Transport in Porous Media **4**: 599–612, 1989.

Capillary Fingering: Percolation and Fractal Dimension★

ROLAND LENORMAND★★
Dowell Schlumberger, BP 90, 42003 Saint Etienne Cedex 1, France

and

CESAR ZARCONE
Institut de Mécanique des Fluides, 2 rue C. Camichel, 31071 Toulouse Cedex, France

(Received: 11 April 1989)

Abstract. We present experimental and theoretical results concerning immiscible displacements (drainage) in 2-dimensional permeable media. When capillary forces are predominant, the injected fluid presents very thin fingers and the 'Representative Elementary Volume' concept cannot be used for describing the partial saturations. The purpose of this paper is to show how this classical concept can be replaced by a statistical approach based on 'fractal' geometry.

Key words. Fractal, fingering, capillarity, percolation, fractal dimension, volume averaging, micromodel, multiphase flow, drainage, stochastic.

1. Introduction

The displacement of one fluid by another nonmiscible fluid in a porous medium is of importance in many processes, especially petroleum recovery. In this domain, an understanding of the relevant mechanisms is very important, because fingering leads to very inefficient recoveries.

The classical approach to describe monophasic or multiphasic flows in porous media is based on the *Representative Elementary Volume* concept (Bear, 1972). For instance, a property such as the porosity ϵ can be measured as a function of the radius l of a sphere centered on a point M (Figure 1). For values of l smaller than l_0, of the order of the grain size, we see the microscopic effects due to the pores and for values larger than l_0, the porosity becomes constant if the medium is homogeneous (for heterogeneous media, the porosity varies at the macroscopic scale L). Consequently, the porous medium can be studied as a *continuum* in the range l_0–L. Generally, the same kind of assumption is made for the study of multiphasic flows, the REV concept being extended to the partial saturations of the various fluids (Marle, 1981; Scheidegger, 1974). With this assumption, one

★ Communication presented at the International Symposium on the Stochastic Approach to Subsurface Flow, Montvillargenne 4–7 June 1985.
★★ Now at Institut Français du Pétrole, BP 311, 92506 Rueil Malmaison, France.

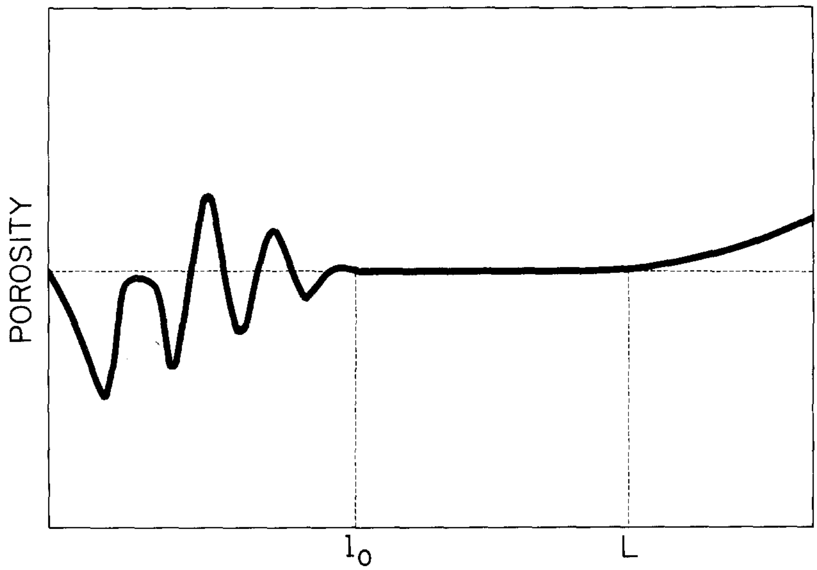

Fig. 1. Porosity and representative elementary volume.

can define any property (partial saturations, pressures) at each mathematical point. This approach leads to the classical notions of relative permeabilities, generalised Darcy's laws and capillary pressure curves.

This approach, however, does not seem suitable for describing some kinds of two-phase displacements in porous media. For instance, the scaling of various free imbibition experiments does not fit (Lefebvre du Prey, 1978), and visualisations in transparent media show large-scale heterogeneities (or fingers) in the fluid patterns (Chuoke *et al.*, 1959; Paterson, 1983; Paterson *et al.*, 1984). This fingering may be due to various causes, especially viscous instabilities (Saffman and Taylor, 1958; Lenormand and Zarcone, 1985a) or capillary mechanisms.

The purpose of this paper is to describe experimental displacements in a two-dimensional permeable medium when capillary forces are important, and then to discuss a theoretical model. In this case, all the displacement mechanisms are linked to capillary forces and to the random sizes of the pores in a porous medium.

The first part of this study presents the set-up (a two-dimensional etched network) and the experimental results. In the second part, we show how these results can be described by a statistical theory known as *invasion percolation* (IP). In this approach, the shape of the fingers can be characterized by one parameter called the *fractal dimension* and, in the final part, we present a technique for measuring the fractal dimension of the injected cluster.

2. Experimental Results

Some experiments have been carried out in transparent three-dimensional media (Chuoke *et al.*, 1959; Paterson *et al.*, 1984), but it is difficult to see the details of the structure. So, as a first approach, we prefer to use a two-dimensional medium which is easier to observe and describe.

We have developed a technique (Bonnet and Lenormand, 1977) using photochemical etching and transparent resin molding. Figure 2 shows our method:

- A negative film is made from a photograph or from a network drawn by a computer. The black parts will be the channels of the micromodel.
- A photosensitive plastic plate (used to make printing plates) is illuminated by ultra-violet light through the negative. The plastic under the white parts of the negative polymerizes and becomes tough (Figure 2a).

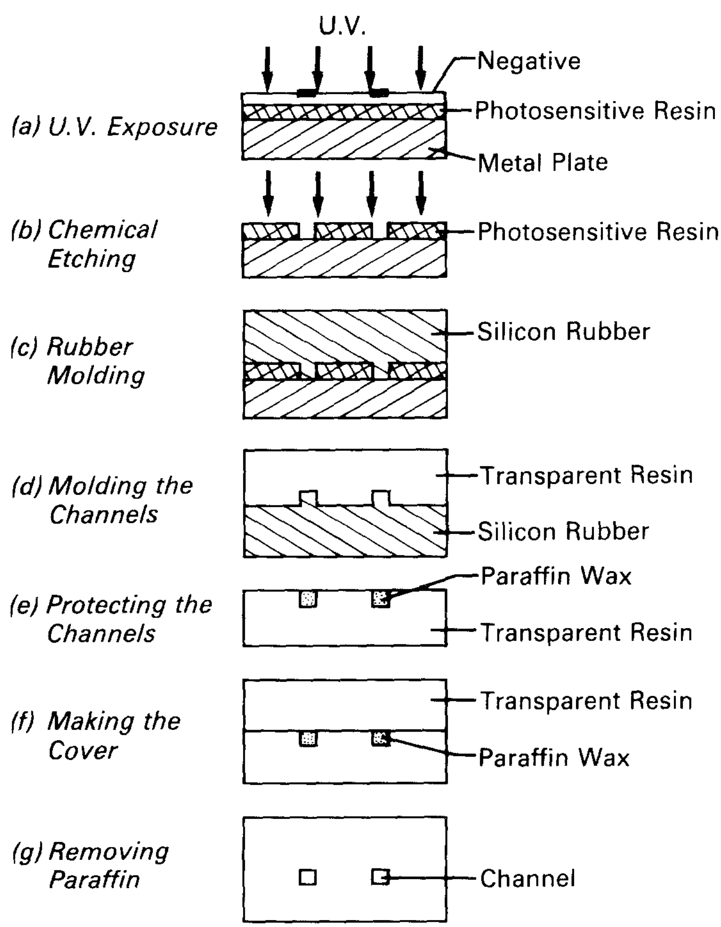

Fig. 2. Procedure for the making of the etched networks.

- The soft part of the plate is washed by a warm sodium hydroxyde solution. The depth of each etched channel is constant (equal to the 1 mm thickness of the photosensitive plate) and its cross-section is rectangular, with a minimum width around 0.1 mm (Figure 2b).
- This plate is not transparent. So, a transparent replica made in polyester resin is cast in a rubber mold of the pattern (Figure 2c and 2d).
- The channels are filled with paraffin wax under vacuum and the surface is carefully cleaned (Figure 2e).
- A thick layer of polyester resin is cast on this plate to make the cover (Figure 2f) and, after polymerisation, we obtain a block of transparent resin, with the inside of the channels filled with the paraffin wax (Figure 2g).

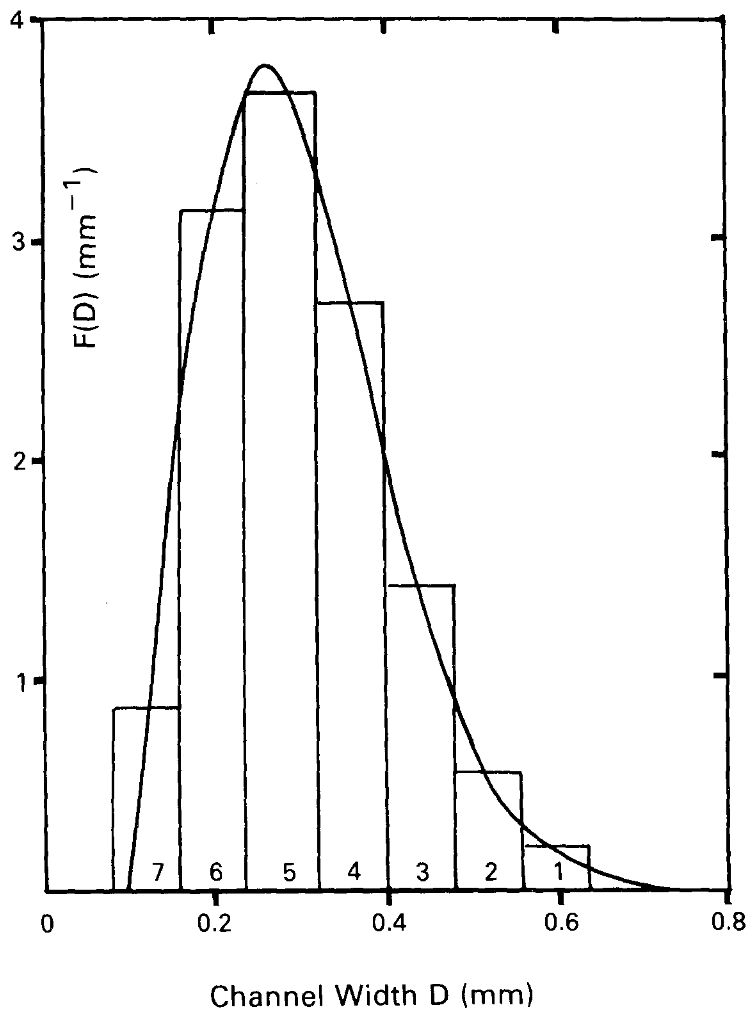

Fig. 3. Distribution of the widths of the capillaries in the 42 000 ducts network.

- The paraffin is removed by heating the micromodel and the channels are cleaned by flowing toluene, which dissolves paraffin.

The cross-section of each duct of the etched network is rectangular with a constant depth $x = 1$ mm and a width d which varies from throat to throat (generally $d > 0.1$ mm). For this study, we used two kinds of networks: (i) a 42 000 duct network (150×150 mm) with seven classes of channels (width from 0.1 to 0.6 mm) distributed with a log-normal law (Figure 3) and a random location; (ii) a very large network (300×300 mm) containing 250 000 ducts, used

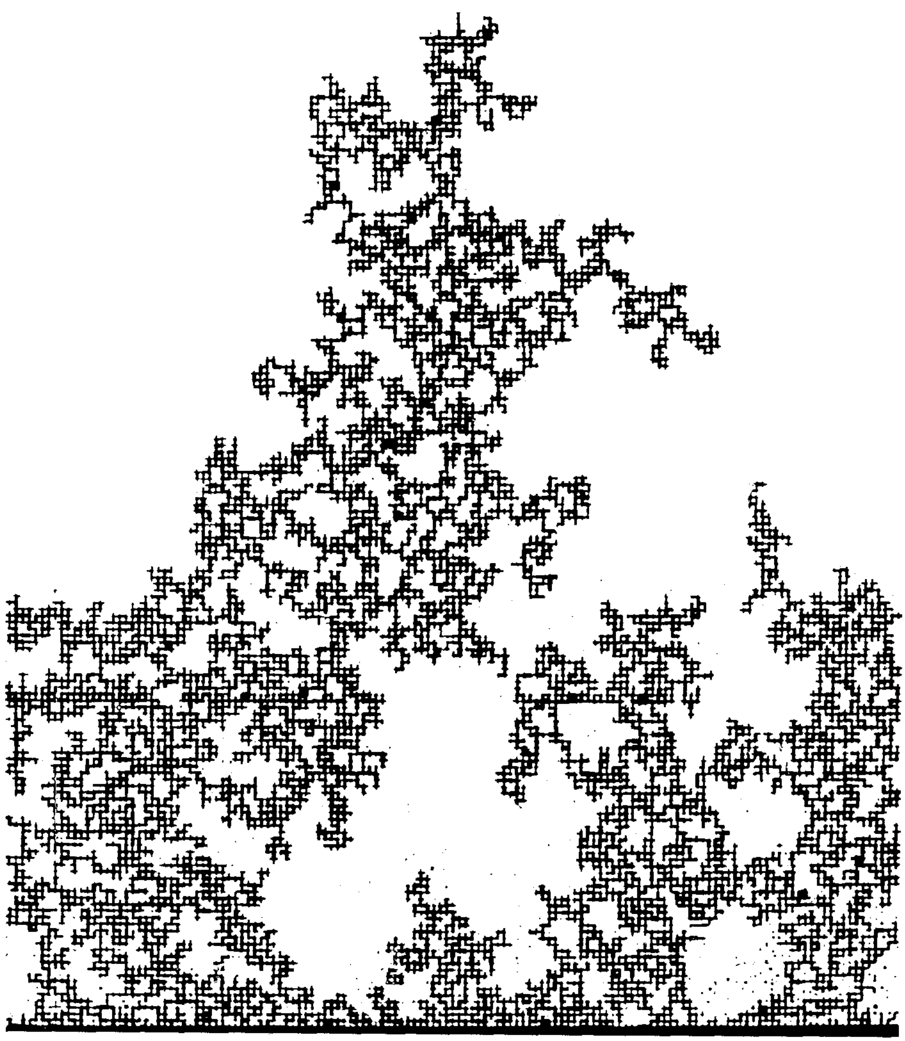

Figure 4. Cluster generated by injecting a non-wetting fluid (air, in white) in a 40 000 duct network filled with a wetting fluid (oil, in black). Situation at the percolation threshold.

for statistical measurement. The distance between two sites of the network is about 1 mm.

Generally speaking, when one fluid (oil, say) is slowly displacing another nonmiscible fluid (water, say) in a capillary tube, the fluid for which the contact angle θ (between the tube and the meniscus) is smaller than $\pi/2$ is called the *wetting fluid*; the other one is the *nonwetting fluid*.

The wetting fluid is paraffin oil (viscosity $\mu = 20$ cP or 0.020 SI), the nonwetting fluid is air ($\mu = 0.02$ cP), the contact angle is zero and the surface tension $\gamma = 20$ dyne/cm (0.020 SI). The nonwetting fluid is injected by slowly decreasing the pressure in the wetting fluid (constant level container) and different experiments are run from 1 to 96 h. This time scale is characterized by the capillary number (calculated for the wetting fluid), which is a dimensionless form of the flow rate q:

$$Ca = \frac{q\mu}{\sum \gamma}. \tag{1}$$

In this equation, μ is the viscosity of the wetting fluid. In a three-dimensional medium $\sum$ is the cross-section area of the sample and, for the two-dimensional network, we will take the product of the total width (150 mm) by the channel depth (1 mm), $q/\sum$ being a mean velocity of the fluid in the channels. This capillary number characterizes the ratio between viscous and capillary forces. The fluids are pumped through the micromodel using constant flow rate syringe

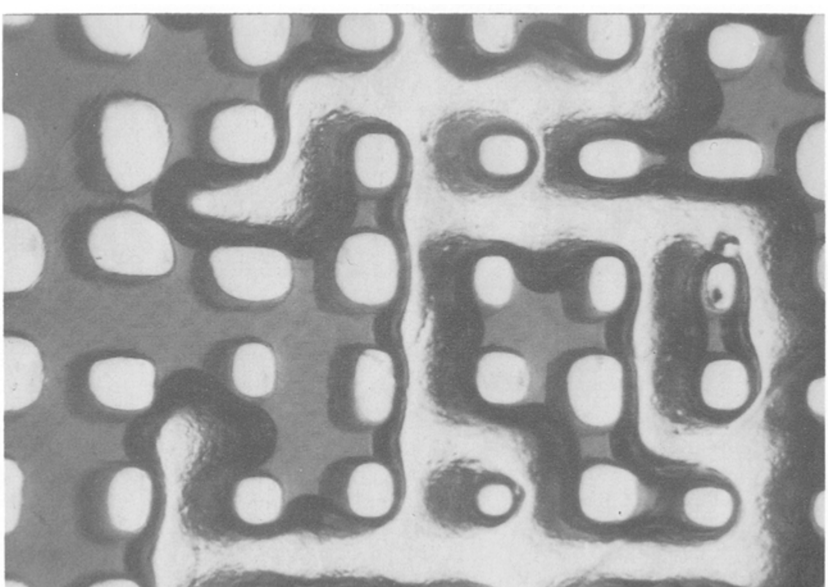

Fig. 5. Close-up of the situation of the wetting fluid (black).and the nonwetting fluid (white) in the ducts of the etched network.

pumps. The micromodel is held horizontal to avoid gravity effects and the wetting fluid, which initially fills the network, is displaced by the nonwetting fluid (drainage).

For a given capillary number, the experiments are reproducible. During the displacement, the nonwetting fluid presents very thin and ramified fingers (Figure 4) and at the end of the experiment, the cluster size of the trapped phase varies from the pore scale (Figure 5) to the network scale (large clusters in black, Figure 4).

3. Theoretical Model

Capillary forces prevent the nonwetting fluid from spontaneously entering a porous medium. It can only enter a throat (diameter D_0) when the pressure exceeds the pressure in the wetting fluid by a value P, called the capillary pressure and linked to the surface tension γ by the Laplace law: $P = 4\gamma \cos(\theta)/D_0$. Assuming that the porous medium can be described by a network of pores (nodes or intersections of the lattice) connected by ducts (bonds), from a statistical point of view, a duct with $D > D_0$ is an *active* or *conductive* bond and a duct with $D < D_0$ an *inactive* bond. The fraction p of active bonds can easily be deduced from the throat size distribution (Lenormand, 1981).

The geometrical and transport properties of a lattice with active and nonactive bonds can be described by *percolation theory*. This approach is described in detail, e.g., Guyon *et al.* (1984) but it is perhaps appropriate to mention a few important features of percolation theory here. Let us imagine a regular electrical network containing a fraction p of resistances and a fraction $(1-p)$ of open bonds at random locations. When the fraction p is small, the network as a whole is nonconductive: all the clusters of connected resistances are smaller than the size of the network. For a given value of the fraction p (the *percolation threshold* p^*) the network becomes conductive because a large cluster of resistances (the *infinite cluster* in percolation theory) joins the opposite faces of the network. Theoretical calculations and computer simulations can predict the value of p^*, the geometrical properties of the clusters and also the transport properties of the whole network near the threshold. For instance:

- For a square network the value of p^* is 0.5.
- Above the threshold, the conductance σ of the whole network varies as $(p-p^*)^{\mu}$.
- The fraction of bonds Y belonging to the infinite cluster is proportional to $(p-p^*)^{\beta}$.

In these equations, μ and β are called *critical exponents* and do not depend on the details of the network (shape of the mesh) but only on the dimensionality of the Euclidean space. For instance, in a two-dimensional network, $\mu = 1.1$ and $\beta = 0.14$.

Another important property of the infinite cluster near the percolation threshold is that it has a *fractal structure* (fractal geometries are described in Mandelbrot, 1977). If we consider a box of dimension $L \times L$ centered on a point of the cluster, the number N of bonds belonging to the cluster and lying within this box scales as a power law

$$N \propto L^D, \tag{2}$$

where D, the *fractal dimension* is not an integer ($D = 1.89$). For comparison a cluster is homogeneous (in a two-dimensional space) if the value of the exponent is 2.

It has been proved that the injected fluid invades all the percolation clusters connected to the injection face (De Gennes and Guyon, 1978; Lenormand and

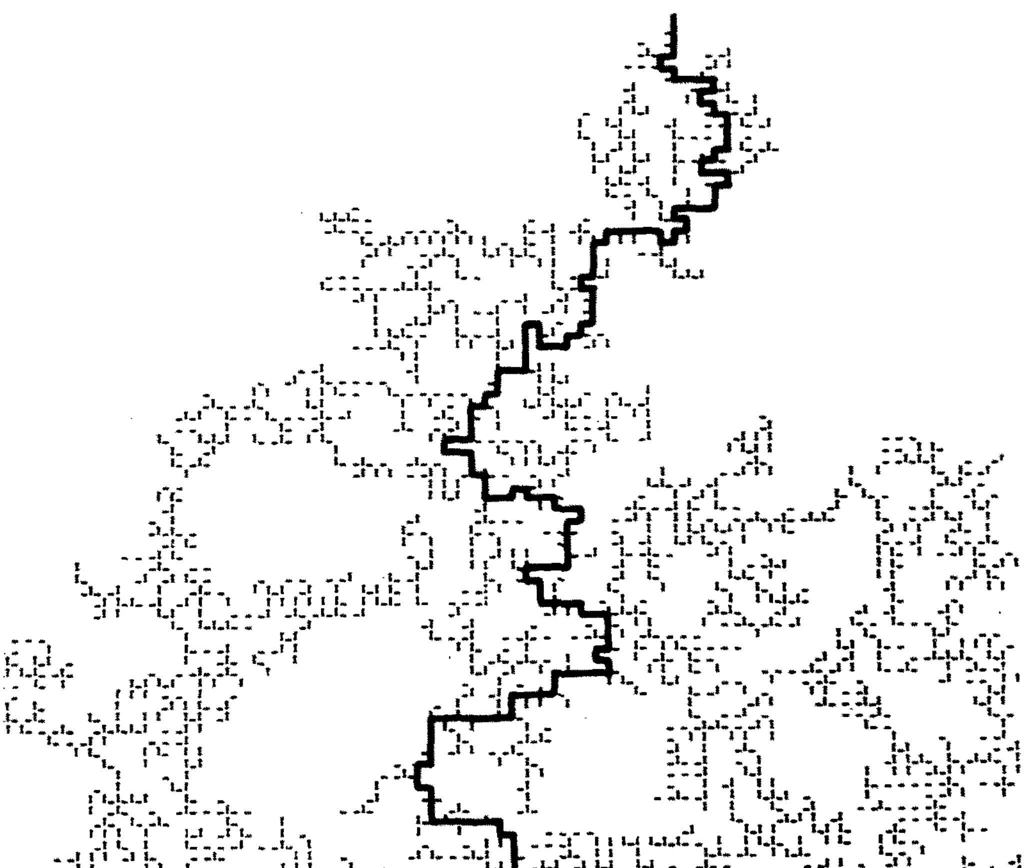

Fig. 6. Computer simulations of invasion percolation (after Lenormand and Bories, 1980). The bold line shows the continuous path between the opposite faces.

Bories, 1980; Chandler *et al.*, 1982). This mechanism has been called *invasion percolation*.

This percolation theory, however, does not take into account the trapping mechanism which occurs when the displaced fluid is incompressible. During the displacement, the wetting phase is trapped in the network when the invading nonwetting fluid breaks the continuous path toward the exit. Computer simulations of this invasion percolation with trapping using two-dimensional networks (size up to 100×100) are shown in Figure 6 and our experiments qualitatively agree with this simulation (cf. with Figure 4). Computer simulations also show a fractal behavior at the percolation threshold and at the end of the displacement, when all the bonds are active ($p = 1$) but not necessarily displaced (effect of trapping). In both cases, the fractal dimension D is found to be 1.82 in a two-dimensional network (Wilkinson and Willemsen, 1983) and the difference with ordinary percolation ($D = 1.89$) seems significant.

To check the validity of this approach, we have measured the fractal dimension of the injected cluster on the experiments.

4. Fractal Dimension

This study has been published with more details in Lenormand and Zarcone, (1985b).

The experiments were run in the large network (250 000 ducts) at various capillary numbers and photographs were taken at the end of the displacement (Figure 7). Digitization of the photographs was quite impossible because of the black meniscus which surrounds the nonwetting phase in each pore and we had to use a simple but laborious technique: from an origin O roughly at the center of the network, we count the number N of invaded ducts in an $L \times L$ square centered on O. A duct is counted only when the nonwetting fluid has invaded both the duct and the pore (intersection) next to this duct (ducts where the meniscus remains at one or both ends are not counted). At the end of the displacement, the variation of N as a function of L (measured in units of the mesh size) for different capillary numbers was plotted on a log–log scale (Figure 8). The curves are linear when the size L is greater than about 40 meshes and a least square fit for the slope leads to $D = 1.83 \pm 0.01$ for the three slowest displacements ($Ca = 3.3 \times 10^{-8}$, 6.5×10^{-8}, 1.2×10^{-7}) and $D = 1.80$ for $Ca = 6.2 \times 10^{-7}$. These measurements are in good agreement with the theoretical value $D = 1.82$.

We can conclude that experimental displacements of a wetting fluid by a nonwetting fluid in a two-dimensional random network are consistent with computer simulations of invasion percolation. In particular, the measured fractal dimension of the injected cluster is close to the simulation value of 1.82. While it is true that the range of L-values used is less than one decade, this is also the case for the computer experiments, the latter being limited by the time required at

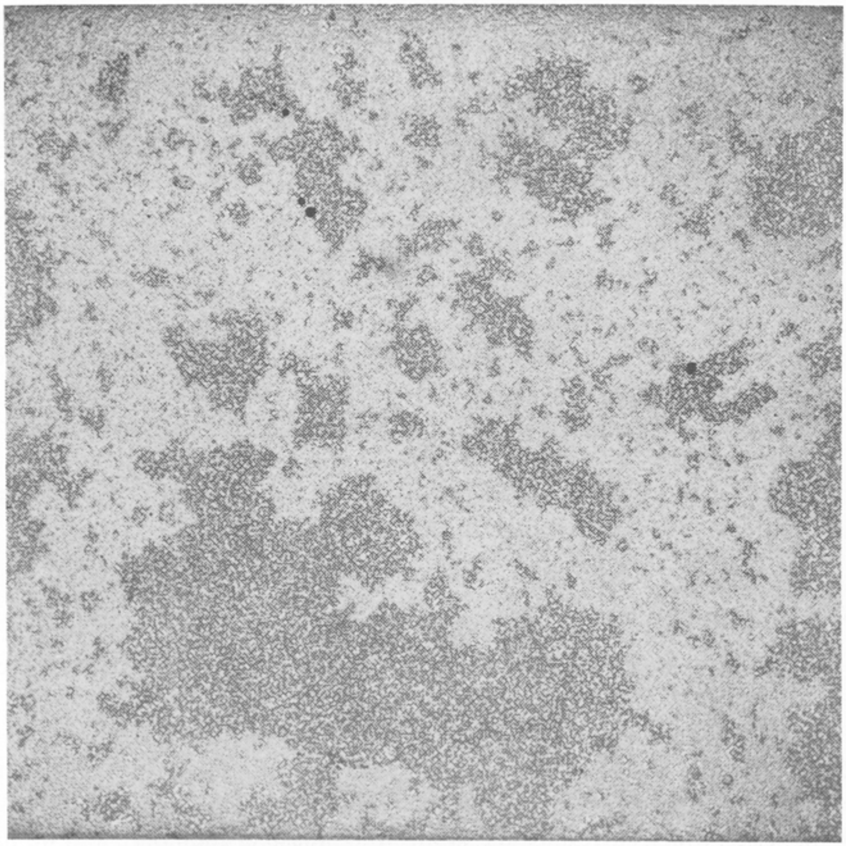

Fig. 7. Displacement of the wetting fluid (black), which initially fills the 250 000 duct network, by the nonwetting fluid. Situation at the end of drainage.

each step to check if part of the displaced fluid has been trapped. Thus, we cannot strictly exclude the possibility that, in both the experiment and simulation, the fractal dimension should really be the same as the value 1.89 for classical percolation. Nevertheless, these experiments do show that the saturation is not *homogeneous* at the scale of the network and that this result is strongly linked to capillary effects.

5. Discussion

Now, we can compare this fractal approach with REV concept described in the introduction.

First of all, we have measured the size of the REV corresponding to the porosity in the etched network. Figure 9 shows that for three points, chosen at random locations, the porosity ϵ is constant when the radius r of the circle is

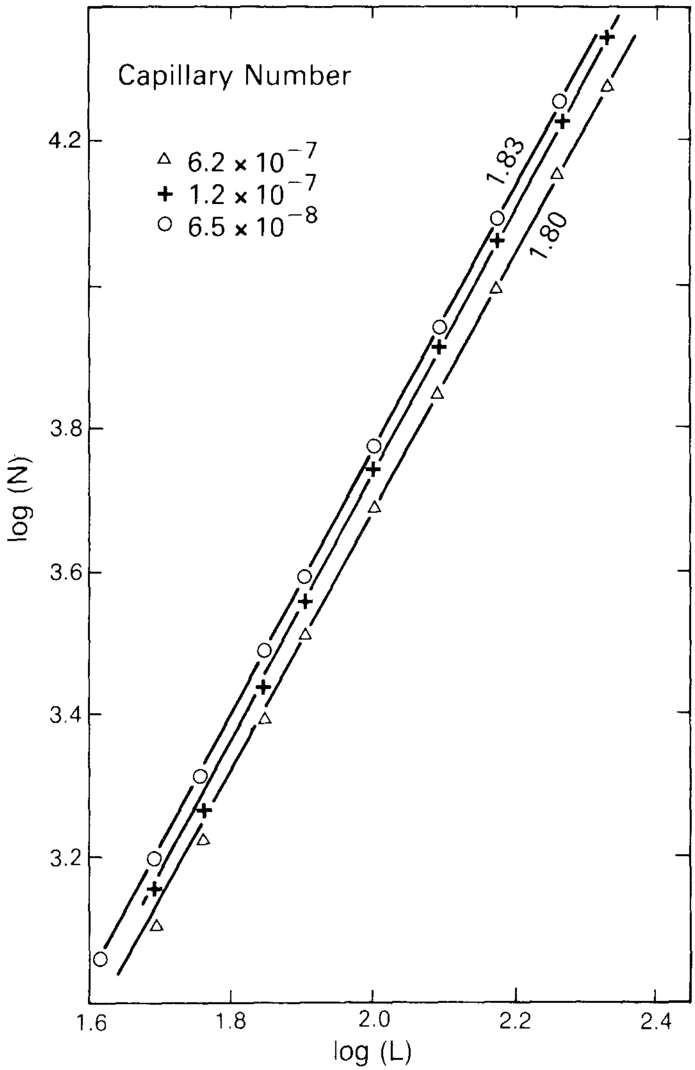

Figure 8. Number N of filled ducts versus size of the $L \times L$ bos for different capillary numbers Ca. The data for $Ca = 3.3 \times 10^{-8}$ are exactly the same as for $Ca = 6.5 \times 10^{-8}$ and are not shown in this picture.

larger than about five mesh sizes a. Now, we can do the same calculation for the saturation of the injected fluid for one of the displacements described previously ($Ca = 1.2 \times 10^{-7}$). We measure the fraction S of displaced ducts (property very close to the saturation) in a $L \times L$ square ($2r = L$) and Figure 10 shows that S is slowly decreasing when r increases.

The fraction S is defined by $S = N/L^2$, so we can deduce the variation of S

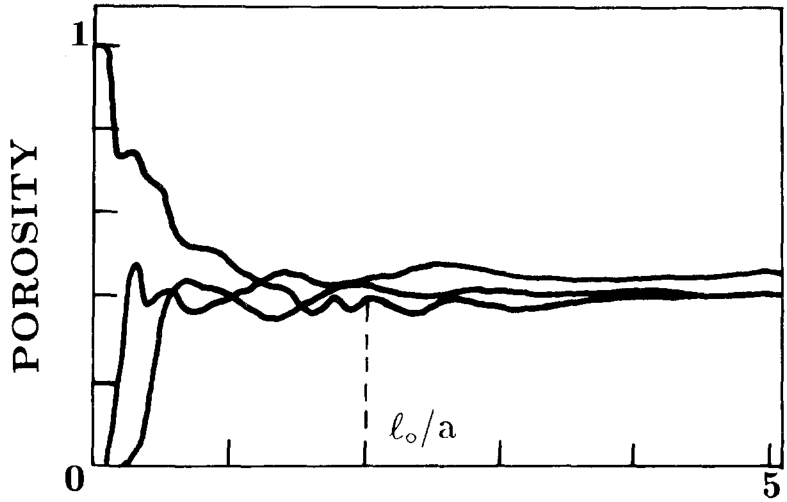

Fig. 9. Porosity ϵ and REV in the 40 000 duct etched network.

from Equation (2):

$$S \propto L^{(D-2)} \tag{3}$$

and, consequently, the exponent $(D-2)$ is negative when $D<2$.

Thus, the fractal approach can be understood as an extension of the REV concept when the property studied (for instance, porosity or saturation) is not

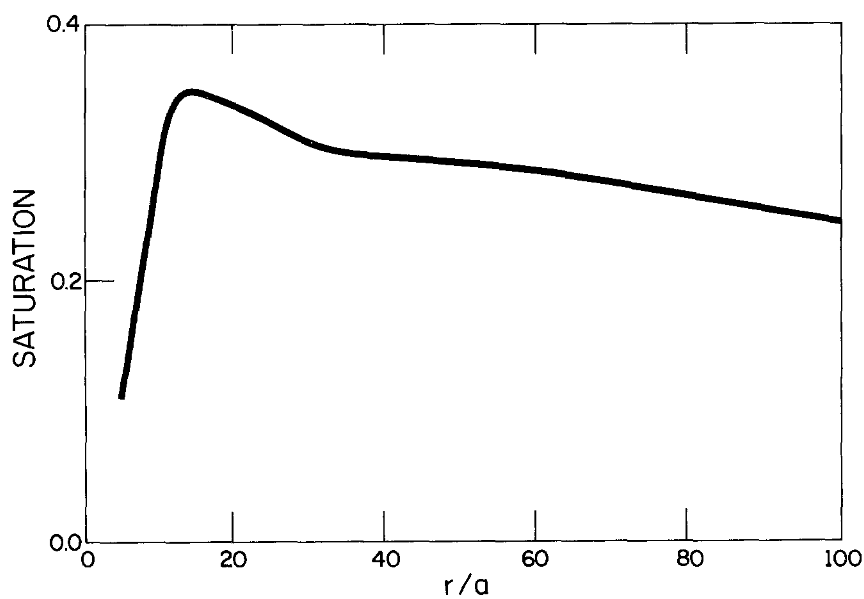

Fig. 10. Fraction of filled ducts S in a $2r \times 2r$ box.

constant but can be approximated by a power law. The classical approach is only a special case, when the exponent is integer. Also notice that the power law is limited to values $r/a > 20$ (or $L > 40$, Figure 8).

This fractal approach can also be very useful to describe the porosity of very inhomogeneous media, the surface area of the pores (Katz and Thomson, 1985) and also viscous fingering in a porous medium (Paterson, 1984) or a Hele Shaw cell (Nittmann *et al.*, 1985).

6. Conclusion

We obtained some results concerning the displacement of a wetting fluid by a nonwetting fluid in a two-dimensional etched network. At low flow rate (quasi-static displacement), the nonwetting fluid forms very thin fingers. This kind of *capillary fingering* is well described by *invasion percolation* theory and the measured fractal dimension is consistent with computer simulations $D = 1.82$.

Consequently, it seems to us that a statistical approach would be more suitable for describing capillary displacement in porous media than the classical equations based on the continuum approach.

7. Note

Since the presentation of this paper at the International Symposium on the Stochastic Approach to Subsurface Flow, in June 1985, some new results have been published in this domain: (i) calculation of injection conditions (capillary number and viscosity ratio) required for 'pure' capillary fingering (Lenormand, 1985 and 1986a); (ii) development of a technique for digitizing the pictures by using a photographic substraction of an image taken before the displacement (see results in Lenormand, 1986b); (iii) publication of an overview on applications of fractals by Williams and Dawe.

Acknowledgement

This research was supported in part by Centre National de la Recherche Scientifique (ATP Milieux Aléatoires Macroscopiques).

References

Bear, J., 1972, *Dynamics of Fluids in Porous Media*, Elsevier, New York.

Bonnet, J. and Lenormand, R., 1977, Réalisation de micromodèles pour l'étude des écoulements polyphasiques en milieu poreux, *Rev. Inst. Franc. Petr.* **42**, 447–480.

Chandler, R., Koplik, J., Lerman, K., and Willemsen, J. F., 1982, Capillary displacement and percolation in porous media, *J. Fluid Mech.* **119**, 249–267.

Chuoke, R. L., Van Meurs, P., and Van der Poel, C., 1959, The instability of slow, immiscible, viscous liquid–liquid displacement in permeable media, *Trans. AIME* **216**, 188–194.

De Gennes, P. G., and Guyon, E., 1978, Lois générales pour l'injection d'un fluide dans un milieu poreux aléatoire, *J. Mécanique* **17**, 403–442.

Guyon, E., Hulin, J. P., and Lenormand, R., 1984, Application de la percolation à la physique des milieux poreux, *Ann. Mines* **5–6**, 17–40.

Katz, A. J. and Thompson, A. H., 1985, Fractal sandstone pores: Implications for conductivity and pore formation, *Phys. Rev. Let.* **54**, 1325–1328.

Lefebvre du Prey, E., 1978, Gravity and capillary effects on imbibition in porous media, *Soc. Petr. Eng. J.*, 195–206.

Lenormand, R., 1981, Déplacements polyphasiques en milieu poreux sous l'influence des forces capillaires. Modélisation de type percolation, Thesis, University of Toulouse.

Lenormand, R., 1985, Différents mécanismes de déplacements visqueux et capillaires en milieux poreux: Diagramme de phase. *CR Acad. Sci. Paris* **301**, 247–250.

Lenormand, R., 1986a, Scaling laws for immiscible displacements with capillary and viscous fingering, Soc. Petr. Eng. paper 15390, presented at the 61st annual conference, New Orleans, October 1986.

Lenormand, R., 1986b, Pattern growth and fluid displacement through porous media, in *Statistical Physics*, edited by H. E. Stanley, North-Holland, Amsterdam.

Lenormand, R. and Bories, S., 1980, Description d'un mécanisme de connexion de liaison destiné à l'étude du drainage avec piégeage en milieu poreux, *CR Acad. Sci. Paris* **291B**, 279–280.

Lenormand, R. and Zarcone, C., 1985a, Two-phase flow experiments in a two-dimensional medium, *Phys. Chem. Hydro. J.* **6**, 215, 497–506.

Lenormand, R. and Zarcone, C., 1985b, Invasion percolation in an etched network: measurement of a fractal dimension, *Phys. Rev. Lett.* **54**, 2226–2229.

Mandelbrot, B. B., 1977, *Fractals, Form, Chance and Dimension*, W. H. Freeman, San Francisco.

Marle, C. M., 1981, *Multiphase Flow in Porous Media*, Technip, Paris.

Nittmann, J., Daccord, G., and Stanley, H. E., 1985, Fractal growth of viscous fingers: quantitative characterization of a fluid instability phenomenon, *Nature* **314**(14), 141–144.

Paterson, L., 1983, Dispersion and fingering in miscible and immiscible fluids within a porous medium, *Powder Tech.* **36**, 71–78.

Paterson, L., 1984, Diffusion-limited aggregation and two-fluid displacements in porous media, *Phys. Rev. Lett.* **52**(18), 1621–1624.

Paterson, L., Hornof, V., and Neale, G., 1984, Visualization of a surfactant flood of an oil-saturated porous medium, *SPE J*, 325–327.

Paterson, L., Hornof, V., and Neale, G., 1984, Water fingering into an oil-wet porous medium saturated with oil at connate water saturation, *Rev. Inst. Franc. Petr.* **39**(4), 517–522.

Saffman, P. G. and Taylor, G. I., 1958, The penetration of a fluid into a porous medium or Hele Shaw cell containing a more viscous liquid, *Proc. R. Soc. Lond.* **A245**, 311–329.

Scheidegger, A. E., 1974, *The Physics of Flow through Porous Media*, Univ. Toronto Press.

Wilkinson, D. and Willemsen, J. F., 1983, Invasion percolation: a new form of percolation theory, *J. Phys. A* **16**, 3365–3376.

Williams, J. K. and Dawe, R. A., 1986, *Transport in Porous Media* **1**, 201–209.