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Asymmetrical trajectory formation in cyclic forearm movements in man

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Summary. Predictions of the minimum-jerk model for a human cyclic motion were given in terms of asymmetry in movement trajectories. A detailed kinematic analysis of cyclic forearm motion, i.e., extension/flexion movements around the elbow joint in a horizontal plane ranging in frequency from 2–5.5 Hz, was made to examine the validity of the predictions. The kinematics of the trajectories were described in terms of deviation from symmetry in velocity and acceleration profiles, and jerk cost. The asymmetry could be accounted for by the solution of the minimum-jerk model using the boundary condition differences between extension and flexion during a movement cycle. The trajectory was asymmetrical at relatively low frequencies, and symmetrical at higher frequencies; the frequency boundary from asymmetrical to symmetrical trajectories differed among subjects with a range of 3–4.3 Hz. It was suggested for the asymmetrical trajectory formation that consecutive extension and flexion in a cycle could be processed as a unit in which speed and acceleration in each direction were differentiated. The shift from asymmetrical to symmetrical trajectories with increasing frequency was accompanied by a reduction in jerk cost and mechanical energy. The oscillators underpinning the high frequency movements were mainly non-linear. The results suggested a shift of control from the "rhythmic" sequencing of extension and flexion which resulted in trajectory asymmetry, to non-linear oscillation with no directional difference.

Key words: Forearm movement – Kinematic analysis – Trajectory asymmetry – Minimum jerk model – Human

Introduction

In order to specify the organizational principle of skilled human movements, a number of theoretical and experimental studies have been conducted using the minimum-jerk model, including single-joint arm movements (Hogan 1984; Nagasaki 1989), multi-joint arm (Flash and Hogan 1985; Schneider and Zernicke 1989; Uno et al. 1989) and leg (Itoh and Nagasaki 1991) movements; ellipse-drawing movements (Wann et al. 1988) and handwriting (Edelman and Flash 1987). This model assumes that for skilled movements, the movement trajectory selected is the one that minimizes "jerk cost." Jerk cost is defined as $\frac{1}{2}\int_0^T J^2(t)dt$, where $J(t)$ is a time function of jerk, the acceleration change rate, and T the movement time. The minimum-jerk model is appealing because it is mathematically simple; also, it predicts the optimally smooth trajectory that is an important index of skilled motor acts.

The minimum-jerk model in its simplest form, however, deviated considerably from the experimental data for discrete arm motion at extremely slow and fast movement velocities (Nagasaki 1989); in these slow and fast movements, velocity did not exhibit a bell-shaped, symmetrical profile as predicted by the simple minimum-jerk model (Hogan 1984). Asymmetrical (skewed) velocity profiles were also reported by a number of researchers for discrete single-joint movements (see Nagasaki 1989), by Wann et al. (1988) for a cyclic ellipse-drawing motion, and by Uno et al. (1989) for a class of multi-joint arm movements. Because of the discrepancy between the predictions of the simple minimum-jerk model and the empirical data, the third derivative of the viscoelastic muscle distortion was added to jerk as an alternative movement objective (Wann et al. 1988), or an entirely different cost function (i.e., the torque-change) was introduced (Uno et al. 1989). I on the other hand, explained the asymmetrical trajectory formation of the discrete arm movements within the framework of the minimum-jerk model by introducing a constraint upon the control input (i.e., jerk) at the start and end of the movement (Nagasaki 1989). I also suggested that for the trajectory formation of continuous cyclic movements, with no discrete beginning or end, the jerk cost per unit movement time and extent may be reduced compared with discrete movements, and a closer agreement of the model with the observed trajectories would be obtained. Whether the asymmetrical trajectory formation that was observed could be accounted for by the minimum-jerk model now appears critical to its relevance to the modeling of human motor control. In the present study, I will provide detailed kinematic data of a cyclic forearm motion (extension/flexion movements) within a wide range of cycle frequencies (2–5.5 Hz), and examine whether the asymmetrical movements observed could be accounted for by the minimum-jerk model for the cyclic motion.

Application of the minimum-jerk model to cyclic motions is also interesting in regard to the frequency-dependence of the control mechanisms underlying the cyclic motions. Cyclic movements often exhibit temporal grouping or chunking of the elements composing movement sequences, and the structure of the grouping is dependent on the movement frequency. In finger tapping, for instance, three types of temporal patterning (i.e., rhythm) occur in the force and inter-tap interval sequences, even when subjects are asked to tap in synchrony with the sound of a perfectly isochronous pulse train composed of identical elements; in the type I rhythm, two consecutive taps are grouped so that a relatively longer inter-tap interval and a more stressed tap precede an unstressed tap with a shorter interval. The relationship between interval and force in this group is reversed in type II rhythm, and four consecutive taps are grouped in type II rhythm. Each type of tapping rhythm is predominant in its specific region of tapping frequency; type I below 4 Hz, type II between 4 and 5 Hz, and type III above 5 Hz (Nagasaki 1987a, b). It is suggested that the structures of the rhythm transform each other in the sense that tapping with one type of rhythm is the most stable in its intrinsic frequency region because of the least "cost" to tap; but in other regions, the cost of tapping with other type of rhythms is optimal and a transition in the rhythmic structure thereby takes place. In the present study, I also relate trajectory asymmetry, jerk cost and other kinematic variables of the cyclic forearm movements to cycle frequency. It is shown that trajectory asymmetry of the movements observed in a low frequency region can be attributed to a "rhythmic" sequencing of extension and flexion in a cycle of the movements, which disappears at higher frequencies.

Predictions of the minimum-jerk model for cyclic movements

In this section, the minimum-jerk model is applied to a cyclic human motion, i.e., the extension/flexion forearm movements around the elbow joint along a horizontal plane. An example of the profiles of angular displacement, velocity, acceleration, and jerk during one cycle of the movements can be seen in Fig. 1. Some of the notations are indicated in the figure, and further details are listed below, along with other variables discussed in this paper. The variables are defined for both extension and flexion and, if necessary, suffix "e" or "f" will be added to the notation:

t	time (s);
X(t)	angular displacement (deg);
V(t)	velocity (deg/s);
A(t)	acceleration (deg/s^2);
J(t)	jerk (deg/s^3);
D	movement extent (deg);
T	movement time (s);
VP	peak velocity (deg/s), absolute value;
k	relative timing of peak velocity, i.e., (peak velocity time)/(movement time);
A0, A1	acceleration (deg/s^2) at the boundary from flexion to extension (A0) and vice versa (A1), absolute value;
JP	peak jerk (deg/s^3), absolute value;
JC	jerk cost (deg^2/s^5);
EN	mechanical energy divided by the moment of inertia of the forearm and the device (deg^2/s^2);

Normalized kinematic variables are defined as follows;

$vp = VP/(D/T)$;
$a_0 = A0/(D/T^2)$, $a_1 = A1/(D/T^2)$;
$jp = JP/(D/T^3)$;
$jc = JC/(D^2/T^5)$;
$en = EN/(D^2/T^2)$;

Angular displacement of a single-joint motion was predicted to be a fifth-order polynomial in time under the constraint to minimize jerk cost defined by $\frac{1}{2}\int_0^T J(t)^2\,dt$, where $J(t) = d^3X(t)/dt^3$ and T is movement time (Hogan 1984; Nelson 1983). Nelson (1983) gave the coefficients of the polynomial for a cyclic motion in which the trajectory had a perfectly symmetrical (bell-shaped) velocity profile during both halves of the cycle. Human limb motions, however, are necessarily performed in a workspace closely related to the body coordinates, e.g., toward or away from the body. The cyclic forearm movements, therefore, may exhibit different trajectories during movement phases toward and away from the body. It is also probable that the trajectory exhibits an asymmetrical (skewed) velocity and acceleration profiles during each phase (Wann et al. 1988). Considering these possibilities of the asymmetrical trajectory formation for cyclic forearm movements, the following boundary conditions for angular displacement, velocity, and acceleration were assumed here for the extension and flexion phases separately (see Fig. 1);

For extension:

$$\begin{aligned} &X(0) = 0 && X(Te) = D \\ &V(0) = 0 && V(Te) = 0 \\ &A(0) = a_0 D/Te^2 = A0 && A(Te) = -a_1 D/Te^2 = A1 \end{aligned} \quad (1)$$

For flexion:

$$
\begin{aligned}
&X(0)=D \quad X(Tf)=0\\
&V(0)=0 \quad V(Tf)=0\\
&A(0)=-a_1D/Tf^2=A1 \quad A(Tf)=a_0D/Tf^2=A0 \qquad (2)
\end{aligned}
$$

The boundary conditions for acceleration are written above in terms of a_0 and a_1, i.e., normalized accelerations at the boundaries of movement phases, from flexion to extension and from extension to flexion, respectively. The boundary conditions must satisfy a continuity in trajectories at the boundaries of the two phases, e.g., A(Te) = A(0) in flexion. Thus, a_0 and a_1 may differ slightly between extension and flexion, if Te is not equal to Tf. When the trajectory is not symmetrical between two phases, Te and peak velocity in extension may differ from Tf and peak velocity in flexion, respectively. Asymmetry during each phase, on the other hand, could be represented (i) by the relative timing of peak velocity during a phase, i.e., k_e for extension and k_f for flexion and (ii) by the difference between a_0 and a_1. When the profile is symmetrical, acceleration at the phase boundary equals peak acceleration.

Under boundary conditions (1) and (2), the smoothest movement trajectories in extension and flexion are determined as follows;

Extension:

$$
\begin{aligned}
X(t) = {} & D[a_0t^2+(20-3a_0-a_1)t^3+(3a_0+2a_1-30)t^4\\
& +(12-a_0-a_1)t^5]/2,\\
& 0\leq t\leq 1 \qquad (3)
\end{aligned}
$$

Flexion:

$$
\begin{aligned}
X(t) = {} & D[2-a_1t^2+(a_0+3a_1-20)t^3+(30-2a_0-3a_1)t^4\\
& +(a_0+a_1-12)t^5]/2,\\
& 0\leq t\leq 1 \qquad (4)
\end{aligned}
$$

Here, time t is normalized by movement time. By differentiating equations (3) or (4), velocity V(t), acceleration A(t), and jerk J(t) are obtained. Integration of $J(t)^2/2$ between $t=0$ and 1 gives the minimum-jerk cost:

$$
\begin{aligned}
JC = {} & D^2[(a_0+3a_1-20)^2/2+4(a_0-5)^2+60]/T^5\\
T=Te \quad & \text{for extension}\\
Tf \quad & \text{for flexion} \qquad (5)
\end{aligned}
$$

The mean mechanical energy consumed during extension or flexion can be defined as $I\int_0^T V(t)^2\,dt/2T$, where V(t) is instantaneous angular velocity during movement time T, and I is the moment of inertia of the forearm and the apparatus. The mechanical energy measurement can be computed using the formula $\int_0^T V(t)^2\,dt/2T$.

The symmetrical trajectory

The jerk cost in equation (5) is further minimized when $a_0=a_1=5$. By substituting $a_0=a_1=5$, equations (3) and (4) give the symmetrical trajectories of extension and flexion; in both cases, the velocity and acceleration profiles are also symmetrical. For the symmetrical trajectory, the normalized kinematic variables may be invariant for movements with different speeds and extents;

$$
\begin{aligned}
&k=0.5;\\
&v=1.56;\\
&a_0=a_1=5;\\
&j=15;\\
&jc=60;\\
&en=1.23; \qquad (6)
\end{aligned}
$$

These 'invariant' relationships of the normalized kinematic variables indicate that trajectories under different conditions of speed and extent could be represented by a single trajectory when the time and distance axes are rescaled. Note that the normalized jerk cost for the cyclic motion is 1/6 the cost of the discrete motion (Nagasaki 1989; Nelson 1983).

The asymmetrical trajectory

The minimum-jerk cost for asymmetrical trajectory is higher than symmetrical trajectory with the same speed and extent. Equation (5) indicates that the increment in the normalized jerk cost due to the asymmetry is a function of a_0 and a_1, which is represented by an ellipse with the long axis $a_0+a_1 = 10$, as depicted in Fig. 4. Due to asymmetry, the kinematic variables characterizing the trajectory may deviate from those given in equation (6) for the symmetrical trajectory. Those values can be numerically computed by use of equation (3) or (4) when the values or a function of a_0 and a_1 are given.

Experimental methods

Subjects

Five male subjects ranging from 21–51 years old participated in the study.

Apparatus and procedure

The subject was seated with his right forearm on a light, horizontally rotating handle (moment of inertia 0.024 kg · m^2) of an arm-rotator. The axis of the elbow joint was aligned with the handle pivot. When the task began, the shoulder was at 90° flexion, and 20° horizontal abduction, and the elbow at 80° flexion. The subject closed his eyes, held a vertical rod attached to the handle, and performed extension/flexion forearm movements cyclically. Angular displacement of the elbow joint, which was defined as increasing during extension and decreasing during flexion, was measured by a DC potentiometer aligned with the pivot of the handle. The angular displacement data were stored in a computer (NEC 9800) via an A/D converter with a sampling frequency of 512 Hz. The sampling duration was 4 sec per trial.

Metronome-paced movements were recorded, then preferred movements. In paced trials, the subjects were told to follow the sounds (1 kHz, 50 ms duration) of a metronome to produce one full cycle of movement for each sound. Pacing was provided for 13

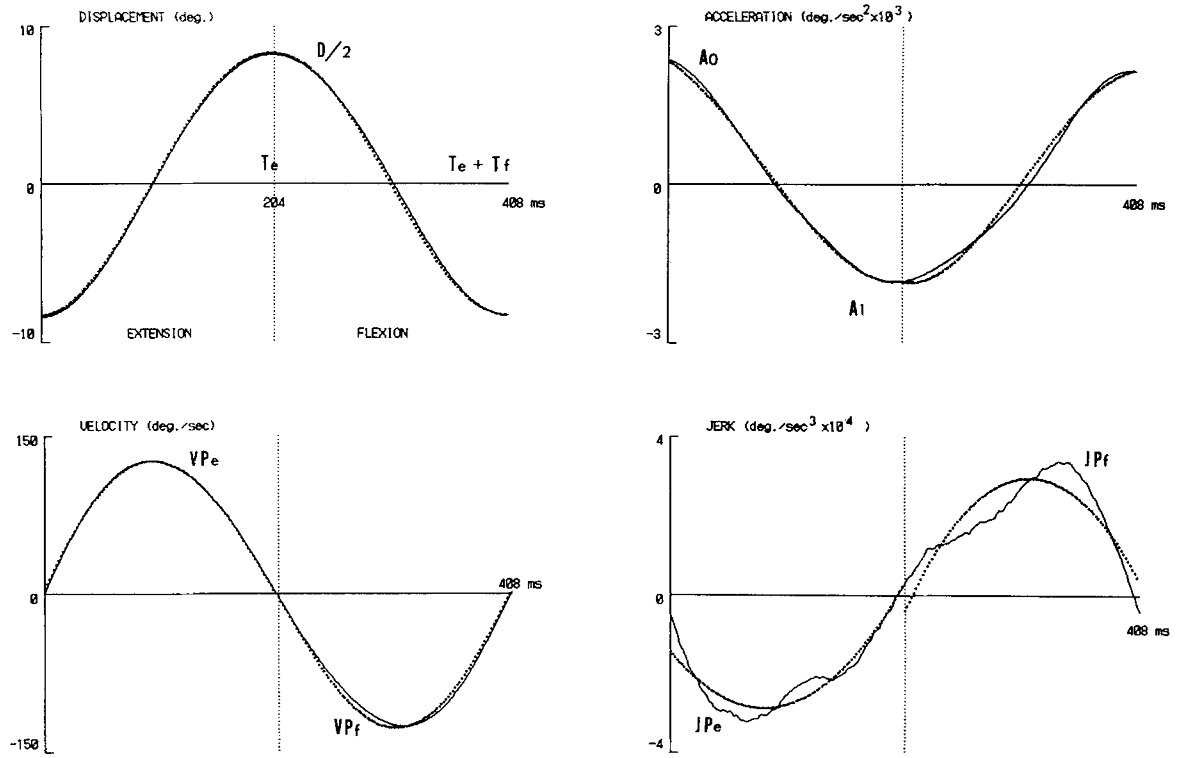

Fig. 1. An example of angular displacement, velocity, acceleration and jerk for the cyclic forearm movements of one subject at 2.5 Hz. Relative position of the velocity peak is 0.46 during extension and 0.55 during flexion. The *dotted lines* are simulations using the minimum-jerk model (equations (3) and (4), $D = 16.6$ degrees, $a_0 = 5.53$, $a_1 = 4.57$)

different frequencies ranging from 2–5.5 Hz and presented in a random order. Sound pulses produced by the metronome were sampled simultaneously and displayed on the computer's CRT, together with the displacement data, in order to ascertain the sound position during a cycle of movement. In the preferred trials, the subjects were told to perform the movement at a comfortable rate. For both the paced and preferred conditions, the subjects were given no explicit instructions concerning the extent of movement. Two trials were conducted for each condition.

Data analysis

The angular displacement data sampled at 4 s for each trial were smoothed using a second-order, zero-lag digital filter. The cutoff frequency of the low-pass filter was four times each cycle frequency, e.g., 8 Hz for the movements at 2 Hz. Instantaneous angular velocity was computed from the smoothed displacement data by means of a two-point central difference algorithm. One movement cycle was defined as the period between two consecutive points at which the velocity crossed zero from negative (flexion) to positive (extension). The angular displacement data from successive movement cycles thus defined, were then ensemble averaged across all cycles in a movement trial, taking the first zero-crossing of velocity as the start point. Numbers of the cycle averaged in a trial were in the approximate range of 7 at 2 Hz, to 21 at 5.5 Hz.

By applying the two-point central difference algorithm to the averaged displacement data, averaged angular velocity, acceleration, and jerk (change of acceleration over time) during one movement cycle were computed. The mean cycle period was again defined as the point where the mean velocity crossed zero. In Fig. 1, an example of the trajectory and its derivatives is illustrated for the trial of one subject. The extension and flexion movement phases were defined as the periods from first to second zero-crossing of velocity, and from second to third zero-crossing, respectively. Te and Tf are the movement times for extension and flexion, respectively. The relative location of the velocity peak, k_e for extension and k_f for flexion, were calculated using the zero-crossing in an acceleration curve. The other kinematic variables defined in the previous section were also measured using the averaged trajectory for the separate trials of each subject, and their normalized values were thereby computed. Data for the two trials at each movement frequency were averaged for each subject.

Results

Three of five subjects accurately matched one cycle of movements with the metronome. One subject's response was rushed, while another subject's response was delayed, about 0.5 Hz from the metronome cycle at a frequency greater than 3.5 Hz.

Predictions of the minimum-jerk model for the cyclic forearm movements were described in the previous section in terms of trajectory asymmetry. Accordingly, results from this experiment are described below, first in regard to the trajectory asymmetry dealing implicitly with movement frequency and then the asymmetry's frequency-dependence.

Asymmetry of the trajectory

The kinematic variables obtained by the experiment were subjected to an ANOVA (frequency × direction). The direction (extension and flexion) had a significant main

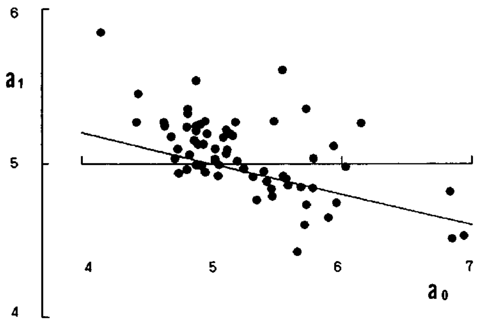

Fig. 2. Normalized boundary acceleration, a_0 from flexion to extension and a_1 from extension to flexion (data for extension from each subject and frequency)

effect on T and k; F(12,1) = 7.30 for T ($P < 0.01$) and F(12,1) = 7.42 for k ($P < 0.01$). This indicates that the trajectory was not symmetrical in regard to the movement direction; the movement time was shorter for flexion than extension. k_e was smaller than k_f, indicating that a velocity profile was skewed toward the beginning of the movement in extension, and toward the termination in flexion. The ANOVA also indicated that the normalized peak velocity and jerk were higher for flexion than extension (F(12,1) = 11.87 for vp and 8.59 for jp, $P < 0.01$). The trajectory asymmetry for extension and flexion, as well as between both movement directions, is typically represented by the difference between normalized accelerations at the boundaries from flexion to extension (a_0) and vice versa (a_1); a_0 was higher than a_1, F(12,1) = 28.48, $P < 0.01$.

In Fig. 2, a_1 is plotted against a_0 for the data in extension which include every subject and frequency. Note that the acceleration profile is symmetrical when $a_0 = a_1 = 5$. This figure shows that the trajectory exhibited considerable asymmetry (skewed acceleration profile), and that a trade-off relationship between a_0 and a_1 exists. Linear regression of a_1 to a_0 with a constraint of $a_1 = a_0 = 5$ was significant: $(a_1 - 5) + 0.20(a_0 - 5) = 0$ ($r = 0.46$, $P < 0.01$). In terms of individual subjects, the regression was significant for three of the subjects ($r = 0.66 - 0.92$, $P < 0.01$). The regression slope was negative in the case of four subjects, at a range of -0.12 to -0.70. A similar relationship between a_0 and a_1 was also found in flexion; $(a_1 - 5) + 0.48(a_0 - 5) = 0$ ($r = 0.51$, $P < 0.01$) for all subjects, and r ranged from 0.71 to 0.73 for three of the subjects. The slope ranged from -0.13 to -1.00 for four subjects.

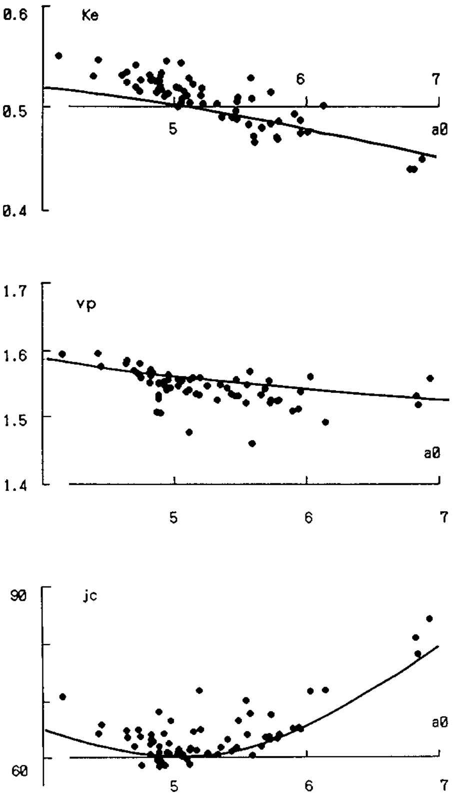

Fig. 3. Deviations in relative timing of peak velocity (k_e), normalized peak velocity (vp), and normalized jerk cost (jc) from the values of symmetrical trajectory, depicted as a function of normalized acceleration at the boundary from flexion to extension (a_0; data for extension from each subject and frequency). The line superimposed over each figure was predicted by the minimum-jerk model based on a relationship of $(a_1 - 5) + 0.2(a_0 - 5) = 0$

In Fig. 3, the relative timing of peak velocity, normalized peak velocity and jerk cost are plotted against a_0 for extension. The figure shows that these normalized variables also deviated from the values for symmetrical trajectory as a function of a_0. In fact, k and peak velocity related linearly with a_0 ($P < 0.01$); ke: $r = 0.88$, kf: $r = 0.74$, vp_e: $r = 0.48$. In addition, a second-order regression of jerk cost (jc) to a_0 was significant ($P < 0.01$) in extension ($r = 0.85$) and in flexion ($r = 0.45$). By using the linear relation between a_0 and a_1 that was found empirically, and equations (3) – (4), k, vp or jc can be simulated as a function of a_0, which was depicted in Fig. 3 by superimposing the data. The results of the simulation and the experiment can be considered to be in agreement. The results were similar for flexion. Given the numerical values of a_0 and a_1 along with the movement time and extent (T and D), an individual trajectory and its derivatives were simulated by equation (3) or (4). An example is given in Fig. 1 for the trial of one subject with an asymmetrical trajectory. The agreement of the simulation with the data was satisfactory, as seen in the figure. Thus, the detailed kinematic features of the asymmetrical trajectories in forearm movements may be explained by the minimum-jerk model applied to the cyclic motion.

In Fig. 4, the theoretically obtained increment in normalized minimum-jerk cost due to the trajectory asymmetry is shown as a function of a_0 and a_1 (5). This figure predicts that the additional cost is minimized when both variables deviate from symmetry along the long axis of the ellipses, i.e., $a_0 + a_1 = 10$, while the maximum occurs along the short axis, $a_0 - a_1 = 0$. This raises the question of whether the trajectory of our movements deviated from symmetry with the result of minimizing jerk cost which was additionally consumed by the asymmetry. The linear regressions of a_1 to a_0, which were empirically obtained, are superimposed on the elliptical equicost plane in Fig. 4 (see also Fig. 2). These figures suggest that most subjects shifted the trajectory from symmetry in

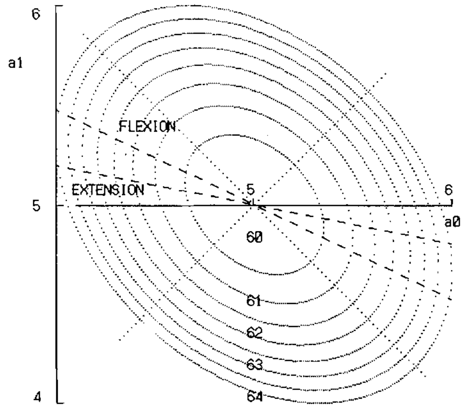

Fig. 4. The increment in normalized jerk cost due to trajectory asymmetry predicted by the minimum-jerk model (equation (5)). The cost is depicted as a function of a_0 and a_1, normalized accelerations at the boundary from flexion to extension and vice versa, respectively. Linear regressions of a_1 to a_0 *(broken lines)* obtained experimentally are superimposed to show the flexion and extension

such a way that the additional jerk cost due to asymmetry did not greatly exceed the minimum predicted by the minimum-jerk model for asymmetrical movements.

Frequency-dependence of asymmetry in the trajectories

The asymmetrical trajectory formation described above was then related to cyclic frequency. The frequency-dependence of a_0 and a_1 is depicted in Fig. 5 for the individual subjects in extension. This figure suggests that asymmetry occurred at relatively low frequencies, while the trajectory became more symmetrical at higher frequencies. The ANOVA showed a significant interaction of the movement direction with frequency for a_0 and a_1, $F(12,1)=4.45$ ($P<0.01$), along with the main effect of the direction. The interaction was also significant for k, $F(12,1)=3.42$ ($P<0.01$). These figures suggested that the deviation of a_0 and a_1 from symmetry toward the opposite directions tended to diminish as the frequency increased. The boundary of frequency between the asymmetrical to symmetrical trajectories appeared to differ considerably among the subjects as seen in Fig. 5. Nevertheless, it can be generalized that the trajectory for each subject was asymmetrical at a frequency less than 3 Hz, and symmetrical at more than 4.3 Hz. It has been reported (Nagasaki 1989; Wann et al. 1988) that the relative position of peak velocity, k, deviates from symmetry ($k=0.5$) depending on movement speed – $k<0.5$ and >0.5 at slow and fast speeds, respectively. But in the present study, this directional reversal of the position of the velocity peak was not detected except in the case of one subject (k_e for NAGA). Rather, it was indicated in the present, as well as in previous (Nagasaki 1989), studies that k is a direct function of normalized acceleration, not of speed perse. It is questionable whether asymmetry should be attributed to the fact that the average number of the displacement data was smaller for the trials at low frequencies than at high frequencies, e.g., 7 at 2 Hz and 21 at 5.5 Hz (see Results). However, the displacement data, when averaged across an equal number of consecutive cycles (i.e., 7 cycles) and reanalyzed, resulted in the same asymmetry with the frequency-dependency described above.

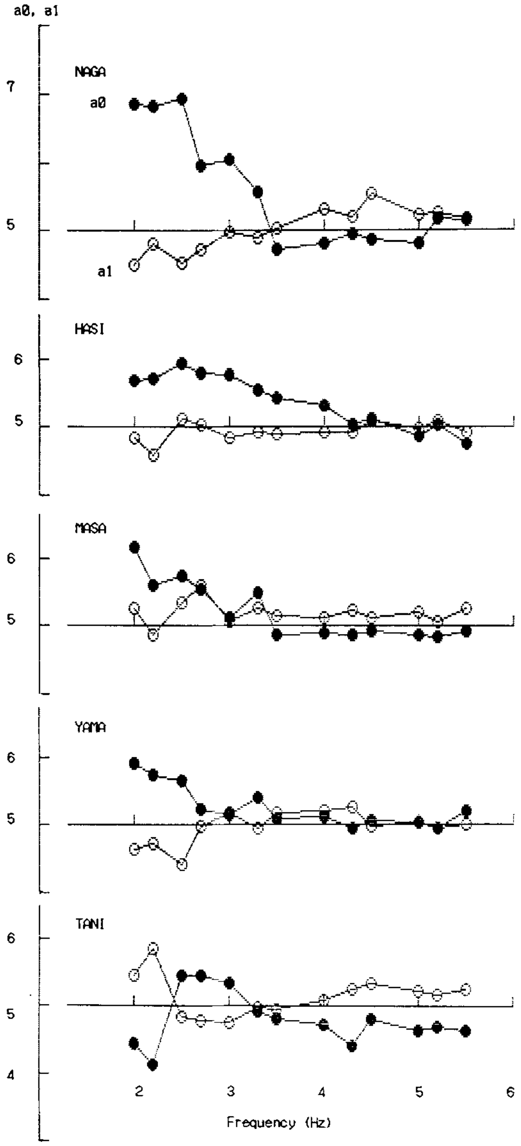

Fig. 5. The frequency-dependence of the boundary acceleration from flexion to extension (a_0: ●) and from extension to flexion (a_1: ○) for individual subjects

At relatively low frequencies, four of five subjects matched their responses with the sound stimuli during flexion, and one subject (TANI in Fig. 5) during extension. This may have brought about movement profile differences between the extension and flexion phases at these frequencies. In other words, the movement duration was shorter and peak velocity higher when the subject matched the responses with the sounds than during the other phase. The rate of acceleration for the directional change from extension to flexion (a_0) and from flexion to extension (a_1) were differentiated accordingly. Note that the deviation from symmetry for a_0 and a_1 was reversed in the subject TANI at 2 and 2.5 Hz. At a low frequency, however, the trajectory was also asymmetrical in movements at a preferred rate without the sound stimuli. This suggests that the differentiation of extension and flexion movements was not due to the sounds matched by the responses, but to the movement direction toward or away from a subject's body. During symmetrical movements above 4.3 Hz, on the other hand, the subjects no longer matched responses of a specified direction with the sounds.

Frequency-dependence of mechanical energy

While peak acceleration and jerk increased significantly with an increase of movement frequency, the main effect of frequency was not significant on peak velocity, movement extent, and mechanical energy (ANOVA). Figure 6 illustrates the frequency-dependence of mechanical energy for individual subjects, where energy was normalized at the lowest examined frequency, 2 Hz. The energy change based on frequency exhibited a similar profile for four of the subjects: the energy was reduced when the frequency exceeded a certain level. For subject NAGA, for example, the energy increased up to 4.3 Hz, decreased at 4.5 Hz, and above this frequency the energy increased again. The frequency of the possible energy reduction differed among the subjects (3.3–4.5 Hz, mean = 4.1 Hz), and coincided roughly with the frequency at which the trajectory became symmetrical for each subject (see Fig. 5). These results suggest that the control structure of the cyclic forearm movements, in terms of energy expenditure, may differ above 4.5 Hz compared with a level below 3.3 Hz.

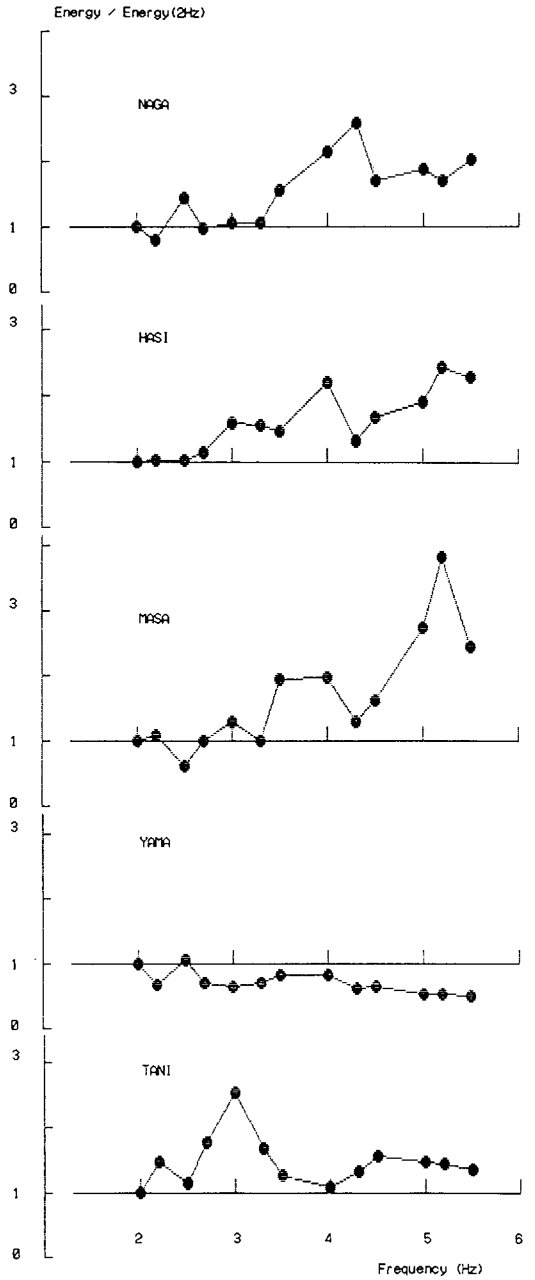

Fig. 6. The frequency-dependence of mean mechanical energy during one half-cycle (averaged across extension and flexion) for individual subjects, divided by the moment of inertia and normalized at 2 Hz

Discussion

Asymmetrical trajectory and the minimum-jerk model

The detailed kinematic analysis in the present study reveals considerable asymmetry in the trajectory formation of the cyclic forearm movements. Asymmetry here refers specifically to the skewed acceleration profile or the differentiated boundary accelerations, a_0 and a_1. Trajectory asymmetry should not be attributed to variability in the empirical data, because the index of the asymmetry, i.e., a_0 or a_1, was consistently interrelated to a deviation from symmetry in k, vp and so on. Asymmetrical trajectory has also been reported for a variety of discrete motions (see Nagasaki 1989) and for a cyclic motion (Wann et al. 1988). The asymmetrical trajectory formation demonstrates a complexity or diversity of human movements which should be addressed by any theory of human motor control. For cyclic motion, in particular, a model which simulates the symmetrical trajectories of a given motion should not necessarily be regarded as the only theory relevant to that motion, because simulated trajectories are virtually indistinguishable from conventional approximations using harmonic (sinusoidal) oscillators. Whether a proposed model can illustrate asymmetrical trajectory formation within a class of cyclic movements is one criterion for substantiating its relevance to human motor control.

The present study shows that asymmetry which is observed in the cyclic forearm movements can be explained quantitatively by the minimum-jerk model for the asymmetrical cyclic motion. I have also suggested that a shift from symmetrical to asymmetrical trajectory formations occurred in most subjects in such a way that the additional jerk cost due to the trajectory asymmetry was minimized. The asymmetry in the trajectory was suggested to have arisen from the differentiated control between extension and flexion during one cycle of the movements. Four of five subjects performed extension faster than flexion, and vice versa for the remaining subject, at a given frequency below 3 Hz. In the other words, the subjects performed the cyclic forearm movements with rhythm or temporal grouping of the movement duration and/or peak acceleration during consecutive extension and flexion in a cycle. Since the difference between extension and flexion was observed in the movements at the preferred rate, it is possible that asymmetry may not result from matching the metronome sounds with the specified position in a cycle. Rather, the cue for the rhythmic sequencing of extension and flexion in a cycle may have been given in terms of the direction of the movements toward a subject's body. Thus, cyclic forearm movements at low frequencies were controlled by processing one cycle as a unit in which the extension and flexion were segmented in terms of the movement directions, i.e., toward and away from a subjects' body. Sequential motions are often controlled rhythmically in terms of inter-response interval and response force, even if the motions are performed under instructions to match responses with perfectly isochronous stimuli (Nagasaki 1987a).

The difference between consecutive extension and flexion was expressed most clearly by the difference in the acceleration at the boundaries between extension and flexion – that is, by the muscular force for directional change of the movements. The different boundary conditions for acceleration between extension and flexion that were introduced to the present minimum-jerk model represented this rhythmic sequencing of the cyclic extension/flexion movements, and thus the model could explain the asymmetrical trajectory formation at low frequencies. At frequencies greater than 4.3 Hz, the subjects were unable to maintain rhythmic control. Therefore, solving the minimum-jerk model with a symmetrical boundary condition can explain the symmetrical trajectory that was observed.

The above discussion, however, does not exclude movement objectives other than jerk cost in order to account for the trajectory asymmetry. Wann et al. (1988) used a combined cost function comprised of jerk and the third derivative of the visco-elastic muscle distortion, and thereby explained the skewed velocity profiles which were observed in a cyclic ellipse-drawing motion. Uno et al. (1989) contrasted their minimum torque-change model with minimum-jerk model for a discrete multijoint motion. One of their arguments is that the double-peaked velocity profiles observed in a via-point movement could not be explained by the minimum-jerk model. However, in the trajectory formation model proposed by Wann et al. (1988), and Uno et al. (1989), the simplicity which is a characteristic of the minimum-jerk model, is lost because the minimum-jerk model is independent of any of the neuromuscular dynamics involved in movement. Though it may sound strange that the optimal trajectory formation of human voluntary movements is determined independently of the muscle dynamics (Uno et al. 1989), the minimum-jerk model may be closely related to the concept of the generalized motor program proposed for both simple and skilled human motions (Schmidt et al. 1979). The generalized motor program theory assumes that adult humans have acquired an internal representation of the motor program for a class of movements, and execute a movement by specifying movement parameters (movement time, extent, etc.) which depend on a given movement situation. The minimum-jerk model also assumes that the optimal trajectory has been internally formed as the general quintic polynomial in the work-space, and that the movement trajectory may be computed for each intended movement by specifying the boundary conditions (Nagasaki 1989). Conversely, the minimum-jerk model, as well as the generalized motor program theory, does not apply to complex movements which are unfamiliar in our daily life. The present study suggests that trajectory formation in the cyclic extension/flexion movements which are simple and familiar for every adult humans, could be accounted for within the context of the minimum-jerk model.

Symmetrical trajectory formation at high frequencies

The trajectory of the cyclic forearm movements was symmetrical for each subject at frequencies higher than 4.3 Hz. Interestingly, mechanical energy appears to have been reduced at a frequency greater than 3.3 Hz. The rhythmic sequencing or asymmetrical trajectory formation of extension and flexion in a movement cycle seems to become energetically unstable at high frequencies, and is thereby replaced by the other control mechanism which requires less mechanical energy at the same frequency. The jerk cost was also very close to the minimum predicted by the minimum-jerk model for cyclic motion.

What control mechanisms could be used for cyclic motion at these high frequencies? The increase in joint stiffness with an increased speed of movement (Feldman 1980; Hasan 1986) may be relevant to the mechanism underpinning movements at high frequencies. The modified minimum-jerk model of Wann et al. (1988) related the symmetrical trajectory formation and a decrease in jerk cost to increased joint stiffness in fast movements, compared with slow and relaxed movements with asymmetrical trajectories. Although the prediction of Wann et al. (1988) that trajectory becomes perfectly symmetrical with increasing speed and stiffness, did not hold true for their ellipse-drawing movements, their model suggests that neuromuscular dynamics at high frequencies could affect the empirical results of the present study.

Symmetrical trajectory formation at high frequencies does not mean that the motor system resembles a mass-spring which could be approximated simply by a sinusoidal linear oscillator. Kay et al. (1987) examined a cyclic extension/flexion wrist motion and argued that changes in the movement extent and peak velocity with increasing frequency can be explained by assuming a limit-cycle oscillation of a hybrid oscillator composed of the van der Pol and Rayleigh types. A contribution of the non-linearity to the forearm movements of the present study was evaluated by assuming the following equation of motion:

$$\begin{aligned} \ddot{x} + w^2 x &= G(x, \dot{x}) \\ G(x, \dot{x}) &= a\dot{x} + b\dot{x}x^2 + c\dot{x}^3 + dx\dot{x}^2 + ex^3 + fx \end{aligned} \tag{7}$$

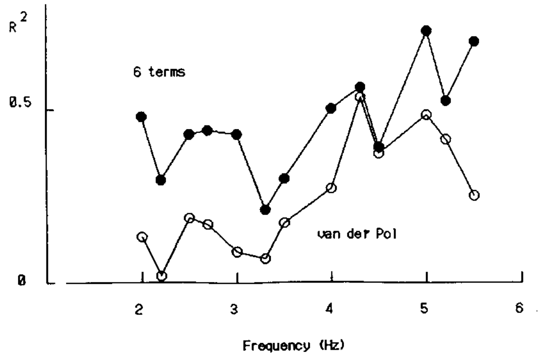

Fig. 7. Contribution ratio of the non-linear oscillators to the cyclic forearm movements (the ratio was based on the average of the five subjects at each frequency). ○ : the contribution from six non-linear terms in equation (7), ○: the contribution from a van der Pol oscillator

Here, x, $\dot{x}$ and $\ddot{x}$ are the angular displacement, velocity, and acceleration, respectively, and w is the frequency of the movement, $w = 2\pi/T$. The left side of equation (7) represents a harmonic oscillator and $G(x, \dot{x})$ is a function of the non-linear terms in which the second, third, and fifth terms are the van der Pol, Rayleigh and Duffing oscillators, respectively (Beek and Beek 1989). By using experimental data during one period of the movement, the multiple regression procedure gave an estimate of the contribution of the non-linear terms in $G(x, \dot{x})$ for every trial (Fig. 7). In every subject, the contribution in terms of R^2 by all terms in $G(x, \dot{x})$ of equation (7) increased more than 50% at high frequencies (above 4.3 Hz); at these frequencies, the contribution was attributed almost exclusively to a single van der Pol or Rayleigh oscillator. Thus, the limit-cycle oscillator model of Kay et al. (1987) may be valid only for fast cyclic motions, and the symmetrical trajectory formation at high frequencies may be largely non-linear, though the trajectory is seemingly sinusoidal. The non-linear oscillator controlling the fast cyclic movements may show that the movements were "mechanical", as contrasted with the rhythmic control of the movements at low frequencies.

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