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Learning and Recall in a Dynamic Theory of Coordination Patterns

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Abstract. A dynamic theory of learning and recall of coordination patterns is developed in the context of relative timing skills. Characterizing the coordination patterns in such skills by the collective variable, relative phase, we choose a model system in which the intrinsic pattern dynamics as well as the influence of environmental and memorized information are well understood from previous experimental and theoretical work. To describe learning we endow memorized information with dynamics which is determined by a phenomenological strategy. Similarly, additional degrees of freedom must be introduced to understand recall. As such recall variables we choose the relative strengths with which each memorized pattern acts on the pattern dynamics and model their dynamics phenomenologically. The resulting dynamical system that resembles models used in pattern recognition theory is shown to adequately describe the learning and recall processes. Moreover, due to the operational character of the theory, several predictions emerge that are open to experimental test. In particular, we show under which conditions phase transitions occur in the dynamics of the coordination patterns during learning and during recall. Considering different time scales and their relations we demonstrate how these phase transitions can be identified and observed. Other predictions include the influence of the intrinsic pattern dynamics on the recall process and the existence of history and hysteresis effects in recall. We discuss different forms of "forgetting" and differentiation of memorized information. The results show how a new theoretical view of learning and recall as change of behavioral dynamics can lead to a different understanding of these processes by providing testable predictions.

1 Introduction

Learning and recall are concepts used in a variety of senses within a wide field of study. In the present theoretical investigation learning is defined as a change of behavior in the direction of a to-be-learned behavior that is specified by the environment. Recall designates the process of activating a memorized behavior. (More precise definitions will be given below.) Here the word "behavior" is used in the specific sense of behavioral or coordination pattern, defined by stable and reproducible relationships among the components of a behaving system. More specifically, we will deal with coordination of rhythmic movement, where such relationships are expressed as relative timing.

We are building on a theoretical framework (Kelso et al. 1987; Kelso and Schöner 1987, 1988; Schöner and Kelso 1988a) that is based upon theories of self-organization and pattern formation in non-equilibrium systems, in particular synergetics (Haken 1983a, b). Central ideas of the theoretical, but operational language that has been developed to understand coordination patterns are the characterization of coordination patterns by *collective variables*, the determination of the *dynamics of coordination patterns* and the study of their *stability*. A second building block of the present theory is the concept of *behavioral information* (Schöner and Kelso 1988b–e) that allows to express the influence of environmental demands, of learned behavioral patterns or even of an intention to change behavioral pattern in the form of required behavioral patterns. Behavioral information in this sense can be made part of the pattern dynamics attracting the behavioral pattern toward the required pattern and is measured by the same collective variables that characterize the coordination patterns. In this framework, learning can be viewed as the process by which environmentally defined behavioral information gives rise to memorized behavioral information. To account for this change of the pattern dynamics memorized information itself is endowed with dynamics that can be modelled phenomenologically. Similarly, recall can be viewed as the process by which a particular behavioral information is activated to influence the coordination pattern dynamics. The relative strengths with which different memorized patterns act on the pattern dynamics are introduced as new degrees of freedom and their dynamics are likewise modelled phenomenologically.

We treat a concrete experimentally accessible model system (Kelso 1984; Tuller and Kelso 1989a; Yamanishi et al. 1980) that involves the coordination of rhythmic movements of two limbs in the presence of relative timing requirements. The system is chosen because we know both its intrinsic dynamics (in the absence of relative timing requirements) as well as the role of behavioral information from detailed experimental (Kelso et al. 1986, 1988; Scholz et al. 1987; Tuller and Kelso 1989a; Yamanishi et al. 1980) and theoretical (Haken et al. 1985; Schöner et al. 1986; Schöner and Kelso 1988b, c, e) work. Furthermore, rhythmic movement characterizes a broad class of motor behaviors including many basic behaviors such as locomotion and feeding, but also relating to such non-trivial, "higher" behaviors like human speech (e.g. Stetson 1951). For such rhythmic activities environmental constraints and skills can often be expressed as demands on the relative timing order (see, e.g. Shaffer 1982 for review). In this investigation we address the learning of such relative timing skills. Possible generalizations of the theoretical approach are discussed in Sect. 7.

The article is structured as follows: We first review briefly the theoretical models capturing the intrinsic dynamics and its adaptation to behavioral information (Sect. 2). By endowing behavioral information with dynamics we describe learning, first in the limit case of a capacity to learn only a single relative timing pattern (Sect. 3). We show by analytical and numerical means under which conditions phase transitions occur during learning and how they can be observed in two time scale régimes (Sect. 4). Generalizing to learning of multiple patterns we address the question of recall. The new degrees of freedom and their dynamics are determined in Sect. 5. The predictions of the theory are established by numerical study and their experimental relevance is discussed (Sect. 6). In Sect. 7 we discuss possible generalizations of the present approach and place the present theoretical attempt into a broader perspective.

2 The Coordination System: Relative Phase Dynamics

By using relative phase, ϕ, as the collective variable or order parameter to characterize the coordination patterns of two rhythmically moving limbs we choose a level of description for the systems under study. According to the dynamic pattern strategy (Kelso and Schöner 1978, 1988; Schöner and Kelso 1988a) stable and reproducible patterns of coordination are then mapped onto attractors of a relative phase dynamics. Based on which patterns of relative timing a given system can perform without additional behavioral information and on the observation of multi-stability and transitions among such patterns a concrete model of the intrinsic dynamics can be determined. Here we employ a particular form of the intrinsic relative phase dynamics that was identified for Kelso's (1984) movement system following his discovery of a phase transition in that system. The basic experimental observation is as follows: Human subjects are asked to move two limbs rhythmically at a common frequency. Two patterns of coordination, in-phase (homologous muscles groups contracting together) and anti-phase (homologous muscles contracting alternately) are found to be stably performed at various frequencies. However, when frequency of movement is increased, a spontaneous switch from the anti-phase to the in-phase pattern of coordination is observed, while the in-phase pattern remains stable at all reachable frequencies.

A simple form of the intrinsic pattern dynamics that fulfills basic symmetry requirements was found by Haken et al. (1985):

$$\dot{\phi} = -a\sin(\phi) - 2b\sin(2\phi)\,, \tag{1}$$

where a, b are parameters. This model captures the observed phase diagram: for $0 < a < 4b$ two stable states (fixed point attractors) $\phi = 0$ and $\phi = \pm\pi$ exist, while for $a > 4b > 0$ only $\phi = 0$ remains stable. By analyzing the nature of the transitions from anti- to in-phase and taking fluctuations into account (Schöner et al. 1986):

$$\dot{\phi} = f_{\text{intr}}(\phi) = -a\sin(\phi) - 2b\sin(2\phi) + \sqrt{Q}\,\xi_t \tag{2}$$

(where ξ_t is gaussian white noise of unit variance and Q is the noise strength parameter) it can be shown, that these dynamics capture the behavior of the system in detail. In particular, loss of stability is predicted to be observable as the transition is approached in the form of enhancement of the relative phase fluctuations and enhancement of the *(local) relaxation time*, τ_{rel} (time to return to the coordination pattern from a small perturbation). Both quantities were measured experimentally and loss of stability was indeed found (Kelso et al. 1986; Scholz et al. 1987). Furthermore, relations among measurable time scales are established in the theory. The *equilibration* (or *global relaxation*) *time*, τ_{equ}, defined as the time it takes the system to achieve a stationary probability distribution from a typical initial distribution is determined in the bistable régime of (2) mostly by the typical time it takes to traverse from one basin of attraction into another. Only when observed on a time scale, τ_{obs}, intermediate between local and global relaxation times

$$\tau_{\text{rel}} \ll \tau_{\text{obs}} \ll \tau_{\text{equ}} \tag{3}$$

is the system in a stationary state described by one of the attractors *(local stationarity)*. In experiments (Scholz et al. 1987) it was indeed found that switching occurred as this relationship was violated. In the other limit:

$$\tau_{\text{obs}} \gg \tau_{\text{equ}} \tag{4}$$

the system is governed by the stationary probability distribution of the order parameter *(global stationarity)*. In the bistable case this is a bimodal, non-gaussian distribution. Characteristic features of the actual transient switching process (like the mean duration of this process, the so-called mean switching time, and the distribution of the switching times) were predicted and found experimentally to be in good agreement with theory. Thus theory and experiment convergently show that the coordination patterns of this system obey the intrinsic dynamics (2).

Adaptation of these coordination patterns to environmental (Tuller and Kelso 1989a) and memorized (Yamanishi et al. 1980) requirements have been studied in two experiments both involving bimanual rhythmic finger tapping at a common frequency [below a transition frequency in the sense of Kelso's (1984) experiment]. In Tuller and Kelso's experiment a temporally structured environment was present in the form of two metronomes which paced the two index fingers at the same frequency. By changing the relative phase of the two metronomes, the environmentally-required relative phase, was varied. In Yamanishi et al.'s (1980) experiment subjects practiced several such relative phasing conditions until a certain criterion level of performance was reached. Feedback methods were used to enhance learning. Then a given relative phasing pattern was elicited, using two metronomes, which were turned off for the actual measurement. Thus, in this case, the requirement was a memorized relative phase. In both experimental studies two robust findings emerged. First, the mean performed relative phasing deviated systematically from the required task such that it was closer to the nearest intrinsic pattern, either in-phase or anti-phase. Second, the variability of the performed phasing was minimal in the two intrinsic patterns and larger at intermediate conditions.

A theoretical description of these system is possible by employing the concept of behavioral information, defined as a required behavioral pattern (Schöner and Kelso 1988b–e). This requirement may result from a perceptual process (environmental information) or from the activation of a learned behavior (memorized information) or even from an intention to change behavior. That behavioral information in this sense is specific to the biological system and function is evident from the fact that this information is measured by the same type of collective variables that characterize the behavioral patterns, in the present case, by the relative phase required by the environment, ψ_{env}, or by memory, ψ_{mem}. Behavioral information acts on the pattern dynamics as a perturbation of the intrinsic dynamics, attracting the order parameter toward the required pattern. Taking certain symmetry requirements into account a simple functional form of the resulting dynamics is (Schöner and Kelso 1988b)[1]:

$$\dot{\phi} = f_{\text{intr}}(\phi) + c_{\text{env}} f_{\text{info}}(\phi, \psi_{\text{env}}) \tag{5}$$

for the case of environmental information and

$$\dot{\phi} = f_{\text{intr}}(\phi) + c_{\text{mem}} f_{\text{info}}(\phi, \psi_{\text{mem}}) \tag{6}$$

for the case of memorized information where $f_{\text{info}}(\phi, \psi) = -\sin(\phi - \psi)$. Here $c_{\text{env}} > 0$ and $c_{\text{mem}} > 0$ are constants that measure the strength of the perturbation of the intrinsic dynamics by the respective type of information.

Several predictions arising from these pattern dynamics with behavioral information can be compared to experiment (Schöner and Kelso 1988b, d). In particular, the influence of the intrinsic dynamics on the behavioral patterns in the presence of behavioral information accounts for the two main experimental results mentioned above: When intrinsic dynamics and behavioral information cooperate the resulting state is close to the requirement and very stable, while in the case where they compete the resulting state is shifted from the requirement toward the instrinsic patterns (here in-phase and anti-phase) and is less stable (larger fluctuations). Qualitative change of the dynamics can occur as behavioral information or the strength of the corresponding perturbations change. An aspect of this instability has been seen in the two experiments: Although for most required relative phases a monomodal distribution of the performed relative phase was observed, at some intermediate relative phase values close to 90 deg this distribution was bimodal, reflecting the underlying bistability under these conditions. In summary, we find that the system is governed by the dynamics of (5) and (6) when performing under the influence of environmental or memorized requirements.

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[1] For convenience we use slightly different conventions than in the reference

3 Learning a Single Relative Phase: Theoretical Model

In the present theoretical framework, the problem of learning a relative timing skill can be expressed very simply: How do the relative phase dynamics with environmental information, (5), evolve in time to assume the form with memorized information, (6), that describes the skilled performance? We define learning as the temporal evolution of the pattern dynamics in the presence of environmental information. For example, feedback (Yamanishi et al. 1980) or a structured environment (cf. discussion in Schöner and Kelso 1988c) may effectively define the to-be-learned pattern. As a first step we consider the case where only one coordination pattern is learned, corresponding to a memory with capacity one.

We assume that both environmental and memorized information act on the pattern dynamics:

$$\dot{\phi} = f_{\mathrm{intr}}(\phi) + c_{\mathrm{env}}(t) f_{\mathrm{info}}(\phi, \psi_{\mathrm{env}}) + \hat{c}_{\mathrm{mem}}(\psi_{\mathrm{mem}}) f_{\mathrm{info}}(\phi, \psi_{\mathrm{mem}}) \,. \tag{7}$$

However, the strength of memorized information must now fulfill the limit cases $\hat{c}_{\mathrm{mem}} \approx 0$ before learning and $\hat{c}_{\mathrm{mem}} = c_{\mathrm{mem}} > 0$ after learning [where c_{mem} is the constant of (6)]. The strength, $c_{\mathrm{env}}(t)$, of environmental information can be explicitly time dependent reflecting the presence or absence of feedback. A potential form of this dynamics will be useful later:

$$\dot{\phi} = -\frac{\mathrm{d}V_{\mathrm{total}}(\phi)}{\mathrm{d}\phi} + \sqrt{Q}\xi_t \tag{8}$$

with

$$V_{\mathrm{total}} = -a\cos(\phi) - b\cos(2\phi) - c_{\mathrm{env}}(t)\cos(\phi - \psi_{\mathrm{env}}) - \hat{c}_{\mathrm{mem}}(\psi_{\mathrm{mem}})\cos(\phi - \psi_{\mathrm{mem}}) \,.$$

It is clear that additional degrees of freedom have to be introduced to account for the change of the pattern dynamics during learning. The obvious idea is to endow memorized information with dynamics. Thus the parameter, ψ_{mem}, now becomes a memory variable, $\psi_{\mathrm{mem}}(t)$. To constrain models of the learning dynamics we use the basic phenomenological strategy of mapping the stable, reproducible result of learning, the to-be-learned pattern, ψ_{env}, onto an attractor of these dynamics. In principle, information on constraints that are intrinsic to the learning process (e.g., are certain patterns learned more quickly than others?) should be used in the modelling. At present, we lack such information for our model system. We therefore choose a simple 2π-periodic form of the learning dynamics that has a single attractor at the to-be-learned relative phase, ψ_{env}:

$$\dot{\psi}_{\mathrm{mem}} = -\tau_{\mathrm{learn}}^{-1} \sin(\psi_{\mathrm{mem}} - \psi_{\mathrm{env}}) \,. \tag{9}$$

The parameter, $\tau_{\mathrm{learn}} > 0$, defines the relaxation time of the attractor at the to-be-learned pattern and can be viewed as the typical time scale of the learning process which is clearly much larger than the relaxation times, τ_{rel}, of the behavioral patterns:

$$\tau_{\mathrm{rel}} \ll \tau_{\mathrm{learn}} \tag{10}$$

so that the above definition of learning is consistent. The assumption that a single time scale is sufficient to characterize the learning process is actually rather non-trivial and possibly not generally correct. Some experimental evidence indicates that in a late phase of skill learning performance continues to improve on all time scales (see Fitts 1964, for review). That means, that if the behavior is characterized by some performance criterion (typically an error or precision variable or the speed at which certain stereotype movements can be performed), its time dependence is of the form of a power law for large times. This interesting aspect which hints at an essentially nonlinear nature of the learning dynamics, has not yet been addressed in the present framework. Experimentally, an early phase of skill learning often exhibits exponential improvement with respect to typical performance measures. For the sake of simplicity we have assumed here that the learning dynamics are independent of the actual performance, ϕ.

The limit cases for the strength of memorized information, $\hat{c}_{\mathrm{mem}}$, can be accomodated by choosing this strength as a match function that measures the distance between environmentally required and memorized relative phase. We adopt the simple normalized and periodic function:

$$\hat{c}_{\mathrm{mem}}(\psi_{\mathrm{mem}}) = c_{\mathrm{mem}} \cos^2\left(\frac{\psi_{\mathrm{env}} - \psi_{\mathrm{mem}}}{2}\right) \tag{11}$$

avoiding the introduction of additional parameters. Here $c_{\mathrm{mem}} > 0$ is the constant of (6) and denotes the maximal value of the strength function, $\hat{c}_{\mathrm{mem}}$, when memory variable and required pattern coincide. When memorized and required relative phase have the largest possible distance, π, the strength is zero. Note, that distance between different types of behavioral information can be operationally defined because this information is measured in terms of the collective variables. For example, the typical variability of performance may provide a metric for such a distance.

4 Learning a Single Relative Phase: Theoretical Results

Because the limit cases before learning (5) and after learning (6) are well known theoretically and experimentally, we can determine all parameters of the intrinsic dynamics and of the contributions by behavioral information based on earlier work (cf. Schöner et al. 1986; Schöner and Kelso 1988b), which leads to parameter values of the following orders: $a \sim 1.0$ Hz, $b \sim 1.0$ Hz, $Q \sim 0.25$ Hz, $c_{\mathrm{env}} \sim 5$ Hz, $c_{\mathrm{mem}} \sim 5$ Hz. Note, that the time axis in our system is endowed with physical units (seconds) that can be compared to real experimental times. The relaxation times of the intrinsic patterns are then in the order of $\tau_{\mathrm{rel}} \sim 0.5$ s, but increase strongly as a state loses stability (see Scholz et al. 1987). In mono-stable cases the equilibration time, τ_{equ}, is of the same order of magnitude, while it is much larger in bi-stable cases. Estimates can be based on the mean first passage time from one state to another (Schöner et al. 1986).

The only new parameter to be estimated is the time scale of learning, τ_{learn}. In this context a conceptual subtlety needs to be addressed. Most experiments on skill learning are not actually performed with a continuously behaving system as conceptualized in our time scales argument, but over a series of "trials" or "sessions". How can the time scale of learning be estimated in such paradigms? If the different "sessions" are not temporally separated very much, we can view them as discrete segments of a continuous learning process. The present model accomodates such a view, because the initial condition of the memory variable allows to express previous experience (see below). The situation is, however, complicated by such questions as interference and decay, which can be addressed theoretically only when learning of multiple patterns is considered (Sect. 5). Obviously, from the present theoretical viewpoint, it seems desirable to observe learning in a more continuous or temporally controlled fashion.

If we observe the system on the time scale of learning, then an important question is, whether local or global stationarity is fulfilled. This is non-trivial only in the case of multistability, that is, if several different patterns can be performed within the same task and in the presence of the same environmental information as is the case, for example, for patterns close to 90 deg relative phase in the present system. Two different methodologies can be related to these time scales relations in paradigms involving multiple learning "sessions". If the system can be prepared to behave in a well-defined pattern during each of the trails, then the data can be analyzed as locally stationary ones. If, on the other hand, the system is distributed over the different patterns from trial to trial (or within trial), then we may view the ensemble of such trials as an estimate of a stationary probability distribution of the different states and the time scale relation of global stationarity effectively applies.

For the purpose of concrete simulations we choose the parameter time scale of learning of the order $\tau_{\mathrm{learn}} \sim 20$ s, which is much larger than all local relaxation times, but probably still too fast to be realistic. However, the qualitative results do not depend on the exact value of this time scale as long as its relation to other time scales is correct. In direct numerical solutions of (8) and (9) (employing a fourth-order Runge-Kutta algorithm), that we present first, the limit of local stationarity is realized. Subsequently we address the limit of global stationarity by calculating the evolution of the stationary probability distribution of relative phase.

To illustrate how the model discribes the learning of a relative timing skill we show in Fig. 1 the change of the behavioral dynamics as the memory variable learns by plotting the potential, V_{total}, that represents the dynamics of relative phase (8), as a function of time. While the initial conditions of the performed relative phase do not usually matter (except in bistable cases, see below), the initial condition of the memory variable affects the transient learning process (for example, its duration) and thus the performance during learning. Experimental paradigms involving learning in "sessions" may be described consistently by viewing the final state of a previous learning "session" as such an initial condition for the next learning "session". The final result of the learning process, the to-be-learned relative phase, ψ_{env}, is an attractor of the memory dynamics and does not depend on initial conditions. Thus, perturbing the memory variable (for example, through a temporary change of the required relative phase) leads to relaxation of the memory variable and thus performance within the time scale, τ_{learn}. This

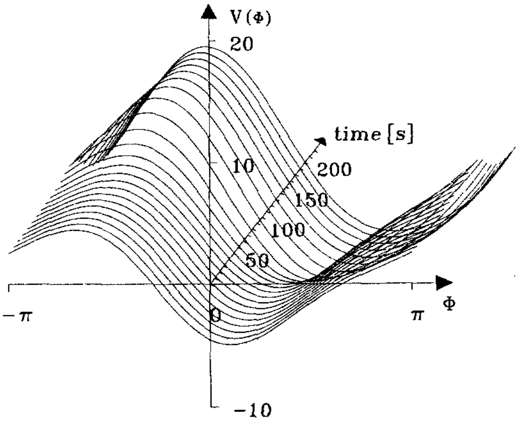

Fig. 1. The potential, $V_{\mathrm{total}}(\phi)$, of (8) is plotted as a function of time while the memory variable, ψ_{mem}, evolves from an initial value of $\psi_{\mathrm{mem}}=0$ to the required relative phase, $\psi_{\mathrm{env}}=\pi/2$, according to (9). Note how the strength of the potential increases due to the growing match factor, $\hat{c}_{\mathrm{mem}}$ (11), and the minimum shifts closer to the required relative phase as learning proceeds

suggests an experimental method to determine the time scale of learning, τ_{learn}, as well as to experimentally test the validity of the basic theoretical strategy of mapping the to-be-learned behavior onto an attractor.

The most interesting result arising from the theory is that phase transitions can occur in the pattern dynamics during learning. There are two scenarios for such phase transitions, both of which can be illustrated in a single simulation of the dynamics shown in Fig. 2. The first abrupt pattern change in this simulation is due to the initial bistability of the pattern dynamics. As mentioned before, the pattern dynamics may remain bistable (both in theory and in the pertinent experiment) if a relative phase, here $\psi_{\mathrm{env}} \approx 114$ deg, is imposed that is in conflict with the intrinsic tendencies. After sufficient learning, however, only one stable state remains. In the theory, this change is due to the increasing strength of memorized information and takes the form of a phase transition. Because the time scales of ϕ and ψ_{mem} are well separated (10) we can analyze this instability by treating the memory variable as an adiabatic constant on the time scale τ_{rel}. We calculate the stationary states of (7) as a function of the value of the memory variable in this learning sequence by solving:

$$0 = f_{\mathrm{intr}}(\phi_{\mathrm{stat}}) + c_{\mathrm{env}} f_{\mathrm{info}}(\phi_{\mathrm{stat}}, \psi_{\mathrm{env}}) + \hat{c}_{\mathrm{mem}}(\psi_{\mathrm{mem}}) f_{\mathrm{info}}(\phi_{\mathrm{stat}}, \psi_{\mathrm{mem}}) \qquad (12)$$

for ϕ_{stat}, where ψ_{mem} is treated as a parameter. The relaxation time is then given as:

$$\tau_{\mathrm{rel}} = -\left[\frac{d}{d\phi}\bigg|_{\phi=\phi_{\mathrm{stat}}} \{ f_{\mathrm{intr}}(\phi) + c_{\mathrm{env}} f_{\mathrm{info}}(\phi, \psi_{\mathrm{env}}) + \hat{c}_{\mathrm{mem}}(\psi_{\mathrm{mem}}) f_{\mathrm{info}}(\phi, \psi_{\mathrm{mem}}) \} \right]^{-1} . \qquad (13)$$

In Fig. 3 these stationary states and their relaxation time are plotted as a function of the memory variable, ψ_{mem}, which varies as in the aforementioned simulation (Fig. 2). Note the increase in the relaxation time of the pattern near anti-phase at the first instability.

The second instability in the simulation of Fig. 2 is typical of a different scenario for phase transitions during learning. Here, no argument can be made about qualitative difference of the pattern dynamics before and after learning. The mechanism can be understood intuitively as follows: Initially memorized information is still acting only weakly on the pattern dynamics due to poor match, so that the intrinsic dynamics lead to an "unskilled" performed pattern quite far from the required pattern (and close to an intrinsic pattern, here $\phi = 0$). As memorized information increases in strength during learning, a new attractor emerges close to the required pattern. The bifurcation analysis of Fig. 3

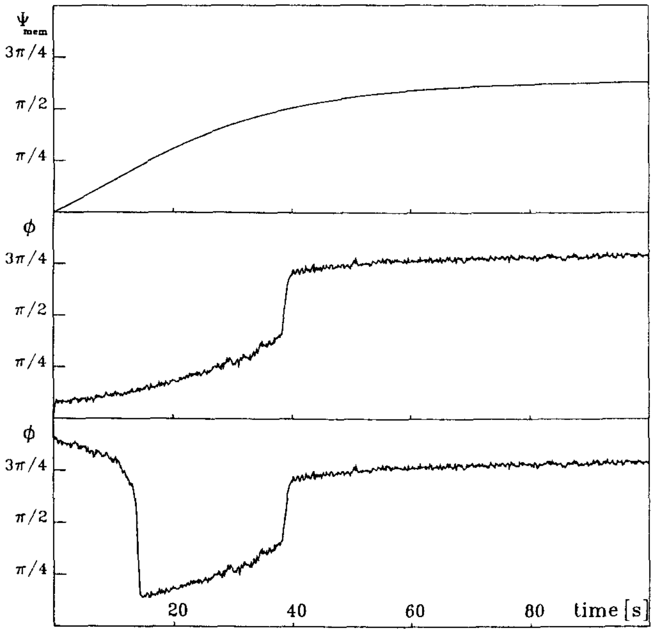

Fig. 2. Simulations of the single-memory learning and behavioral dynamics (7) and (9) show two phase transitions in the relative phase dynamics during learning. In the top panel the evolution of the memory variable from zero to the required relative phase, $\psi_{\mathrm{env}} = 2.0$ rad is shown. The middle panel shows the evolution of the relative phase if the initial condition is $\phi = 0$, while the lower panel shows this evolution if the initial condition is $\phi = \pi$. Note the bistability in the first ~15 s, followed by an instability of the state close to anti-phase. A second instability occurs around $t \sim 40$ s leading to the final "skilled" state close to the requirement

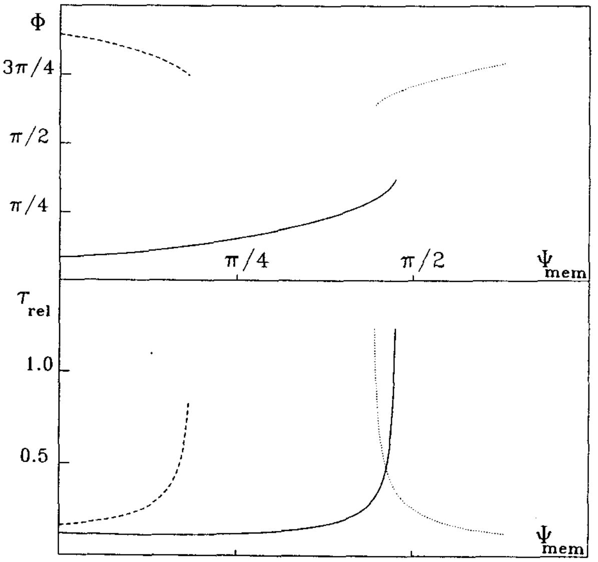

Fig. 3. A bifurcation analysis of the phase transitions of Fig. 2: the top panel shows the stable stationary solutions of the relative phase dynamics when the memorized relative phase is treated as a parameter (12) as a function of that parameter, which varies from zero to 2.0 rad, as in Fig. 2. In the lower panel the corresponding relaxation times (13) are displayed. Note, the divergence of relaxation time at each instability

shows, that this "skilled" pattern coexists bistably with the "unskilled" pattern, before the "unskilled" pattern loses stability. No smooth blending of the two patterns occurs, because the distance between them is large compared to their the local stabilities (e.g. as measured by the standard deviation of the relative phase fluctuations). This competition between "unskilled" and "skilled" behavior is at the origin of the phase transition.

Under the condition of local stationarity (3) such phase transitions can be observed experimentally as abrupt jumps in performance during learning. At such jumps, loss of stability is experimentally observable through the usual stability measures, fluctuations and relaxation time, because the learning dynamics can be viewed as quasi-stationary on the time scale of the behavioral dynamics. Under conditions of global stationarity (4) experimental results must be compared to the stationary probability distribution of relative phase. In this case phase transitions can be observed as qualitative changes of this distribution (for example, from bi-modal to mono-modal) or as abrupt shifts of the peaks of the distribution as will be shown now.

Solving the learning dynamics, (9), as

$$\psi_{\text{mem}}(t) = \psi_{\text{env}} - \arctan\{\tan[\psi_{\text{env}} - \psi_{\text{mem}}(t=0)] \times \exp[-t/\tau_{\text{learn}}]\} \qquad (14)$$

we obtain the non-autonomous Fokker-Planck equation for the time-dependent probability distribution, $P(\phi, t)$, of relative phase (cf., e.g. Gardiner 1983):

$$\partial P(\phi, t) \partial t = -\frac{\partial}{\partial \phi}\{V'_{\text{total}}(\phi, \psi_{\text{mem}}(t)) P(\phi, t)\} + \frac{Q}{2}\frac{\partial^2}{\partial \phi^2} P(\phi, t), \qquad (15)$$

where the prime indicates the derivative by ϕ. Because we are interested in what happens on the time scale of learning: $\tau_{\text{obs}} \sim \tau_{\text{learn}}$, we can treat ψ_{mem} adiabatically due to the condition of global stationarity, (4). The stationary distribution, $P_{\text{stat}}(\phi)$, is then obtained from:

$$0 = -\frac{\partial}{\partial \phi}\{V'_{\text{total}}(\phi, \psi_{\text{mem}}) P_{\text{stat}}(\phi)\} + \frac{Q}{2}\frac{\partial^2}{\partial \phi^2} P_{\text{stat}}(\phi) \qquad (16)$$

in which ψ_{mem} is treated as a parameter. The general solution with periodic boundary conditions is given as (Gardiner 1983, Chap. 5.2.2):

$$P_{\text{stat}}(\phi) = \mathcal{N} \exp[-2V_{\text{total}}(\phi, \psi_{\text{mem}})/Q], \qquad (17)$$

where $\mathcal{N}$ is a normalization constant. Obviously, this stationary probability distribution reflects the underlying dynamics through its potential.

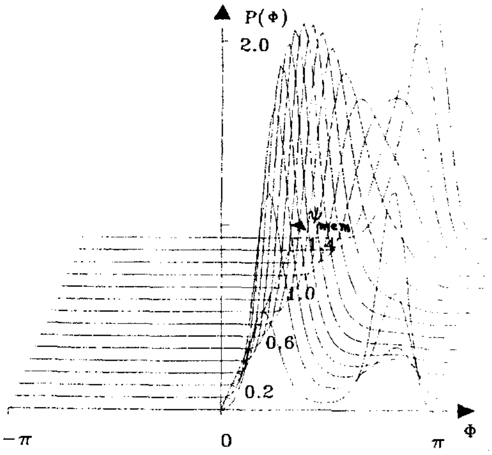

Fig. 4. The phase transitions during learning analyzed in the régime of global stationarity. The stationary probability distribution of relative phase, $P_{\text{stat}}(\phi)$ (17) is plotted as a function of relative phase, ϕ, as the memorized relative phase, ψ_{mem}, varies from zero to 1.4 rad (third axis). Note that the distribution is initially bimodal – reflecting the underlying bistability of the dynamics – and changes to monomodal at the first phase transition ($\psi_{\text{mem}} \sim 0.4$ rad). It changes shape again at the second phase transition ($\psi_{\text{mem}} \sim 1.3$ rad)

In Fig. 4 this distribution is plotted as a function of the memory variable, ψ_{mem}, which varies from 0 to 2. rad as in the bifurcation diagram of Fig. 3. Initially, the bistable relative phase dynamics induce a bimodal distribution that changes to monomodal at the first phase transition. The qualitative change at the second phase transition is not resolved as bimodality, because the two peaks merge at the chosen noise level ($Q = 0.5$ Hz), but is clear in the abrupt change of the peak position. This picture illustrates how phase transitions can be observed in experimental paradigms that realize global stationarity (4). On a more speculative note we may think of behavioral changes during development, which clearly have a very slow time scale, τ_{learn}, as being observable in this way.

Because phase transitions as qualitative changes of the order parameter dynamics can be identified independently of detailed modelling assumptions, the experimental observation of a phase transition during learning would provide strong evidence for the validity of the conceptual framework of this theory. Moreover, such phase transitions could be viewed as evidence for constraints or lawful aspects in a learning system and thus provide insight into the nature of the learning process (e.g., relating to such concepts as stages of learning or classification of phases of learning).

5 Learning and Recall of Multiple Relative Phases: Theoretical Model

The subjects of Yamanishi et al. (1980) experiment were clearly able to learn several relative timing patterns. To capture the learning of multiple patterns we may consider multiple memory variables representing these multiple required patterns. However, not all memorized patterns can influence the pattern dynamics at the same time (in which case the system would, de facto, be reduced again to a single memorized pattern). Therefore, learning multiple patterns can be given meaning only if the notion of *recall* is introduced, defined as the selective activation of a learned coordination pattern under the influence of the same or similar environmental information as was present during learning. Insofar as the environmental information need not be exactly identical to one of the memorized patterns, recall in this sense resembles pattern recognition in models of perception. In a straightforward generalization we define *learning as* the process, occurring in the presence of environmental information, by which the system changes such that a required pattern can be recalled that could not be recalled before. This definition is consistent with the one given previously (Sect. 3). The capacity, N, of memory is defined as the number of patterns that can be learned.

To derive a model for the learning and recall of multiple relative timing patterns we introduce a set of N memory variables, ψ_i $(i=1,2,\ldots,N)$, that represent memorized relative phases. Each memory variable acts as a perturbation of the form, f_{info} (6) onto the pattern dynamics which expresses the functional equivalence of the different memory variables. However, not all memory variables act with the same strength. To express the selective activation of a memorized pattern we introduce relative strength factors, w_i, that are normalized and positive. Combined into a vector, $\mathbf{w}$, these relative strength factors live in a space

$$\mathbf{S}_N=\left\{(w_1,w_2,\ldots,w_N),\ w_i>0,\ \sum_{i=1}^{N} w_i=1\right\},$$

called the N-simplex, which can be visualized as a finite hypersurface in an N-dimensional cube that connects all points that have only one non-vanishing component, which is equal to one. Activation of one particular memorized pattern means that the vector, $\mathbf{w}$, is in one of these corners. The relative phase dynamics thus generalize to:

$$\begin{aligned}\dot{\phi}&=f_{\text{intr}}(\phi)+c_{\text{env}}(t)f_{\text{info}}(\phi,\psi_{\text{env}})\\&\quad+\sum_{i=1}^{N} w_i\hat{c}_{\text{mem}}(\psi_i)f_{\text{info}}(\phi,\psi_i),\end{aligned}\tag{18}$$

wherein all functions and constants are defined as in (5), (6), and (11) above. Anywhere on the N-simplex, the total strength of memorized information is bounded by c_{mem}. Note that each memory variable acts onto the pattern dynamics only as far as its match function, $\hat{c}_{\text{mem}}$, prescribes.

It is clear that additional degrees of freedom have to be introduced because neither memory variables nor the order parameter account for the change of the pattern dynamics during recall. The obvious idea is to endow the relative strengths, $\mathbf{w}$, with dynamics. Such recall dynamics can be constrained by mapping the corner of the N-simplex, that corresponds to recall of the "best-fitting" (see below) pattern, onto an attractor. A concrete functional form can be found by assuming first order dynamics of $\mathbf{w}$ and exanding its vectorfield into a power series. The functional equivalence of the different memory variables can be used as a symmetry to reduce the coefficients. Here we use, for convenience, unnormalized variables, $n_i\in\mathbf{R}$, from which we project according to

$$w_i=\frac{|n_i|}{\sum_{j=1}^{N}|n_j|}\qquad(i=1,2,\ldots,N).\tag{19}$$

A simple model fulfilling the symmetry requirements is:

$$\dot{n}_i=m(\phi,\psi_i)n_i-r\left(\sum_{j=1}^{N}n_j^2\right)n_i+q\xi_i,\tag{20}$$

where ξ_i are independent gaussian white noise processes and q is a noise strength parameter. The function, $m(\phi,\psi_i)$, plays the role of a match function similar to $\hat{c}_{\text{mem}}$: When this function is positive, the corresponding recall variable, n_i, is repelled from zero, while when this function is negative, n_i relaxes to zero. The nonlinear part of the dynamics, parametrized by r, introduces a competition among recall variables, favoring inhomogenous distributions along one of the axes of the N-dimensional coordinate system. One can show that asymptotically the recall variable with the largest match, m, is attracted to a non-zero value, while all others relax to zero. A simple functional form that takes the angular character of the phase variables into account is:

$$m(\phi,\psi_i)=\tau_{\text{rec}}^{-1}[\cos(\phi-\psi_i)+\cos(\psi_{\text{env}}-\psi_i)]/2\,.\tag{21}$$

Here we have assumed that the performed pattern would influence the recall dynamics in the same way as the required pattern, and thus have assigned a role to performance in recall. The new parameter, τ_{rec}, can be shown to determine the time scale of recall asymptotically (close to recall) in the case of optimal match. For consistency the time scale of recall must be faster than that of learning, so that at least:

$$\tau_{\text{rec}}\ll\tau_{\text{learn}}\tag{22}$$

is valid. In principle the recall dynamics should be modelled based on insights into specific intrinsic tendencies of the recall process, although we do not at present see a practical way of doing that. The dynamics in (21) are similar to the multi-mode laser equations (Haken 1970), which capture mode competition, and resemble typical models used to describe selective dynamics in pattern recognition (e.g. Shimizu and Yamaguchi 1987), micro-evolution (Eigen and Schuster 1979) and elsewhere.

Generalizing the learning dynamics to several memory variables the question arises of how such memorized patterns interact. We make an additional assumption by imposing an interference type of "forgetting": The recalled (and thus best matching) memory variable has a finite learning rate, while all other memory variables are marginally stable:

$$\dot{\psi}_i = \tau_{\text{learn}}^{-1} w_i \sin(\psi - \psi_i) . \qquad (23)$$

In principle, this assumption can be directly tested in experiment.

6 Learning and Recall of Multiple Relative Phases: Theoretical Results

Three new parameters were introduced for recall: The time scale of recall, τ_{rec}, must be chosen much smaller than the learning time scale. In the absence of detailed experimental information we choose $\tau_{\text{rec}} = 1$ s, which makes recall slower than the relaxation of the performed patterns. The results do not depend strongly on the parameter, r, that measures the nonlinearity in the recall dynamics, and is fixed somewhat arbitrarily at $r = 5$ Hz. The noise level for the recall dynamics is kept at $q = 0.01$ Hz, which relative to the deterministic part, garantees a good compromise of stability and flexibility (see below). In numerical simulations of the dynamics we choose the capacity of memory as $N = 3$ for simplicity, but larger values are possible. Which aspects of the learning, recall and pattern dynamics are observable depends, of course, on the relations of the various time scales. Here we present numerical simulations that represent the limit of local stationarity. As discussed before the stationary probability distribution representing the stability structure of relative phase through its modality is the relevant measure in the case of global stationarity.

6.1 Recall Without Learning

We first study the system on the time scale of recall by setting $\tau_{\text{learn}} \rightarrow \infty$, which is a relevant limit case because of time scale relation (22). Thus the memory variables are held fixed, in most numerical examples at $\psi_1 = 0$, $\psi_1 = \pi/2$, and $\psi_3 = \pi$. Subsequently the interplay of both learning and recall is studied on the time scale of learning (Subsect. 6.2).

In the simplest case, when a pattern is environmentally required that is identical to one of the memorized patterns, say $\psi_i = \psi_{\text{env}}$, this pattern is recalled, that is, the recall variables relax to the corresponding corner of the N-simplex: $w_i = 1$ and $w_j = 0$ for $j \neq i$. The initial conditions of the performed relative phase, ϕ, do not normally play a role (except if the pattern dynamics is initially multistable, see below). The initial conditions of the recall variables are accessible to experiment, if different patterns are required sequentially. The last successfully recalled pattern then defines these initial conditions of the recall variables. We may thus ask, how the previous condition affects recall. If one memorized pattern clearly matches best a new requirement, the attractor of the recall dynamics is globally stable and the initial conditions affect only the transient recall process. (More dramatic effects of previously recalled pattern are predicted if an ambiguous pattern is required, that matches several memorized patterns equally well, see below.) Because the performed pattern may depend on the nature of this transient, such dependence may be experimentally observable as illustrated by the simulation shown in Fig. 5. Note, that due to the fact that the recall process is slower than the relaxation of the pattern dynamics the performed pattern is actually in a stable state during the transient of recall. This stability could be experimentally established in systems in which the times scales of recall and pattern relaxation are sufficiently different (for example, by perturbing the performed pattern during recall). Because of this pattern stability during recall, the analysis predicting phase transitions during learning carries over to the present case. For example, the system may be bi-stable before recall and mono-stable after recall as shown in Fig. 6. Dependent on the preparation of the system, an abrupt pattern switch may be observed during recall, possibly accompanied by indicators of loss of stability. (A corresponding change of the probability distribution of relative phase would be observed in the limit of global stationarity.)

What happens when a pattern is required that does not match exactly one memorized pattern? Generally, the memorized pattern that best matches the requirement [as defined by the function, m, of (21)] is recalled. The corresponding memory variable then learns the new required pattern (see Subsect. 6.2). What, however, if the required pattern is *ambiguous*, that is, several memorized patterns match the required pattern equally well? This symmetry can be broken in two ways: In a first scenario the initial condition of the recall variable, that is, the previously recalled pattern, can break this symmetry in favor of one memorized pattern which is closer to this previously recalled pattern. Such a *history* effect is illustrated in the

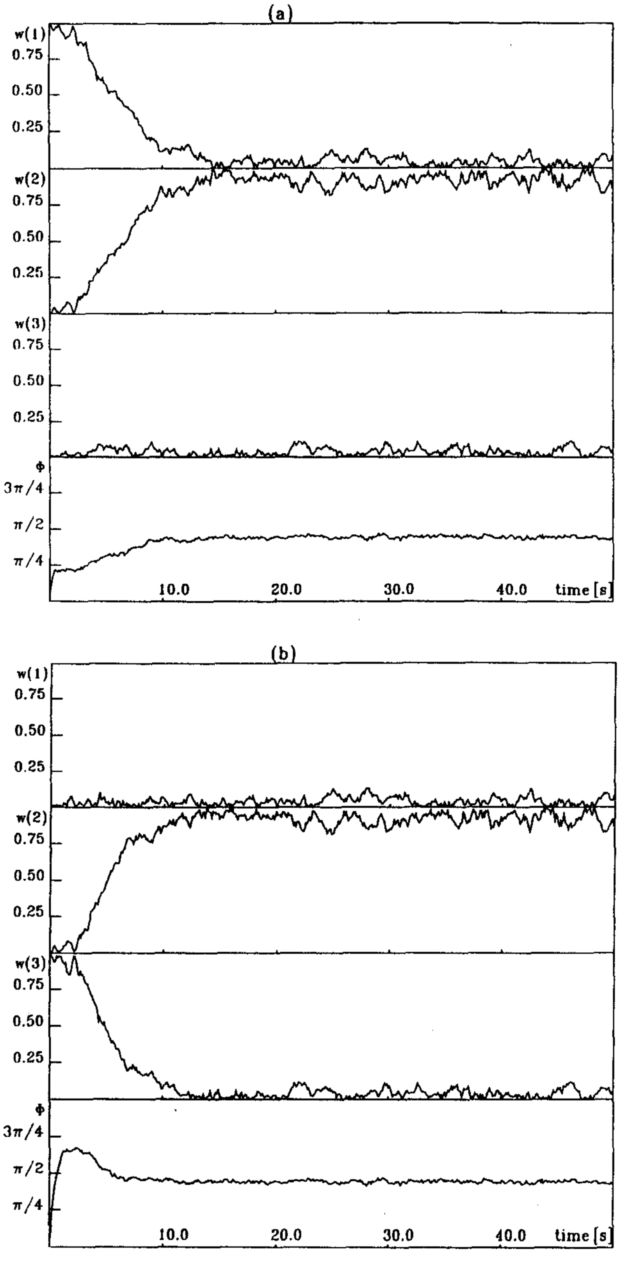

Fig. 5a and b. The rôle of the initial conditions of the recall variables is illustrated in two simulations of (18)–(23). In each panel the three recall variables, w_1, w_2, w_3, corresponding to three memory variables, $\psi_1 = 0$, $\psi_2 = \pi/2$, and $\psi_3 = \pi$ are shown together with the performed relative phase, ϕ. In panel **a**, ψ_1 is initially active, in panel **b** ψ_3. While in both cases the memorized pattern ψ_2 is quickly recalled, the performance during the transient does depend on these initial conditions

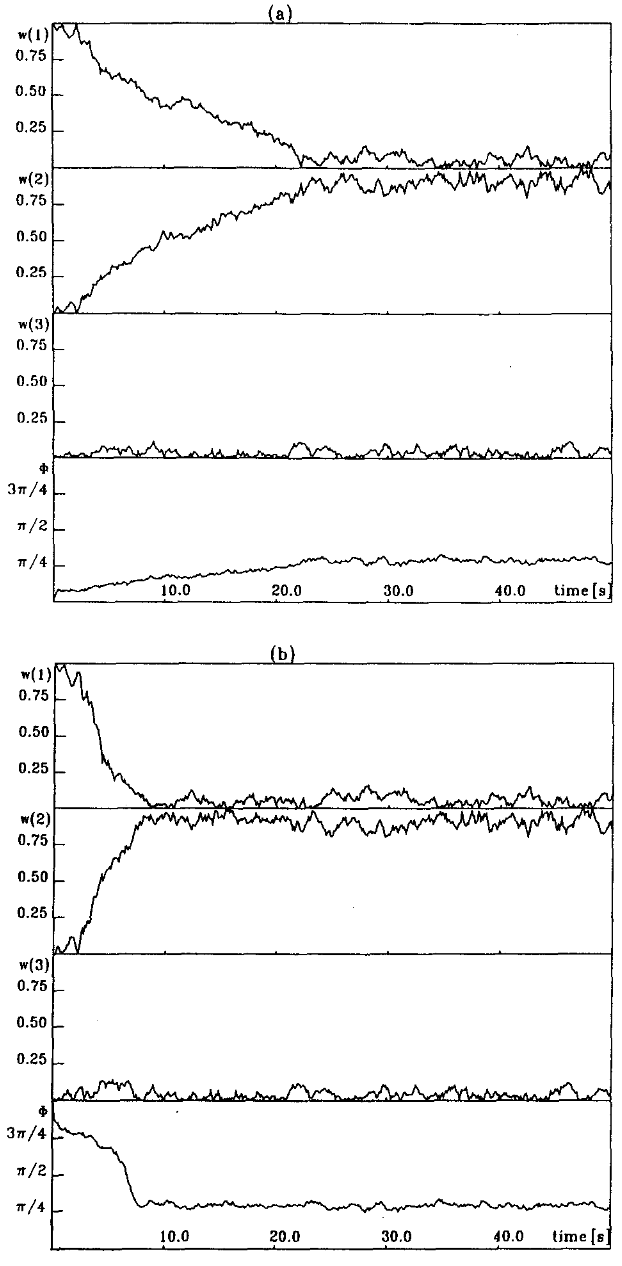

Fig. 6a and b. A phase transition in the behavioral dynamics during recall: Panels **a** and **b** differ only in the initial condition of relative phase, which is $\phi = 0$ for **a** and $\phi = \pi$ for **b**. In the initial phase the behavioral dynamics is bistable until an instability occurs at $t \sim 8$ s leading to performance according to the recalled memorized pattern, ψ_2. Note that the recall dynamics are influenced by the performance: When prepared in a state far from the requirement (panel **a**), recall is slower because the overall match of the recalled memorized pattern is poorer

simulation shown in Fig. 7. If a series of patterns is required sequentially, then the recalled pattern depends on the order in which the series is presented (*hysteresis* effect). Such history and hysteresis effects reflect the essentially nonlinear character of the recall dynamics and their experimental detection would provide an important insight into the nature of the recall dynamics. Similar phenomena are known from perception and pattern recognition (cf., e.g., Vernon 1970; Julesz 1971, for review).

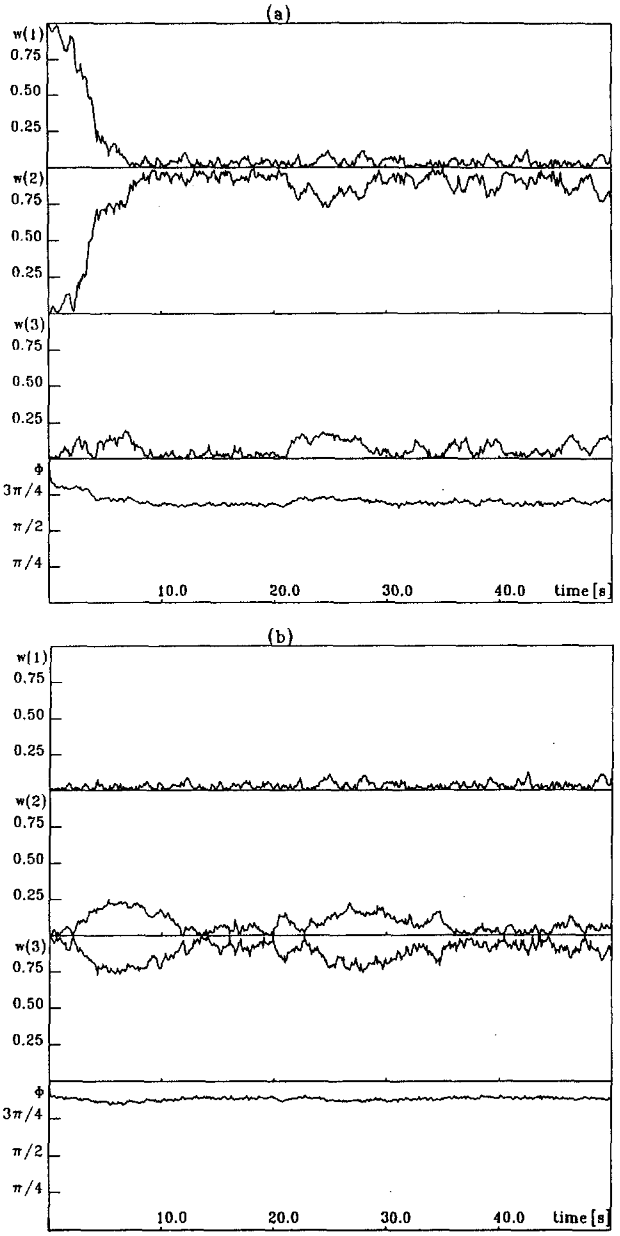

Fig. 7a and b. History effect in recall: When an ambiguous pattern is required (here: $\psi_{\text{env}} = 3\pi/4$ but $\psi_2 = \pi/2$ and $\psi_3 = \pi$) the initial conditions of the recall variables (last pattern recalled) can decide the selective activation process. In panel **a**, the memorized pattern $\psi_1 = 0$ is initially active leading to recall of ψ_2, which is closer to ψ_1 (the same result is found if ψ_2 itself is initially active). In panel **b** the memorized pattern ψ_3 is initially active and stays activated. The effect can, in principle, be experimentally observed because the deviation of the performance from the requirement depends on which pattern is recalled (lowest trace in both panels)

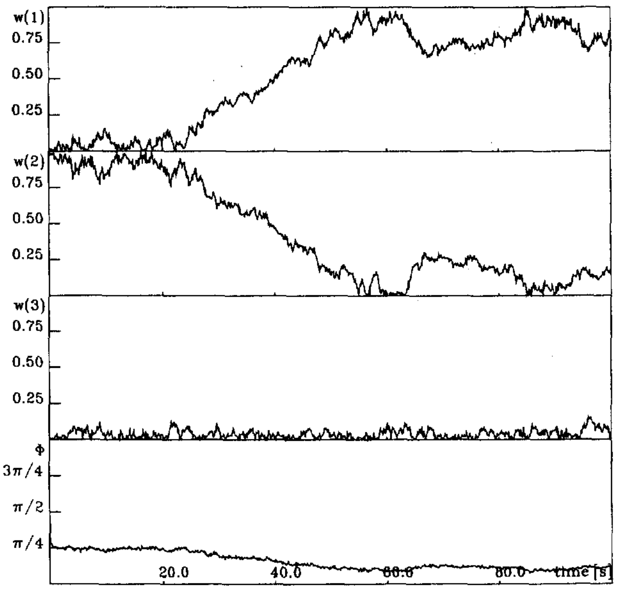

Fig. 8. Influence of the intrinsic dynamics on the recall process when an ambigous pattern ($\psi_{\text{env}} = \pi/4$, where $\psi_1 = 0$ and $\psi_2 = \pi/2$) is required. Although the memorized pattern ψ_2 is initially active the attraction of the performance toward the intrinsically stable in-phase pattern leads to recall of ψ_1. In this case the performance actually deteriorates after recall because memorized information and intrinsic tendencies no longer almost cancel as initially

In a second scenario, the symmetry among the different memorized patterns matching the requirement similarly well may be broken by the actual performance. During the transient the performed pattern deviates from the environmentally required state in the direction of an intrinsically stable state (cf. Schöner and Kelso 1988b). Thus, the match of a memorized pattern that is closer to this intrinsically stable pattern, is enhanced over the other matches due to the influence of performance on the match (21). As a result, this memorized pattern may be recalled even if not favored by the initial conditions. The simulation shown in Fig. 8 exhibits such an effect. (The difference between this case and that shown in Fig. 7 is that here the intrinsic dynamics lead to a stronger deviation from the required pattern.) Although it remains to be seen, whether such somewhat subtle effects can be detected experimentally, the question of whether the intrinsic pattern dynamics can constrain the process of recall seems conceptually relevant.

6.2 Learning and Recall

To study both learning and recall we consider the situation where a pattern is environmentally required that does not match exactly one of the memorized patterns. As shown above, the memorized pattern with best match is recalled. Due to the learning dynamics, the corresponding memory variable learns the new pattern, and thus "forgets" its previous information. Because the "forgotten" and the new pattern are close to each other, this type of "forgetting" might be viewed as an effect of interference. Another type of "forgetting" simultaneously takes place: The memory variables that have not been activated (that is, those ψ_j with $w_j \approx 0$) are not asymptotically, but only marginally stable: $\dot{\psi}_j \approx 0$. Due to the noise in the recall variables, a slow drift of these memory variables occurs, which can be viewed as an unspecific slow decay of memorized information. Whenever one of these memorized patterns is recalled again, it reestablishes its value within the time scale, τ_{learn}. In this sense the memorized patterns are stable. The two types of "forgetting" are illustrated in the simulation shown in Fig. 9. It seems difficult, however, to actually define a notion of "forgetting" operationally. If one thinks of "forgetting" as a change in the system dynamics such that a pattern that had been learned cannot be recalled anymore, it remains open, how to determine experimentally that a pattern has *not* been recalled.

A more technical modelling question concerns, how new memory variables are "created" when the number of learned patterns increases. In the present formulation the potential for differentiating degenerate memory variables exists: When several memory variables have the same value and a pattern sufficiently close to this value is required, then one of these memory variables is activated. The stochastic forces in the recall dynamics lead to such a decision even when initially neither of the degenerate memory variables is favoured over the others. The recalled memory variable proceeds to learn the new pattern and thus differentiates itself from the other degenerate memory variables. In this way "new" memory variables (with respect to their contents) can be created through fluctuations which shows how noise is conceptually important in the recall dynamics. Several questions remain open, however, for example, how degenerate memory variables arise to begin with and how this effect relates to "forgetting" by interference. An example for differentiation of memory variables is

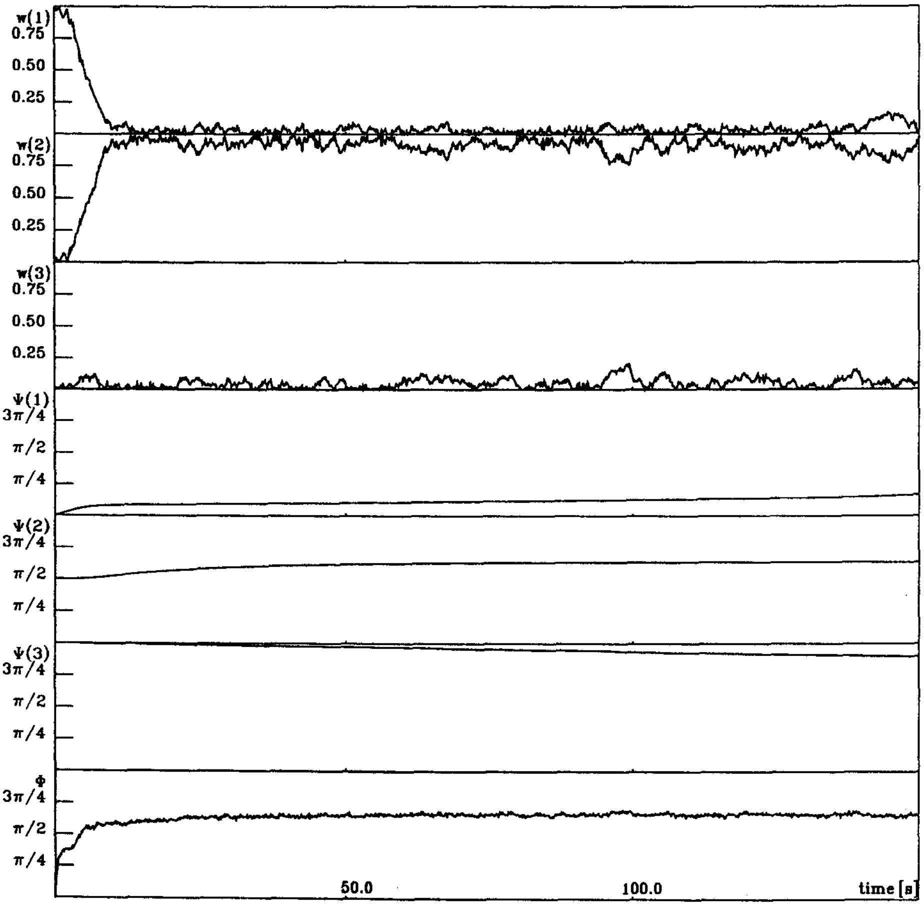

Fig. 9. A general example of learning and recall: A pattern, $\psi_{\text{env}} = 2.0$ rad, close to memorized pattern ψ_2 is required leading to recall of this memorized pattern within a few seconds. The corresponding memory variable learns the new pattern and thus unlearns its previous contents, $\psi_2 = \pi/2$. The other memory variables drift under the influence of noise. Note the stronger drift of ψ_1 during the initial phase, when it is active, as well as the two transients in the performed relative phase, ϕ, during recall and during learning

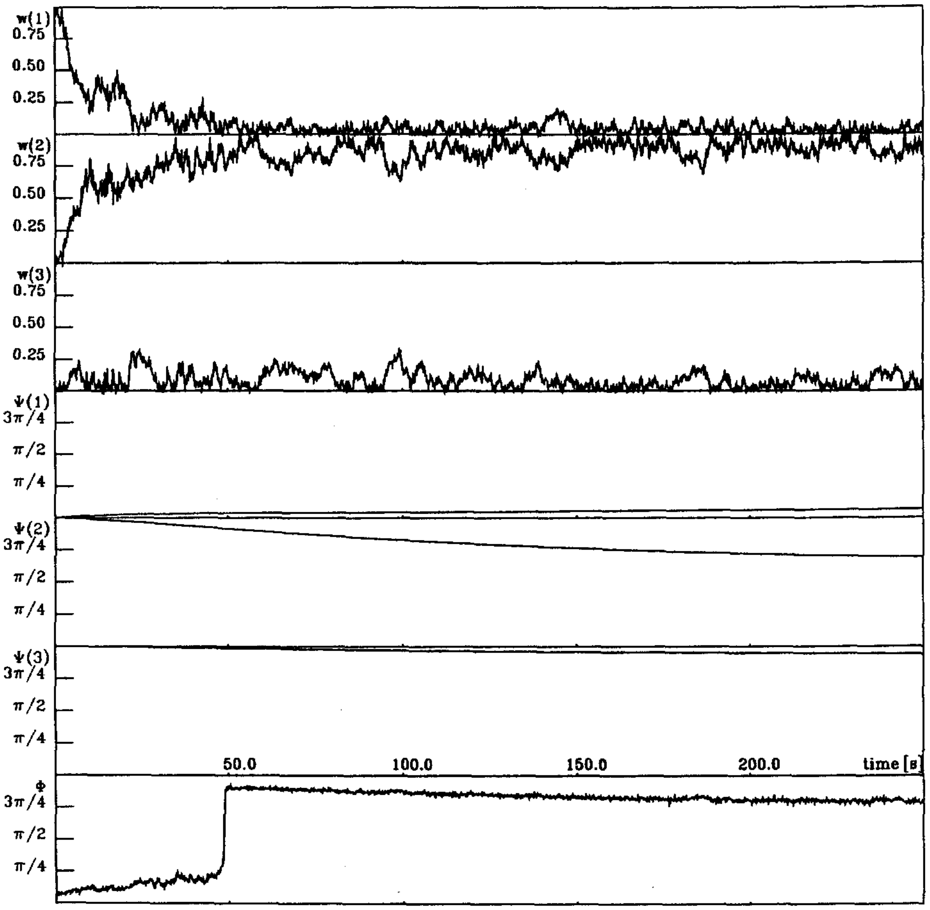

Fig. 10. Differentiation of memory variables and a phase transition during learning in the case of multiple memory variables: Initially both memory variables $\psi_2 = \psi_3 = \pi$ contain the same pattern, while $\psi_1 = 0$ is active. The requirement, $\psi_{env} = 2.0$ rad, leads to activation of one (here: ψ_2) of the memory variables by chance, and this memory variable subsequently learns the new pattern. In this case, the behavioral dynamics is initially bistable (only one state closer to in-phase is shown). As the memorized pattern ψ_2 increases in strength, $\hat{c}_{mem}(\psi_2)$, an instability occurs analogous to the first phase transition in Fig. 2. To prepare this case in a numerically stable way the contribution of the behavior to the match function (21) was reduced to 20% (cf. discussion Sect. 7)

shown in Fig. 10. In this example we also find a phase transition during learning, which illustrates that such phase transitions persist in the case of multiple memory variables. The analysis of such phase transitions of Sect. 4 remains valid, because the system is essentially a system with a single memorized pattern on the time scale of learning due to the time scales relation (22).

7 Discussion

In this article we have developed a dynamic theory of learning and recall of coordination patterns for a class of model systems involving rhythmic movement coordination. Characterizing the coordination patterns by the collective variable relative phase we could build on previous work in which both the intrinsic pattern dynamics as well as the adaptation of the coordination patterns to environmental and memorized requirements were understood. Memorized information, which is part of the pattern dynamics attracting to the pattern required by memory, was endowed with dynamics to account for the change of behavior during learning. These learning dynamics were modelled by mapping the stable, reproducible result of learning, the to-be-learned pattern, onto an attractor. Similarly, the relative strengths with which different memorized patterns influence the pattern dynamics were endowed with dynamics to account for the behavioral change during selective activation or recall of a memorized pattern. The recall dynamics were modelled by imposing functional equivalence of all memory variables as a symmetry and mapping the reproducible endresult of recall onto an attractor. Note, that the functional nature of the collective variables that are used to characterize both the coordination patterns and behavioral information, makes it possible to express generalization in learning.

For the present model case involving the learning of relative timing skills we established explicitly several predictions of the theory. In particular, we showed under which conditions phase transitions in the dynamics of the coordination patterns occur during the processes of learning and of recall. These are observable as abrupt changes of behavior accompanied by loss of stability in the time scale régime of local stationarity, and as qualitative changes in the shape of the probability distribution of the collective variable in the time scale régime of global stationarity. Two methodologies of observing learning over a series of "sessions" are related to these two time scale relations. The experimental observation of such phase transitions would not only represent an important test of the present theoretical framework, but also provide insight about lawful constraints on learning and recall. The essentially nonlinear nature of the recall dynamics could be experimentally tested by looking for history or hysteresis effects predicted by this theory in cases of ambiguous behavioral information. Furthermore, in these experimental situations the intrinsic dynamics can influence the recall process favouring recall of intrinsically stable patterns. Experimental observation of this effect would also provide insight into how recall is constrained by the behavioral dynamics of the system. Other, more specific properties of the theoretical model are "forgetting" by interference, unspecific decay of memorized information, and differentiation of memory variables, which may be subjected to experimental test separately.

We would like to stress that the specific functional form of our dynamic model was determined on the basis of simplicity. For example, the mathematical performance of the model (e.g., number of patterns that can be stably recalled, range of parameter values over which the different effects are observed, etc.) can be vastly improved by introducing more sharply peaked match functions and more pronounced attractor landscapes. We convinced ourselves, that sharpening the match criteria with:

$$match = \{\tanh(h(m - m_0) + 1\}/2\,,$$

(where m_0 is a threshold and h a sharpness factor, cf. Shimizu and Yamaguchi 1987), leads to a more robust performance in above sense. However, our purpose here was to point out the general, experimentally revelant consequences of the theoretical framework that do not depend very strongly on these mathematical details.

More generally, one may view the present approach as aiming at a phenomenological theory of learning and recall, in which a theoretical language is developed whose basic concepts and varaibles are *operationally* defined and thus, at least in principle, measurable. In this language, laws or constraints can be formulated that reveal organizational principles of a behavioral system capable of learning and recall. More microscopic or neurophysiological models of learning might aim to derive such laws. Through the concept of behavioral information we avoid introducing arbitrary representation of information and instead provide a language with which to identify relevant information by its effect on behavior itself. In this language learning and recall affect the dynamics of coordination patterns, including their stability.

To use such a theoretical approach in other experimental systems the language has to be "filled with life" in each case. That means, collective variables, environmental parameters and intrinsic dynamics have to be identified, time scales have to be determined, and stability measures to be implemented. Clearly, not every experimental situation lends itself to such a description and allows for prediction. Beyond the examples treated here, a number of systems have been studied within such a theoretical framework, including coordination of movement with a metronome (Kelso et al. 1989), locomotory gaits of quadrupeds and gait changes (Schöner et al. 1989) and coordination of rhythmic movement among two human subjects (Schmidt 1988). Several other behavioral systems appear open to this theoretical approach, for example, interlimb coordination of discrete movement (Kelso et al. 1979), coordination of discrete movement with the visual field (see Lee and Young 1986, for recent review), tracking (Poulon 1974), posture (Feldman 1966), spontaneous leg movement in infants (Thelen et al. 1987), repetitive speech (Tuller and Kelso 1989b), and centrally generated patterns in neural networks (see Schöner and Kelso 1988a, for a qualitative discussion). A generalization of the present theory of learning and recall to such systems seems possible.

In most formal learning and pattern recognition theories a whole layer of dynamics, namely, that of the intrinsic pattern dynamics, is missing. Because the presence of non-trivial intrinsic dynamics on the behavioral level are ultimately responsible for the predicted phase transitions as well as the various qualitative effects in recall, ignoring this layer may be missing an essential aspect. Several recent models of the learning and control of sensory-motor behavior (e.g., Kawato et al. 1987; Kupferstein 1988; Bullock and Grossberg 1988) have taken the step of including a behavioral component in the system dynamics, but fail to identify such intrinsic constraints on the behavioral level and thus do not arive at the predictions discussed in this article. As an aside we note that here we have shown that phase transitions may occur in the behavioral dynamics during learning, for example, from multiple attractors to a single attractor. It is sometimes argued that learning corresponds to developing more attractors. This refers, however, to attractors in a representation space (as for recall), not of the behavioral dynamics.

Beyond the concrete predictions and descriptions we hope to have illustrated how the theoretical language used here may provide a fresh view of learning and recall, leading to a different perspective and new questions. As a next step it may be interesting to express dynamic properties of the learning and recall processes themselves in this language.

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References

Bullock D, Grossberg D (1988) The VITE model: A neural command circuit for generating arm and articulator trajectories. In: Kelso JAS, Mandell AJ, Shlesinger MF (eds) Dynamic patterns in complex systems. World Scientific, Singapore, pp 305–326

Eigen M, Schuster P (1979) Hypercycle – a principle of natural self-organization. Springer, Berlin Heidelberg New York

Feldman AG (1966) Functional tuning of the nervous system during control of movement or maintenance of a steady posture. III. Mechnographic analysis of the execution by man of the simplest motor tasks. Biofizika 11:667–675

Fitts PM (1964) Perceptual-motor skill learning. In Melton AW (ed) Categories of human learning. Academic Press, New York

Gardiner CW (1983) Handbook of stochastic methods. Springer, Berlin Heidelberg New York

Haken H (1970) Laser theory. In: Flügge S (ed) Encyclopaedia of physics, vol XXV/2c. Springer, Berlin Heidelberg New York

Haken H (1983a) Synergetics – an introduction. Springer, Berlin Heidelberg New York

Haken H (1983b) Synopsis and introduction. In: Basar, E, Flohr H, Haken H, Mandell AJ (eds) Synergetics of the brain. Springer, Berlin Heidelberg New York

Haken H, Kelso JAS, Bunz H (1985) A theoretical model of phase transitions in human hand movements. Biol Cybern 51:347–356

Julesz B (1971) Foundations of cyclopean perception. University of Chicago Press, Chicago

Kawato M, Furukawa K, Suzuki R (1987) A hierarchical neural-network model for control and learning of movement. Biol Cybern 57:169–185

Kelso JAS (1984) Phase transitions and critical behavior in human bimanual coordination. Am J Physiol: Reg Integ Comp 15:R1000-R1004

Kelso JAS, Schöner G (1987) Toward a physical (synergetic) theory of biological coordination. Springer Proc Phys 19:224–237

Kelso JAS, Schöner G (1988) Self-organization of coordinative movement patterns. Hum Mov Sci 7:27–46

Kelso JAS, Southard DL, Goodman D (1979) On the nature of human interlimb coordination. Science 203:1029–1031

Kelso JAS, Scholz JP, Schöner G (1986) Nonequilibrium phase transitions in coordinated biological motion: critical fluctuations. Phys Lett A118:279–284

Kelso JAS, Schöner G, Scholz JP, Haken H (1987) Phase-locked modes, phase transitions and component oscillators in biological motion. Phys Scr 35:79–87

Kelso JAS, Scholz JP, Schöner G (1988) Dynamic governs switching among patterns of coordination in biological movement. Phys Lett A134:8–12

Kelso JAS, DelColle JD, Schöner G (1989) Action-perception as a pattern formation process. In: Jeannerod M (ed) Attention and performance XIII. Erlbaum, Hillsdale, NJ

Kupferstein M (1988) Neural model of adaptive hand-eye coordination for single postures. Science 239:1308–1311

Lee DN, Young DS (1986) Gearing action to the environment. In: Heuer H, Fromm C (eds) Generation and modulation of action patterns. Exp Brain Res Ser 15. Springer, Berlin Heidelberg New York, pp 217–230

Poulton EC (1974) Tracking skills and manual control. Academic Press, New York

Schmidt RC (1988) Ph.D. Thesis. University of Connecticut, Storrs CT, USA (unpublished)

Schöner G, Kelso JAS (1988a) Dynamic pattern generation in behavioral and neural systems. Science 239:1513–1520

Schöner G, Kelso JAS (1988b) A synergetic theory of environmentally-specified and learned patterns of movement coordination. I. Relative phase dynamics. Biol Cybern 58:71–80

Schöner G, Kelso JAS (1988c) A synergetic theory of environmentally-specified and learned patterns of movement coordination. II. Component oscillator dynamics. Biol Cybern 58:81–89

Schöner G, Kelso JAS (1988d) Dynamic patterns in biological coordination: Theoretical strategy and new results. In: Kelso JAS, Mandell AJ, Shlesinger MF (eds) Dynamic patterns in complex systems. World Scientific, Singapore, pp 77–102

Schöner G, Kelso JAS (1988e) A dynamic theory of behavioral change. J Theor Biol 135:501–524

Schöner G, Haken H, Kelso JAS (1986) A stochastic theory of phase transitions in human hand movement. Biol Cybern 53:247–257

Schöner G, Jiang WY, Kelso JAS (1989) A synergetic theory of quadruped gaits and gait transitions. J Theor Biol (submitted for publication)

Scholz JP, Kelso JAS, Schöner G (1987) Nonequilibrium phase transitions in coordinated biological motion: critical slowing down and switching time. Phys Lett A123:390–394

Shaffer LH (1982) Rhythm and timing skill. Psych Rev 89:109–122

Shimizu H, Yamaguchi Y (1987) Synergetic computer and holonics – information dynamics of a semantic computer. Phys Scr 36:970–985

Stetson RH (1951) Motor phonetics: a study of speech movements in action. North Holland, Amsterdam (republished in: Kelso JAS, Munhall K (eds) College-Hill Press, Boston 1988)

Thelen E, Kelso JAS, Fogel A (1987) Self-organizating systems and infant motor development. Dev Rev 7:39–65

Tuller B, Kelso JAS (1989a) Environmentally-specified patterns of movement coordination in normal and split-brain subjects. Exp Brain Res 75:306–316

Tuller B, Kelso JAS (1989a) Phase transitions in speech production and their perceptual consequences. In: Jeannerod M (ed) Attention and performance XIII. Erlbaum, Hillsdale, NJ

Vernon MD (1970) Perception through experience. Methuen, London

Yamanishi J, Kawato M, Suzuki R (1980) Two coupled oscillators as a model for the coordinated finger tapping by both hands. Biol Cybern 37:219–225

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