WAVES AND INSTABILITIES IN DUSTY SPACE PLASMAS

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Abstract. Astrophysical dust occurs in many circumstances, like interstellar and circumstellar media, and between and around planets and comets. Typical Solar System applications include planetary rings, asteroid zones, cometary comae and tails, and regions of Earth's lower magnetosphere. Dust grains immersed in ambient plasmas are electrically charged by various processes and interact with electromagnetic fields. Intriguing phenomena observed in the 1980s by *Voyager* cameras and attributed to charged dust are radial spokes in the B-ring and braids in the F-ring of Saturn. Collective effects become important when the dust intergrain distance is smaller than the plasma Debye length, and start from observations that micron-sized dust grains can have very high negative charges and in proportion even higher masses. Characteristic dust frequencies are considerably smaller than corresponding electron or ion quantities, giving rise to new low-frequency eigenmodes, which could explain some of the low-frequency noise in space and astrophysical plasmas. Repelling electrostatic forces between charged dust grains prevent planetary rings from collapsing to very thin sheets, and oscillations in transverse ring thickness give rise to resonant phenomena, held responsible for gaps in the rings of Jupiter and Saturn. Further features are connected with fluctuating dust charges, which imply highly nontrivial source and/or sink terms in the description, and those in turn lead to new electrostatic and electromagnetic instabilities. Many different papers are reviewed which discuss waves and instabilities in dusty space plasmas, both with fixed and variable dust charges, at the linear level and, at the nonlinear level, involving double layers, solitons, vortices and other waves. These studies are at present far ahead of what observations can corroborate, a situation not likely to change soon due to the paucity of coming solar system missions concerned with planetary or cometary phenomena.

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1. Introduction

Dusty plasmas contain charged dust grains besides the electrons and ions we find in normal plasmas. It is often said that 99% of the universe is in the plasma state, and then dust would make up most of the remainder. The presence of dust in astrophysical environments has been known since a long time, from different type of remote observations, as for the dust around and between stars. Dust in the solar system, between and around planets and comets, has been detected also *in situ*. Closer inspection indicates that the transition from gas to large dust particles in astrophysics is almost continuous, from electrons and ions through macromolecules, clusters of molecules, very small or sub-micron sized grains, micron-sized grains, larger grains, boulders, asteroid remnants, etc. The most typical applications in the Solar System include planetary rings, different asteroid zones, cometary comae and tails, and even closer home certain regions of the Earth's lower magnetosphere.

Such dust grains are more often than not immersed in ambient plasmas and radiative environments, which lead to electric charging of the grains by various processes. Although this idea was put forward a long time ago by Spitzer (1941), it was only given much thought when the very successful space missions to planets and comets brought compelling evidence for that to happen, or at least threw up phenomena which could not otherwise be explained. The most intriguing from this point of view are the radial spokes in the B-ring of Saturn (Figure 1) and the braids in the F-ring (Figure 2), as observed in the 1980s by the *Voyager* cameras (Smith *et al.*, 1981, 1982). We will return to these later.

Once the dust is charged, it starts to react to electromagnetic as well as to ordinary gravitational effects. Hence there is a consequent coupling to the ambient plasma through the ubiquitous electromagnetic fields, pre-existent or self-induced, and this coupling tends to get stronger as the grain size decreases. Boulders in orbit around Saturn, even when charged, would still have their Keplerian motions

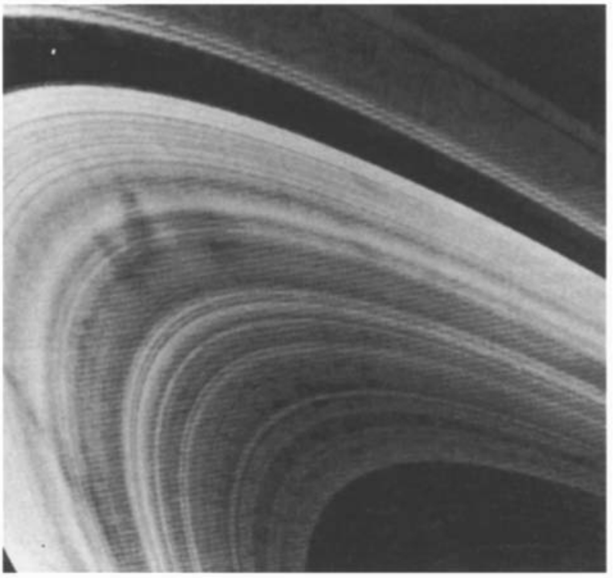

Figure 1. Images of spokes in the B-ring of Saturn. *Voyager 2* photograph. (Reprinted from Smith *et al.* (1982), Figure 42 on p. 534, copyright NASA.)

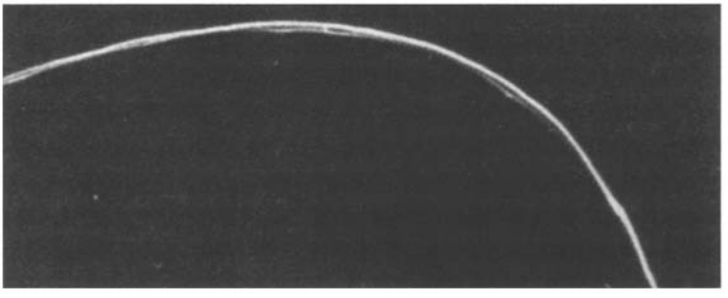

Figure 2. Braiding of the F-ring of Saturn. *Voyager 1* photograph. (Reprinted from Smith *et al.* (1981), Figure 43 on p. 189, copyright NASA.)

determined exclusively by gravitation, as electromagnetic forces are then far too weak to counterbalance these.

One can think of three characteristic length scales for such a combined dust and plasma mixture. First of all, there is a typical dust grain size a, then the plasma Debye length λ_D, and finally an average intergrain distance d, roughly related to the dust density by $d \simeq n_d^{-1/3}$. It turns out that for cosmic plasmas with dust there are two regimes, depending on the concentration of the dust grains. In any case the size of the dust grains is the smallest of the three lengths. One can treat the dust from a particle dynamics point of view, provided $a \ll \lambda_D < d$, and in that case we speak of a plasma containing isolated screened dust grains, or of dust-in-plasma. On the other hand, collective effects of the charged dust become important when

$a \ll d < \lambda_D$, and for this latter case we would prefer to reserve the name of dusty plasmas, as now the dust is an essential constituent.

Concerning dusty space plasmas as such and various other aspects of these, besides collective waves and instabilities, we would refer to recent and eminent overviews like those by Goertz (1989), de Angelis (1992), Northrop (1992) and Mendis and Rosenberg (1994). It is certainly not the aim of the present review to repeat much of what was said in those papers, but to expand upon the area of collective phenomena which was only a minor part in their treatment. Nevertheless, some of the notions exposed in more detail by Goertz (1989), de Angelis (1992), Northrop (1992) and Mendis and Rosenberg (1994) will have to be recalled here, in order to fix the ideas and to let the reader follow without too many difficulties. This will be done in Sections 2 and 3, before we address collective modes and the many papers which have described aspects of these.

Detailed references to these papers will be given in the relevant sections, as they are far too numerous to be listed at this place. We would also like to remark that for the intricacies of the mathematical and numerical treatments we refer the reader to the original papers themselves. We will only give the minimal analytical expressions needed to set the stage and follow the ideas, so as to concentrate on the physical mechanisms contained in these papers. Attention is also drawn to several conference reports devoted to dusty plasmas (de Angelis *et al.*, 1990, and the topical section of *Physica Scripta* **45**, 465–544, 1992).

2. Charging of the Dust

The first question to be tackled is how the dust grains get their charges, because that will have important consequences for the wave phenomena we would like to describe. Taking a single grain at the time, the customary answer is to write down an equation of the form

$$\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} C(\varphi - \overline{\varphi}) = I, \tag{1}$$

where q is the charge of the dust grain under consideration, C the grain capacitance, φ the grain surface potential, to be compared then with the average plasma potential $\overline{\varphi}$, and I the total current to the grain. The latter is itself a sum over many possible physical processes. In decreasing order of importance for space plasmas these are: primary electron and ion collection, photo emission, secondary electron current, electric field emission, ion induced secondary current, electric field emission, electrostatic disruption, and others (Goertz, 1989; de Angelis, 1992; Northrop, 1992). The results of this exercise differ considerably, whether one talks of isolated grains or of grains which participate in the plasma Debye shielding as a whole. It turns out that whereas isolated grains can collect a lot of electrons, the charges are much reduced if there are many grains inside a Debye sphere. As the electrons are much more mobile, we get in first instance negative grain charges.

One should immediately add at this point that for the treatment of waves and instabilities in space plasmas, as far as the papers we will review are concerned, the description of the charging is usually restricted to the primary electron and ion currents. These can be modelled in a fairly straightforward way by viewing the dust grain as a spherical probe stuck in the plasma, and then applying standard probe theory. Specifically for grains imbedded in an electron-ion plasma the primary currents are (see, e.g., de Angelis, 1992)

$$I_e = -\pi a^2 e n_e \sqrt{\frac{8\kappa T_e}{\pi m_e}} \exp \frac{e\varphi}{\kappa T_e},$$

$$I_i = \pi a^2 e n_i \sqrt{\frac{8\kappa T_i}{\pi m_i}} \left(1 - \frac{e\varphi}{\kappa T_i}\right). \tag{2}$$

Here e is the unit charge, n_e and n_i are the densities, m_e and m_i the masses, and T_e and T_i the temperatures of the electrons and ions, respectively. In what follows, subscripts e and i will refer to the corresponding electron and ion (protons usually) quantities, with the label d reserved for the dust quantities, when we restrict ourselves to the simplest model for a dusty plasma. By setting the sum of the electron and ion currents equal to zero, we find the equilibrium grain surface potential φ_{eq}. This then yields the equilibrium dust charge through the capacitance relation (Goertz and Ip, 1984; Whipple *et al.*, 1985; Houpis and Whipple, 1987)

$$q = 4\pi\varepsilon_0 a \varphi_{\text{eq}}. \tag{3}$$

Little or no attempt has been made to include other currents, always speaking of the description of waves in dusty plasmas as found in the papers discussed in Sections 6–11. Hence the reader is referred to the already cited general reviews (Goertz, 1989; de Angelis, 1992; Northrop, 1992; Mendis and Rosenberg, 1994), where expressions for other charging currents are given.

In addition, one could accept, out of necessity, the standard probe model as a reasonable first approximation for electrostatic waves in unmagnetized plasmas. Unfortunately for the modelling, almost all space plasmas are magnetized, and one would need to adapt probe theory to include a static magnetic field, especially when dealing with electromagnetic waves. None of the papers reviewed further on have done that, presumably because of its intrinsic difficulties.

3. Grain Dynamics, Planetary Rings, and Comets

Coming back to the usual picture, we are now discussing grains with rather more charges than ions usually carry. Hence, electrostatic repulsion of like charges on the same grain could lead to disruption of the grain itself when this repulsion exceeds the tensile strength of the grain. More important, since there tend to be many grains with similar charges, we also have electrostatic repulsion between the grains themselves. When we are discussing the structure of planetary rings or of cometary tails, this effect could lead to electrostatic levitation out of the equatorial plane and to a finite thickness for the rings.

For the motion of a single grain near a planet with an intrinsic magnetic field one starts from the equation of motion

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{q(t)}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{GM_p}{r^3}\mathbf{r}. \tag{4}$$

Here $\mathbf{v}$ refers to the velocity of a grain with position vector $\mathbf{r}$ from the centre of the planet, $q(t)$ and m to the (variable) charge and mass, $\mathbf{E}$ and $\mathbf{B}$ to the electric and magnetic fields and M_p to the mass of the planet. G is the gravitational constant. Only the Lorentz and gravitational forces have been written down. One could easily add additional forces like collisions between grains and plasma, radiation pressure (which seems to be negligible in Jovian and Saturnian magnetospheres, but could be important for parts of the terrestrial magnetosphere), collisions with other dust grains, but these are usually not the dominant factors (Northrop, 1992).

When we transform the equation of motion to a frame which is co-rotating with the planet, such that the velocity becomes $\mathbf{w} = \mathbf{v} - \boldsymbol{\Omega} \times \mathbf{r}$, we find that

$$\frac{\mathrm{d}'\mathbf{w}}{\mathrm{d}t} = \mathbf{w} \times \left(\frac{q(t)\mathbf{B}}{m} + 2\boldsymbol{\Omega}\right) + \boldsymbol{\nabla}\left(\frac{1}{2}|\boldsymbol{\Omega} \times \mathbf{r}|^2 - \frac{GM_p}{r}\right). \tag{5}$$

The electric field was assumed to be generated by the rotation of the planetary magnetic dipole field, and hence dropped out of the transformed equation.

Nevertheless, this equation cannot hold in the same form for planets where the dipole field is a poor approximation to the real field, as the quadrupole or higher order contributions will invalidate this too rosy picture, or when the magnetic dipole is not aligned with the rotation axis. So for Saturn we would assume that the above description would be a fair approximation, as Saturn's magnetic field may adequately be represented by a dipole aligned with the rotation axis but with a small offset to the north, whereas Jupiter's magnetic field is tilted by about 10° from the rotation axis. Moreover, electric currents along field lines connecting Jupiter and Io and in a current sheet around Jupiter's equator distort the magnetic field sufficiently so as to make a simple offset dipole field a poor representation. Even worse, both Uranus and Neptune have magnetic fields which are highly tilted with respect to their rotation axes and offset from their centres by large fractions of the planetary radii. In particular, the 60° tilt of the Uranian dipole field gives rise to a rather messy picture of the magnetosphere with a marked rotational asymmetry, especially as the rotation axis of Uranus lies itself almost in the ecliptic (Anderson and Kurth, 1989).

It is clear from the equations of motions for a single grain that there will be a competition between gravitational and electromagnetic effects. For too small grains, the trapping by the magnetic field lines will dominate, much as it does for whatever plasma particles are present. Too large grains on the other hand will follow gravitationally bound orbits, without being distracted from them by electromagnetic forces. Only for medium-sized grains will there be a balance between both effects.

A related reasoning will hold for different grains, where there will be competition also between their gravitational self-attraction and their electrostatic self-repulsion. A simple estimate for this to occur would be when for like grains the Newtonian and Coulomb forces balance,

$$Gm_d^2 \sim \frac{q_d^2}{4\pi\varepsilon_0}, \tag{6}$$

giving an idea for the charge-to-mass ratios, which we compare to the proton or electron quantities:

$$\frac{q_d}{m_d} \sim \sqrt{4\pi G\varepsilon_0} \simeq 10^{-18}\frac{q_p}{m_p} \simeq 5 \times 10^{-22}\frac{q_e}{m_e}. \tag{7}$$

As the spokes in the B-ring of Saturn have been observed as being darker than the ring plane itself in back scattered light, but brighter in forward scattered light (Smith *et al.*, 1981), the conclusion has been that these could be imputed to charged dust grains of micron size. Moreover, the apex of the spokes is at synchronous orbit. This would tend to corroborate the idea that the dust grains essentially follow Keplerian orbits around the planet, whereas the plasma corotates with the planet, being trapped in the planetary magnetosphere. At the synchronous distance both the Keplerian and corotation frequencies would be equal, and then the $\mathbf{E} \times \mathbf{B}$ drift on the dust grains forces these grains away, inward inside and outward outside the corotation distance (Northrop, 1992).

Another effect which has been invoked is that repelling electrostatic forces between dust grains with like charges mean that planetary rings cannot collapse to very thin sheets, but would need retain a finite thickness transverse to the equatorial plane. The equilibrium thickness due to this effect would be important only for Jupiter's ring and some of Uranus' rings, and maybe for others too, if the uncertainties in the relevant parameters are taken into account. The dust charge depends on the dust properties, the plasma conditions and the radiation field. For many grains, the charges are reduced compared to the single grain picture, and the thickness of the rings is determined by a balance between the gravity component toward the central plane and the expanding electrostatic force on the dust grains. If the ambient plasma conditions are changed, natural oscillations occur in the thickness profiles, which become increasingly complicated for denser rings, and resonances may be found between these oscillations and the Doppler-shifted Keplerian frequencies.

These were studied by Melandsø and Havnes (1991) in electrostatically supported dust rings around planets. The description is essentially based upon a single test-particle approach to determine the charge on each grain, but uses an equilibrium vertical dust density distribution. It is found that such oscillations occur at a frequency $\Omega_K(L)\sqrt{3}$, with $\Omega_K(L)$ being the purely Keplerian frequency at distance L from the planetary centre. For the tenuous clouds under consideration, the oscillation frequency is practically unaffected by charging delays, so that the dust charges can be supposed to remain constant. There is now scope for resonant phenomena to occur between the ring oscillations and plasma features in the magnetosphere which corotate with the planet. We need integers ℓ and n such that

$$\mp n(\Omega_K(L) - \Omega_{CR}) = \ell\sqrt{3}\,\Omega_K(L). \tag{8}$$

This leads to specific distances

$$L_{n,\ell} = L_{CR}\left(1 \pm \frac{\ell}{n}\sqrt{3}\right)^{2/3}. \tag{9}$$

Here $\pm$ refer to whether we are outside or inside the corotation distance L_{CR}. Such resonances are claimed to be responsible for certain gaps or prominent features in the ring structures at Jupiter and Saturn, as shown in Figure 3. The whole treatment, of course, relies upon the dust being effectively charged rather than neutral. A more definite observational answer will have to wait until the Cassini–Huygens satellite is in orbit around Saturn.

As there is little primordial dust, there is need for a constant regeneration of it by impact of meteoroids, comets... Here the predictions are that the impact of Comet P/Shoemaker–Levy 9 on Jupiter should produce a new dust ring around the planet, but it is of course too early to really know!

On the other hand, cometary missions to 21P/Giacobini–Zinner and 1P/Halley have shown that the smaller grains are not distributed in a symmetrical fashion with respect to the orbital plane of the comet, as we would expect if only pure radiation pressure is taken into account, but that the grains tend to gather preferentially to one side. This could be due to a downward push on negative grains, induced because the magnetic field and velocity of the solar wind are not aligned. Furthermore, a turbulent drag in high velocity solar wind streams could produce narrower dust tails, whereas in lower velocity streams gravity, radiation pressure and expansion dominate to give wider tails (Havnes, 1988).

Dust is also supposed to play a crucial role in the formation of stars, solar or planetary systems. In particular, the recently deceased Alfvén (1981) was a great advocate of a solar system formed out of a dusty plasma. If that is truly the case, then there is a pressing need to self-consistently adapt Jeans' instability criterion to the presence of charged grains. We refer to Section 11 where the interest in self-gravitational effects is reviewed in more detail, although there are at present not enough theoretical works dealing with this very important question.

4. Collective Phenomena

When we now turn to collective phenomena, we start from observations which suggest that typical micron-sized dust grains can have very high negative charges,

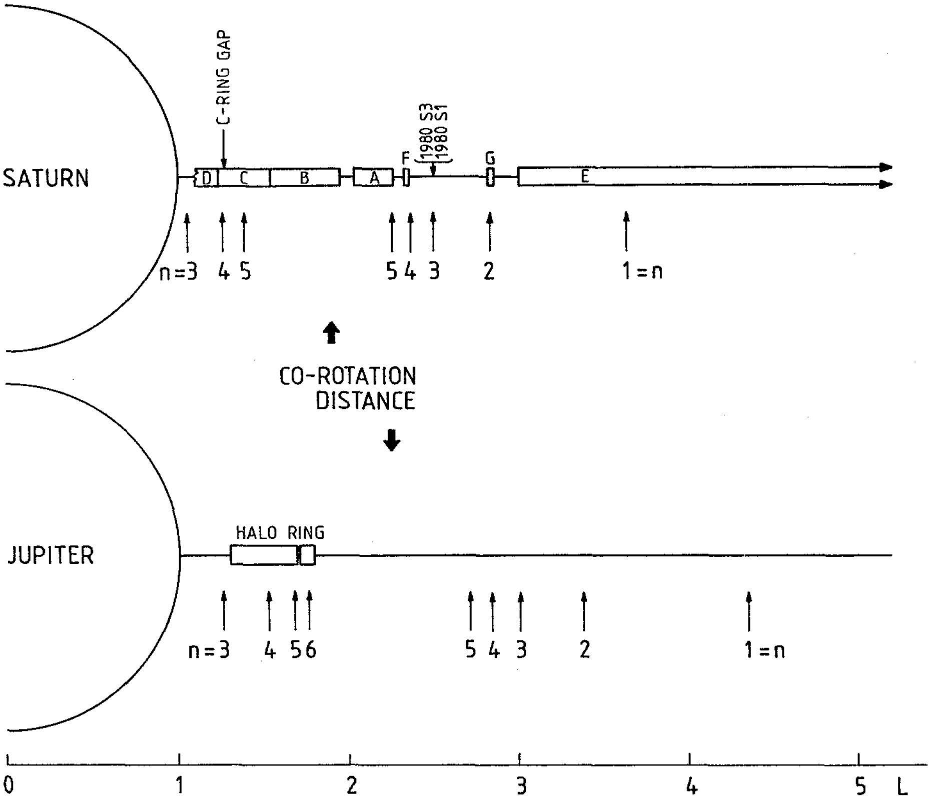

Figure 3. Radial distances of the major ($m = 1, , n = 1, 2, \ldots$) inner and outer resonances of Jupiter and Saturn, shown together with major ring features. (Reprinted from Melandsø and Havnes (1991), Figure 3 on p. 5842, with permission of the American Geophysical Union.)

up to 10^4 e, and in proportion to electrons and ordinary ions even higher masses, some $10^6 \sim 10^{18}$ m_p (de Angelis, 1992). We will, in what follows, use averages for dust grains with similar characteristics, which is of course far from being true but can serve to fix the ideas. We also have to point out that a self-consistent treatment of charge and mass distributions of grains of different sizes is not yet available. There are several immediate consequences of such typical grain charges and masses.

One is that the frequencies characteristic for the dust components are considerably smaller than the corresponding electron or ion quantities. There is the dust plasma frequency ω_{pd} defined through

$$\omega_{pd}^2 = \frac{N_d Q_d^2}{\varepsilon_0 m_d} = \frac{Z_d(N_i - N_e)e^2}{\varepsilon_0 m_d} = \frac{Z_d m_i}{m_d}\omega_{pi}^2 - \frac{Z_d m_e}{m_d}\omega_{pe}^2, \tag{10}$$

with $Q_d = Z_d e$ and N_d referring to the equilibrium charges, charge numbers and number density of the dust. We will later have to look at the variations in the charges themselves due to fluctuations of different kinds in the plasma as a whole. Similarly, there is the dust gyrofrequency in absolute value

$$\Omega_d = \frac{|Q_d|B_0}{m_d} = \frac{Z_d e B_0}{m_d} = \frac{Z_d m_i}{m_d}\Omega_i. \tag{11}$$

For both frequencies, the main reason why they are so much lower than in an ordinary hydrogen plasma is the unusually small charge-to-mass ratio of the charged dust. Such low frequencies will give rise to new low-frequency eigenmodes of the combined plasma, besides effects on existing eigenmodes.

A second change from ordinary plasmas comes from the fact that in order to maintain charge and current neutrality in equilibrium, there are more free protons than electrons, and this will also affect wave phenomena, sometimes to a considerable extent. At strong enough electron depletion, the electron and ion plasma frequencies could become of the same order of magnitude.

Finally, and this has proved to be the main stumbling block for the theoreticians to arrive at proper self-consistent treatments, the outstanding distinction with ordinary plasmas is that the dust charges are not fixed at all but can fluctuate with perturbations in the plasma potentials responsible for the charging. So the efforts have gone towards describing the charged dust in first instance as one or more negative-ion fluids, sometimes with variable charges, besides the classic electron and proton fluids. Of course, that electrons and protons can be given up or captured by the dust means that source and/or sink terms have to incorporated in the different electron and proton equations, amounting to changes in densities, momenta and energies. The precise forms of these source/sink terms are highly nontrivial, but imply new electrostatic and electromagnetic instabilities, both at the linear and the nonlinear level.

5. General Framework

We will review most of what has been done during the last decade in respect to wave phenomena in dusty space plasmas, and then try to filter out of all the predictions made those which might really be relevant to particular astrophysical situations. Since most of the relevant work has been carried out in a fluid description, we shall have first of all need of the continuity equations per species

$$\frac{\partial}{\partial t} n_\alpha + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = S_\alpha, \tag{12}$$

where the index α gives the species under consideration, with density n_α and fluid velocity $\mathbf{u}_\alpha$. For this and other basic equations, we refer to any of the many textbooks on plasma physics, like the one by Booker (1984). We will use capital letters to refer to the equilibrium quantities of these. The source or sink terms S_α are not further specified for the time being, but are assumed to vanish in equilibrium for all species, and for the dust always. The argument for the last statement is that the dust number density is not affected by the dust loosing or picking up some charges. Coalescing or breaking up of the dust could be important but have not been incorporated in the treatments discussed further on.

Similarly, the equations of motion are

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\alpha \cdot \boldsymbol{\nabla}\right) \mathbf{u}_\alpha + \frac{1}{n_\alpha m_\alpha} \boldsymbol{\nabla} p_\alpha = \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) + \mathbf{M}_\alpha. \tag{13}$$

New quantities introduced here are the pressures p_α and the exchanges of momentum $\mathbf{M}_\alpha$ due to the variability of the dust charges, supposed zero in equilibrium. Obviously, such momentum exchanges will have to be retained also for the dust.

To close the set of equation, we turn to Maxwell's equations for multispecies plasmas

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} = \mathbf{0}, \tag{14}$$

$$c^2 \boldsymbol{\nabla} \times \mathbf{B} = \frac{\partial}{\partial t}\mathbf{E} + \frac{1}{\varepsilon_0} \sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha, \tag{15}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \sum_\alpha n_\alpha q_\alpha, \tag{16}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0. \tag{17}$$

Global charge and current neutrality in equilibrium would mean that

$$\sum_\alpha N_\alpha Q_\alpha = 0, \qquad \sum_\alpha N_\alpha Q_\alpha \mathbf{U}_\alpha = \mathbf{0}. \tag{18}$$

There is now also, due to the variability of the dust charges, an equation expressing conservation of charge in the plasma as a whole,

$$\frac{\partial}{\partial t} \sum_\alpha n_\alpha q_\alpha + \boldsymbol{\nabla} \cdot \sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha = 0, \tag{19}$$

which can be rewritten with the help of the continuity equations (12) as

$$\sum_{\alpha \neq \text{dust}} q_\alpha S_\alpha + \sum_{\text{dust}} n_{\text{dust}} \left(\frac{\partial}{\partial t} + \mathbf{u}_{\text{dust}} \cdot \boldsymbol{\nabla}\right) q_{\text{dust}} = 0. \tag{20}$$

In what follows, it is worth stressing that we will primarily be looking at those wave phenomena which have the most chances of clearly showing the influence of the dust. That is, either modes at the lower end of the frequency spectrum, where the dust can give some distinctive contribution, by modifying existing or creating novel waves, or else particular effects which have to do with the variability of the dust grains. We will try to distinguish each time between these approaches.

6. Linear Electrostatic Modes with Constant Dust Charges

6.1. Ion- and Dust-Acoustic Waves

To set the ideas, we recall that for the lowest-frequency electrostatic modes in ordinary plasmas the phase velocity is of the order of the ion-acoustic velocity, defined through (Stix, 1962)

$$\left(\frac{\omega}{k}\right)^2 = \frac{\kappa T_e}{m_i}. \tag{21}$$

The electrons provide the pressure and the ions the inertial response. An intuitive adaptation of this model will lead to dust-acoustic modes with phase velocities given by

$$\left(\frac{\omega}{k}\right)^2 = \frac{\kappa T_{\text{eff}}}{\langle m_d \rangle}. \tag{22}$$

Hence the dust-acoustic velocity incorporates both an effective temperature for the lighter species, now electrons and protons, and an average dust grain mass, representing the inertia of the much heavier species. In space plasmas the conditions (de Angelis, 1992) mostly are such that

$$T_e \gg T_i \gg \frac{T_d}{Z_d}, \tag{23}$$

$$m_e \ll m_i \ll \frac{m_d}{Z_d}, \tag{24}$$

$$N_e \sim N_i \sim Z_d N_d. \tag{25}$$

One of the first to treat collective modes in what was then called a microparticle plasma cloud were James and Vermeulen (1968), elaborating on earlier theoretical work for linear multispecies waves (see, e.g., Verheest, 1967). Since the particle sizes vary over quite a range, a cold plasma multispecies model is used, where each fluid represents particles of a particular radius and with like equilibrium streaming. Plasma waves studied in this framework lead to the possibility of wave damping in a multistream model akin to macroscopic Landau damping. No particular space applications were intended.

Much later the attention returned to dusty plasmas, at first for linear electrostatic modes where the dust species are assumed to have constant charges. The dust is treated either as an immobile neutralizing background (as the protons were in the first descriptions of electron plasma oscillations) or as one or more additional species of negative ions, but with very much lower characteristic frequencies.

In the first category, with constant dust charges, we find work by de Angelis *et al.* (1988) on ion-acoustic waves in an unmagnetized plasma. The massive, immobile but charged dust grains are surrounded by a statistical distribution of plasma particles. The choice of ion modes for this first look at the plasma response was motivated by low-frequency electrostatic noise enhancement associated with Halley's comet, as observed by the *Vega* and *Giotto* space probes in the region of increased dust, with a peak below the ion plasma frequency. Although the data could also be interpreted in terms of dust impact on the electrical probes, the experiments were sufficiently different on both *Vega* satellites to make it clear that at least part of the noise enhancement is really due to electrostatic modes, trapped in the dust region with increased amplitudes.

The authors did not try to get quantitative agreement with the observations, but only looked for a possible mechanism for the observed phenomena, a remark which can be addressed to many papers in the field. Clearly the motivation for the work comes from astrophysical observations, but the work in itself is only qualitatively and in a rather intuitive way connected with dusty plasmas in space.

Goertz briefly touched upon electrostatic modes in the presence of drifting dust grains in planetary environments in his general review (1989), leading to the familiar two-stream instability. In particular, there is a reference to previous work by Bliokh and Yaroshenko (1985) on electrostatic waves in Saturn's rings. These authors had considered a multistream model to account for the very many narrow rings and gaps in the B-ring. Waves of dust charge density are at the same time waves of material density, leading to a possible connection with the observed spokes. The model is very close to the pioneering but apparently forgotten paper by James and Vermeulen (1968), as Bliokh and Yaroshenko (1985) did not mention this reference. Although the modes considered cannot be strong, the idea of dust-driven plasma waves provides an intriguing mechanism for ion heating at the expense of the gravitational energy of the orbiting dust grains.

The real start of dust-acoustic mode research was given by Rao *et al.* (1990), who included the dynamics of a tenuous dust fluid but used Boltzmann distributions for the electrons and the ions. They coined the name for the novel dust-acoustic modes at low enough frequencies and with phase velocities far below the ion-acoustic velocity. Due to the wide gap in frequencies, these waves are quite different from the familiar ion-acoustic modes, which can also exist in dusty plasmas, with modifications due to the resulting imbalance between the number of free electrons and ions. The nonlinear development as compressive and rarefactive supersonic solitons is discussed in Section 10.

Similar results were obtained by d'Angelo (1990), who looked at the electrostatic ion and dust cyclotron and ion-acoustic modes. The connection to earlier results on wave modes in plasmas containing negative ions or in plasmas with two positive ion species is given. Later, Shukla (1992) reviewed linear low-frequency electrostatic and electromagnetic modes. The possible relevance of these results to astrophysical and cometary plasmas is pointed out, without going into specific details. In a companion paper, Shukla and Silin (1992) describe yet another new wave, due to electron depletion and high dust charges, in a model where the dust grains form an immobile background.

By averaging over a random distribution of dust particles, Forlani *et al.* (1992) have shown how plasma fluctuations are modified and point to the possibility of wave damping due to beating of the wave with the dust density fluctuations. This has nothing to do with damping due to variable dust charges, which was not included but will be discussed in the next section. Similar reasoning hold for the electromagnetic part of the dispersion law. Also Salimullah and Sen (1992a) examined the dielectric properties of a plasma studded with charged dust. The resulting inhomogeneous electric field significantly influences the dispersion properties of the plasma, even though the plasma Debye length is much smaller than the average grain separation. At low frequencies important modifications of the ion-acoustic branch occur, as well as a new ultra low-frequency mode arising from ion oscillations in the static dust distribution.

Further to the same, Salimullah *et al.* (1992) then include a static magnetic field. It is found that the combined presence of dust and magnetic field has a significant effect for perpendicular wave propagation, leading to an electrostatic ion Bernstein mode in a dusty plasma.

A standard Vlasov approach to ion- and dust-acoustic instabilities was given by Rosenberg (1993). By considering some weighting of the dust at different sizes and charges, one can find that the charge goes as the radius a of the dust grains, whereas the mass goes as a^3. Hence if $N_d \sim a^{-p}$, then the plasma frequency squared goes as a^{-p-1}, with a consequent weighting toward grains of smaller radius. This is one of the few papers in which grains of different sizes are discussed. In a later paper, Chow and Rosenberg (1995) investigate the electrostatic ion cyclotron instability, and show that the critical drift, needed to excite the instability, decreases as the negative dust charge density increases. As an example, the current system that connects Jupiter with its satellite Io is given (for details of the system see, e.g., Grün *et al.*, 1993). In the absence of dust measurements at Io, the detection of the electrostatic ion cyclotron instability would signal the presence of negative dust, as without it the mode would be stable under the conditions prevailing near Io.

6.2. Two-Stream Instabilities

The two-stream instability has also received attention, and Havnes (1980) already examined the conditions for the onset of the instability in a plasma model in which two charged dust distributions stream relative to each other, akin to the Buneman instability. Small grains (below micrometer) are brought to rest with respect to the surrounding gas in very short distances, while larger grains are practically unaffected. This braking mechanism is applied to interstellar gas clouds, for which one observes a large depletion of several elements at low cloud velocities, a depletion which decreases with cloud velocity. Hence this two-stream instability may be important for grain destruction in high velocity clouds, as it effectively brakes the small grains but leaves the larger ones intact, thereby creating two populations of grains which stream relative to each other with the cloud velocity. Collisions between the grains in the two populations may thus become sufficiently frequent and energetic to ensure the destruction of a large fraction of the total grain content. This could be important when shocks travel through interstellar gas, setting up grain separation and subsequent destruction. The destructed grains are returned to the cloud in gas form, explaining why some elements are observed to be underabundant in the gas of low-velocity clouds but have near normal abundances in clouds of high velocity.

In a later paper (Havnes, 1988), streaming is considered between the solar wind and cometary dust grains. At the high dust densities which may be found in cometary comae, the dust grains can be important charge carriers, leading to electron depletion. The resulting instability drastically enhances the coupling between the solar wind and the dust, which favours small dust grains to be swept along with the solar wind plasma. A high neutral gas density prevents the onset of this instability. Turbulent drag could form dust striae in cometary tails, leading to narrower ones in a high velocity solar wind, whereas one would expect wider tails in lower velocity solar winds, where gravity, radiation pressure and expansion dominate. It is estimated that dust-solar wind interaction would be higher for comets at larger heliocentric distances, outside the inner part of the solar system.

Bharuthram *et al.* (1992) have investigated whether the dust grains just influence two-stream instabilities between electrons and ions, or else can themselves form a drifting beam. This could occur in planetary rings. In all cases the presence of dust grains enhances the growth of the instabilities, as well as the velocity ranges over which the instability can occur. Stationary dust, as in the magnetosphere of Neptune, also modifies the propagation properties, although then the planetary magnetic field might have to be taken into account.

In related papers, Rosenberg and Krall discuss drift instabilities (Rosenberg and Krall, 1994), when there is a local electron density gradient opposite in sign to a dust density gradient, or two-stream instabilities (Rosenberg and Krall, 1995). Different equilibria are reviewed, according to whether the dust gyroradius is smaller or larger than the density scale length. Electrons and ions are magnetically confined, whereas the dust grains would be electrostatically confined and a zeroth order electric field appears. Variation of dust charges has been omitted. As an application, it would seem that the edges of the ringlets in the F-ring of Saturn are stable against the excitation of this high-frequency mode, whereas the low-frequency two-stream instability could be of importance in the E-ring.

Finally, Winske *et al.* (1995) give a extensive numerical treatment of a dust-acoustic instability, caused by a drift of plasma ions with respect to the charged dust. The instability saturates by trapping some of the ions, and is slightly weaker when the dust grains have a range of sizes, charges and masses. It is argued that this process could contribute to ion heating and diffusion observed in the inner magnetosphere of Saturn.

7. Linear Electrostatic Waves with Variable Dust Charges

7.1. Fluid Description

The first to include variations in the dust grain charge due to collective modes in the plasma were Melandsø *et al.* (1993a). With hindsight, it is easy to sketch in general terms what will happen. For the continuity equations (12) the source or sink term can be supposed in first instance to depend on the density of the species under consideration, plus the dust charge, $S_\alpha = S_\alpha(n_\alpha, q_d)$, with in equilibrium no changes $S_\alpha(N_\alpha, Q_d) = 0$. If one linearizes and Fourier transforms the continuity equations (12) in such a model, one finds that

$$\delta n_\alpha = \frac{N_\alpha \mathbf{k} \cdot \delta \mathbf{u}_\alpha}{\omega - \mathbf{k} \cdot \mathbf{U}_\alpha + i\nu_\alpha} - i \frac{\mu_\alpha \delta q_d}{\omega - \mathbf{k} \cdot \mathbf{U}_\alpha + i\nu_\alpha}. \tag{26}$$

Besides the fact that there appear some *ad hoc* coefficients which eventually will have to be determined from a proper kinetic theory, there occur complex quantities, whereas in the absence of charge fluctuations everything is real. The equations of motion (13) similarly give rise to complex quantities and hence wave damping or unstable growth becomes the rule, if one analyzes the dispersion law. The phase difference between the dust charge variation and the wave potential for electrostatic modes can lead to strong damping. At sufficiently low frequencies, the phase shift is small, and hence there is little damping, even though the grain charge fluctuations could be important, as very little energy is exchanged between waves and grains. At the highest frequencies, although the phase shift is large, there is hardly any damping as the charging or decharging of the dust cannot follow! It is at the middle frequencies, of the order of the charging frequencies, that the strongest damping occurs.

The treatment of Melandsø *et al.* (1993a) deals with dust-acoustic waves, but is not yet fully self-consistent, as the issue of sink and source terms in the electron and ion continuity equations is sidestepped by assuming these species to be Boltzmann distributed. Nevertheless, they were able to define a charging frequency, the inverse of the charging time of a dust particle. The damping of the wave is then due to the delay in the charging of the dust particles, and in later papers by other authors referred to as Tromsø damping.

Several papers then follow and embroider on these ideas. Varma *et al.* (1993) study electrostatic oscillations and instabilities, by treating the dust charge as an additional dynamical variable. Dust charge fluctuations give rise to two purely damped modes, in addition to causing a collisionless damping of existing normal modes, much as in the recombination process in partially ionized plasmas. Furthermore, some interesting electrostatic instabilities arise in the presence of an equilibrium drift of the charged particles. At the end, a brief allusion is made to the incorporation of loss terms in the ion and electron momentum equations, although no explicit results are presented. Similarly, Jana *et al.* (1993) discuss charge fluctuations in response to oscillations in the plasma currents flowing to the grains, but also not fully self-consistently. Dissipative and instability mechanisms for ion and dust waves in the plasma lead to interesting applications in laboratory and astrophysical situations. One of these is that there could be anomalous transport of grains due to a turbulent spectrum of waves. Related phenomena are described by quantum chromodynamics in quark-gluon plasmas (Kaw, 1992).

Besides Tromsø damping, d'Angelo (1994) includes another mechanism, that of creation damping due to the continuous injection of fresh ions in the plasma, to replace those that are lost on the dust grains. Such fresh ions do not share initially in the wave motion of the existing ones and hence lower the average momentum of the ion population. Creation damping had been considered much earlier by d'Angelo (1967) in discussing recombination instabilities in ordinary plasmas. Creation damping is usually dominant, except for such low-frequency motions that the grain dynamics becomes important. Then inertia is provided by the dust grains, and ion momentum matters little.

Losses in electron and ion number densities are represented by Bhatt and Pandey (1994) through two attachment frequencies, represented schematically by ν_α and μ_α in (26). Charge variations occur, due to changes not only in electron and ion currents falling onto the grains, but also in electron and ion densities themselves. Various approximations are given for dust- and ion-acoustic modes, as well as for the streaming instability. The results are consistent with other earlier papers and lead to mode damping.

Similar self-consistent descriptions were given by Ma and Yu (1994a), when the dust is an immobile background but still fluctuating in charge. The associated damping could easily surpass Landau damping. One of the drawbacks of the treatment, as with many similar ones which rely on standard probe models for the charging of the dust, is that it corresponds to a thermodynamically open system in which the dust grains can absorb electrons and ions indefinitely. Although charge is conserved in this way, the total number of electrons and ions is not, and a source for these particles must exist. High-frequency Langmuir waves become unstable when the ions and electrons stream with respect to the dust, but on rather long timescales (Ma and Yu, 1994b). This resembles the ionization instability, and because of the open nature of the system it is not clear how the instability saturates.

7.2. Kinetic Theory

Melandsø *et al.* (1993b) have given a kinetic model for dust-acoustic waves applied to planetary rings, which can have thicknesses of up to 100 km. They include both Landau and Tromsø damping or growth, in addition to velocity differences between dust and plasma, as dealt with by Rosenberg (1993) who used constant dust charges, however. As in their previous paper (Melandsø *et al.*, 1993a), the electron and ion Vlasov equations do not contain sink or source terms. The instability criterion of this dust-acoustic mode is applied to planetary ring systems, leading in the most likely case to mode damping. For certain parameter regimes there is competition between Landau and Tromsø damping. Resonances with the satellite or corotational perturbations would lead to clumping of the rings.

Also Li *et al.* (1994) looked at longitudinal waves in plasmas where the dust forms an immobile background but has variable charges. There is damping for ion-acoustic and electrostatic ion cyclotron waves, whereas high-frequency Langmuir waves grow. The damping or growth rates are proportional to the dust charging frequency. One would also have to include friction between charged and neutral dust for certain planets, and later also magnetic effects on the charging of the grains.

An interesting new development comes from Aslaksen and Havnes (1994), who assume the dust grains to be of one size but with a discrete distribution of electric charges, in other words as heavy ions with a large number of ionization levels. Dust charge thus becomes an additional phase space variable. Classically, microscopic velocity fluctuations around an average result in pressure terms in the macroscopic fluid equations. Similarly, fluctuations in dust charges will give additional terms. The absorption frequency of electrons and ions by the dust is comparable to and often even larger than the frequency of typical dust-acoustic modes. Hence, transitions between the dust ionization levels are of great importance, and described by a Balescu–Lenard collision term. Sink or source terms in the momentum equations were omitted, in order not to burden the computations too much. Analytical and numerical estimates are given for the relaxation times both towards a Maxwellian distribution and towards an equilibrium distribution for the ionization levels. Summing over the different ionization levels gives a hierarchy of charge-moment equations for dust density function, and yields estimates of the importance of terms coming from the ionization distribution. As an aside, in plasmas with large dust grains the coupling between the grains becomes very strong and might lead to crystallization of the plasma, as observed in recent experiments (Thomas *et al.*, 1994). The work discussed here is one of the first to tackle the problem of fluctuating dust charges at the microscopic level, and while far from complete gives already estimates of what can happen. In particular for space plasmas, there will be need to extend the treatment to include the interplay between a nonzero electromagnetic field and the distribution of the dust grains.

Finally in this section, Vladimirov (1994b) investigates the propagation of electromagnetic and Langmuir waves, taking the effect of capture of plasma electrons and ions into account, extending thereby the recent kinetic theory of Tsytovich and Havnes (1993). The new wave damping leads to a lowering of the frequencies.

8. Linear Electromagnetic Waves with Constant Dust Charges

For electromagnetic waves the most characteristic velocity is the Alfvén speed V_{A} defined through (Stix, 1962)

$$V_{\mathrm{A}}^2 = \frac{B_0^2}{\mu_0 \sum_\alpha N_\alpha m_\alpha}. \tag{27}$$

For ordinary plasmas the mass is in the ions, and the Alfvén velocity is essentially determined by them. However, for dusty plasmas the mass density is much higher, as the dust is so massive, even at small number densities. There is a consequent lowering of the Alfvén velocity, which is the main effect of including charged dust, if variations of the dust charges are not taken into account. One of the first to treat the effect of charged dust grains on the propagation and dissipation of Alfvén waves in interstellar clouds were Pilipp *et al.* (1987), as Alfvén waves are the slowest decaying, showing no nonlinear steepening. Although their model included collisional coupling between the different fluids, the dust grains were assumed to be either neutral or singly charged, without wave induced fluctuations in the charges themselves. The collisional coupling is of the standard form (Booker, 1984), with in the equations of motion (13)

$$\mathbf{M}_\alpha = -\sum_\beta \nu_{\alpha\beta}(\mathbf{u}_\alpha - \mathbf{u}_\beta). \tag{28}$$

The two dust populations interact through the conversion of ionized grains to neutral ones by recombination of ions on the grain surfaces, or on the contrary by conversion of neutral to ionized grains by electron impact. The main effect of introducing charged dust then is to reduce the minimum wavelengths and increase the maximum frequencies at which the waves remain coupled to the neutrals.

de Angelis *et al.* (1992) analyze the scattering of electromagnetic waves by a distribution of fixed charged dust. When the intergrain separation is of the order or below the Debye length, the grains are not independent scatterers, so that the scattering cross section has to be calculated via their electrostatic coupling by statistical averaging. Some results shown in Figure 4. For typical dusty plasmas in the solar system, there is an enhancement with respect to scattering by free electrons, even in the regime of relatively small grain charges. The electrical nonneutrality of the plasma ($N_e \neq N_i$) means that the scattering cross section can be smaller for negative grains than for positive ones at the same absolute charge. As a consequence, negative grains experience a lower incident radiation pressure, which could lead to separation of the two charge components under the Sun's radiation and hence to cometary tail filamentation. There could also be enhanced radar backscattering from noctilucent clouds.

In his general discussion of dust modes, Shukla (1992) also studied low-frequency electromagnetic modes, showing that modes purely driven by the dust occur, as well as modified shear Alfvén waves. This theme was then picked up by Rao (1993a), for hydromagnetic waves and shocks. Electron inertia is neglected, but ion and dust motion taken into account, albeit at fixed dust charges. The modes considered are typical low-frequency ones such as Alfvén and magnetosonic waves, with suitable redefinitions. Shocks of magnetosonic waves driven by either upper-hybrid or O-mode electromagnetic waves are shown to exist, allowing a transition in the dust flow velocity from sub- to supermagnetosonic values. Such transitions

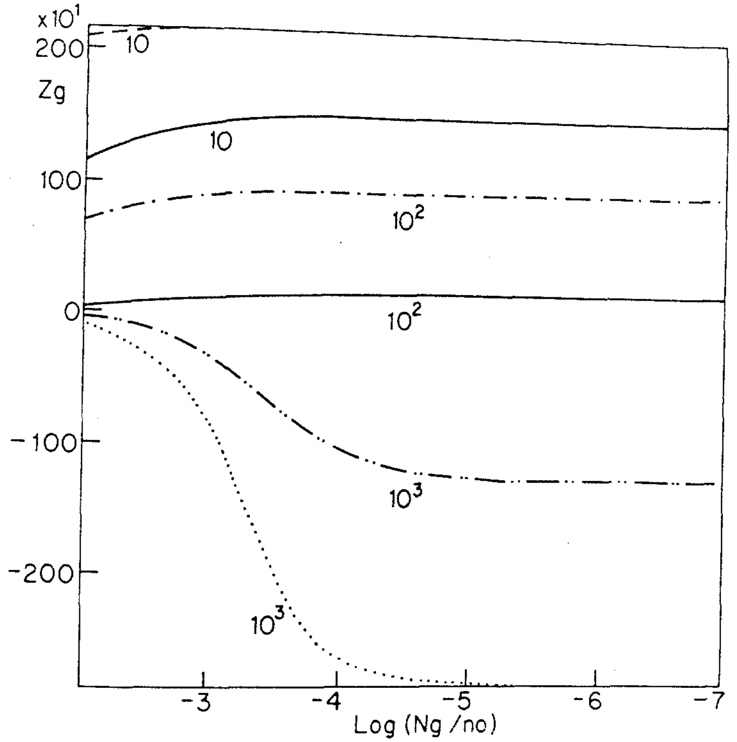

Figure 4. Grain charge number as a function of the ratio of grain to plasma density. The curves are labelled by the value of the plasma total density in cm^{-3}. The plasma temperature is 5 eV, the grain radius 10^{-6} m. (Reprinted from de Angelis *et al.* (1992), Figure 1 on p. 6265, with permission of the American Geophysical Union.)

could occur when charged dust clouds act as obstacles to the plasma flow, as near comets.

Next comes a more systematic treatment (Rao, 1993b), showing a new class of dust-magneto-acoustic waves, both of fast and slow type. The fast branch generalizes electrostatic dust-acoustic modes to magnetized plasmas. The wave frequencies for oblique modes are between the dust gyrofrequency (frozen-in dust) and the ion and electron gyrofrequencies. The departure from the frozen-in field approximation for the ordinary ions is given in a subsequent paper (Rao, 1993c). This is finally rounded off by a general overview of the different modes, including two new higher-frequency types of dust-magnetoacoustic waves, generalizations of dust-acoustic modes when a static magnetic field is included (Rao, 1995).

9. Linear Electromagnetic Waves with Variable Dust Charges

It then fell to Verheest (1994b) to look at charge fluctuation instabilities for electromagnetic waves, by taking into account the momentum changes due to the capture or release of plasma electrons and ions by the dust, albeit not in a fully self-consistent way. Damping of different modes is found, and especially for Alfvén waves equilibrium drifts between the dust and the ordinary plasma species can sometimes balance the charging losses. Verheest and Meuris (1995) extend the previous model in a self-consistent way, by describing the plasma species as moving in a static background of dust particles with variable charges. Use is made of momentum exchanges modeled on what happens for ordinary collisions (28). This leads to a kind of whistler wave with damping, but at lower frequencies than in the normal regime, with the ions responsible for the motion and the dust grains for the background.

Dust dynamics are included in the description by Reddy *et al.* (1996). When the charging and discharging effects conserve momentum between any two species as if they were ordinary collisions, the modes become unstable if there is enough global drift in the system. However, when the charging processes are more complicated, drift dissipative type modes are excited. Both type of modes could generate spatial structures over a wide range of hundreds to tens of million kilometres in dusty cometary tails.

10. Nonlinear Developments

10.1. Solitons

In their seminal paper, Rao *et al.* (1990) not only point out a novel dust-acoustic mode, but give its nonlinear development through a generalized Boussinesq equation, which reduces for unidirectional, slightly supersonic propagation to a generalized Korteweg–de Vries equation, with both quadratic and cubic nonlinearities. The solutions are compressive and rarefactive solitons, and the potential-dip solutions are unique to the influence of the dust, as ordinary ion-acoustic waves propagate as localized potential humps only. In an extension of the previous analysis, Verheest (1992b) considers a number of different dust fluids, showing that whereas rarefactive solutions can readily be found, compressive ones are not likely to exist.

Bharuthram and Shukla (1992c) investigate large amplitude ion-acoustic solitons, by treating the dust in two different ways, once as a fixed background, and once fluid-like. Numerical criteria for finite solitons lead to maximum Mach numbers for the soliton speeds. Both amplitudes and speeds of compressional solitons are much larger than in the absence of dust. In addition, rarefactive solitons have a comparatively larger amplitude still, as indicated in Figure 5.

A review of double layers and solitons in plasmas with constant dust charges has been given by Verheest (1993a). Canonical model equations include (modi-

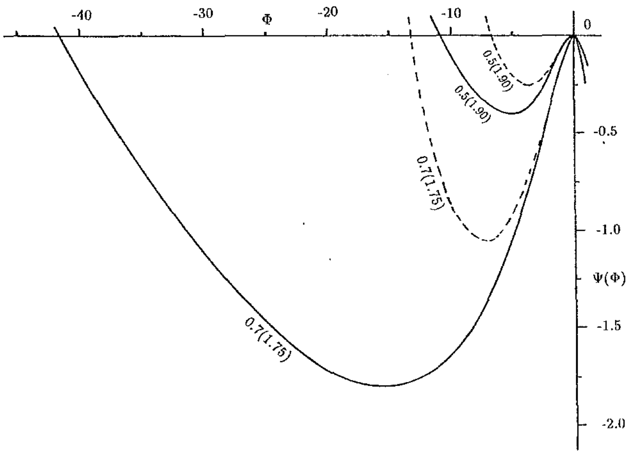

Figure 5. Sagdeev potential for rarefactive solitons. The parameter labelling the curves is the normalized electron density N_e/N_0, with the corresponding Mach number M shown in parentheses. The continuous (broken) curves are for stationary (moving dust) particles. (Reprinted from Bharuthram and Shukla (1992c), Figure 4 on p. 976, with permission of Elsevier Science, copyright 1992.)

fied) Korteweg–de Vries equations for electrostatic waves and derivative nonlinear Schrödinger equation for electromagnetic waves. The nonlinear propagation of ion-acoustic modes is investigated by Mofiz *et al.* (1993), when the intergrain distance is large compared to the plasma Debye length. The plasma perturbations leave the dust essentially as it is, thus avoiding the complications due to dust charge variations. An interesting conclusion is that ion-acoustic solitons are found for small grain charges, but increasing the grain charges render these solitons more unlikely and at large charges no solitons are possible.

Turning now to papers which include fluctuating grain charges, Rao and Shukla (1994) extend their original work (Rao *et al.*, 1990) on nonlinear dust-acoustic waves. For small but finite amplitudes the waves are shown to be governed by a driven Boussinesq-like nonlinear equation coupled to the charge fluctuation equation, similar to what was obtained by Varma *et al.* (1993):

$$\frac{\partial^2 \varphi}{\partial t^2} + A\frac{\partial^2 \varphi}{\partial z^2} + B\frac{\partial^2 \varphi^2}{\partial z^2} + C\frac{\partial^4 \varphi}{\partial z^4} = D\frac{\partial^2}{\partial z \partial t}\,\delta q_d. \tag{29}$$

For unidirectional propagation it reduces to a Korteweg–de Vries equation with a source term. The picture given here is not fully self-consistent, as the dust charges fluctuate but there are no corresponding sink or source terms in the plasma continuity equations. Localized solutions are found to be damped due to the fluctuations in the dust charges.

For nonlinear electromagnetic modes the treatment by Verheest (1994a) leads for low-frequency motions, typical for the dust components, to a derivative nonlinear Schrödinger equation, of the form

$$\frac{\partial \varphi}{\partial t} + A\frac{\partial}{\partial z}\left(|\varphi|^2\varphi\right) + iB\frac{\partial^2 \varphi}{\partial z^2} = 0. \tag{30}$$

At constant dust charges one can fall back on existing multispecies descriptions (Verheest, 1990, 1992a; Verheest and Buti, 1992; Deconinck *et al.*, 1993a, b). Equilibrium drifts and a first attempt to deal with fluctuating dust charges are included (Verheest, 1994a), although not in a truly self-consistent way, leading to extra terms in the equation. Fully nonlinear ion-acoustic solitary waves in impurity-containing plasmas are given by Yinhua and Yu (1994a, b) and by Popel and Yu (1995). The impurities can be either massive ions or charged grains, in plasmas without or with an external magnetic field. The permitted range of soliton speeds and widths is increased by the presence of grains with constant charges.

Lakshmi and Bharuthram (1994) return to the problem of large amplitude, rarefactive dust-acoustic solitons in a plasma with Boltzmann distributed electrons, ion species at different temperatures and dust grains with constant charges. The main difference with the previous treatment by Bharuthram and Shukla (1992c) is in the description of the ions, which are treated as isothermal and inertialess, as was done already by Bharuthram and Shukla (1992a) in their description of double layers, showing no really qualitative differences with earlier results.

At the end of this subsection, we would like to emphasize that the nonlinear evolution of electromagnetic modes in dusty plasmas with variable charges has not yet been satisfactorily investigated!

10.2. Double Layers

The interest in double layers and vortices is that the former could play a large role in the acceleration of particles, much as occurs in the Earth's auroral regions where double layers are believed to be responsible for the observed energetic electrons and ions (Raadu, 1989). Furthermore, models for solar flares have been advanced in which the triggering mechanism is a double layer related phenomenon. Hence the question whether the intense dust jets of comet 1P/Halley could be explained this way. Another question is if dust gets trapped in double layers or better still in vortices and possibly transported over large distances.

Large-amplitude electrostatic double layers are treated by Bharuthram and Shukla (1992a). As in many such papers, the equations have been written in the appropriate nondimensional units, relating to the dust, which makes quantitative comparisons between papers uneasy. Criteria for the existence of finite double layers are obtained numerically, as typified in Figure 6. As an extreme case, one could consider that most of the electrons have been collected onto the dust grains, as is believed to be a acceptable first approximation for the F-ring of Saturn. In that case, only compressive double layers are possible, supported by the ion non-isothermality, the dust providing the necessary inertia.

At the same time, Mace and Hellberg (1993a, b) discussed the effects of ion inertia on the existence of dust-acoustic double layers. All species of the plasma are described as fluids, instead of having the lighter species Boltzmann distributed and effectively inertialess. The regimes in which dust-acoustic double layers can

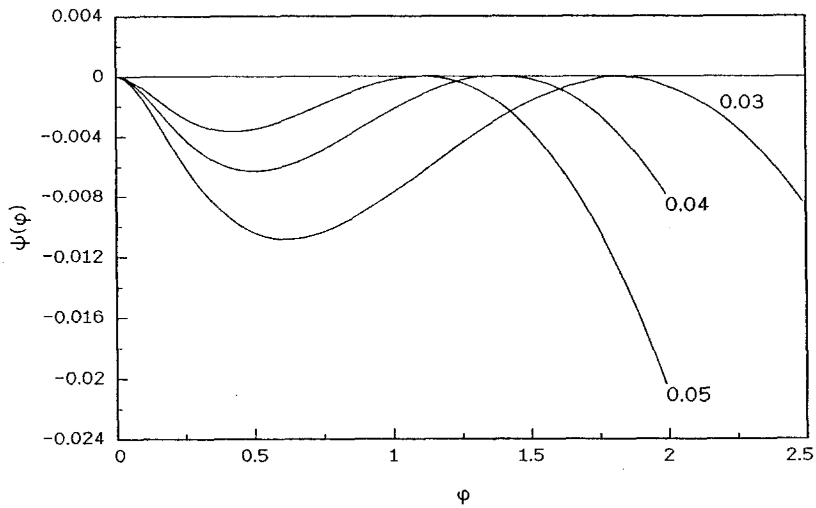

Figure 6. Typical form for the Sagdeev potential for $N_e = 0$ and $N_{i,cool}/N_{i,hot} = 0.11$. The parameter labelling the curves is the ratio of the cool to hot ion temperatures. (Reprinted from Bharuthram and Shukla (1992a), Figure 1 on p. 468, with permission of Elsevier Science, copyright 1992.)

occur are found to be much more restricted than with Boltzmann distributions. Significantly, highly nonlinear double layers are ruled out, placing rather strict restrictions on the neglect of ion inertia. Interestingly, the strongest double layer profiles can be expected when most of the electrons have been absorbed by the dust grains. This disagrees with conclusions drawn by Bharuthram and Shukla (1992a) for a similar problem.

Verheest (1993b) then tried to find out how well weak dust-acoustic double layers are described by modified Korteweg–de Vries equations, and more in particular whether the existence ranges correspond to what can be inferred from exact solutions. It transpires that weak double layer theory only gives valid descriptions in a very limited range. Sah and Goswami (1994) give a similar theory of weak dust-acoustic double layers, and include the case of nonisothermal electrons, described by modified Schamel equations. Only compressive double layers are found.

10.3. Vortices in Inhomogeneous Plasmas

Linear and nonlinear properties of low-frequency motion in inhomogeneous, magnetized, dusty plasmas are investigated by Shukla *et al.* (1991) and Bharuthram and Shukla (1992b). A new dust-drift wave is shown to exist, similar to impurity-drift modes. The nonlinear mode coupling equations indicate the possibility of solitary vortex structures, as might occur in astrophysical and cometary plasmas, in view of the claims that very low frequency drift-like waves have been observed near comet Halley (Grard *et al.*, 1986; Klimov *et al.*, 1986). The Kelvin–Helmholtz instability leads to circular dipolar vortices. These can trap and transport particles over large distances.

Purely damped convective cell modes acquire a real frequency in the presence of static charged dust grains (Shukla and Varma, 1993). This frequency is induced by the plasma density gradient and corresponds to charge density waves, the dynamics of which is governed by a generalized Navier-Stokes equation. The dust inhomogeneity provides the possibility of a two-dimensional dipolar vortex. In a further paper, Shukla *et al.* (1993b) show that Alfvén waves can be coupled to finite-frequency convective cell modes, leading again to dipolar vortices, with frequencies proportional to the gradient of the dust.

Shukla and Rao (1993) derive nonlinear equations for short-wavelength electrostatic fluctuations in the presence of density and magnetic-field aligned dust flow gradients. This is an extension of the work by Rawat and Rao (1993). Solutions in the form of dipolar vortices could be important in cometary tails where the cometary dust flows through the solar wind. Similarly, Lakshmi *et al.* (1993) investigate nonlinear potential structures, keeping the grain charges constant. The model follows that of Shukla *et al.* (1991), with the inclusion now of magnetic curvature effects. In addition to solitary dipole vortex solutions, modified convective cell vortex structures are shown to be possible in the complementary region of parameter space.

10.4. Modulational Instabilities and Plasma Masers

The modulational instability of dust-acoustic waves was investigated by Salahuddin (1993) with the help of a Krylov–Bogoliubov–Mitropolsky method, yielding a nonlinear Schrödinger equation of the form

$$i\frac{\partial\varphi}{\partial t} = A|\varphi|^2\varphi + B\frac{\partial^2\varphi}{\partial z^2}. \tag{31}$$

The instability criterion critically depends on the ratio of the ion to electron densities, whereas the dust charges, supposed to be constant, do not play a direct role. A plasma maser effect was described by Nambu *et al.* (1993) for stationary charged particulates, arising from the coupling between two types of waves. One is a resonant mode in Cherenkov resonance with the plasma, whereas the second is a nonresonant one for which neither the Cherenkov nor the scattering conditions hold. Dust-acoustic turbulence leads to more efficient acceleration of electrons, with much enhanced Langmuir and electromagnetic radiation, which might be important for high-frequency waves in space and astrophysical plasmas.

A different nonlinear interaction between a large-amplitude electron plasma wave and low-frequency perturbations is given by Shukla *et al.* (1993a), who obtain three-wave decay, as well as modulational and filamentation instabilities. de Angelis *et al.* (1994) describe the charged grains as a statistical background, where the electron plasma frequency could be substantially reduced, becoming even equal to the ion plasma frequency. The model of Forlani *et al.* (1992) is extended by looking at high-frequency waves, compared to the ion plasma frequency. By including the nonlinear current, nonlinear Landau damping or induced scattering is produced, as had been noted before by de Angelis *et al.* (1989) and Forlani *et al.* (1992). Beat waves are formed with phase velocities much lower than that of the propagating wave, and which can resonate with the electrons in the bulk of the distribution, thereby producing a very effective damping of the wave. Similar conclusion can be drawn for transverse waves with frequencies not much larger than the local electron plasma frequency. Effects due to variable dust charges are included by Vladimirov (1994a), so that capture of electrons by the dust amplifies the electromagnetic waves, rendering the plasma-maser instability more effective. Popel and Yu (1994) return to the modulation of finite amplitude waves by much lower frequency motions, which is the basic mechanism for the transition from weak to strong turbulence in ordinary plasmas, as plasma is expelled from regions of high wave energy by the ponderomotive force, due to much slower density perturbations. Now in dusty plasmas, even lower frequency motions are possible, at the level of dust-acoustic oscillations. The conditions studied by Popel and Yu (1994) are fairly typical and it is found that unstable Langmuir waves can resonate with thermal ions, yielding a mechanism for heating the bulk of the ions. Corresponding ion Langmuir solitons are also discussed. Resonances between electromagnetic waves and plasma slow motion incorporating dust charge perturbations are investigated by Ma *et al.* (1995) and Ma and Shukla (1995), who show that there is a large downshift of the resonance frequency for the slow dust-acoustic wave, but a much smaller one for the fast dust-acoustic wave. For the modulational and filamentation instabilities the thresholds are reduced.

10.5. Self-Similar Expansion

Several papers purport to address self-similar expansion of a dusty plasma in vacuum, starting with the one by Lonngren (1990), based upon the model proposed by Rao *et al.* (1990). Such expansions are essentially governed by the mass of the dust and the pressure of the plasma particles. Later, Luo and Yu (1992) gave a kinetic description, showing that the velocity distribution of the dust quickly becomes highly asymmetrical and narrow, leading to a vanishing dust density. Moreover, if dust grains of different masses and charges were present in an expanding plasma, they would rapidly be separated into beams of similar charge-to-mass ratios. By looking at the scaling effects, one can see that the plasma expansion is hindered by the massive dust grain inertia.

While previous papers address one-dimensional or planar expansion, Yu and Bharuthram (1994) work in a cylindrical geometry, to essentially reach similar conclusions. Later, Yu and Luo (1995) include charge variations, to find that these can significantly alter the expansion process. Also, the cooling effect arising from pressure variations is an important feature. In two related papers (Bharuthram and Rao, 1995; Rao and Bharuthram, 1995), not only temperature but also static magnetic field effects are investigated. Isothermal rather than adiabatic pressures lead to expansions over larger distances. As expected, the magnetic field constrains the expansion perpendicular to the field.

11. Other Modes

11.1. Kelvin–Helmholtz Instability

The Kelvin–Helmholtz instability is treated by d'Angelo and Song (1990), when adjacent plasma layers are in relative motion, but the dust is stationary. For negative dust grains, the critical ion shear increases without upper bound, whereas for positively charged grains it is reduced, so that excitation of the instability becomes easier. This could be important for wavelike motions in cometary tails, as it would seem that the sign of the charged dust grains is a function of the distance to the cometary nucleus. For comet 1P/Halley the grain potential varies from -8 V at a distance of 15 000 km, changes sign at about 30 000 km, and continues to increase with distance up to a value of about $+7$ V in the undisturbed solar wind (Ellis and Neff, 1990). Thus the stability conditions of various regions of cometary plasmas can be assessed.

Sheared positive or negative dust rather than ion flows are discussed by Rawat and Rao (1993), by also taking the dust dynamics fully into account, contrary to what was done earlier by Bharuthram and Shukla (1992b). The relative velocity between adjacent layers of dust should be at least equal to the dust-acoustic rather than to the ion-acoustic velocity, and the instability would hence be easier to excite. This could arise in comets where dust grains flow through the solar wind plasma. Nonlinear aspects of the instability, which might lead to vortex structures, were not investigated.

11.2. Rayleigh–Taylor Instability

A first treatment of the Rayleigh–Taylor instability was given by d'Angelo (1993). The presence of negatively charged dust considerably reduces the range of unstable wave numbers, while the opposite is true for positive dust grains. The dust charges were assumed constant, an assumption which is valid for wave frequencies much smaller or larger than the charging frequency of the dust. Selfconsistent charge fluctuations coupled to dust dynamics influence the Rayleigh–Taylor instability (Jana *et al.*, 1995), leading to a rapid decrease of the unstable regime. On the other hand, charge fluctuations can drive drift waves unstable, under conditions that might prevail in the tail of comet 21P/Giacobini–Zinner.

Varma and Shukla (1995a) then show that the dust grains directly respond to gravity, not really hindered by magnetic fields, whereas electrons and ions, which are magnetized, can do so only through the Rayleigh–Taylor instability. Static dust grains reduce not only the increment of this instability, but also the frequency of convective cells. For the nonlinear development similar conclusions hold, also when dust inertia is properly taken into account. In a companion paper, Varma and Shukla (1995b) study a novel flute-like instability that is different from the usual Rayleigh–Taylor instability. It comes about because in a nonuniform plasma, held in equilibrium by a magnetic field against gravity, there is a polarization electric field in the direction of gravity for negatively charged dust grains. The familiar $\mathbf{E} \times \mathbf{B}$ drift then induces this new instability, in combination with the positive current and compressible dust dynamics.

11.3. Self-Gravitation

The magnetogravitational instability of self-gravitating dusty plasmas is an important problem in understanding star formation. Alfvén (1981) already expressed the idea that the solar system was formed out of dusty plasma. While Jeans' instability criterion for the fragmentation of interstellar matter has been discussed including magnetic fields and rotation, the effects of charged grains had not been incorporated before. Whereas the dynamics of large astrophysical bodies like stars, planets and satellites is controlled overwhelmingly by gravitation alone, the behaviour of electrons and ions in ordinary plasmas is almost exclusively determined by electromagnetic forces. It is only for micron or submicron sized charged grains that both classes of interactions become comparable to within an order of magnitude, as noted already in the context of spoke formation in the B-ring of Saturn. Hence Jeans' instability is to be reconsidered for plasmas containing such grains, and we would expect unusual and interesting deviations from the ordinary rules. One can indeed no longer assume that astrophysical objects are electrically neutral in the traditional sense, as they contain a significant amount of charged dust.

A first analysis of this problem is given by Chhajlani and Parihar (1994) and indicates how charged dust grains in interstellar clouds affect the fragmentation process. For low-frequency modes the effect of the grains is important, decreasing the region of instability and the critical Jeans' wave number and rendering star formation more difficult. Pandey *et al.* (1994) obtain conditions for stable electrostatic levitation, condensation and dispersion of grains in a plasma background. As the grains start to condense due to self-gravitation, an electrostatic field is set up due to charge separation between grains, electrons and ions. The electrons and ions rush to shield this field, creating density perturbations which if fast enough will tend to smooth out the effects and inhibit the condensation. However, the nonlinear evolution shows a condensation of grains, even when the effect of self-gravitation is annulled by electrostatic repulsion.

More generally, Avinash and Shukla (1994) have studied a purely growing instability, generalizing Jeans' instability, which can play a decisive role in levitation/
condensation of grains in planetary rings as well as in the formation of galaxies

and stars. The description includes a mixture of dust-acoustic and self-gravitational modes and leads to a rather robust, purely growing instability with a larger growth rate than given by Pandey *et al.* (1994). The treatment, however, does not include fluctuations in the dust grain charges and it remains to be seen what this important modification will give.

11.4. Surface Waves

This interesting topic has been neglected, except by Bharuthram and Shukla (1993), dealing with low-frequency surface waves in a plasma with warm electrons and ions, in addition to cold dust, and occupying a half space with a sharp vacuum-plasma interface. This is a valid approximation when the wavelengths are larger than the transition zone. A novel surface dust-acoustic mode is derived, in which the electron and ion pressures provide the tension, and the massive dust the inertia to maintain the wave. This could apply to interstellar clouds, if a sharp boundary exists between vacuum and an expanding plasma.

12. Comparison with Observations

Before coming to the conclusions, it is worth to recapitulate from the above discussion of different modes and instabilities those elements which invite comparisons with observations, albeit mostly in qualitative form rather than quantitative. Besides the usual gravitational theories, several explanations have been advanced for the spokes in the B-ring of Saturn, involving charged dust grains following Keplerian orbits around the planet, and imbedded in a plasma corotating with the planetary magnetosphere. Either the resulting $\mathbf{E} \times \mathbf{B}$ drift on the dust forces them away from corotation distance (Goertz, 1989), or multistream dust charge density waves are also waves of material density (Bliokh and Yaroshenko, 1985), or charged dust enhances the growth and the velocity ranges over which two-stream instabilities can occur (Bharuthram *et al.*, 1992).

Another feature of planetary rings is that repelling electrostatic forces between charged dust grains prevent collapse to very thin sheets. Resonances between oscillations in transverse thickness and Keplerian frequencies can produce gaps and prominent features in ring structures at Jupiter and Saturn (Melandsø and Havnes, 1991). In the plasma that connects Jupiter with its satellite Io, the detection of the electrostatic ion cyclotron instability would signal the presence of negative dust, as without it the mode would be stable (Chow and Rosenberg, 1995).

Turning to comets, electrostatic noise enhancement below the ion plasma frequency, observed by the *Vega* and *Giotto* space probes at comet 1P/Halley, is partly due to modes trapped in the dust region (de Angelis *et al.*, 1988). Turbulent drag forms dust striae in cometary tails, leading to narrower ones in a high velocity solar wind, whereas wider tails occur in lower velocity solar winds, where gravity, radiation pressure and expansion dominate (Havnes, 1988). Variable dust charges influence electromagnetic waves, the instabilities of which could generate spatial structures over a wide range of hundreds to tens of million kilometres in dusty cometary tails (Reddy *et al.*, 1996). Double layers and vortices can play a large role in particle acceleration and transport in auroral and solar plasmas (Raadu, 1989), hence may also be responsible for the intense dust jets of comet 1P/Halley (Shukla *et al.*, 1991). Kelvin–Helmholtz like motions in cometary tails (d'Angelo and Song, 1990) depend on the sign of the charged grains as a function of distance to the cometary nucleus. Near comet 1P/Halley the grain potential varies from negative to positive, changing sign at about 30 000 km (Ellis and Neff, 1990). Related very low-frequency drift-like waves have been observed near comet Halley (Grard *et al.*, 1986; Klimov *et al.*, 1986).

Inclusion of charged dust decreases the region of instability and the critical Jeans' wave number, rendering star formation more difficult (Chhajlani and Parihar, 1994). A two-stream instability between two charged dust distributions indicates that small grains are braked in very short distances, while larger grains are practically unaffected. Collisions between grains of the two populations return material to the cloud in gas form, hence some elements are underabundant in the gas of low-velocity clouds but have near normal abundances in clouds of high velocity (Havnes, 1980).

The only paper which really sets out to give a detailed and quantitative discussion of the braids and kinks observed in the F-ring of Saturn (Smith *et al.*, 1981) is the one by Avinash and Sen (1994). They show them to be a consequence of the electromagnetic equilibrium of moving charged grains, and involve collective rather than single particle effects. The electrostatic pressure on the grains is balanced by the electromagnetic pinch produced by the dust ring current. Helical current filaments, with three strands as observed, are easily produced. Similar arguments allow to reproduce the ringlets within the B-ring. From stability considerations it is further argued that the formation of these braids and filaments is a dynamic process, which critically depends upon local plasma conditions prevailing in the ring. This would explain why such braids and kinks are not seen in all rings all the time.

13. Conclusions

To sum up, the typical characteristics of dusty space plasmas are that charged grains, massive compared to electrons and ions, have variable charges which usually are highly negative. The lower characteristic frequencies for the dust components modify existing wave spectra and introduce novel eigenmodes at the ultra low-frequency end. These new eigenmodes could help in the interpretation of low-frequency noise observed in space and astrophysical plasmas. Also, variable charges induce new damping or growth mechanisms for different kind of modes.

The main applications, which really stimulated the growth of this field of space plasma physics, are thought to be phenomena like the spokes and braids in planetary rings, observed certainly in the rings of Saturn by the *Voyager* cameras. Similarly, certain features of comet tails are difficult to explain in a satisfactorily manner without the presence of charged dust. Nevertheless, reality commands us to note that at present, barring the proverbial exception, there is no quantitative agreement between the observed phenomena and the theoretical explanations, nor are there many theoretical papers which specifically purport to give such detailed explanations.

So we have reached a situation where the theoretical studies concerning waves and instabilities in dusty (space) plasmas are far ahead of what observations can corroborate at present. Due to the paucity of coming solar system missions specifically concerned with planetary or cometary phenomena, the situation is unlikely to change much in the near future. About the only cometary mission planned is the Rosetta mission, which will not reach comet 46P/Wirtanen before 2011, if anything goes according to present plan. Delays would even worse this already not rosy picture... We expect the Cassini/Huygens mission to yield additional information about the rings of Saturn, but again that will take time.

While this dearth of observational data might for the time being point to a theoretician's heaven, one would like to see a consensus between different views, if only to design on board experiments which might be able to discriminate between different opinions. However, there is now also renewed and vigourous interest in laboratory dusty plasmas, and help from there might be able to fill gaps in our understanding and description before space observations can do that. In particular, the work on plasma dust crystal structures looks very promising in furthering our knowledge about dust charging. It could be worth speculating whether plasma dust crystal structures could occur in space, and where we would first presume to chance upon these.

Among the theoretical problems still rather widely open at present are a better description of how a static magnetic field influences the charging processes of grains, the need for a fully self-consistent treatment of linear modes from the point of view of kinetic theory for variable charges, in order to determine parameters and coefficients which are introduced in a rather *ad hoc* fashion at present, and finally, proper nonlinear descriptions of electrostatic and, most of all, of electromagnetic waves in plasmas with variable dust charges.

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References

Alfvén, H.: 1981, *Cosmic Plasma*, D. Reidel Publ. Co., Dordrecht, Holland, p. 118–121.

Anderson, R. R. and Kurth, W. S.: 1989, 'Ultra-Low Frequency Waves at Comets', in B. T. Tsurutani and H. Oya (eds.), *Plasma Waves and Instabilities at Comets and in Magnetospheres*, Geophys. Monograph Series 53, AGU, Washington, 81–117.

Aslaksen, T. K. and Havnes, O.: 1994, 'Kinetic Theory for a Distribution of Ionized Dust Particles', *J. Plasma Phys.* **51**, 271–290.

Avinash, K. and Sen, A.: 1994, 'A Model for the Fine Structure of Saturn's Rings', *Phys. Letters* **A194**, 241–245.

Avinash, K. and Shukla, P. K.: 1994, 'A Purely Growing Instability in a Gravitating Dusty Plasma', *Phys. Letters* **A189**, 470–472.

Bharuthram, R. and Shukla, P. K.: 1992a, 'Large Amplitude Double Layers in Dusty Plasmas', *Planetary Space Sci.* **40**, 465–471.

Bharuthram, R. and Shukla, P. K.: 1992b, 'Vortices in Non-Uniform Dusty Plasmas', *Planetary Space Sci.* **40**, 647–654.

Bharuthram, R. and Shukla, P. K.: 1992c, 'Large Amplitude Ion-Acoustic Solitons in a Dusty Plasma', *Planetary Space Sci.* **40**, 973–977.

Bharuthram, R. and Shukla, P. K.: 1993, 'Low-Frequency Surface Waves on a Warm Dusty Plasma', *Planetary Space Sci.* **41**, 17–19.

Bharuthram, R. and Rao, N. N.: 1995, 'Self-Similar Expansion of a Warm Dusty Plasma – I. Unmagnetized Case', *Planetary Space Sci.* **43**, 1079–1085.

Bharuthram, R., Saleem, H., and Shukla, P. K.: 1992, 'Two-Stream Instabilities in Unmagnetized Dusty Plasmas', *Physica Scripta* **45**, 512–514.

Bhatt, J. R. and Pandey, B. P.: 1994, 'Self-Consistent Charge Dynamics and Collective Modes in a Dusty Plasma', *Phys. Rev.* **E50**, 3980–3983.

Bliokh, P. V. and Yaroshenko, V. V.: 1985, 'Electrostatic Waves in Saturn's Rings', *Soviet Astron.* **29**, 330–336.

Booker, H. G.: 1984, *Cold Plasma Waves*, Nijhoff, Dordrecht. 2–12.

Chhajlani, R. K. and Parihar, A. K.: 1994, 'Magnetogravitational Instability of Self-Gravitating Dusty Plasma', *Astrophys. J.* **422**, 746–750.

Chow, V. W. and Rosenberg, M.: 1995, 'Electrostatic Ion Cyclotron Instability in Dusty Plasmas', *Planetary Space Sci.* **43**, 613–618.

d'Angelo, N.: 1967, 'Recombination Instability', *Phys. Fluids* **10**, 719–723.

d'Angelo, N.: 1990, 'Low-Frequency Electrostatic Waves in Dusty Plasmas', *Planetary Space Sci.* **38**, 1143–1146.

d'Angelo, N.: 1993, 'The Rayleigh–Taylor Instability in Dusty Plasmas', *Planetary Space Sci.* **41**, 469–474.

d'Angelo, N.: 1994, 'Ion-Acoustic Waves in Dusty Plasmas', *Planetary Space Sci.* **42**, 507–511.

d'Angelo, N. and Song, B.: 1990, 'The Kelvin–Helmholtz Instability in Dusty Plasmas', *Planetary Space Sci.* **38**, 1577–1579.

de Angelis, U.: 1992, 'The Physics of Dusty Plasmas', *Physica Scripta* **45**, 465–474.

de Angelis, U., Formisano, V. and Giordano, M.: 1988, 'Ion Plasma Waves in Dusty Plasmas: Halley's Comet', *J. Plasma Phys.* **40**, 399–406.

de Angelis, U., Bingham, R., and Tsytovich, V. N.: 1989, 'Dispersion Properties of Dusty Plasmas', *J. Plasma Phys.* **42**, 445–456.

de Angelis, U., Forlani, A., Tsytovich, V. N., and Bingham, R.: 1992, 'Scattering of Electromagnetic Waves by a Distribution of Charged Dust Particles in Space Plasmas', *J. Geophys. Res.* **97**, 6261–6267.

de Angelis, U., Forlani, A., Bingham, R., Shukla, P. K., Ponomarev, A., and Tsytovich, V. N.: 1994, 'Damping and Absorption of High-Frequency Waves in Dusty Plasmas', *Phys. Plasmas* **1**, 236–244.

Deconinck, B., Meuris, P., and Verheest, F.: 1993a, 'Oblique Nonlinear Alfvén Waves in Strongly Magnetized Beam Plasmas. Part 1. Nonlinear Vector Evolution Equation', *J. Plasma Phys.* **50**, 445–455.

Deconinck, B., Meuris, P., and Verheest, F.: 1993b, 'Oblique Nonlinear Alfvén Waves in Strongly Magnetized Beam Plasmas. Part 2. Soliton Solutions and Integrability', *J. Plasma Phys.* **50**, 457–476.

Ellis, T. A. and Neff, J. S.: 1991, 'Numerical Simulation of the Emission and Motion of Neutral and Charged Dust from P/Halley', *Icarus* **91**, 280–296.

Forlani, A., de Angelis, U., and Tsytovich, V. N.: 1992, 'Waves in Dusty Plasmas', *Physica Scripta* **45**, 509–511.

Goertz, C. K.: 1989, 'Dusty Plasmas in the Solar System', *Rev. Geophys.* **27**, 271–292.

Goertz, C. K. and Ip, W.-H.: 1984, 'Limitations of Electrostatic Charging of Dust Particles in a Plasma', *Geoph. Res. Letters* **11**, 349–352.

Grard, R., Pedersen, A., Trotignon, J.-G., Beghin, C., Mogilevsky, M., Mikhaïlov, Y., Molchanov, O., and Formisano, V.: 1986, 'Observations of Waves and Plasma in the Environment of Comet Halley', *Nature* **321**, 290–291.

Grün, E., Zook, H. A., Baguhl, M., Balogh, A., Bame, S. J., Fechtig, H., Forsyth, R., Hanner, M. S., Horanyi, M., Kissel, J., Lindblad, B.-A., Linkert, D., Linkert, G., Mann, I., McDonnell, J. A. M., Morfill, G. E., Phillips, J. L., Polanskey, C., Schwehm, G., Siddique, N., Staubach, P., Švestka, Z., and Taylor, A.: 1993, 'Discovery of Jovian Dust Streams and Interstellar Grains by the Ulysses Spacecraft', *Nature* **362**, 428–430.

Havnes, O.: 1980, 'On the Motion and Destruction of Grains in Interstellar Clouds', *Astron. Astrophys.* **90**, 106–112.

Havnes, O.: 1988, 'A Streaming Instability Interaction Between the Solar Wind and Cometary Dust', *Astron. Astrophys.* **193**, 309–312.

Houpis, H. L. F. and Whipple, E. C., Jr.: 1987, 'Electrostatic Charge on a Dust Size Distribution in a Plasma', *J. Geophys. Res.* **92**, 12057–12068.

James, C. R. and Vermeulen, F.: 1968, 'A Microparticle Plasma', *Canadian J. Phys.* **46**, 855–863.

Jana, M. R., Sen, A., and Kaw, P. K.: 1993, 'Collective Effects Due to Charge–Fluctuation Dynamics in a Dusty Plasma', *Phys. Rev.* **E48**, 3930–3933.

Jana, M. R., Sen, A., and Kaw, P. K.: 1995, 'Influence of Grain Charge Fluctuation Dynamics on Collective Modes in a Magnetized Dusty Plasma', *Physica Scripta* **51**, 385–389.

Kaw, P. K.: 1992, 'Non-Abelian Screening and Colour Oscillations in a Quark Gluon Plasma', *Plasma Phys. Contr. Fusion* **34**, 1795–1802.

Klimov, S., Savin, S., Aleksevich, Ya., Avanesova, G., Balebanov, V., Balikhin, M., Galeev, A., Gribov, B., Nozdrachev, M., Smirnov, V., Sokolov, A., Vaisberg, O., Oberc, P., Krawczyk, Z., Grzedzielski, S., Juchniewicz, J., Nowak, K., Orlowski, D., Parfianovich, B., Woźniak, D., Zbyszynski, Z., Voita, Ya., and Triska, P.: 1986, 'Extremely-Low-Frequency Plasma Waves in the Environment of Comet Halley', *Nature* **321**, 292–293.

Lakshmi, S. V. and Bharuthram, R.: 1994, 'Arbitrary Amplitude Rarefactive Dust-Acoustic Solitons', *Planetary Space Sci.* **42**, 875–881.

Lakshmi, S. V., Bharuthram, R. and Yu, M. Y.: 1993, 'Nonlinear Potential Structures in a Dusty Plasma', *Astrophys. Space Sci.* **209**, 71–78.

Li, F., Havnes, O., and Melandsø, F.: 1994, 'Longitudinal Waves in a Dusty Plasma', *Planetary Space Sci.* **42**, 401–407.

Lonngren, K. E.: 1990, 'Expansion of a Dusty Plasma Into a Vacuum', *Planetary Space Sci.* **38**, 1457–1459.

Luo, H. and Yu, M. Y.: 1992, 'Kinetic Theory of Self-Similar Expansion of a Dusty Plasma', *Phys. Fluids* **B4**, 1122–1125.

Ma, J.-X. and Yu, M. Y.: 1994a, 'Self-Consistent Theory of Ion Acoustic Waves in a Dusty Plasma', *Phys. Plasmas* **1**, 3520–3522.

Ma, J.-X. and Yu, M. Y.: 1994b, 'Langmuir Wave Instability in a Dusty Plasma', *Phys. Rev.* **E50**, R2431–R2434.
Ma, J.-X. and Shukla, P. K.: 1995, 'Compact Dispersion Relation for Parametric Instabilities of Electromagnetic Waves in Dusty Plasmas', *Phys. Plasmas* **2**, 1506–1509.
Ma, J.-X., Shukla, P. K., and Yu, M. Y.: 1995, 'Nonlinear Dielectric Function of a Dusty Plasma in the Presence of Electromagnetic Fields', *Phys. Letters* **A198**, 357–363.
Mace, R. L. and Hellberg, M. A.: 1993a, 'The Effects of Ion Inertia on Dust-Acoustic Double Layers', in R. W. Schrittwieser (ed.), *Double Layers and Other Nonlinear Potential Structures in Plasmas*, World Scientific, Singapore, pp. 370–375.
Mace, R. L. and Hellberg, M. A.: 1993b, 'Dust-Acoustic Double Layers: Ion Inertial Effects', *Planetary Space Sci.* **41**, 235–244.
Melandsø, F. and Havnes, O.: 1991, 'Oscillations and Resonances in Electrostatically Supported Dust Rings', *J. Geophys. Res.* **96**, 5837–5845.
Melandsø, F., Aslaksen,T. K., and Havnes, O.: 1993a, 'A New Damping Effect for the Dust-Acoustic Wave', *Planetary Space Sci.* **41**, 321–325.
Melandsø, F., Aslaksen, T. K., and Havnes, O.: 1993b, 'A Kinetic Model for Dust Acoustic Waves Applied to Planetary Rings', *J. Geophys. Res.* **98**, 13315–13323.
Mendis, D. A. and Rosenberg, M.: 1994, 'Cosmic Dusty Plasma', *Ann. Rev. Astron. Astrophys.* **32**, 419–463.
Mofiz, U. A., Islam, M., and Ahmed, Z.: 1993, 'Nonlinear Propagation of Ion-Acoustic Waves and Low-Frequency Modes in a Dusty Plasma', *J. Plasma Phys.* **50**, 37–44.
Nambu, M., Shukla, P. K., and Vladimirov, S. V.: 1993, 'Plasma-Maser Instability in Dusty Plasmas', *Phys. Letters* **A180**, 441–443.
Northrop, T. G.: 1992, 'Dusty Plasmas', *Physica Scripta* **45**, 475–490.
Pandey, B. P., Avinash, K., and Dwivedi, C. B.: 1994, 'Jeans Instability of a Dusty Plasma', *Phys. Rev.* **E49**, 5599–5606.
Pilipp, W., Hartquist, T. W., Havnes, O., and Morfill, G. E.: 1987, 'The Effects of Dust on the Propagation and Dissipation of Alfvén Waves in Interstellar Clouds', *Astrophys. J.* **314**, 341–351.
Popel, S. I. and Yu, M. Y.: 1994, 'Modulational Interaction of Short-Wavelength Ion-Acoustic Oscillations in Impurity-Containing Plasmas', *Phys. Rev.* **E50**, 3060–3067.
Popel, S. I. and Yu, M. Y.: 1995, 'Ion Acoustic Solitons in Impurity-Containing Plasmas', *Contrib. Plasma Phys.* **35**, 103–108.
Raadu, M. A.: 1989, 'The Physics of Double Layers and Their Role in Astrophysics', *Phys. Reports* **178**, 25–97.
Rao, N. N.: 1993a, 'Hydromagnetic Waves and Shocks in Magnetized Dusty Plasmas', *Planetary Space Sci.* **41**, 21–26.
Rao, N. N.: 1993b, 'Low-Frequency Waves in Magnetized Dusty Plasmas', *J. Plasma Phys.* **49**, 375–393.
Rao, N. N.: 1993c, 'Dust-Magnetoacoustic Waves in Magnetized Dusty Plasmas', *Phys. Scripta* **48**, 363–366.
Rao, N. N.: 1995, 'Magnetoacoustic Modes in a Magnetized Dusty Plasma', *J. Plasma Phys.* **53**, 317–334.
Rao, N. N. and Bharuthram, R. N.: 1995, 'Self-Similar Expansion of a Warm Dusty Plasma – II. Magnetized Case', *Planetary Space Sci.* **43**, 1087–1093.
Rao, N. N. and Shukla, P. K.: 1994, 'Nonlinear Dust-Acoustic Waves with Dust Charge Fluctuations', *Planetary Space Sci.* **42**, 221–225.
Rao, N. N., Shukla, P. K., and Yu, M. Y.: 1990, 'Dust-Acoustic Waves in Dusty Plasmas', *Planetary Space Sci.* **38**, 543–546.
Rawat, S. P. S. and Rao, N. N.: 1993, 'Kelvin–Helmholtz Instability Driven by Sheared Dust Flow', *Planetary Space Sci.* **41**, 137–140.
Reddy, R. V., Lakhina, G. S., Verheest, F., and Meuris, P.: 1996, 'Alfvén Modes in Dusty Cometary and Planetary Plasmas', *Planetary Space Sci.* **44**, 129–135.
Rosenberg, M.: 1993, 'Ion- and Dust-Acoustic Instabilities in Dusty Plasmas', *Planetary Space Sci.* **41**, 229–233.

Rosenberg, M. and Krall, N. A.: 1994, 'High Frequency Drift Instabilities in a Dusty Plasma', *Planetary Space Sci.* **42**, 889–894.
Rosenberg, M. and Krall, N. A.: 1995, 'Modified Two-Stream Instabilities in Dusty Space Plasmas', *Planetary Space Sci.* **43**, 619–624.
Sah, O. P. and Goswami, K. S.: 1994, 'Theory of Weak Dust Acoustic Double Layers', *Phys. Letters* **A190**, 317–322.
Salahuddin, M.: 1993, 'Dust Acoustic Wave Instability', *Phys. Scripta* **48**, 478–480.
Salimullah, M. and Sen, A.: 1992, 'Low Frequency Response of a Dusty Plasma', *Phys. Letters* **A163**, 82–86.
Salimullah, M., Hassan, M. H. A., and Sen, A.: 1992, 'Low-Frequency Electrostatic Modes in a Magnetized Dusty Plasma', *Phys. Rev.* **A45**, 5929–5934.
Shukla, P. K.: 1992, 'Low-Frequency Modes in Dusty Plasmas', *Physica Scripta* **45**, 504–507.
Shukla, P. K. and Rao, N. N.: 1993, 'Vortex Structures in Magnetized Plasmas with Sheared Dust Flow', *Planetary Space Sci.* **41**, 401–403.
Shukla, P. K. and Silin, V. P.: 1992, 'Dust Ion-Acoustic Wave', *Physica Scripta* **45**, 508.
Shukla, P. K. and Varma, R. K.: 1993, 'Convective Cells in Nonuniform Dusty Plasmas', *Phys. Fluids* **B5**, 236–237.
Shukla, P. K., Yu, M. Y., and Bharuthram, R.: 1991, 'Linear and Nonlinear Dust Drift Waves', *J. Geophys. Res.* **96**, 21343–21346.
Shukla, P. K., Feix, G., and Rao, N. N.: 1993a, 'Decay and Modulational Instabilities of Electron Plasma Waves in Unmagnetized Susty Plasmas', *Planetary Space Sci.* **41**, 693–695.
Shukla, P. K., Varma, R. K., Krishan, V., and McKenzie, J. F.: 1993b, 'Alfvén Vortices in Nonuniform Dusty Magnetoplasmas', *Phys. Rev.* **E47**, 750–752.
Smith, B. A., Soderblom, L., Beebe, R., Boyce, J., Briggs, G., Bunker, A., Collins, S. A., Hansen, C. J., Johnson, T. V., Mitchell, J. L., Terrile, R. J., Carr, M., Cook, A. F., II, Cuzzi, J., Pollack, J. B., Danielson, G. E., Ingersoll, A., Davies, M .E., Hunt, G. E., Masursky, H., Shoemaker, E., Morrison, D., Owen, T., Sagan, C., Veverka, J., Strom, R., and Suomi, V. E.: 1981, 'Encounter with Saturn: Voyager 1 Imaging Science Results', *Science* **212**, 163–191.
Smith, B. A., Soderblom, L., Batson, R., Bridges, P., Inge, J., Masursky, H., Shoemaker, E., Beebe, R., Boyce, J., Briggs, G., Bunker, A., Collins, S. A., Hansen, C. J., Johnson, T. V., Mitchell, J. L., Terrile, R. J., Cook, A. F., II, Cuzzi, J., Pollack, J. B., Danielson, G. E., Ingersoll, A., Davies, M. E., Hunt, G. E., Morrison, D., Owen, T., Sagan, C., Veverka, J., Strom, R., and Suomi, V. E.: 1982, 'A New Look at the Saturn System: The Voyager 2 Images', *Science* **215**, 504–537.
Spitzer, L.: 1941, 'The Dynamics of the Interstellar Medium. I. Local Equilibrium', *Astrophys. J.* **93**, 369–379.
Stix, T. H.: 1962, *The Theory of Plasma Waves*, McGraw–Hill, New York. 33–42.
Thomas, H., Morfill, G. E., Demmel, V., Goree, J., Feuerbacher, B., and Möhlmann, D.: 1994, 'Plasma Crystal: Coulomb Crystallization in a Dusty Plasma', *Phys. Rev. Letters* **73**, 652–655.
Tsytovich, V. N. and Havnes, O.: 1993, 'Charging Processes, Dispersion Properties and Anomalous Transport in Dusty Plasmas', *Comm. Plasma Phys. Contr. Fusion* **15**, 267–280.
Tsytovich, V. N., Morfill, G. E., Bingham, R., and de Angelis, U.: 1990, 'Dusty Plasmas (Capri Workshop, May 1989)', *Comm. Plasma Phys. Contr. Fusion* **13**, 153–162.
Varma, R. K. and Shukla, P. K.: 1995a, 'Linear and Nonlinear Rayleigh–Taylor Modes in Nonuniform Dusty Magnetoplasmas', *Physica Scripta* **51**, 522–525.
Varma, R. K. and Shukla, P. K.: 1995b, 'A New Dust-Dynamics-Induced Interchange Instability in Dusty Plasmas', *Phys. Letters* **A196**, 342–345.
Varma, R. K., Shukla, P. K., and Krishan, V.: 1993, 'Electrostatic Oscillations in the Presence of Grain-Charge Perturbations in Dusty Plasmas', *Phys. Rev.* **E47**, 3612–3616.
Verheest, F.: 1967, 'General Dispersion Relations for Linear Waves in Multicomponent Plasmas', *Physica* **34**, 17–35.
Verheest, F.: 1990, 'Nonlinear Parallel Alfvén Waves in Cometary Plasmas', *Icarus* **86**, 273–282.
Verheest, F.: 1992a, 'Parallel Solitary Alfvén Waves in Warm Multispecies Beam-Plasma Systems. Part 2. Anisotropic Pressures', *J. Plasma Phys.* **47**, 25–37.
Verheest, F.: 1992b, 'Nonlinear Dust-Acoustic Waves in Multispecies Dusty Plasmas', *Planetary Space Sci.* **40**, 1–6.

Verheest, F.: 1993a, 'Double Layers and Solitons in Dusty Plasmas', in R. W. Schrittwieser (ed.), *Double Layers and Other Nonlinear Potential Structures in Plasmas*, World Scientific, Singapore, pp. 162–173.

Verheest, F.: 1993b, 'Are Weak Dust-Acoustic Double Layers Adequately Described by Modified Korteweg–de Vries Equations?', *Phys. Scripta* **47**, 274–277.

Verheest, F.: 1994a, 'Nonlinear Dust Alfvén Modes', *Space Sci. Rev.* **68**, 109–114.

Verheest, F.: 1994b, 'Charge Fluctuation Instabilities in Dusty Plasmas', *Proc. 1994 Int. Conf. Plasma Phys.* **2**, 286–289.

Verheest, F. and Buti, B.: 1992, 'Parallel Solitary Alfvén Waves in Warm Multispecies Beam-Plasma Systems. Part 1', *J. Plasma Phys.* **47**, 15–24.

Verheest, F. and Meuris, P.: 1995, 'Whistler-Like Instabilities Due to Charge Fluctuations in Dusty Plasmas', *Phys. Letters* **A198**, 228–232.

Vladimirov, S. V.: 1994a, 'Amplification of Electromagnetic Waves in Dusty Nonstationary Plasmas', *Phys. Rev.* **E49**, R997–R999.

Vladimirov, S. V.: 1994b, 'Propagation of Waves in Dusty Plasmas with Variable Charges on Dust Particles', *Phys. Plasmas* **1**, 2762–2767.

Whipple, E. C., Northrop, T. G., and Mendis, D. A.: 1985, 'The Electrostatics of a Dusty Plasma', *J. Geoph. Res.* **90**, 7405–7413.

Winske, D., Gary, S. P., Jones, M. E., Rosenberg, M., Chow, V. W., and Mendis, D. A.: 1995, 'Ion Heating in a Dusty Plasma Due to the Dust/Ion Acoustic Instability', *Geophys. Res. Letters* **22**, 2069–2072.

Yinhua, C. and Yu, M. Y.: 1994a, 'Exact Ion Acoustic Solitary Waves in an Impurity-Containing Magnetized Plasma', *Phys. Plasmas* **1**, 1868–1870.

Yinhua, C. and Yu, M. Y.: 1994b, 'Fully Nonlinear Dust Acoustic Solitary Waves in a Impurity-Containing Magnetized Plasma', *Physica Scripta* **50**, 298–300.

Yu, M. Y. and Bharuthram, R.: 1994, 'Self-Similar Cylindrical Expansion of Impurity Particles in a Plasma', *J. Plasma Phys.* **52**, 345–352.

Yu, M. Y. and Luo, H.: 1995, 'Adiabatic Self-Similar Expansion of Dust Grains in a Plasma', *Phys. Plasmas* **2**, 591–593.