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A theory of cross-sectional shrinkage distortion and its experimental verification

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Summary. A theory has been developed for calculating the cup and edge distortion that will occur when green boards are dried, or the moisture content of dry boards changes in service. The parameters on which the calculations are based are the annual ring orientation coordinates R and θ of the boards, and the transverse shrinkage factors. For boards of square-cross-section the theory agrees very closely with the shrinkage predicted by the older theory of Greenhill, MacLean and Keylwerth. For *Beilschmiedia tawa* specimens 50×50 mm in cross-section the experimental and calculated width and thickness shrinkage were in excellent agreement. For radiata pine boards 200×50 mm in cross-section the experimental cup, edge distortion and shrinkage in width and thickness agreed very closely with the values predicted by the theory.

Introduction

It has been known for at least a century that the cupping and edge distortion experienced by boards during drying are caused by differential radial and tangential shrinkage. Textbooks such as Kollmann and Côté (1968) describe this in a very generalised way, but no accurate theory exists at present that allows the complete cross-sectional deformation of a board cross-section to be calculated prior to drying. The aim of this article is to develop a theory for calculating cross-sectional shrinkage deformation of boards from a knowledge of radial and tangential shrinkage coefficients and the direction of the annual rings, and to verify this theory experimentally.

Previous investigators such as Greenhill (1940), MacLean (1945) and Keylwerth (1948) developed shrinkage equations based on the assumption that the annual rings in a board can be effectively approximated by straight lines at an angle ϕ to the width axis (Fig. 1). This was a reasonable assumption while boards were still being sawn from large diameter trees from virgin forests. However, this assumption is poor for

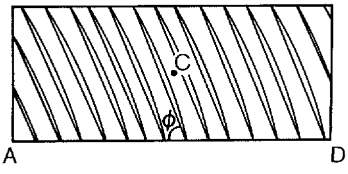

Fig. 1. Board with straight line annual rings at angle ϕ

boards from the smaller diameter plantation-grown logs that have been increasingly replacing those from virgin forests since about 1950.

The theories based on the assumption that annual rings in boards can be approximated by straight lines allow the calculation of edge distortion, but not cupping. Formulae describing edge distortion derived with the assumption of straight line annual rings are given in Appendix 1.

Development of a theory for calculating shrinkage distortion

A realistic theory of cross-sectional shrinkage distortion must take into account a number of facts. Firstly, that annual rings in boards are approximately arcs of a circle and not straight lines, and secondly that shrinkage occurs tangentially along these rings and radially perpendicular to the rings. Hence to calculate the shrinkage distortion of a board the orientation and curvature of its annual rings must be known.

Booker (1987) has recently shown that the direction of the annual rings in a board can be uniquely defined by two coordinates, a radial coordinate R and an angle θ. The (R, θ) coordinates of a board correspond to the polar coordinates of the centroid (C) of the board cross-section in the tree before sawing (Fig. 2).

When the polar coordinates of the centroid of a board are known, as well as the board's width and thickness, the board circumference and the orientation of the annual rings can be graphically or mathematically reconstructed. The above system of representation would be of little practical use if it were necessary to measure the board coordinates in the log before sawing. This is not the case. The coordinates can be measured with a transparent overlay as shown in Fig. 3. This has been fully described by Booker (1987).

When a green tree disc dries, it normally develops a large number of small radial checks because the tangential shrinkage is greater than the radial shrinkage. However, if such a disc has a single sawcut made from bark to pith, this sawcut widens into a slit during drying, and the two sides of the slit are effectively straight lines (Fig. 4). Moreover, the remainder of the disc is practically free from radial checks. This indicates that the residual stresses developed in the slotted disc are small and that the disc moves as a whole. This means that in Fig. 4 we may assume that one side of the slit remains fixed in position while the other rotates. In fact, any point on the disc such as P can be assumed to rotate towards the fixed side of the slit during drying. In other words, the point P in the board cross-section represented in Fig. 5 will during drying move tangentially along the annual ring towards Ox and radially towards O to take up the final position P′.

The chord PL has length $R\theta$, where θ is in radians. If the radial and tangential shrinkage fractions are given by r and t respectively, the point P moves a distance $R\theta t$ along the arc PL towards L, and a distance $R r$ along the radius OP towards O. After drying, the radial distance from P′ to O will hence be OP $(1-r)$, and the tangential distance of P′ from Ox will be PL $(1-t)$. Hence the angle P′Ox is given by:

$$\theta' = \frac{R\theta(1-t)}{R(1-r)} = \left(\frac{1-t}{1-r}\right)\theta .$$

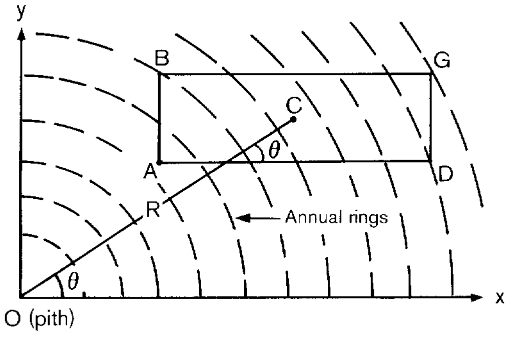

Fig. 2. Orientation in the log of a rectangular board with coordinates (R, θ). The edge A that is closest to the pith 0 is the leading edge

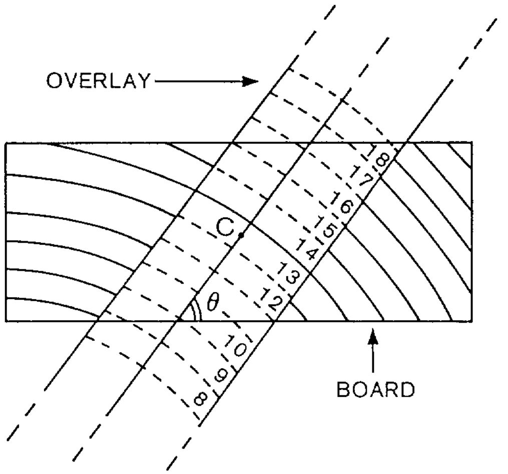

Fig. 3. Use of a transparent overlay to determine the coordinates of the centroid, C. The overlay has a large number of arcs of concentric circles printed on it. These are 10 mm apart and have their radius of curvature clearly marked. By matching the concentric circle segments with the annual rings as shown the (R, θ) coordinates can be determined. In this example the 150 mm × 60 mm board has coordinates R = 13.5 cm; θ = 54°

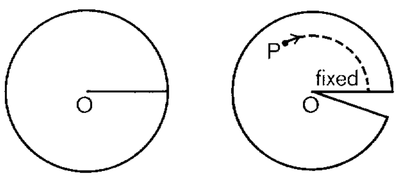

Fig. 4. a Disc with a sawcut prior to drying; **b** same disc after drying

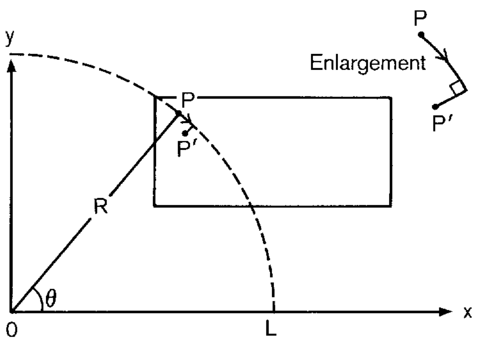

Fig. 5. Movement of a point P in a board along and perpendicular to the annual ring direction during drying

Hence any point P with polar coordinates (R, θ) will after shrinkage occupy a position P′ whose polar coordinates (R′, θ') are given by:

$$R' = R(1-r) \tag{1}$$

$$\theta' = \left(\frac{1-t}{1-r}\right)\theta \tag{2}$$

where t and r are the tangential and radial shrinkage fractions.

These equations will be referred to as the transformation equations. Mathematically they are equivalent to a uniform stress-free shrinkage in all directions by a fraction r, followed by fractional shrinkage (t − r) in the tangential direction.

A computer program has been developed that calculates the shrinkage distortion of a board in a series of steps:

1. It is assumed that for a given board the polar coordinates of the centroid (R_c, θ_c) have been determined, either in the log or more commonly with a transparent overlay. The thickness (p) and width (q) are also assumed known. The program calculates the coordinates of a large number of points along the edges of the board cross-section using 100 intervals along each edge. First the coordinates (x_c, y_c) of the centroid are calculated; the points on the top and bottom surfaces BG and AD (Fig. 2) have y-coordinates $y_c \pm p/2$, while the points on the sides have an x-coordinate given by $x_c \pm q/2$.
2. For each of the points on the lines AB, BG, GD and DA (Fig. 2) the (x, y) coordinates are converted into polar coordinates.
3. The transformation equations are applied.
4. The transformed polar coordinates are converted into (x, y) coordinates.
5. The (x, y) coordinates of all the points along the edges are plotted to represent the cross-section of the board after drying.
6. Further calculations are carried out as described later.

Experimental verification

Square tawa specimens

A single length of fresh *Beilschmiedia tawa* sapwood 100 mm × 100 mm in cross-section was obtained. It was carefully selected to avoid spiral grain and other defects. A set of seven specimens each just over 300 mm long was cut. The samples were machined so that a full range of ring angles from zero to 45 degrees was obtained. Each of the specimens was then cut in two to yield a total of 14 specimens of cross-sectional area 50 × 50 mm and 150 mm long. Each sample was weighed (green weight), end-waxed to prevent end-splitting, and the ring angle was determined at each end. This was then averaged to give the ring angle at the centre of the sample. The ring angle was taken to be the angle subtended by the chord connecting the ends of the annual ring that passed closest to the centre of the cross-section (Fig.1).

The width (q) and thickness (p) of each specimen was measured by placing the specimen on a 38 mm pedestal and measuring with a dial gauge the distance between H or K (Fig. 6) and the opposite side AB or BG resting on the pedestal. However, if

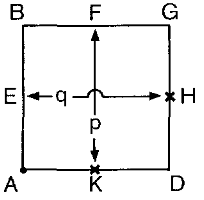

Fig. 6. Measurement positions H and K on the tawa specimens. H and K lie in the centre of DG and AD. The edge A that is closest to the pith is the leading edge

after drying the surface AB or BG was slightly concave, the specimen was turned over and the was measured from E or F to the opposite side.

The specimens were held in a 12% equilibrium moisture content (emc) cabinet for 7 months. When emc was reached the specimens were re-weighed and re-measured on the shrinkage points with a dial gauge. Eventually the specimens were oven-dried and reweighed so that the basic wood density and the true moisture content values could be calculated.

Rectangular radiata pine boards

A pruned butt log of *Pinus radiata* D. Don was selected which had the pith close to the geometric centre. The flared region at the butt was cut off, yielding a pruned log 6.4 m long with little taper. The log was cut into two 3.2 m long sections. The average radius of the two log sections was 263 ± 24 mm. A reference disc was cut from each end of the two log sections, after which each was cut into manageable 55 mm thick slabs with a chainsaw mill. These slabs were transported to the laboratory for further machining. Twenty seven boards 600 mm long and 200 × 50 mm in cross-section were sawn so that they spanned a large range of (R, θ) values. In addition ten boards of smaller cross-sectional dimensions were produced. A large number of flatsawn boards (i.e. $\theta = 90°$) with a large range of R values were manufactured. Nodal areas around the branch stubs with their annual ring and spiral grain deviations were excluded from the sample material.

The (R, θ) values of the boards were measured in the log, with a transparent overlay, and by comparison with two reference discs cut from the end-discs. Each board was weighed (green weight) and painted at its ends to prevent end-splitting. The thickness was measured in the centre of each board, halfway between its ends. The width was measured 5 mm from the top surface of each board. The boards were air-dried without restraint and equilibrated in a 12% emc cabinet. The cross-sectional dimensions were remeasured and a one inch thick slice cut from the centre of each board. On this cross-section the (R, θ) coordinates were measured. The cross-section was also photocopied and digitised for analysis. It was then computer corrected for the small amount of paper stretch that occurs during photocopying.

From the centre of the 6.4 m log a series of shrinkage specimens was cut 150 mm long and 50 mm × 50 mm in cross-section. This yielded a series of shrinkage specimens from pith to bark in four directions at right angles to each other.

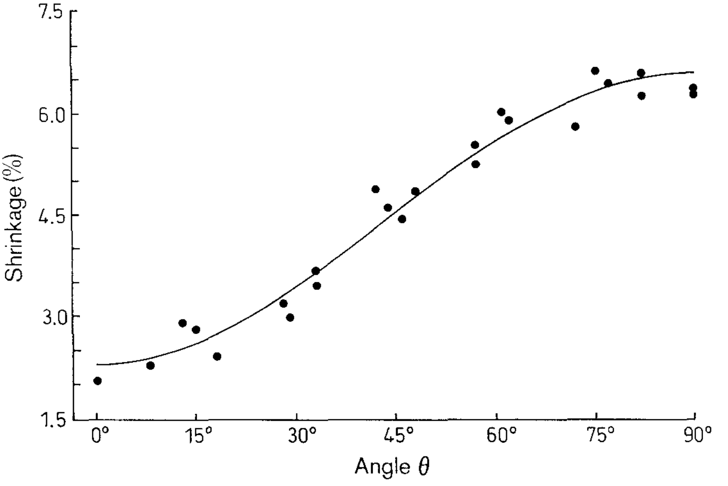

Fig. 7. % Shrinkage in width as a function of ring angle θ from 0° to 90° for square *B. tawa* specimens. The solid line represents both the Keylwerth formula and the new theory

Results

The shrinkage of square B. tawa specimens

Shrinkage data for square specimens are ideally suited to test both the old and the new theories and to compare them, as the square cross-section becomes effectively a rhomboid with parallel sides. Because of symmetry considerations only angles from 0° to 45° needed to be considered, as for complementary angles it is only necessary to interchange height and width. It can also be shown that the angles ϕ of Fig. 1 and θ of Fig. 2 are complementary, so that $\phi = 90° - \theta$.

Experimental results for the square tawa specimens are shown in Table 1. Results for specimen 6 had to be rejected as in spite of all precautions it possessed a very distorted annual ring pattern. The data are also plotted in the form of a graph of percentage shrinkage in width (q) as a function of ring angle θ from 0° to 90° (Fig. 7). A curve of best fit was drawn through the points to calculate the values of r and t according to Keylwerth's formula (Eq. A4):

$$\text{width shrinkage} = r \sin^2 \phi + t \cos^2 \phi \, .$$

The best fitting values for the radial and tangential shrinkage were found to be 2.28% and 6.61% respectively, and the fit of the equation was excellent with a correlation coefficient r^2 of 0.997.

The Greenhill formulae A8 and A9 were used to calculate the theoretical shrinkage values, which are shown as a continuous graph in Fig. 7 and are tabulated in Table 1. In practice the values calculated with the Greenhill formulae A8 and A9 are almost indistinguishable from those calculated with the Keylwerth formulae A3 and A4 (Table 1).

The new transformation Eqs. (1) and (2) above were used to calculate the positions of the four points E′, F′, H′ and K′ (Fig. 6) after drying. From the coordinates

Table 1. Dimensions and shrinkage of the *B. tawa* specimens

Specimen	R mm	θ degree	% Width shrinkage				% Thickness shrinkage				Angle A′ degrees		Angle G′ degrees		Angle ω
			exptal.	Keylwerth	Greenhill	Booker	exptal.	Keylwerth	Greenhill	Booker	exptal.	Booker	exptal.	Booker	Greenhill
1	115	0°	2.05	2.28	2.28	2.28	6.31	6.61	6.61	6.61	90.0	90.0	90.0	90.0	90.0
2	128	0°	2.03	2.28	2.28	2.28	6.26	6.61	6.61	6.62	90.0	90.0	90.0	90.0	90.0
3	90	8°	2.27	2.36	2.37	2.37	6.26	6.52	6.53	6.49	88.0	89.0	90.0	89.6	89.3
4	92	8°	2.27	2.36	2.37	2.37	6.58	6.52	6.53	6.50	88.2	89.1	90.0	89.5	89.3
5	82	13°	2.88	2.49	2.51	2.51	6.42	6.39	6.40	6.39	88.1	88.6	89.0	89.2	88.9
7	80	15°	2.79	2.57	2.59	2.58	6.60	6.32	6.33	6.32	88.0	88.3	89.0	89.1	88.7
8	118	18°	2.39	2.69	2.72	2.70	5.79	6.19	6.22	6.21	88.0	88.1	89.0	88.9	88.5
9	73	28°	3.16	3.23	3.28	3.31	5.89	5.65	5.70	5.67	87.0	87.1	89.0	88.9	87.9
10	80	29°	2.97	3.29	3.35	3.36	6.00	5.59	5.64	5.62	87.0	87.0	89.0	88.8	87.8
11	89	33°	3.66	3.56	3.62	3.62	5.23	5.32	5.38	5.36	86.0	86.8	88.0	88.6	87.7
12	108	33°	3.44	3.56	3.62	3.60	5.51	5.32	5.38	5.36	86.0	86.9	88.0	88.4	87.7
13	88	42°	4.87	4.21	4.29	4.28	4.83	4.67	4.74	4.73	86.0	86.3	88.0	88.5	87.5
14	74	44°	4.59	4.37	4.44	4.46	4.42	4.52	4.59	4.61	86.5	86.2	87.0	88.7	87.5
Average difference between theory and expt.				−0.01	0.03	0.03		0.01	0.04	0.03		0.4		0.2	
Standard deviation				0.29	0.28	0.28		0.27	0.25	0.26		0.4		0.4	

The experimental error involved in the measurement of angles is of the order of half a degree the lengths E′H′ and F′K′ were calculated, as they are equivalent to the width q′ and the thickness p′ after drying respectively. Values are listed in Table 1. It was expected that the values calculated according to the new theory would be closer to the experimental values than those for the Greenhill/Keylwerth theory, as the new theory takes into account differences in curvature of the annual rings. Instead, Table 1 shows that as far as shrinkage is concerned the values predicted by the new theory and the Greenhill theory are practically identical. However, the new theory is slightly better at predicting the size of the angles A′ and G′ (Fig. 6 and 13 and Table 1). The main advantage of the new theory is, however, that it can predict the magnitude of cupping in boards, while theories based on straight line annual rings predict that board surfaces remain flat during shrinking or swelling.

The shrinkage of radiata pine boards

No perceptible difference was found between the shrinkage behaviour of the four log quadrants or the two sections of the log. Consequently the results for all 38 shrinkage specimens (50 mm × 50 mm × 150 mm) have been combined into a single graph of radial and tangential shrinkage versus distance from the pith (Fig. 8). Both radial and tangential shrinkage decline slightly towards the pith. The average tangential shrinkage was found to be $(3.69 \pm 0.43)\%$ and the average radial shrinkage $(1.61 \pm 0.37)\%$. Converted to a percentage the standard deviation of t and r is 12% and 23% respectively. The difference $(t-r)$ is $(2.10 \pm 0.15)\%$, with a standard deviation of only 7%. This is clearly evident in Figure 8; in general whenever the tangential shrinkage for a specimen is high or low, the radial shrinkage is high or low by a similar amount. The shrinkage calculations were performed for the boards assuming that:

$$r(\%) = 1.3262 + 0.00198\,R$$

and

$$t(\%) = 3.1813 + 0.00355\,R$$

However, if a constant radial shrinkage value of 1.61% and a tangential shrinkage fraction of 3.69% were used, irrespective of radial position, this made very little difference to the results.

The range of (R, θ) coordinates for the 38 boards is shown in Fig. 9. The coordinates for 200 mm × 50 mm boards are limited to the area enclosed by the dotted lines. The top line indicates the limit set by the maximum log diameter, and the bottom line the requirement that the theory requires boards to be free from enclosed pith. Boards with smaller cross-section can lie outside the enclosed area. A reasonable spread of (R, θ) values was obtained.

It has been a problem to decide how to display the experimental and theoretical data in such a way that they could be readily compared. While it is a simple matter to plot the digitised experimental cross-section and the calculated cross-section for each board onto a transparent overlay and to compare theory and experiment by superimposing these, it would occupy too much space to display all the results in this form. Figure 10a and b show the experimental and calculated distortion for board no. 27 which has coordinates (71 mm, 61.5°). When transferred to a transparent

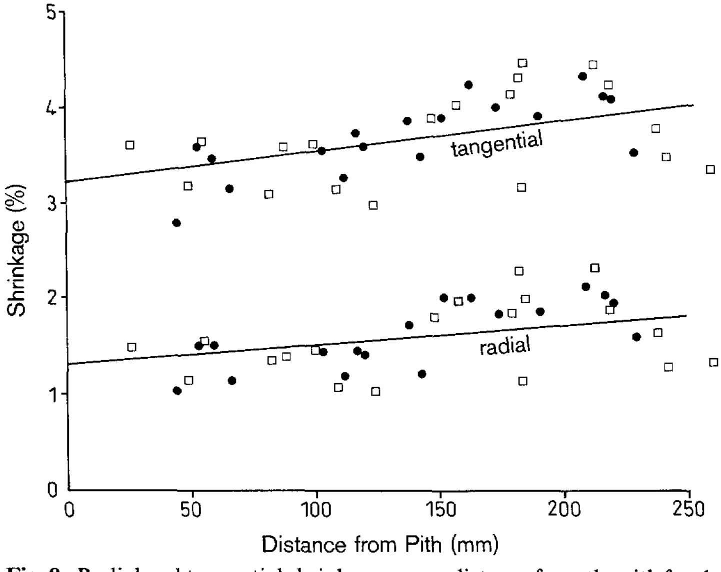

Fig. 8. Radial and tangential shrinkage versus distance from the pith for the radiata pine log. The wood was dried to 12% mc

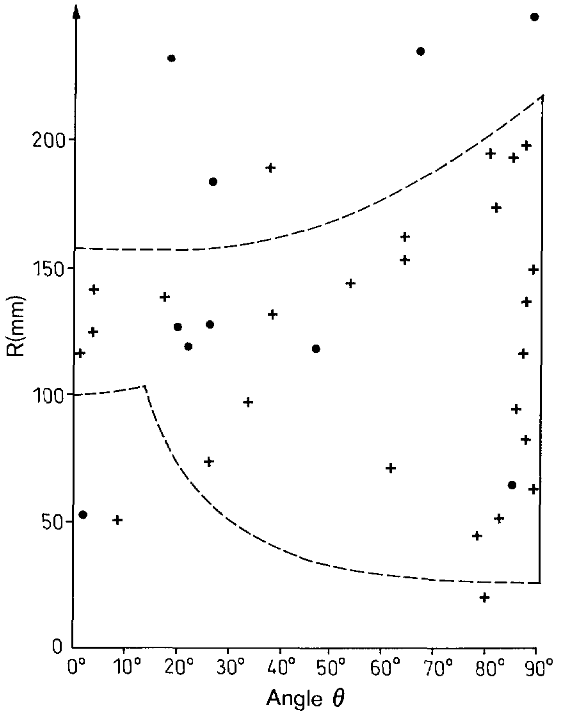

Fig. 9. Range of polar coordinates for the radiata pine boards. The coordinates for 200 mm × 50 mm boards are limited to the area enclosed by the dotted lines. The log diameter sets an upper limit to R. The lower limit is set by the fact that the theory is limited to boards free from pith

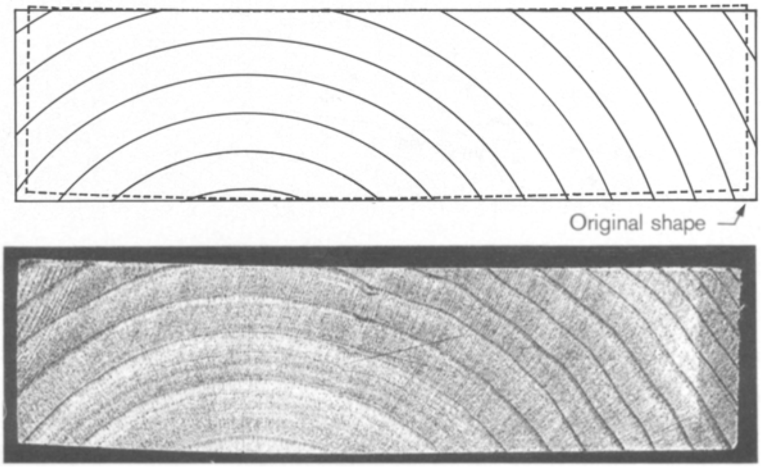

Fig. 10. Cross-sectional distortion of board 27 (half size) as predicted by the theory (top) and observed experimentally (bottom). Close agreement is indicated

overlay the two cross-sections overlap closely, showing excellent agreement between theory and experiment.

Cupping

The experimental results for the 38 boards are listed in Table 2. Figure 11 shows a three dimensional graph of cup (in mm) as a function of the (R, θ) coordinates for all 200 mm wide boards. The largest values of cup occur in the region with low R and high θ values, i.e. for flatsawn boards close to the pith. In addition, for a given value of R the cup steadily decreases from $\theta = 90°$ to $\theta = 0°$, i.e. from flatsawn to quartersawn.

The correlation coefficient r between experimental and theoretical cup for the 32 two hundred mm wide boards without pith was 0.84 ($r^2 = 0.71$). In general the calculated and experimental cup were in good agreement. As the amount of cup is small for radiata pine because of the low values of radial and tangential shrinkage, considerable scatter in the measurements associated with cup was introduced into the results by small surface imperfections. These were caused by the planer and by the fact that the latewood at the surface was slightly depressed below the level of the earlywood by differential shrinkage. Board number 11 (Table 2) showed less cup and shrinkage than expected because of severe compression wood, and boards 18 and 28 possessed non-concentric rings in one corner of the cross-section. Whenever reasons for differences between theory and experiment are known they are indicated in Table 2.

Shrinkage in width

The lowest shrinkage in width occurred for quartersawn boards for which θ is close to 0°. (Table 2). It then gradually increased with increasing ring angle to a maximum

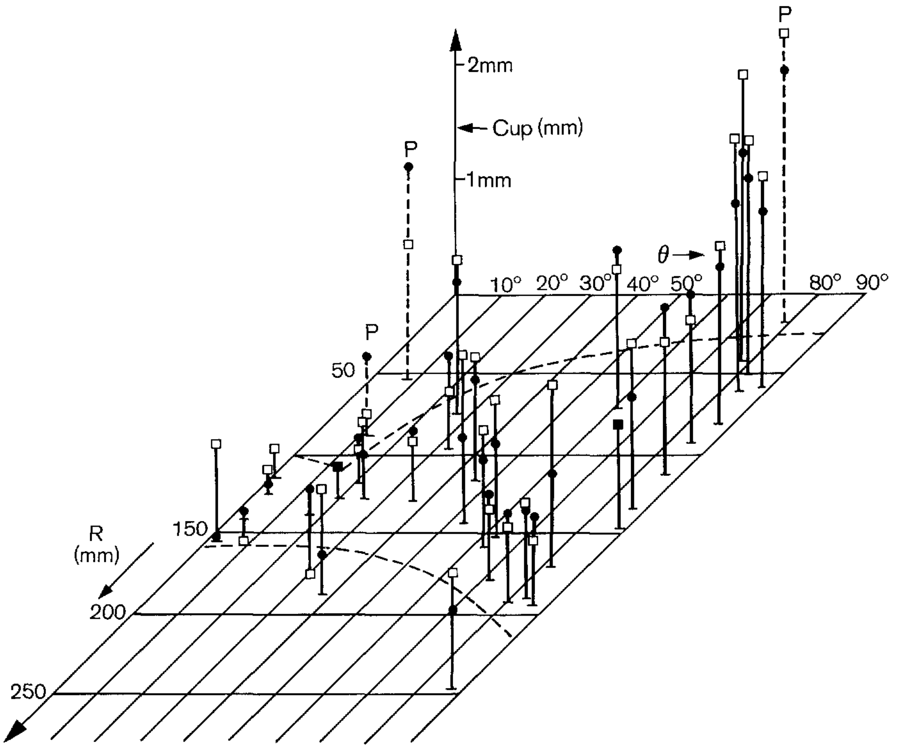

Fig. 11. Cup (in mm) as a function of (R, θ) coordinates for 35 radiata pine boards 200 mm wide. (R, θ) coordinates; ● theoretical values; □ experimental values; ■ identical theoretical and experimental values; --- boundary lines for 200 mm × 50 mm boards; P boards containing pith

at $\theta = 90°$ (flatsawn boards). For flatsawn boards the width shrinkage increased with R for three reasons:

(i) the tangential shrinkage increases slowly with distance from the pith (Fig. 8),

(ii) cup and edge distortion decrease with increasing distance from the pith so that the distance by which the top corners are bent towards each other decreases,

(iii) as R increases the curvature of the annual rings decreases, so that the radial contribution to the width shrinkage decreases.

As a result the maximum calculated shrinkage value of 4.1% occurred for the flatsawn board with the highest value of R, board number 31. The maximum experimental width shrinkage occurred for boards 17, 18 and 22 which are flatsawn and have large R values.

The correlation coefficient r between experimental and theoretical width shrinkage for the 35 boards not containing pith is 0.84 ($r^2 = 0.71$). The main differences between experiment and theory are attributable to the effect of non-concentric annual rings and the presence of compression wood. The relationship between experimental and theoretical shrinkage was calculated to be:

experimental shrinkage in width (%) $= -0.326 + 0.998 \times$ theoretical shrinkage (%)

Table 2. Comparison of theoretical and experimental shrinkage

Board	R mm	θ degrees	Green width mm	Green thickness mm	Thickness shrinkage % theory	Thickness shrinkage % expt.	Width shrinkage % theory	Width shrinkage % expt.	Cupping, mm theory	Cupping, mm expt.	Comments
1	132	38	198.16	50.14	2.98	2.61	2.90	2.97	0.62	0.59	
2	189	37.5	193.10	50.18	3.21	3.31	3.00	2.67	0.36	0.93	Rings not concentric
3	51	82.5	199.62	49.34	1.55	1.48	2.98	3.04	1.77	2.06	
4	142	4.0	195.72	49.49	3.65	3.66	2.03	1.70	0.05	−0.24	
5	117	1.0	199.24	50.29	3.56	3.78	1.97	1.87	0.10	0.28	
6	125	3.5	198.26	50.72	3.58	3.53	2.02	1.57	0.10	0.25	
7	51	8.5	197.16	49.57	3.21	3.53	2.28	2.11	1.85	1.20	Enclosed pith
8	197	87.5	197.96	50.06	2.09	1.14	3.90	3.12	0.81	0.60	
9	162	64	196.14	50.11	2.32	1.64	3.53	3.06	0.79	1.06	
10	97	33.5	194.32	50.10	2.99	2.53	2.76	2.12	0.82	0.50	
11	180	71.5	197.86	50.03	2.22	1.24	3.71	2.53	0.80	0.63	Compression wood
12	94	85.5	199.10	50.12	1.70	0.96	3.36	3.13	1.33	1.10	
13	44	78	199.32	49.69	1.60	1.33	2.89	2.81	1.85	2.50	
14	20	80	198.58	49.49	1.71	1.84	2.55	2.46	2.21	2.55	Pith in bottom side
15	74	26	198.90	49.47	3.08	3.29	2.60	2.66	1.15	1.36	
16	139	17.5	199.16	50.17	3.50	3.29	2.37	1.29	0.25	−0.58	Rings not concentric
17	149	89	196.90	50.57	1.90	2.18	3.69	3.89	0.98	0.97	
18	173	82.5	195.50	50.13	2.03	2.15	3.78	3.98	0.86	1.66	Rings not concentric
19	82	87.5	196.06	49.55	1.64	1.41	3.28	3.47	1.40	1.58	
20	116	87	199.00	49.13	1.77	1.51	3.50	3.65	1.18	1.48	
21	153	64	197.58	50.49	2.29	2.16	3.50	3.48	0.84	1.26	
22	193	85	199.82	50.20	2.09	2.25	3.87	3.80	0.83	0.88	
23	194	80.5	197.40	50.00	2.14	1.58	3.86	3.34	0.80	0.68	
24	144	53.5	198.74	50.97	2.55	1.71	3.28	3.27	0.78	1.53	Rings not concentric
25	62	89.0	194.48	50.13	1.55	1.28	3.12	2.59	1.57	1.90	
26	136	87.5	193.08	50.54	1.85	1.13	3.62	3.45	1.00	1.50	
27	71	61.5	194.08	50.25	2.06	1.67	3.07	2.88	1.39	1.22	

28	119	22	197.58	42.38	3.35	2.81	2.45	2.35	0.43	0.34	Ring distortion
29	128	26	193.50	41.75	3.29	3.09	2.54	2.53	0.41	0.70	
30	64	85	197.58	36.98	1.57	1.19	3.05	3.24	1.66	2.24	
31	247	88.5	199.32	35.08	2.27	1.00	4.07	3.14	0.71	1.07	
32	127	20	198.02	30.21	3.40	2.88	2.31	1.45	0.30	0.30	
33	153	2.5	199.10	20.30	3.60	3.79	1.94	1.38	0.01	0.90	Rings not concentric
34	87	11.5	199.64	18.30	3.30	3.33	2.02	1.79	0.70	0.18	enclosed pith
35	118	47	199.10	28.97	2.63	1.93	2.96	2.81	0.88	1.09	
36	184	26.5	141.40	50.16	3.49	2.29	2.63	1.75	0.11	−0.23	Compression wood
37	196	13.0	147.80	50.24	3.78	3.96	2.32	1.68	0.04	0.70	not concentric,
38	234	67.5	100.30	50.31	2.51	1.49	3.86	2.95	0.16	0.29	distortion
Average for the 35 boards 200 mm in width					2.5	2.2	3.0	2.7	0.9	1.0	
Standard deviation					0.7	0.9	0.6	0.7	0.6	0.7	
Correlation coefficient r between theory and experiment					0.91		0.86		0.83		

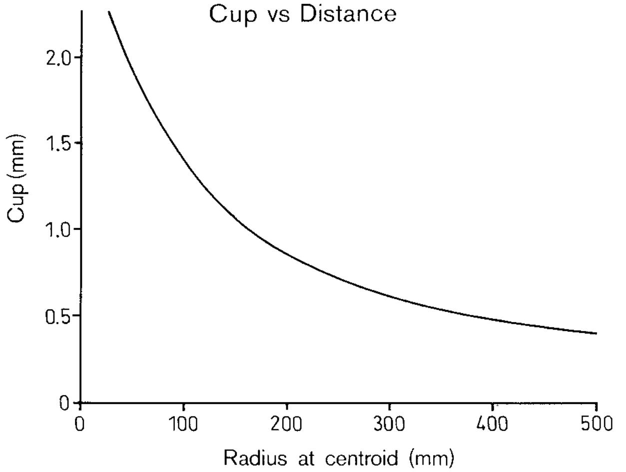

Fig. 12. Cup in a 200 mm × 50 mm flatsawn board as a function of distance from the pith. The calculation assumed radial and tangential shrinkage factors of 1.60% and 3.69% respectively

The slightly lower experimental shrinkage could be caused by the boards having a slightly lower moisture content at the time of measurement than the shrinkage specimens.

Shrinkage in thickness

Calculated and theoretical thickness shrinkage values were in good agreement with a correlation coefficient of 0.89 ($r^2 = 0.79$) for the 38 boards.

Thickness shrinkage decreased gradually from $\theta = 0°$ (quartersawn) to $\theta = 90°$ (flatsawn). For boards with large θ values the thickness shrinkage is essentially independent of R. This can be explained by the fact that at the centre of flatsawn boards the direction of thickness shrinkage is radial, i.e. always perpendicular to the annual rings and hence unaffected by ring curvature. Radial shrinkage changes only slightly with R (Fig. 8), so that thickness shrinkage is almost independent of R for flatsawn boards.

Discussion

It has long been known that boards close to the pith experience much greater distortion than boards further away. This is confirmed by the results in Table 2 and Fig. 11. Many sawmillers believe this is caused by greater transverse shrinkage close to the pith. This is not the case. The amount of cupping to be expected for flat sawn boards 200 mm × 50 mm in cross-section has been calculated for a range of R values, assuming constant shrinkage fractions $r = 1.60\%$ and $t = 3.69\%$. Results are shown in Fig. 12. The amount of cup decreases rapidly with increasing distance from the pith.

The log was chosen because the pith was symmetrically located. It also had very little spiral grain, which is fortunate, as further analysis shows that cross-sections can

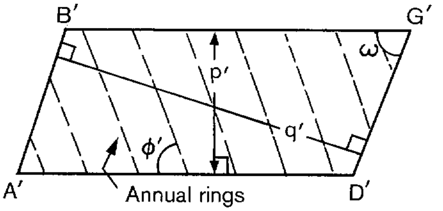

Fig. 13. Distorted cross-section of a board after drying, assuming straight line annual rings

also distort during drying as the result of spiral grain (Booker, unpublished). As most trees are reasonably symmetrical and shrinkage distortion does not vary rapidly with R and θ, it is probable that small deviations of annual rings from circular will in practice not make a large difference to the results. Large differences should be expected however when the rings are badly distorted due to the proximity of knots or when the rings are not concentric. Board 16 is an example of the latter (Table 2).

The new theory can be a useful tool in theoretical studies on sawmilling. Until now only green timber recovery from alternative sawing patterns could be calculated. This can now be extended to recovery of dry dressed timber as the minimum amount of planing required to return the distorted cross section to rectangular can be calculated.

The theory can also be used to predict the cupping that would occur in dry boards of rectangular cross-section on export from an area of high equilibrium moisture content to an area of low emc, or vice versa.

Conclusion

It has been shown that for boards of a given dimension and relatively straight grain cupping and edge distortion are governed by only two pairs of parameters, annual ring orientation and transverse shrinkage fractions r and t. In turn, the shrinkage fractions depend upon the shrinkage coefficients, fibre saturation point and the initial and final moisture content of the wood. Other factors such as density or ring width only affect cupping indirectly through their effect on the shrinkage fractions r and t.

Appendix I

Shrinkage formulae based on straight line annual rings

The following formulae describe the shrinkage of a rectangular board whose annual rings, assumed to be straight lines, make an angle ϕ with the width axis (Fig. 1). After shrinkage the board cross-section becomes a parallelogram (Fig. 13) whose dimensions are given by:

$$A'B' = AB\sqrt{\cos^2\phi(1-r)^2+\sin^2\phi(1-t)^2} \tag{A1}$$

$$A'D' = AD\sqrt{\cos^2\phi(1-t)^2+\sin^2\phi(1-r)^2} \tag{A2}$$

Equations (A1) and (A2) were first derived by Greenhill (1940).

The fractional shrinkage along the thickness direction

$$\frac{AB-A'B'}{AB} = 1-\sqrt{\cos^2\phi(1-r)^2+\sin^2\phi(1-t)^2} = r\cos^2\phi + t\sin^2\phi \quad \text{(Keylwerth 1948)} \tag{A3}$$

The fractional shrinkage along the width direction

$$\frac{AD-A'D'}{AD}=r\sin^2\phi+t\cos^2\phi \qquad \text{(Keylwerth 1948)} \tag{A4}$$

$$\omega=90^\circ-\sin^{-1}\frac{(t-r)\sin\phi\cos\phi}{\sqrt{\cos^2\phi(1-r)^2+\sin^2\phi(1-t)^2}}-\sin^{-1}\frac{(t-r)\sin\phi\cos\phi}{\sqrt{\cos^2\phi(1-t)^2+\sin^2\phi(1-r)^2}}. \tag{A5}$$

Because the board distorts from a rectangle to a parallelogram with angle ω (Fig. 13), thickness and width must both include a term $\sin\omega$:

$$p'=p\sqrt{\cos^2\phi(1-r)^2+\sin^2\phi(1-t)^2}.\ \sin\omega \tag{A6}$$

$$q'=q\sqrt{\cos^2\phi(1-t)^2+\sin^2\phi(1-r)^2}.\ \sin\omega \tag{A7}$$

where p and q are the original thickness and width respectively. Hence the fractional shrinkage in thickness is given by:

$$\frac{p-p'}{p}=1-\sqrt{\cos^2\phi(1-r)^2+\sin^2\phi(1-t)^2}.\ \sin\omega \tag{A8}$$

and the fractional shrinkage in width is given by:

$$\frac{q-q'}{q}=1-\sqrt{\cos^2\phi(1-t)^2+\sin^2\phi(1-r)^2}.\ \sin\omega. \tag{A9}$$

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