda$ ) magnetopause boundary layer (MBL). The fine dynamics of the percolation process – the growth of



Fig. 10. Stochastic percolation model of FTE formation due to the growth of multiple tearing-mode within the magnetopause. (a) The internal structure of the reconnection patchy with characteristic spatial scales  $\lambda \times \lambda \times L$  (the view from the Y-axis). Growth of tearing-modes with different directions of k destruct the magnetopause magnetic surfaces  $X_{sk}$ .  $W^*$  is the internal spatial scale of percolation (the typical step of the random Brownian walk of the field lines). Bold solid lines denote the geomagnetic and interplanetary field lines interconnected in the course of magnetic percolation. (b) Formation of a FTE tube with diameter  $\lambda$ . IMF flux tube  $\mathbf{B}_M$  passing near the nose of the magnetosphere hangs there, so the magnetic fields  $\mathbf{B}_G$  and  $\mathbf{B}_M$  can percolate through the magnetopause. The value of  $\lambda$  is determined by the competition between the convection and percolation. (c) The convection of a new FTE tube pair from the site of its formation with the magnetosheath plasma flow. Thick arrows indicate the approximate direction of convection P, P' – two reconnection patchies, i.e., the holes for the exchange by plasma particles between magnetosheath and magnetosphere. (d) The geometry of the magnetopause configuration (1). The projection on the YZ-plane. The frozen-in condition  $E_{\parallel} = 0$  breaks down at the magnetic surface  $X_{sk}$  due to the growth of the tearing mode with  $\mathbf{K} \perp \mathbf{B}(X_{sk})$ .

magnetic islands, their overlapping, the stochastic wandering of the magnetic field line through the destructed MBL magnetic surfaces – have been discussed in the present paper (Sections 2–7) and are illustrated in Figure 10(a), which shows the increased cross-section of the magnetopause near the site of FTE formation. So, we have considered above the elementary act of magnetic percolation through the MBL and the conditions when it is possible and when it is forbidden.

Figure 10 clarifies the relation between the FTE formation and the multiple tearing-mode growth\*. We have assumed that the characteristic size of the reconnection patchy (or the reconnection hole, as it is sometimes called) is scaled by the length of

<sup>\*</sup> Note that we use the word 'multiple' in a different sense than is implied in the model by Lee and Fu discussed below.

the single magnetic island or their rather short (2-3 islands) chain. Our estimates indicate that very long chains of islands, as shown in Figure 9, can hardly be excited at the magnetopause. Therefore, there are two bounds on the FTE diameter  $\lambda$ . One (internal), that is related to the conditions of the tearing-mode destabilization and growth, limits the value of  $\lambda$  from below  $\lambda > \lambda_{\min} \simeq 2\pi L \simeq 0.5 R_{\rm F}$ . The external (for the magnetopause) conditions of plasma motion within the magnetosheath impose the other (upper) boundary:  $\lambda < \lambda_{max} \simeq 2-3 R_E$ . Experiment and the present discussion suggest that a typical value of  $\lambda$  is of the order of 1–2  $R_{\rm E}$ . The fine structure of the percolation is determined, however, by  $W^*$  (the width of the nonlinear saturation of the growth of magnetic islands), which appears to be rather small (see Appendix 2). We see that the percolation process has three different spatial scales  $W^* \ll L \ll \lambda$ , so it proceeds in many steps and has a diffusive, nonregular character. Figure 10(d), which is the view from the X-axis, supplements the three-dimensional picture of the model of FTE formation. Figure 10(c) (a similar picture was given in the paper by Sonnerup, 1984) shows the formation of two separate patchies at the magnetopause when the threshold conditions within the MBL are satisfied and the reconnection of magnetic flux tubes is accomplished. In the opposite case the IMF flux tube will be draped away to the flanks of the magnetosphere by the solar wind plasma flow without coupling to the geomagnetic field lines.

The estimates given in Section 6 show that even in a very favourable situation the percolation process takes an appreciable time, of the order of few hundred seconds. So this process has a chance to be successfully completed only for flux tubes passing near the stagnation region at the nose of the magnetosphere, where the velocity of the plasma convection is relatively weak. This consideration agrees fairly well with the conclusion made recently by Daly *et al.* (1984), Berchem and Russell (1984) and Russell *et al.* (1984) on the basis of ISEE results that the most probable location of the FTE source lies in the equatorial region. If the external conditions are not too variable on the time-scale of the FTE formation (a few minutes), the percolation process would be repeated with the new flux tubes brought to the magnetopause by the magnetosheath flow after the previously reconnected fluw tube were carried away from the site of its formation. The estimates discussed above give a minimum repetition time-scale of the order of a few minutes. This also, in principle, agrees with the multiple FTE observations by ISEE satellites (4–5 events on one pass) with an average time interval between them of the order of 400 s (Rijnbeek *et al.*, 1984).

Our attempt to imagine the model of the FTE formation in general is shown in Figure 11. FTEs originate at the equatorial magnetosphere near the stagnation region. For simplicity only the north half of the FTE pair is shown. Despite the internal complexity of the process the final picture seems to be remarkably similar to the famous Dungey reconnection model (1961). The principal generalization is that the flux transfer occurs here not in a steady state, but in an impulsive regime by means of the finite magnetic flux 'quants',  $\Delta \phi \simeq 10^7$  Wb. The convection of the reconnected flux tubes in northern and southern hemispheres, and its characteristic features at the dayside, were discussed a few years ago in a review by Cowley (1982). Finally, the magnetosheath



Fig. 11. Schematic picture of FTE formation at the equatorial magnetopause near the stagnation region. For simplicity only the north half of the FTE pair is shown. The figure illustrates the convection of the newly reconnected flux tube and shows the downstream extension of FTE to the magnetotail. Multiple subsequent formation of FTEs is also possible at the nose of the magnetosphere.

plasma motion brings the FTE tubes to the nightside thus increasing the magnetotail magnetic content. This is also illustrated schematically in Figure 11.

A very important question concerning the downstream properties of FTEs have been discussed in a recent paper by Sibeck and Siscoe (1984). The authors have considered the tailward retreat of the magnetosheath part of FTEs and shown that FTEs should be observable both inside and outside the magnetosphere since the characteristic mixture of magnetosheath and magnetosphereic plasma exists within them. The principal conclusion of their study is that the well-known phenomena of multiple crossings of the magnetotail boundary observed by different spacecrafts can be understood as the downstream evidence of the dayside formation of reconnected magnetic flux tubes.

The estimate of the general efficiency of this localized sporadic reconnection made on the basis of ISEE measurements of the size of the FTEs (a few Earth radii) and of their occurrence frequency (a few minutes) shows that 'FTEs are the vehicle for substantial magnetic flux transfer from the dayside to magnetotail and one of the channels by which magnetosheath plasma enters the geomagnetic tail' (Sibeck and Siscoe, 1984).

Very recently, in interesting papers by Lee and Fu (1985) and Lee (1985), a mechanism of the FTE formation also based on the unsteady tearing reconnection was

discussed. Such models as we have seen can explain the impulsive intermittent character of FTE observations. In their simulation studies Fu and Lee (1985) employed the generalized Harris magnetopause model (see Equation (1)). The presence of a guiding  $B_y$ -field is of principal importance for the Lee and Fu model. It is interesting that we came to similar conclusions at the end of Section 7, where the significant role that  $B_y$ plays in the magnetopause reconnection have been discussed.

Nevertheless, the picture shown in Figure 12 (according to Lee and Fu, 1985), despite many similar features, is somewhat different from the picture developed above and shown in Figures 10 and 11. Authors call it 'the multiple X-line reconnection model' assume the formation of 'multiple' (at least two) flux tubes oriented parallel to the y-axis instead of a patchy structure in the y-direction (see Figure 12(a)). In other words, Lee and Fu suggest that the region of reconnection is very elongated in the y-direction,  $L_y \simeq 10 R_E$  (see Figure 12(a)), and that exactly two very large magnetic islands are created in the z-direction along the magnetopause (see Figure 12(b)). Lee and Fu do not treat reconnection as stochastic, but consider it as a purely coherent process and



Fig. 12. The model of FTE formation according to Lee and Fu (1985). The figure represents the schematic sketch of the reconnected field lines at the magnetopause, in which three X-lines are assumed to exist. (a) A three-dimensional perspective view. (b) The projections of flux tubes in the noon-midnight meridian plane. The figure illustrates the formation of two magnetic flux tubes containing twisted field lines. The sense of field twisting is consistent with ISEE observations by Saunders et al. (1984).

assume that the FTE tube can be treated as one very large magnetic island near the threshold of its nonlinear saturation.

In many aspects the idea of wrapping the magnetic surfaces into FTEs, as proposed by Lee and Fu, is very attractive: it enables us to explain the important FTE features, such as the sense of twisting the magnetic field. The model, of course, needs further theoretical treatment and a number of problems must be solved. One of them is the large stretching of the reconnection region in the y-direction because the magnetosheath plasma flow makes the conditions for reconnection unfavourable for large y. The other, more important, problem concerns the nonlinear dynamics. It now seems difficult to imagine that the magnetopause instability can reach a nonlinear stage with the width of the magnetic islands a whole order of magnitude larger than the thickness of the magnetopause itself.

The spatial geometry of the unsteady magnetopause reconnection is really a rather complicated problem and there is no doubt that the work on the development and elaboration of impulsive localized models should be continued.

It is easy to anticipate that the next strong impact to the progress in our knowledge of the magnetopause process will be given by the four-satellite 'Cluster' mission planned by ESA. However, as these measurements are aimed mainly at the study of the local properties and fine structure of the magnetopause, it is hard to overestimate the possible contribution of the low-altitude satellite and ground measurements towards the establishment of a global geometry of magnetopause reconnection. In recent years, some interesting papers have been published on this subject, discussing the search for evidence of reconnection according to the low-altitude satellite measurements (Reiff, 1984; Bythrow et al., 1985) and to study of geomagnetic pulsations under the polar cusps (Bolshakova and Troitskaya, 1982, Kleymenova et al., 1985). This subject, however, is beyond the scope of our paper. We shall mention only the important observations of the irregular long period pulsations (ipcl) made by Bolshakova and Troitskaya (1982). These are special types of pulsations observed at the ground end of the reconnected flux tube. It is very interesting that the period of these pulsations is of the order of a few minutes. This coincides with our estimate of the characteristic time of the FTE formation and experimentally observed repetition time of the FTEs. The twisted FTE magnetic configuration should manifest itself in a specific small-scale (100-200 km) ionospheric current system (Lee, 1985), so it seems reasonable to assume that the magnetic pulsations related to FTEs should have special polarization properties.

### 9. Discussion and Conclusions

The theory of FTE formation developed in Sections 1–7 enables us to find a relation between the characteristic size of the FTEs and the critical thickness of the magnetopause. It is obvious that to establish experimentally the existence of such a relation is very difficult nowadays. So, it is reasonable to assume that the magnetopause should, on average, be close to the threshold of its stability versus the spontaneous localized reconnection. The typical size of reconnection patchies can be obtained from an experiment:  $\lambda \simeq 1-2 R_{\rm E}$  (Saunders *et al.*, 1984; Rijnbeek *et al.*, 1984). After this we may consider the curves in Figure 5 as representing a characteristic dependence of the magnetopause thickness L on the angle of magnetic field rotation within the magnetopause  $\theta_0$ . In preliminary publications (Galeev and Zeleny, 1982; Galeev, 1983) we adopted this assumption. The first set of experimental data concerning the magnetopause thickness has been given by Elphic and Russell (1979) and is shown in Figure 6(a). The theoretical results are in rather satisfactory qualitative agreement with these data. Later the experimental statistics were enriched by new magnetopause crossings (Berchem and Russell, 1982a) and experimental points appear to be distributed rather chaotically (see Figure 6(b)). It seems that it will be more natural to represent these results in dimensionless form, i.e., to study the dependence of  $\beta L/\rho$  on  $\theta_0$ . We can also hope that the scatter of experimental points in such dimensionless variables will be reduced.

The study of magnetic measurements on ISEE 1 and 2 near the magnetopause and measurements on ISEE 3 in the solar wind shows that the southward polarity of the interplanetary magnetic field is the most favourable for the formation of FTEs (Berchem and Russell, 1984). This result seems to be difficult to incorporate into the framework of the proposed theory. According to our calculations (see Figure 5) the angle  $\theta_0$  that is optimal for percolation is of the order of 60-80°. In other words, the maximum thickness of layers subjected to percolation is achieved at these angles. The larger the angle  $\theta_0$  (the more southward the direction of the IMF), the more stringent are the limitations on the magnetopause thickness. There are two reasons, however, that partially relaxes these difficulties. First, the angle entering the calculations above is the angle between the magnetic fields just at the nose of the magnetopause. Interplanetary magnetic flux tubes experience appreciable bending and twisting in the course of their draping around the magnetosphere (Zwan and Wolf, 1976; Luhmann et al., 1984), so the direction of the magnetosheath magnetic field  $\mathbf{B}_{M}$  in the immediate vicinity of the magnetopause stagnation region may differ appreciably from the undisturbed IMF direction. It would be ideal, of course, to measure the magnetopause thickness and the angle  $\theta_0$  just at the magnetospheric nose at the moment of the independent registration of FTEs. The second circumstance is related to our implicit suggestion used in the discussion above that it is more difficult 'for nature' to create a thin magnetopause than a thick one. In principle, this may not be the case. The simple generalized Harris model used here enables us to obtain the self-consistent solutions for the transitional layers with arbitrary thicknesses. Although the applicability of this model for the magnetopause has not yet been seriously objected to, and its use seems rather reasonable, there are examples of other equilibrium models where the presence of an equilibrium electric field normal to the magnetopause is taken into account (Lee and Kan, 1982; Guan Jing et al., 1984; Wang and Sonnerup, 1984). The self-consistent thicknesses of such layers are not already arbitrary but are scaled by the Larmor radius of ions. The study of spontaneous reconnection in the electrostatically nonequipotential configurations represent very important but independent serious problems which should be attacked in the near future. Another line of advance in the theory of spontaneous reconnection seems to be related to the study of the influence of plasma flow effects and the presence of a shear in this flow on the dynamics of tearing modes. These subjects have been discussed, particularly by Bulanov and Sasorov (1978) and Lakhina and Schindler (1983). Although we did not pay to much attention to these effects here as the processes in the stagnation region vicinity were mainly discussed, the study of high-latitude reconnection sites, especially of polar cusps (see Haerendel *et al.*, 1978) makes necessary the exact nonlinear treatment of tearing-mode coupling with hydrodynamic instabilities.

To conclude our discussion we summarize briefly the logical chain of the reasonings develped above. To describe the topological connection of magnetic fields on both sides of the magnetopause, we have employed the most natural mechanism of reconnection which operates due to the free energy contained in the sheared magnetopause magnetic field configuration. The properties of this mechanism that may be discussed in terms of the growth of multiple tearing modes result in a sporadic and localized (patchy) character of the process of reconnection. So it was quite natural to relate our results to the ISEE 1 and 2 observations of FTEs (flux transfer events), i.e., localized magnetopause reconnection. The next problem to investigate was the microstructure of the reconnection process within the magnetopause. We have shown that it has a rather complicated, diffusive type of behaviour and can be described in terms of the percolation of a magnetic field through the magnetopause. We have shown that in order to accomplish the percolation process almost all the magnetic surfaces within the magnetopause layer must be unstable. Correspondingly we have studied the effects leading to the stabilization of tearing modes and preventing thereby the destruction of magnetic surfaces. As a result, the dimensionless value of the magnetopause thickness L, which is critical for FTE formation, was found as a function of the magnetopause rotation angle  $\theta_0$ , pressure ratio  $\beta$ , temperature ratio  $T_e/T_i$ , and FTE spatial scale  $\lambda$ . The results are compared with satellite data on the structure of the magnetopause and the dynamics of nonstationary reconnection at the dayside. The difficulties of such a comparison and some recommendations for an experimental test of the theory of FTE formation are discussed. A number of points where the proposed model may be improved are also discussed in the text. Flux transfer events - the direct evidence of the occurrence of spontaneous reconnection at the dayside magnetopause - remains to be the principal and challenging subject for future experimental and theoretical efforts.

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#### Appendix 1

## Evaluation of the Width of the Interaction Region in the Integral Case $(\rho_i > \delta_i)$

Attempts to determine the structure of the parallel electric field in the case when the characteristic scale of the change of electrostatic potential  $\varphi(x)$  is less than the ion Larmor radius ( $|\hat{k}_x \rho_i| > 1$ ) have been made by Crew *et al.* (1982) and by Farengo *et al.* (1983). This section is aimed at solving Equation (6) in the limit (9b). We shall obtain here the approximate expression (18) used above.

Substituting the asymptotic expansion for the operator  $\hat{\Gamma}_i$  (9b) into Equation (6) and assuming that  $(k^2 + k_x^2)d_i^2 \ll 1$  we come to the integral equation for  $\varphi(x)$ 

$$\frac{\varphi}{\sqrt{\pi}\hat{k}_{x}\rho_{i}} \equiv \frac{i}{\sqrt{\pi}\rho_{i}}\int_{0}^{x}\varphi(x)\,\mathrm{d}x = \frac{\left(\varphi - \frac{\omega}{k_{\parallel}c}Z_{1i}A\right)}{1 - Z_{1i}}.$$
(A1.1)

Let us introduce the new variable

$$\hat{\varphi} = \hat{\Gamma}_i \varphi = \frac{\varphi}{\sqrt{\pi \, \hat{k}_x \rho_i}} \,. \tag{A1.2}$$

Equation (A1.1) now takes the form

$$-i\sqrt{\pi}\rho_i\frac{\partial\hat{\varphi}}{\partial x}-\hat{\varphi}(1-Z_{1i})=\frac{\omega A}{k_{\parallel}c}Z_{1i}.$$
(A1.3)

Expanding  $Z_{1i}$  in two limits ( $x \ge \delta_i$ ) one can find an approximate solution of Equation (A1.3)

$$\hat{\varphi}(x) \simeq \frac{\omega A}{k_{\parallel}c} \frac{\delta_i}{\rho_i} \begin{cases} -\frac{i}{\sqrt{\pi}} \frac{x^3}{4\delta_i^3}, & x \leqslant \delta_i \\ \\ \frac{1}{(1+\delta_i/\rho_i)}, & x \gg \delta_i . \end{cases}$$
(A1.4)

From Equation (A1.4) it is easy to derive the expression for  $\varphi(x)$ 

$$\varphi(x) = -i\sqrt{\pi}\,\rho_i \frac{\partial\hat{\varphi}}{\partial x} \simeq \frac{\omega A}{k_{\parallel}c} \begin{cases} -\frac{1}{2}(x/\delta_i)^2, & x \ll \delta_i \\ i\sqrt{\pi}\,(\delta_i/x), & x \gg \delta_i, \end{cases}$$
(A1.5)

$$|\hat{k}_x \rho_i| = \left| \frac{\partial \varphi}{\partial x} \rho_i / \varphi \right| \sim \rho_i / x.$$
(A1.6)

As far as  $\hat{\varphi}(x) \leq \omega A/k_{\parallel}c$  (see Equation (A1.5)) inside the region  $|x| < \rho_i$  one can approximately assume:

$$\hat{T}_i \varphi \approx 0.$$
 (A1.7)

Outside this region condition (9b) is violated and expressions (A1.3)–(A1.5) become invalid. In order to find  $\hat{\Gamma}_i \varphi$  in the region  $|x| > \rho_i$ , Equation (6) should be solved in the limit (9a) when  $|x| > \rho_i > \delta_i$ . In this limit Equation (6) takes the form

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} = \frac{2ix}{\sqrt{\pi}\,\delta_i\rho_i^2} \left(\varphi - \frac{\omega A}{k_{\parallel}c}\right). \tag{A1.8}$$

It is easy to find the solution of Equation (A1.8) with the boundary condition  $\varphi(x) \rightarrow 0$  at infinity

$$\varphi(x) = \frac{\omega A}{k_{\parallel}c} \frac{\pi x}{x_1} Hi(-x/x_1), \qquad (A1.9)$$

where

$$x_1 = (\rho_i^2 \, \delta_i \sqrt{\pi/2i})^{1/3},$$

and

$$Hi(-y) = \frac{1}{\pi} \int_{0}^{\infty} \exp(-t^{3}/3 - yt) dt$$

is the Airy function, which may be represented asymptotically as  $x_1/\pi x$  when  $|x| \ge x_1$ . As far as  $x_1 \ll \rho_i$  (in the case  $\delta_i < \rho_i$ ) outside the region  $|x| < \rho_i$  one can put that

$$\hat{\Gamma}_i \varphi \simeq \varphi(x) \simeq \frac{\omega A}{k_{\parallel} c} \tag{A1.10}$$

Evaluations (A1.7) and (A1.10) prove the approximate validity of expression (18). Thus, in the case  $\rho_i > \delta_i$  we may estimate the width of the interaction region  $\delta_{\varphi}$  (the cutting distance of the parallel electric field  $E_{\parallel}(x)$ ) as the ion Larmor radius  $\rho_i$ .

#### Appendix 2

#### Nonlinear Stabilization of the Drift Tearing Mode

The aim of this appendix is to obtain the halfwidth  $W^*$  of the saturated magnetic island in the nonlinear regime. The nonlinear evolution of a single tearing mode in configurations of type (1) has been widely discussed during recent years (Biscamp, 1977; Drake and Lee, 1977b; Galeev and Zeleny, 1977; Coroniti and Quest, 1984; Swartz and Hazeltine, 1984; Zeleny and Taktakishvili, 1984; Galeev *et al.*, 1985).

The main effect that determines the growth of a magnetic island is the modification of resonant particle orbits by the growing island. The most accurate calculation of the influence of the perturbation of electron trajectories has been performed recently in the paper by Swartz and Hazeltine (1984). The result obtained in this paper is very similar to those given previously by Biscamp (1977) and Drake and Lee (1977b): if one takes only electrons into account, stabilization occurs when  $W = W^* \sim \delta_e$ . Consideration of the ion influence by Galeev and Zeleny (1977), and in a more detailed manner by Zeleny and Taktakishvili (1984), shows that even in the case  $W > \delta_e$  (when resonant electrons are trapped) the development of a drift tearing mode ( $\omega \simeq \omega_e$ ) is nevertheless possible. As will be shown below (see also Galeev *et al.*, 1985), the nonlinear growth and stabilization of this drift mode are determined by the structure of a parallel electric field and by its influence on ions.

As shown by Zeleny and Taktakishvili (for brevity we shall refer to this paper as Paper I) the integral in the dispersion equation (12) can be written as the sum of the contributions from three kinds of particles:

$$\int_{0}^{\infty} \hat{\mathcal{V}} dx = \sum_{j} \int_{0}^{\infty} \hat{\mathcal{V}}_{j} dx = \sum_{j} U_{j}^{\mathrm{TR}} + \sum_{j} U_{j}^{\mathrm{S}} + \sum_{j} U_{j}^{\mathrm{UT}}.$$
 (A2.1)

 $U^{\text{TR}}$  is the contribution of particles moving along magnetic field lines closed within magnetic islands (trapped particles);  $U^{\text{UT}}$  is the contribution of particles moving outside the island (untrapped particles);  $U^{\text{S}}$  is the contribution of particles moving along field lines in the vicinity of the separatrix that is the island boundary.

When  $\delta_{\varphi} \ge \delta_e$  the reduction of the inductive part of the parallel electric field  $(i\omega/c)A_{\parallel}$  by its electrostatic part  $-ik_{\parallel}(x)\varphi(x)$  weakly influences electrons, trapped ions and ions in the vicinity of the separatrix. Thus, we can employ in Equation (A2.1) the expression for  $U_i^{\text{TR}}$ ,  $U_i^{\text{S}}$  (j = e, i) and  $U_e^{\text{UT}}$  found in Paper I.

Let us evaluate the contribution of untrapped ions  $U_i^{\text{UT}}$ , bearing in mind the real structure of the parallel electric field  $E_{\parallel}(x)$ . Let us consider first the simpler differential case when  $\rho_i < \delta_i$ . It has been shown (see Paper I) that when  $W < \delta_i$  the untrapped ion orbits are almost undistorted by the presence of magnetic islands and can be calculated in a linear approximation. The ion term dominates the equation for electrostatic potential  $\varphi(x)$  in the case of the drift tearing mode. Thus, if the halfwidth of the magnetic island W is smaller than  $\delta_i$ , expression (16) for  $\varphi(x)$  may be used in the region outside the island ( $|x| \ge W$ ). Integrating along unperturbed ion orbits we shall find the general contribution of untrapped ions to Equation (A2.1)

$$U^{\text{UT}} = L \int_{W}^{\infty} V_i \, \mathrm{d}x = \frac{\omega - \omega_i}{\omega} \frac{\delta_{\varphi}}{L \varepsilon_i^2} (F_1 - iF_2), \qquad (A2.2)$$

$$F_{1} = \frac{\pi\Gamma(3/4)}{\sqrt{2}\Gamma(1/4)} - \overline{W} - \frac{\sqrt{\pi}}{4}\Gamma(3/4)2^{3/4} \int_{0}^{\overline{W}^{2}/2} z^{1/4} \mathbf{H}_{1/4}(z) \, \mathrm{d}z, \qquad (A2.3)$$
$$F_{2} = \frac{\pi\Gamma(3/4)}{\sqrt{2}\Gamma(1/4)} \left(1 - \frac{\pi}{4}\overline{W}^{2} \left(J_{1/4}\left(\frac{\overline{W}^{2}}{2}\right)\mathbf{H}_{-3/4}\left(\frac{\overline{W}^{2}}{2}\right) - \right)\right)$$

$$-J_{-3/4}\left(\frac{\overline{W}^2}{2}\right)\mathbf{H}_{1/4}\left(\frac{\overline{W}^2}{2}\right)\right),\tag{A2.4}$$

where  $\overline{W} = W/\delta_{\varphi}$ ;  $J_{1/4}$  and  $J_{-3/4}$  are Bessel functions; and  $\mathbf{H}_{1/4}$  and  $\mathbf{H}_{-3/4}$  are Struve functions. Solving the dispersion equation (A2.1) we can find the growth rate of the nonlinear  $(W > \delta_e)$  drift tearing mode.

$$\gamma_{\rm NL} \simeq \omega_e \frac{10 \,\varepsilon_e^2}{\varepsilon_i^2} \, \frac{\delta_{\varphi}}{W} F_2(W), \quad \delta_i > \rho_i \,.$$
(A2.5)

The nonlinear growth rate (A2.5) is independent of the initial margin of free energy  $\Delta'$ , as if the instability 'forgets' about its linear history. One can see from expression (A2.5) that the nonlinear growth of the mode is determined by the influence of a parallel electric field on the untrapped ions, which occurs within the interaction region  $|x| < \delta_{\varphi}$ . When the magnetic island halfwidth W becomes comparable with the width of the interaction region  $\delta_{\varphi}$  there is no room in the interaction region for such untrapped ions and the nonlinear growth of instability necessarily ceases. The exact numerical solution confirms this assumption

$$\gamma_{NL}|_{W=W^*} \sim F_2(W=W^*) = 0 \quad \text{when } W^* = 1.5 \ \delta_{\varphi} \ .$$
 (A2.6)

In the integral case  $(\rho_i > \delta_i)$  the width of the interaction region  $\delta_{\varphi} \simeq \rho_i$  is greater than  $\delta_i$  and, as has been shown in Paper I, the halfwidth W of the magnetic island can exceed  $\delta_i$  at a nonlinear stage. In this case ions become essentially nonlinear and it is necessary to take into account the nonlinear distortions of the orbits of untrapped ions. The contribution of untrapped ions into integral (A2.1) when  $\rho_i > \delta_i$  takes the form

$$U_i^{\text{UT}} \simeq \frac{\omega - \omega_i}{\omega} \frac{\delta_i}{L\varepsilon_i^2} i \sqrt{\pi} \left(\frac{\delta_i}{W}\right) \frac{\rho_i^2 - W^2}{\rho_i^2}.$$
 (A2.7)

We can find the final expression for the nonlinear growth rate in this limit by substituting Equation (A2.7) into the dispersion equation (A2.1)

$$\gamma_{\rm NL} \simeq \omega_e \frac{10 \,\varepsilon_e^2}{\varepsilon_i^2} \left(\frac{\delta_i}{W}\right)^3 \frac{\rho_i^2 - W^2}{\rho_i^2} \quad \text{when } \rho_i > \delta_i \,.$$
 (A2.8)

Expressions (A2.7) and (A2.8) differ from corresponding expressions given in Paper I by the multiplier  $(\rho_i^2 - W^2)/\rho_i^2$  which reflects the finiteness of the width of the interaction region, i.e., the influence of electrostatic contributions to  $E_{\parallel}(x)$ . Correspondingly, the integration of Equation (A2.1) should be performed up to  $\delta_{\varphi} \simeq \rho_i$  rather than to infinity (see Figure 3).

From Equation (A2.8) one can see that the nonlinear growth of magnetic islands is saturated when

$$W = W^* \simeq \delta_{\varphi} \simeq \rho_i \,. \tag{A2.9}$$

This is, of course, an estimate valid up to some numerical factor. To obtain it one must solve the appropriate nonlinear integral equation. However, Equation (A2.9) is physically rather well justified and reflects the finiteness of the thickness of the interaction

region both in linear and nonlinear regimes which must determine the width of the nonlinear saturation of the growth of islands. Therefore, one can see from Equations (A2.6) and (A2.9) that in both cases the magnetic islands grow to a respectively high (in comparison with previous 'purely electron' stabilization) but finite level  $W^* \simeq \delta_{\varphi}$ . After that one can draw the principal conclusion for this study: if even a very narrow region (but wider than  $\delta_{\varphi}$ ) with stable magnetic surfaces exists within the plasma layer it cannot be overlapped by nearly growing magnetic islands and thus it appears to be impenetrable for the diffusing field lines. This means that the percolation process is interrupted.

#### Appendix 3

## Random Brownian Walk of the Magnetic Field Line Through the Magnetopause Boundary Layer (MBL)

The growth of magnetic perturbations (2) at a given magnetic surface  $X_s$  results in the formation of a chain of magnetic islands along it. Following Galeev (1984) one can easily write the equation of magnetic field lines for a single given **k**-mode

$$\frac{\mathrm{d}s}{B} = \frac{\mathrm{d}x}{B_{\mathbf{k}}\sin(k_{z}z + k_{y}y)} = \frac{\mathrm{d}z}{B_{z}(X)} = \frac{\mathrm{d}y}{B_{y}} \tag{A3.1}$$

where s is the coordinate along the magnetic field and  $B_k$  is the amplitude of the perturbed magnetic field. In the vicinity of the singular surface where  $k_{\parallel}(X_s) = 0$  one can expand the tearing-mode phase  $k_z z + k_y y = k'_{\parallel}(X_s) \int (X - X_s) ds$ . Then the solution of Equation (A3.1) can be easily obtained

$$x = X - X_s = \pm \left\{ \frac{2B_k}{k_{\parallel}'B} \left( 2\varkappa^2 - 1 - \cos\left(k_{\parallel}'\int x(s)\,\mathrm{d}s\right) \right) \right\}^{1/2}.$$
 (A3.2)

Different values of the parameter  $\varkappa^2$  (which is the constant of integration) correspond to closed ( $\varkappa^2 < 1$ ) and open ( $\varkappa^2 > 1$ ) magnetic surfaces. For  $\varkappa^2 = 1$ , Equation (A3.2) gives an equation for the separatrix which determines the value of the half-thickness of the forming magnetic island.

$$W(X_{s\mathbf{k}}) = 2\sqrt{\frac{B_{\mathbf{k}}}{k'_{\parallel}(X_{s\mathbf{k}})B}}.$$
(A3.3)

For the sheared magnetic field configuration (1), magnetic islands with the corresponding directions of wave vector k can, in principle, grow at different locations within the layer (1) (see Figure 8). If the number of growing modes N is sufficiently large the islands  $W_1$  and  $W_2$  growing at the neighbouring magnetic surfaces  $X_{s1}$  and  $X_{s2}$  can overlap. If the condition for overlapping

$$W_1 + W_2 > |X_{s1} - X_{s2}| \tag{A3.4}$$

is satisfied the magnetic surfaces  $X_{s1}$  and  $X_{s2}$  become destructed and magnetic field lines begin to wander stochastically between them (Rosenbluth *et al.*, 1966). One can easily imagine this stochastic process as a random Brownian walk across the magnetopause with the spatial step  $\Delta x \sim W$  and some 'time' step  $\lambda_W$ . For the magnetic field line diffusion we have the s-coordinate instead of real time *t* (see Equation (A3.1). So, the length of the magnetic field line  $\lambda_W$  corresponding to one Brownian step can be easily evaluated from Equation (A3.1):  $\lambda_W \simeq 1/k'_{\parallel}W$ . We then immediately obtain the estimate for the magnetic field line diffusion coefficient  $D_F$ . Omitting the numerical multipliers of the order of unity, we have

$$D_F = \frac{W^2}{\lambda_W} = (k'_{\parallel})^{-1/2} \left(\frac{B_k}{B}\right)^{3/2}.$$
 (A3.5)

It is reasonable to relate  $D_F$  with the general level of magnetic turbulence within the layer

$$b_0^2 = \sum_{\mathbf{k}} \frac{B_{\mathbf{k}}^2}{B^2}.$$
 (A3.6)

For simplicity one can assume that the magnetic islands have approximately equal thicknesses throughout the layer. Then the number of modes required to cover the entire magnetopause layer (-L < X < L) at the margin of their overlap may be estimated as  $N \simeq L/W$ . Accordingly, we have the simple approximate expression

$$b_0^2 \simeq N \frac{B_k^2}{B^2} \simeq W^3 L k_{\parallel}'^2$$
 (A3.7)

The quantity  $b_0^2$  is proportional to the energy released during the development of the instability. It is easy to see from Equation (A3.7) that energetically it is more favourable to 'pave' the MBL by the magnetic islands with maximum possible thicknesses, but without the significant overlapping which can stabilize their growth. This simple consideration indicates the importance of the single mode effects in a study of the magnetopause processes. The nonlinear quenching of the growth of magnetic islands occur, as was shown in the Appendix 2, at the value  $W^* \leq \delta_{\omega}$ .

From Equations (A3.5) and (A3.7) one can easily find a simple, final expression for the magnetic field line diffusion coefficient

$$D_F \simeq b_0^2 \frac{1}{k'_{\parallel}L}.$$
 (A2.8)

It is easy to check that the estimate (A3.8) coincides with the well-known quasilinear expression obtained by Rosenbluth *et al.* (1966)

$$D_F = \sum_{\mathbf{k}} \frac{B_{\mathbf{k}}^2}{B^2} \pi \delta(k_{\parallel}(\mathbf{x}))$$
(A3.9)

were  $\delta$  is the Dirac delta-function.

Finally, taking into account  $W \sim W^* \leq \delta_{\omega} \simeq \rho_i$ , we have from (A3.7) and (A3.8)

$$D_F \simeq \delta_{\varphi}^3 k'_{\parallel} \sim \rho_i^3 \frac{k}{Lb_y} \sim \frac{\rho_i^3}{L\lambda} \operatorname{tg} \frac{\theta_0}{2}.$$
(A3.10)

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