RECURRENCE OF SOLAR ACTIVITY: EVIDENCE FOR ACTIVE LONGITUDES

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Abstract. The autocorrelation coefficients of the daily Wolf sunspot numbers over a period of 128 years reveal a number of interesting features of the variability of solar activity. In addition to establishing periodicities for the solar rotation, the solar activity cycle, and perhaps the 'Gleissberg Cycle', they suggest that active longitudes do exist, but with much greater strength and persistence in some solar cycles than in others. There is evidence for a variation in the solar rotation period, as measured by sunspot number, of as much as two days between different solar cycles.

1. Introduction

Numerous techniques have been used to measure the solar rotation rate as a function of depth, latitude, and time. The rotation of the gas near the photosphere is quite well defined (see Gilman, 1974, for a general review of the observations), but the rotation of the solar interior is still rather uncertain. The only measurement bearing directly on the interior rotation is the solar oblateness, which has only placed an upper limit on the interior rotation rate. The only other accessible phenomenon by which the interior rotation rate may make itself known is the large-scale solar magnetic activity, the dynamo for which is probably located in the lower part of the convection zone. Unfortunately, there is no precise way to measure the large-scale solar activity. Localized manifestations (individual sunspots and groups of sunspots) were the earliest rotational tracers, and have been used for some of the most accurate measurements to date (Newton and Nunn, 1951; Ward, 1966). Measuring the proper-motion of individual spots, however, is beset by numerous difficulties, which have been discussed by Ward, while the locations of spots are undoubtedly affected by the ambient fluid motions. Another tracer of the overall magnetic activity, the large-scale photospheric magnetic field, has been the subject of both proper-motion measurements (Bumba and Howard, 1969) and autocorrelation analysis (Wilcox and Howard, 1970; Wilcox *et al.*, 1970). For solar history, however, the best index of solar magnetic activity is simply the daily Wolf sunspot number R_z, for which a continuous record extends from before 1850 to the present. While this record has been the subject of a massive, continuing effort to identify temporal variations on the scale of the solar cycle and larger, it has surprisingly escaped much scrutiny for short-term periodicity. (In fact, most of the

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longer-term analysis has been in terms of the monthly and annual means; neither of which has any physical significance for the Sun itself.) A recent Fourier analysis of a 122-year segment of Wolf sunspot numbers (Knight *et al.*, 1979) has revealed an interesting feature corresponding to an apparent periodicity of slightly more than 12 days (synodic). In an effort to study the short-term periodicities of the sunspot number further, I examined the autocorrelation function of the daily numbers. Autocorrelation escapes the problem of signals introduced into the Fourier spectrum of a data set being modulated at different frequencies. Furthermore, since the lifetime of an individual sunspot or sunspot group is comparable to a solar rotation period, but much shorter than the strongly-modulated and somewhat irregular eleven-year cycle in total activity, there is presumably less tendency for the autocorrelation function to be skewed toward the representation of short-term phenomena characteristic of only one phase of the eleven-year cycle. A similar analysis of the autocorrelation of daily sunspot areas for the period 1883–1959 has been undertaken by Letfus (1976), who finds evidence of persistence of recurrent groups of sunspots rotating with a period of 27 days and persisting for about 5 rotations. Toman (1967) performed an auto-cross correlation analysis of sunspot number and 10.7 cm solar radio flux for one solar cycle, 1947–1957. Toman's results are discussed in Section 3.

2. Analysis of the Daily Sunspot Number Record

The data analyzed consist of the 46 751 daily relative-sunspot numbers from the period January 1, 1850 through December 31, 1977. The part of the record running through December 31, 1960 is the tabulation by Waldmeier (1961), while the remainder comes from the continuation by the National Geophysical and Solar-Terrestrial Data Center of NOAA. In addition to studying the correlation properties of the whole record, I broke part of it into eleven distinct subsets corresponding to solar cycles 10 through 20, and determined the autocovariance function of the sunspot-number activity for each of these cycles.

The normalized autocorrelation coefficients for a sample N days long are defined by (Southworth, 1960)

$$A_p = \frac{(N-p)\Sigma x_i x_{i+p} - \Sigma x_i \Sigma x_{i+p}}{\sqrt{(N-p)\Sigma x_i^2 - (\Sigma x_i)^2}\,\sqrt{(N-p)\Sigma x_{i+p}^2 - (\Sigma x_{i+p})^2}},$$

where A_p is the autocorrelation coefficient corresponding to a lag of p days, p varying from 0 to (at most) $N-2$; x_i is the sunspot number on day i of the sample, which extends from $i=1$ to N; and all summations extend from $i=1$ through $i=N-p$. Note that this definition includes corrections for the mean and normalization to the product of standard deviations. The autocorrelation coefficients may be treated as the values assumed at integer lags of the autocovariance function of an idealized continuous function representing the data; the autocovariance function

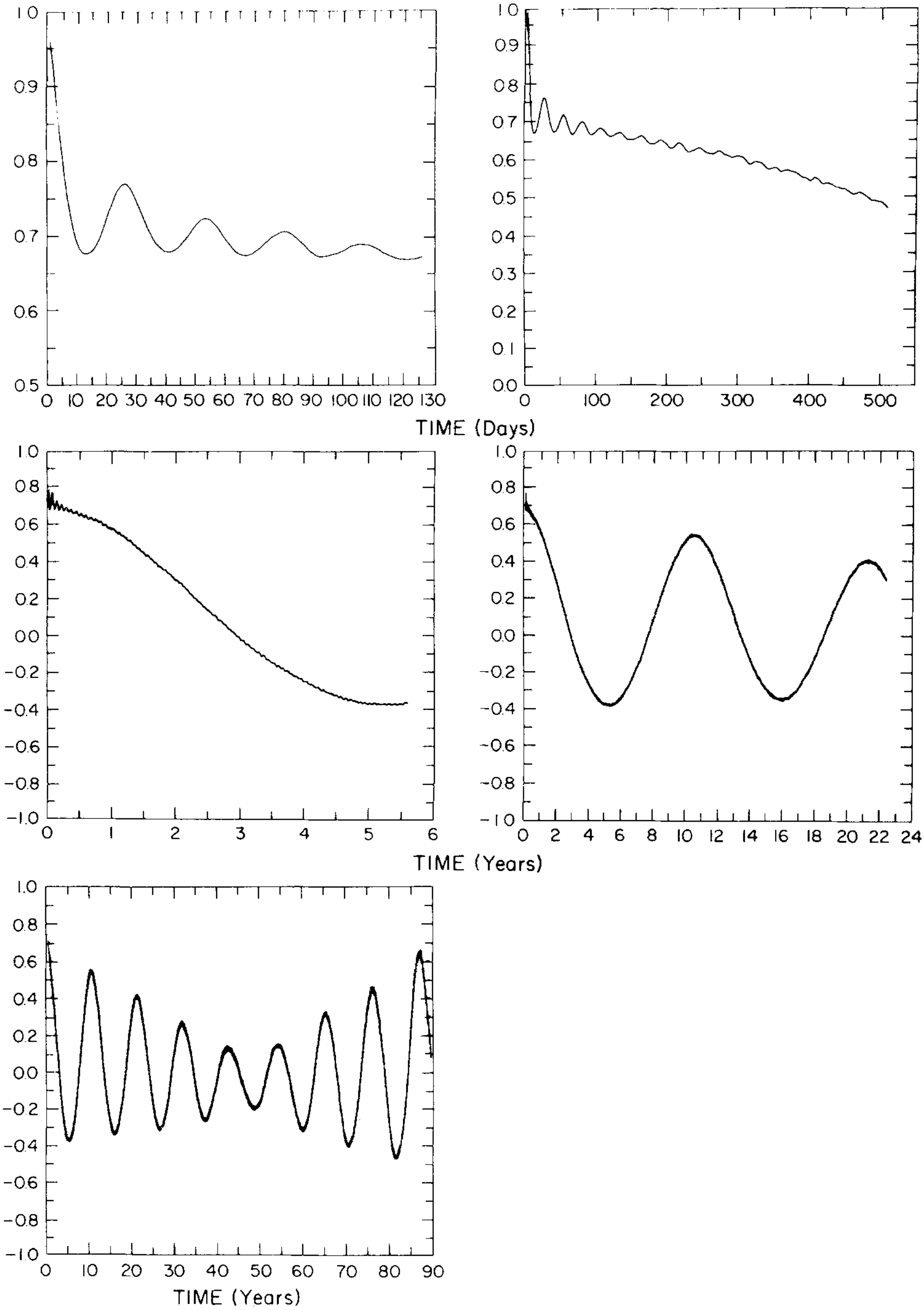


Fig. 1. The autocorrelation coefficients A_p of the Wolf daily relative sunspot numbers for the period January 1, 1850–December 31, 1977, shown as functions over increasing intervals of the lag p. The first two graphs clearly illustrate the effect of rotation combined with persistent longitudinal asymmetries in the level of activity; the third graph shows the striking level of long-term noise at the rotation frequency; and the last two graphs exhibit the solar cycle.

is of course the Fourier transform of the power spectrum of the data, but the two functions reveal quite different things about the recurrence properties of the data.

The autocovariance function for the entire data set (Figure 1) reveals several expected features and some unexpected ones as well. The damped oscillatory behavior with a periodicity of about 27 days is clearly the result of longitudinal patterns of solar activity that persist for longer than one solar rotation. The consistently high positive correlations for lags of at least a year upon which this 27 day oscillation is superimposed are due to the 11 year activity cycle, which itself shows up as a pronounced oscillation in the autocovariance function. The striking envelope of the 11-year oscillations may be a manifestation of a long period (about 8 solar cycles) variation in the solar activity that has been identified as the so-called Gleissberg Cycle, but since less than 1.5 such 'cycle' is present in the data, this identification of a strict periodicity must be made with extreme caution. The absence of any other statistically significant long-term periodicities is in agreement with Ramanuja Rao's (1973) result for the same data set.

The persistence of the 27 day oscillation in the autocorrelation function for at least 8 and perhaps 12 solar rotations is rather surprising, in view of the much shorter sunspot lifetimes and the more rapid damping seen in Letfus' data, and may certainly be taken as evidence for the persistence of active longitudes, as measured by Wolf's index, for periods of up to a year. Even more surprising is the presence of low-level fluctuations near a 27-day period throughout the data, even at lags of many years. This is confirmed by the power spectrum (Figure 2) which shows broad peaks only at the rotation period, 27.5 ± 0.5 day, and at its first harmonic, 13.6 ± 0.25 day, and an otherwise extremely flat spectrum. (The statistical significance of the single-point peak at a 12.07 day period has been discussed by Knight *et al.*). It is difficult to understand this phenomenon except in terms of the persistence of only a few active longitudes over the length of the record that have occasionally coupled to one another in the autocorrelation function; if a large number of such sites had been present, this effect would presumably be washed out unless they recurred at the same absolute longitudes. The 13.6 day peak in particular may be understood in terms of a tendency toward developing simultaneous active sites separated by 180° in longitude, as suggested by Kiepenheuer (1953).

The peaks of the autocorrelation function corresponding to the eleven year cycle of solar activity do not recur at the classic period of 11.1 yr, but rather at intervals of about 10.6 ± 0.1 yr, at least for the first four recurrences. This effect cannot be due to damping: for an exponentially damped cosine curve of the form $f(t) \approx \approx e^{-t/\tau} \cos(2\pi t/P)$, it is easy to show that the maxima recur with period P, being merely phase shifted by an amount $(P/2\pi) \tan^{-1}(P/2\pi\tau)$ with respect to the maxima of the undamped function. (Such a phase shift does appear to be present in the 27-day rotational maxima, advancing them by about 0.5 days, corresponding to a decay time τ of about $1.5 \times P$.) The eleven year cycle is quite irregular, and it is possible that cycles of short recurrence time may be unusually similar to one

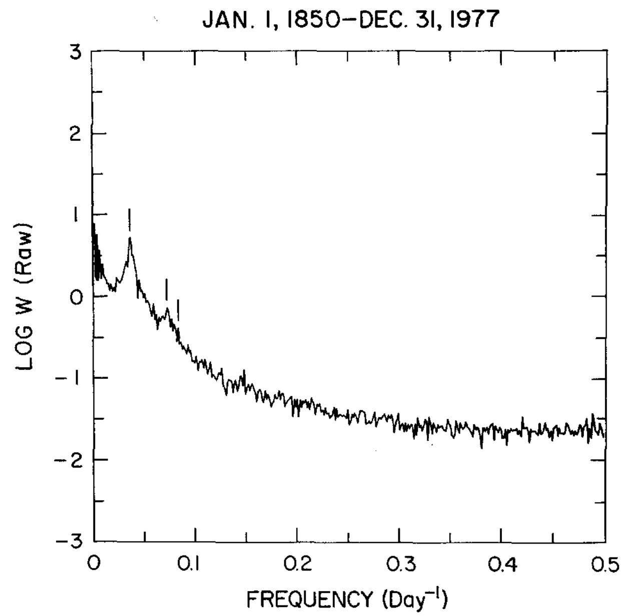
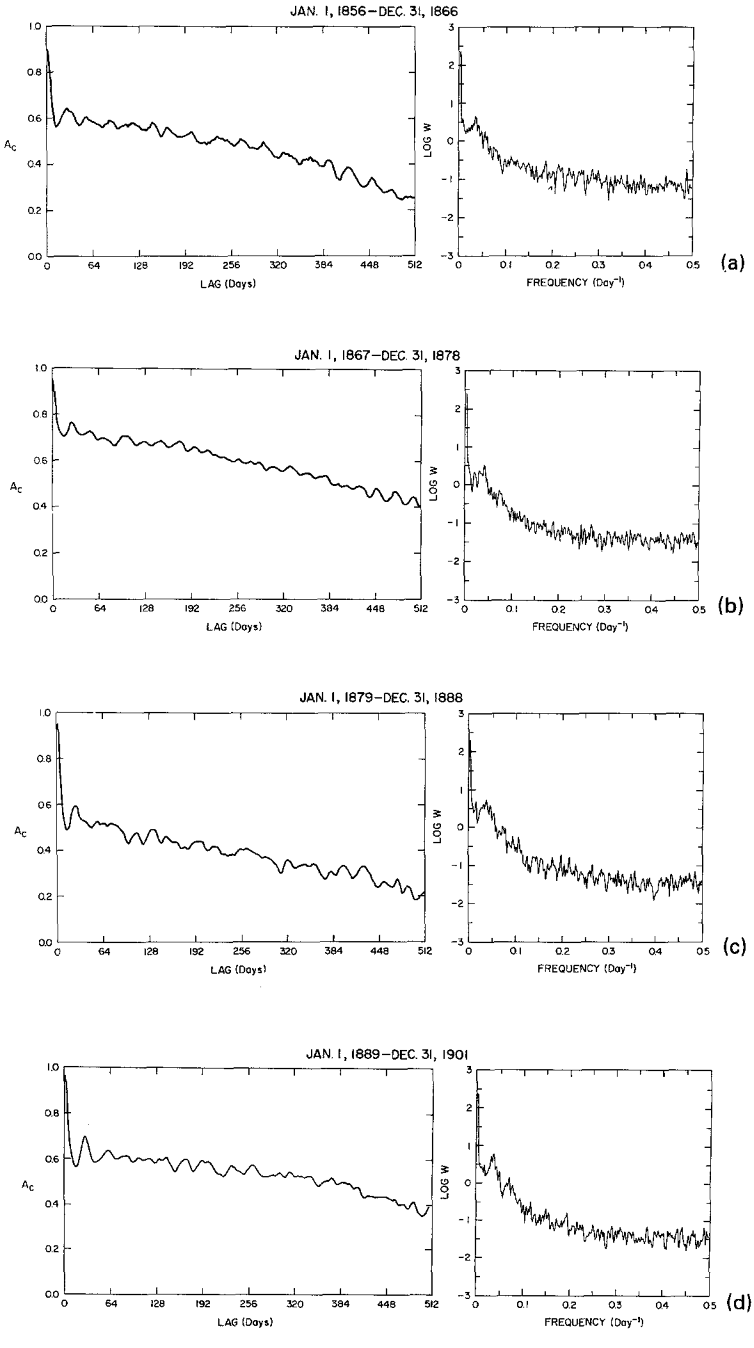


Fig. 2. The raw power spectrum corresponding to the autocovariance function of Figure 1. The power spectrum is the Fourier transform of an even autocovariance function corresponding to all lags between ±29 524 days, with $A_{-p} = A_p$. (The total number of data points is 46 751.) The three frequencies marked correspond to periods of 27.5 days, 13.6 days, and 12.1 days.

another; examination of at least the annual means of the sunspot number, however, does not obviously indicate that such is the case. Maris (1976) has calculated the autocorrelation coefficients for the annual means of the sunspot relative number and finds a period of 11.0 yr, but since the points are one year apart the accuracy is presumably only somewhat less than that. Cole's (1973) findings of both 11.8 yr and 10.45 yr periodicities in the same kind of analysis is based on the tendency of the cycle lengths to drift, but again these identifications seem to depend on peaks in the power spectral density which are only one point wide. As the autocovariance function makes clear, there is probably no simple clock mechanism governing the solar activity variations; large-scale fluctuations in the activity have an important stochastic element. The only time clock seems to be the solar rotation – but how stable is that?

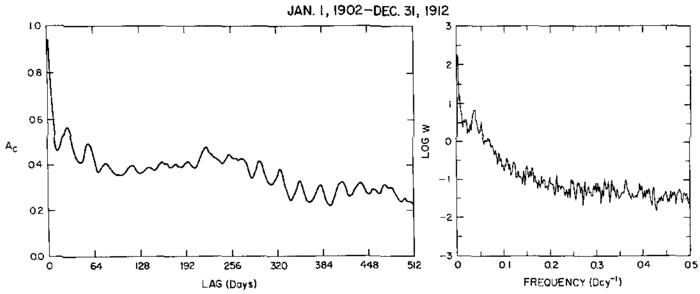
3. Estimation of Variability in the Solar Rotation Rate

To investigate the time-dependence of the rotation rate for the active longitudes, I have calculated the autocorrelation coefficients and power spectra for each of the individual solar cycles 10 through 20, and the set of correlograms and power spectra are shown in Figure 3. The variations in the behavior of the autocorrelation functions from cycle to cycle are far more notable than those of the power spectrum. Active

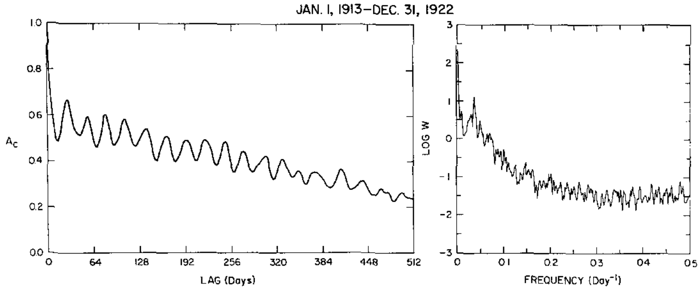


Figs. 3a–d.

Figs. 3a–k. Correlograms and the corresponding (smoothed) power spectra of the daily sunspot numbers for each of the solar activity cycles 10–20. The power spectra are based on lags between ±364 days, and have been Hamming smoothed.



(e)



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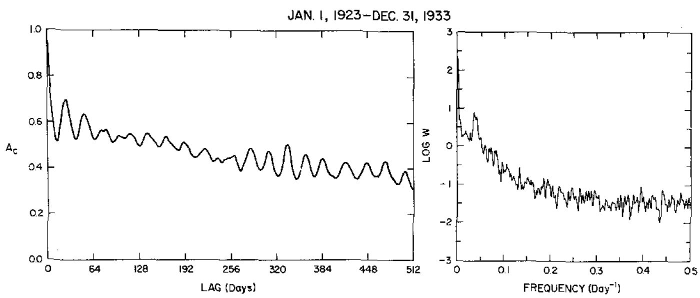
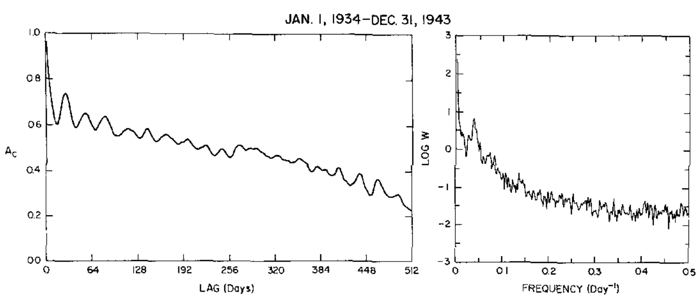
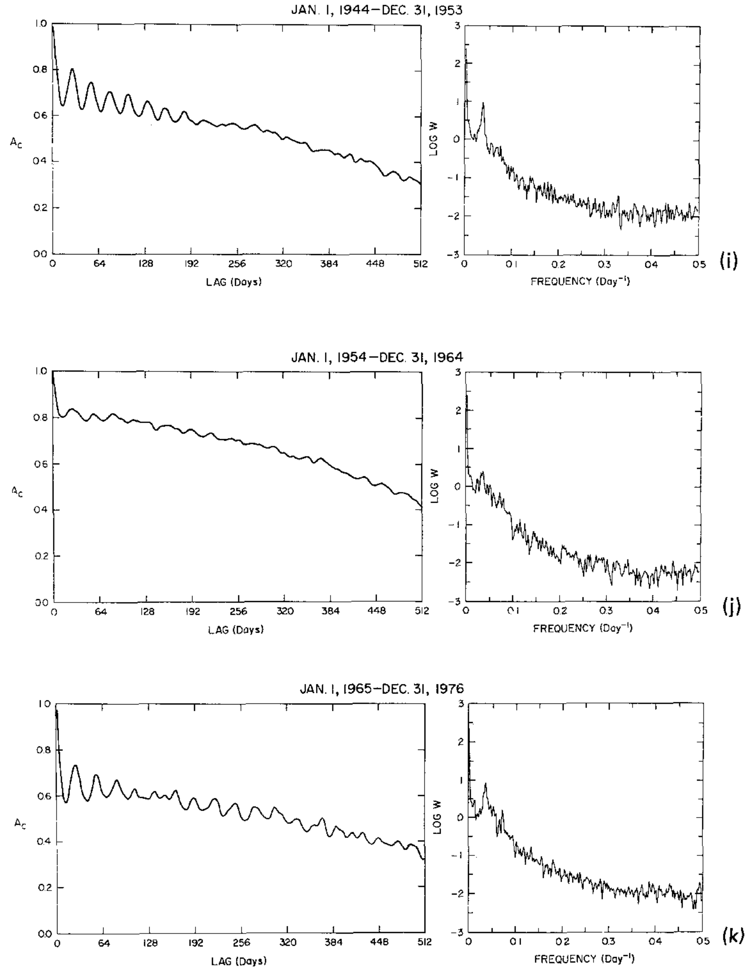


(g)



(h)

Figs. 3e–h.



Figs. 3i–k.

longitudes seem scarcely to have been present in cycle 19 above the general level of activity; cycle 15, by contrast, seems to have been dominated by at least one active longitude which persisted with nearly undiminished strength for as many as sixteen solar rotations. This difference would be very difficult to infer from the power spectra of the two cycles. Even more remarkable than the occasional long lifetimes of the active longitudes, however, is the recurrence of active sites after many rotations, a phenomenon particularly marked in cycles 14 and 16. The fact that the second series of phased oscillations persists longer than the first in these cases seems to argue strongly for the existence of only one or two active longitudes with lifetimes longer than a few rotations during the cycle. The fact that the second series is not necessarily in phase with the first series would appear to favor an explanation in terms of two active longitudes, one being regularly excited sometime before the other. If the first excited longitude were excited more often or to a higher level, but remained excited for a shorter time than the second, the behavior of the autocorrelation function would be explained. Vitinskij (1969) has found evidence for only a few active longitudes, but this work applied only to cycle 19, in which active longitudes appear very weakly in the present data (see Figure 3, Jan. 1954–Dec. 1964). Sawyer (1968) has presented abundant evidence, from data other than sunspot number, of such active longitudes (more precisely active sites) persisting through several solar cycles.

It would be difficult to draw any definite conclusions respecting secular trends in the rotation period from the power spectra of Figure 3. The occurrences of maxima in the correlograms, on the other hand, do seem to indicate significant variations in the rotation period of the active longitudes from cycle to cycle, as illustrated in Figure 4. The time of first maximum, for example, has varied from

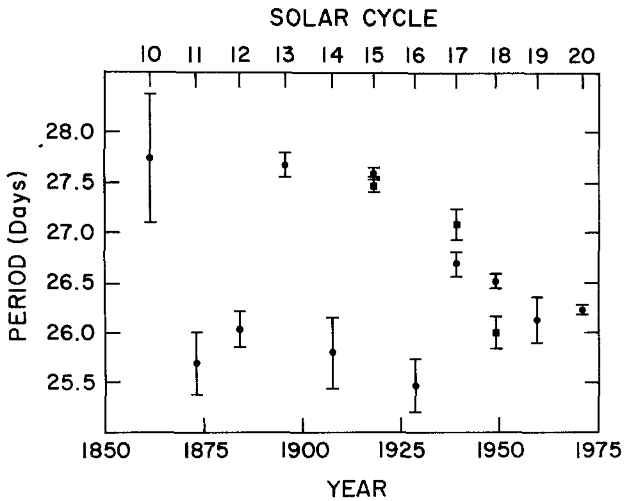


Fig. 4. Estimates of the solar rotation period derived from the location of the first maximum in the autocovariance function for each cycle (circles) and from consistent estimates of the locations of the first four maxima (squares). The 'error bars' are estimates of the uncertainty provided by fitting parabolic curves to different sets of data points near the maximum.

cycle to cycle by as much as two days, an effect which could only be attributed to damping if the mean lifetime of the active longitudes was only about one-third of the rotation period, in which case the oscillations would be very much more strongly attenuated than they are. Furthermore, in the few cases in which the rotation period can be inferred from the spacings of successive maxima, the values derived are in rough agreement with those given by the first maximum and show some of the same large variations from cycle to cycle. An exception is cycle 20, in which successive maxima were spaced more than 28.5 days apart. There is insufficient information in the Wolf sunspot numbers to assess the effect of the equatorward drift of the solar activity during a cycle combined with the differential rotation; the first and second halves of each cycle, however, do not appear to have significant differences in the rotation period. Toman (1967) found variations from 21 to 32 days in the location of first maximum for individual rotations (the correlations were not averaged) for the auto-cross correlation previously mentioned, but with a pronounced clustering around 26 days, in close agreement with the values found here for cycles 18 and 19, which span Toman's data set. Toman's finding of an additional maximum near 29 days was indeed interpreted by Bumba and Howard (1969) as being due to the formation of sunspot groups at high latitudes and on the eastern edges of active regions. In any case, it should be pointed out that the present findings indicate rotation rates that generally exceed the Carrington rate.

4. Conclusion

To summarize, the evidence is strong for the existence of a few sites of unusually strong solar activity in each solar cycle that persist for times of the order of 10 solar rotations and that generally rotate with a period of about 27 days. There is some evidence, however, that the rotation period of these active longitudes varies from cycle to cycle much more than the rotation periods of photospheric features, but also in a different way (Eddy *et al.*, 1978). If this should prove to be the case, it is probably due to variations in the vertical location of the dynamo in an envelope which does possess a radial gradient of angular velocity.

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