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Testing Between Alternative Models of Choice Under Uncertainty: Some Initial Results

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Abstract

Experiments have identified a number of well-known violations of expected utility theory, giving rise to alternative models of choice under uncertainty, all of which are able to explain these violations. In this article, predictions of several prominent rival formulations are examined. No single alternative consistently organizes choices. Among the more important inconsistencies, we identify conditions generating systematic fanning in of indifference curves in the unit probability triangle, and find risk-loving over a number of gambles with all positive payoffs, in cases where prospect theory predicts risk aversion.

Recent years have witnessed increasing dissatisfaction with expected utility (EU) theory as a descriptive model of choice under uncertainty. This has spurred the development of a number of alternative models able to explain many of the most telling violations of EU theory (see Sugden, 1986, and Weber and Camerer, 1987, for reviews). These alternatives to EU theory often hold quite different views of the behavioral processes underlying choice under uncertainty, and make a number of new predictions that have yet to be subject to thorough experimental investigation. This article marks an effort to explore these new predictions.

Responses to four series of questions are reported. Each question required subjects to indicate which of two gambles they preferred. Four primary research questions underlie the questions posed. First, would we observe significant, and systematic, differences between choices over the same questions involving real as opposed to hypothetical choices, since this has important methodological implications for studying choice under uncertainty? Second, is the domain of the utility function the final net outcomes of choices (net asset position), as a number of economic formulations explicitly or implicitly assume, or is it better characterized in terms of gains and losses from a zero change point, as prospect theory

(Kahneman and Tversky, 1979) asserts? Third, will choices display risk aversion over gains and risk-loving over losses, on both the individual subject and aggregate (between group) levels, exclusive of gambles with low probabilities of gains and losses, as prospect theory asserts? Fourth, will hypothesis II, fanning out of the local expected utility function, explain deviations from expected utility theory, as generalized expected utility theory (Machina, 1982) asserts; or will variations in the probability weighting function explain these deviations, as prospect theory and rank-dependent expected utility (RDEU) theory (Segal, 1987; Chew, Karni and Safra, 1987) asserts; or are regret-rejoicing effects partially responsible for these deviations, as regret theory (Loomes and Sugden, 1982) asserts?

Answers to these questions are mixed. We summarize our results as follows. First, there are systematic and statistically significant quantitative differences between real and hypothetical choice questions, but the *qualitative* conclusions reached regarding substantive behavioral issues do not differ between hypothetical and real choices. Second, choices over gambles do *not* reflect asset integration, since choices with the same net balance position tend to produce risk-loving (or risk neutrality) when they involve losses, and risk aversion when they involve gains, in cases where one of the alternatives has a positive probability of a zero payoff. Third, the reflection effect (risk-loving over high-probability losses, risk aversion over high-probability gains, with the pattern reversed for low-probability gains and losses) proposed in prospect theory is present on average (but is far from universal) when one of the alternatives has a positive probability of a zero payoff. However, we also find strong risk-loving over some questions with strictly positive payoffs, in cases where prospect theory predicts risk aversion. This calls into question prospect theory's arguments regarding the extent to which the gains function is concave and/or the probability weighting function exhibits subcertainty. Finally, we do not observe regret effects in our data, we identify conditions generating large systematic violations of hypothesis II in previously unexplored areas of the unit probability triangle. and we observe common ratio effect violations of expected utility theory under conditions where RDEU argues that they should not be observed. In short, none of the rival formulations considered consistently organizes choices. Given this fact, our immediate objective is to provide some general characterizations of the nature of these deviations, so that others may replicate them, and theorists will have some clear stylized facts to organize in their efforts to develop alternatives to EU theory as a descriptive model of choice under uncertainty.

1. Experimental procedures

Responses to four series of questions posed to four different groups of subjects are reported. Each question required subjects to indicate which of two gambles they preferred. In two series of questions, called gains series 1 and 2 (G1 and G2), all gambles involved nonnegative payoffs. In the other two series, called loss series 1 and 2 (L1 and L2), most, but not all, of the gambles involved nonpositive payoffs. G1 and L1 were conducted first, with G2 and L2 designed to pin down results and insights obtained from the initial series of questions.

In the first series (G1 and L1), subjects answered a set of hypothetical questions first, none of which would actually be played out. They returned several days later to answer a second set of questions, one of which, picked at random, would actually be played. Subjects were asked to answer the hypothetical questions as if they involved real payoffs. After answering these questions, each subject received a voucher entitling them to a $30 balance in the loss series, and a $5 balance in the gain series, to be used as their starting balances in the subsequent series of questions with real payoffs.[1] The gambles involving real payoffs consisted of a subset of the corresponding hypothetical gambles so that direct comparisons could be made between choices over hypothetical and real payoffs.

All questions were of the following form:

Example 1:

A: Losing $14 if 1-100   B: Losing $20 if 1-70
                                          Losing $0 if 71-100

Answer: (1) I prefer A. (2) I prefer B. (3) Indifference

The numbers following each dollar amount referred to 100 numbered poker chips in a bowl, sitting in front of the room, one of which would be drawn to determine payoffs. Thus, if a subject chose A, he/she would lose $14 with certainty, since any numbered poker chip drawn gave this result. If, however, the subject chose B, and a chip numbered 1-70 was drawn, he/she would lose $20, but if a chip numbered 71-100 was drawn, he/she would lose $0.

Subjects answered a total of 41 hypothetical questions, administered in two separate blocks with a brief rest period in between.[2] Fifteen questions were answered in the series involving real payoffs. In the case of real payoffs, once all the questions were answered, each subject drew a poker chip numbered 1–15 to determine which question they would be paid off on.[3] They then drew a poker chip from the bowl of 100 chips to determine the actual payoff for that question.[4] Immediately prior to answering questions with real payoffs, subjects answered three hypothetical questions and went through the two-stage chip-drawing procedure, to ensure their understanding of the payoff process.

Procedures in series 2 were similar to series 1 with the following differences: 1) subjects answered a single set of 18 questions; 2) *half* the subjects, chosen at random after all questions were answered, were allowed to play out a randomly chosen gamble from this set of 18 gambles; 3) these subjects received a $5 cash balance in the gains series, and a $30 cash balance in the loss series; and 4) all questions showed the chances of winning the different dollar amounts, expressed in percentage terms, in parentheses immediately following the chip numbers.

Six questions in series 2 were identical to those in series 1. Separate chi-square tests show no significant differences, at the 5% level or better, in answering any of these questions. Combining results through summing over the chi-square values gives a chi-square of 4.72 with six degrees of freedom, which is not statistically significant at conventional levels. Hence, we ignore procedural differences across the two series in analyzing the data.

All subjects were undergraduate students enrolled in introductory or intermediate economics classes at Texas A&M University. Experimental sessions lasted approximately one hour, with half the time being devoted to playing out the gambles.

2. Differences between real and hypothetical payoffs

One of the questions of primary interest was whether we would observe significant, and systematic, differences in choices over hypothetical and real payoffs. The answer to this question is crucial to designing additional experiments and to interpreting previously reported results, most of which have involved choices over hypothetical outcomes.

Loss series 1 repeated seven questions across hypothetical and real-loss questionnaires. Subjects were consistently more risk-averse with real as compared to hypothetical payoffs (see tables 1 and 2 below for representative results). Chi-square tests show significant differences, at the 10% level, in two of the seven questions. Combining results through summing over the chi-square values gives $\chi^2 = 13.26$ with seven degrees of freedom, which is significant at the 5% level.

Between gains series 1 and loss series 1, there was a total of 19 questions involving hypothetical and real gains. As was the case with losses, subjects were generally more risk-averse with real as compared to hypothetical payoffs (see tables 1 and 2 below). Chi-square tests show significant differences in two of the 19 questions at the 10% level, and in one question at better than the 1% level. Summing over chi-square values gives $\chi^2 = 37.12$ with 19 degrees of freedom, which is significant at better than the 1% level.

Since shifts in the direction of more risk-averse choices were reported for both gains and losses, we suspect a general conservatism in choosing real as compared to hypothetical outcomes, rather than any strong tendencies towards asset integration in the domain of losses.[5] Note, however, that despite these systematic and at times significant quantitative differences between responses to real versus hypothetical payoffs, *qualitative* conclusions regarding differences in risk attitudes over gains and losses were quite similar across both real and hypothetical choices; for gambles in which one of the alternatives had a positive probability of a zero payoff, subjects were considerably more risk-loving over high-probability losses as compared to high-probability gains with equal absolute dollar payoffs (see table 1 and the discussion in section 3.2 below); further, for gambles involving strictly positive payoffs, we see risk-loving with both real and hypothetical payoffs. This finding of qualitative similarities, with some real quantitative differences, in comparing questionnaire responses with hypothetical and real payoffs in choice under uncertainty, is similar to results others have reported (Grether and Plott, 1979; Grether, 1981; Camerer, 1989). It suggests some caution in interpreting results from strictly hypothetical choices, but far from complete rejection. In what follows, the analysis concentrates on questions with real payoffs, since we feel more comfortable with the validity of these results, and there were more than enough of these to answer the primary questions of interest.

3. Asset integration/differential treatment of gains and losses

This section examines two closely related questions: Is the domain of the utility function final states, or is it better characterized in terms of gains and losses from the zero change point? Will the predominant choice pattern observed display risk aversion over gains and risk-loving over losses?

In much of the economics literature dealing with risk and uncertainty, the carriers of value, either implicitly or explicitly, consist of final, net asset, positions. This holds for von Neuman–Morgenstern expected utility theory, and carries over into a number of nonexpected utility formulations as well. For example, Machina (1982, p. 307-308) explicitly assumes asset integration, although the techniques of applying concepts from expected utility theory to locally linear functions can be undertaken with or without assuming asset integration. In contrast, in prospect theory (Kahneman and Tversky, 1979), the primary carriers of value are assumed to be *changes* in wealth or welfare, rather than final states. While the value of any particular change in prospect theory is not completely independent of initial wealth levels, the preference order of prospects is not greatly altered by small, or even moderate, changes in asset positions.

Prospect theory goes on to argue that the valuation function over losses, $u(-x)$, is convex, while the valuation function over gains, $u(x)$, is concave. This follows directly from the fact that the carriers of value are changes in wealth or welfare and the psychological principle that the perceived marginal value of gains and losses generally decreases with their magnitude, resulting in a concave function above the zero (no change) reference point, and a convex function below it (see Figure 1). As a result, choices over losses will generally be risk-seeking and choices over gains will be risk-avoiding. Further, it is argued that the loss function is relatively steeper than the gains function in the neighborhood of the zero change point, resulting in a general aversion to fair gambles (a 50–50 chance of winning or losing $\$x$ versus a $0 change alternative). In contrast to prospect theory, most economic formulations postulate a predominant pattern of risk aversion over both gains and losses, with preferences invariant to whether questions are posed in terms of gains or losses, as long as the net balance position is the same.[6]

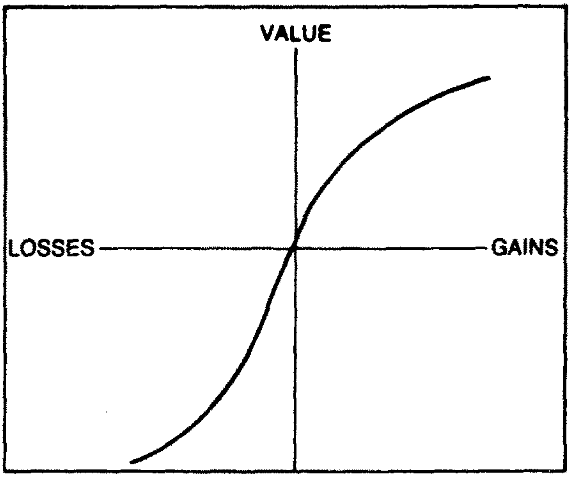

Fig. 1. Concave gain function and convex loss function proposed in prospect theory.

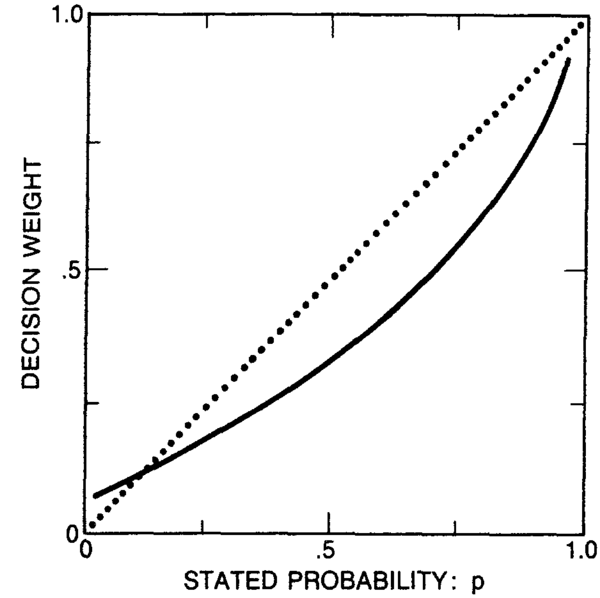

Fig. 2. Typical probability weighting function in prospect theory as reported in Kahneman and Tversky (1979) and Tversky and Kahneman (1986).

Prospect theory permits exceptions to the rule of risk-loving over losses and risk aversion over gains. The primary exception is for prospects involving small probabilities of gains or losses as a result of overweighting of small probabilities (Kahneman and Tversky, 1979, p. 285). The point at which overweighting of probabilities takes effect is an empirical matter, with the only a priori restriction being that $p < .5$. This restriction is necessary to insure subcertainty of the probability weighting function: $w(p) + w(1-p) < 1$ (where w is the probability weighting function). The subcertainty restriction is used within prospect theory to explain Allais-type common consequence violations of EU theory (Kahneman and Tversky, 1979, pp. 281–282). One empirical specification of what constitutes "small" probabilities is suggested by the hypothetical probability weighting function offered by Kahneman and Tversky (1979, figure 4; Tversky and Kahneman, 1986), which shows overweighting of probabilities at $p<.15$ (as shown in figure 2).

3.1. Tests for asset integration

To test whether the domain of the utility function is better characterized in terms of final states, or in terms of gains and losses from the zero change point, we asked paired choice questions for the loss and gain series where the losses, when added to starting balances in the loss series, resulted in the same net balance position as adding gains to starting balances in the gains series. Examples 2 and 3 list one set of paired questions employed.

Example 2: Loss questions starting with positive balance of $30
A: Losing $12 if 1-100 B: Losing $20 if 1-60
Losing $0 if 61-100

Example 3: Gains question starting with positive balances of $5
A: Winning $13 if 1-100 B: Winning $5 if 1-60
Winning $25 if 61-100

Adding gains and losses to the respective starting balances results in the same final net balance position in both cases. Assuming comparable distributions of initial asset positions across the two groups of subjects, and comparable distributions of risk attitudes, we should observe roughly the same distribution of preferences across both series of questions if value functions are defined over final asset positions. In contrast, prospect theory would predict a predominant preference for the certain gain of $13 in example 3 and a preference for the risky alternative (B) in example 2.

Table 1 reports answers to two sets of questions of this sort. In each case, the choice problem is displayed first, with the first number in brackets showing the dollar amount of the loss, followed by its probability (as a convention, we omit displaying zero-loss probabilities). The number of subjects choosing each alternative, with both real and hypothetical payoffs, is shown to the right of each prospect.[7] The results of chi-square tests for differences between hypothetical and real payoff choices are reported below these numbers. The significance level for rejecting the null hypothesis is shown in parentheses below the test statistic. As noted earlier, table 1 shows systematic, and sometimes large,

Table 1. Gains and loss questions with the same net outcomes

Negative outcomes			Positive outcomes		
	Choice frequencies			Choice frequencies	
	Real	Hypoth.		Real	Hypoth.
Question 1:			Question 1′:		
(−$12, 1.0)	11	8	($13, 1.0)	17	20
(−$20, .6)	20	23	($5, .6, $25, .4)	14	9
	$\chi^2 = .68$			$\chi^2 = 1.26$ (.26)	
Question 2:			Question 2′:		
(−$11, 1.0)	16	10	($14, 1.0)	23	24
(−$20, .7)	16	22	($5, .7, $25, .3)	7	7
	$\chi^2 = 2.33$ (.13)			$\chi^2 = .01$	

differences between choices involving real and hypothetical payoffs, with subjects generally showing less tolerance to risk with real payoffs (also see table 2 below).

Comparing choices under real payoffs, the results do not support asset integration, but rather a hypothesis of differential risk attitudes over gains and losses. While the chi-square statistic comparing question 1 and 1′ is not statistically significant at conventional levels ($\chi^2 = 2.34, p = .13$), the one comparing 2 and 2′ is ($\chi^2 = 4.72, p = .03$). Given the size and consistency of these differences, they are not likely to be due to inherent differences in risk attitudes between the two groups in question, but rather to a tendency to evaluate prospects relative to deviations from the zero change point instead of in terms of final net outcomes. This interpretation is supported by the fact that 1) in the real payoff series, we posed the same question (question 4′ in table 2 below) to both groups answering the questions in table 1 and found no significant differences in responses ($\chi^2 = 1.45, p = .23$), and 2) we identified three hypothetical questions posed to both groups, where answers to hypothetical and real payoffs for the gains group were quite similar (each had chi-square values of less than 1.0), and we found no differences in responses between the two groups on any question (each chi-square value was less than 1.0).

3.2. Tests for a reflection effect

Table 2 shows responses to gains and loss questions with equal absolute dollar payoffs and probabilities. Looking at real payoffs, for high-probability losses with equal expected values (questions 3 and 4), subjects had a modest to strong preference for the less certain alternative (risk-loving). In contrast, the corresponding high-probability gains questions (3′ and 4′) display strong risk aversion, consistent with the reflection effect hypothesis.

Preferences over losses varied systematically with the loss probabilities, since a clear majority of subjects preferred the more certain alternative (risk aversion) for the low-probability loss questions with equal expected values (questions 5 and 6). Of the two corresponding gains questions, only 5′ showed risk-loving, as the reflection effect suggests. Further, to explain the shift from risk-loving to risk aversion in going from high- to low-probability losses on the basis of prospect theory would require overweighting probabilities of .20 or better, somewhat above the threshold of the probability weighting function offered in Kahneman and Tversky (1979; see also figure 2 above).

Questions 7 and 7′, whose alternatives involved unequal expected values, and high probabilities of loses and gains, do not display a reflection effect of the kind specified in prospect theory, since subjects prefer the certain but lower expected loss alternative in question 7, and are evenly split between alternatives in the corresponding gains question (7′).[8] Preferences for the more certain, lower expected loss alternative persist in question 8, with strong preference for the less certain but higher expected value alternative in question 8′, consistent with the reflection effect specified in prospect theory.

The data provide qualified support for the reflection effect hypothesis. The support is there, but it is far from universal. Hershey and Shoemaker (1980), employing hypothetical choices, provide a much more extensive investigation of the reflection effect, finding that it is most prevalent when either small or large payoffs and extreme probabilities are involved.[9] Our results are consistent with theirs.

Table 2. Tests of the reflection effect hypothesis

Negative outcomes	Choice frequencies		Positive outcomes	Choice frequencies	
	Real	Hypoth.		Real	Hypoth.
Question 3:			Question 3′:		
(−$12, 1.0)	11	8	($12, 1.0)	25	19
(−$20, .6)	20	23	($20, .6)	6	10
	$\chi^2 = .68$			$\chi^2 = 1.75$ (.19)	
Question 4:			Question 4′:		
(−$14, 1.0)	15	7	($14, 1.0)	24	16
(−$20, .7)	16	23	($20, .7)	7	13
	$\chi^2 = 4.15$ (.04)			$\chi^2 = 3.34$ (.07)	
Question 5:			Question 5′:		
(−$12, .20)	22	16	($12, .20)	24	
(−$20, .12)	11	15	($20, .12)	6	
	$\chi^2 = 1.50$ (.22)				
Question 6:			Question 6′:		
(−$14, .20)	23	19	($14, .20)	13	14
(−$20, .14)	10	13	($20, .14)	16	15
	$\chi^2 = .76$			$\chi^2 = .07$	
Question 7:			Question 7′:		
(−$18, 1.0)	28		($18, 1.0)	17	
(−$27, .80)	6		($27, .80)	18	
Question 8:			Question 8′:		
(−$18, .20)	29		($18, .20)	5	
(−$27, .16)	7		($27, .16)	27	

3.3. Further results on gains and losses

While the data in section 3.2 provide qualified support for the reflection effect and, consequently, for the underlying shape of the gains and loss functions hypothesized in prospect theory (figure 1), answers to other questions do not. In particular, we observed risk-loving over gains questions with all positive outcomes, and a willingness on the part of a number of subjects to accept fair gambles involving gains and losses. These results call into question prospect theory's predictions regarding the conditions under which 1) the gains function is concave, and 2) the loss function is relatively steeper than the gains function.

Questions 9–11 in table 3 show choices on three gains questions with all positive outcomes. In all three cases, choices would result in gains irrespective of the alternative

Table 3. Choice over positively valued alternatives with large probability of gain

Gamble	Choice frequencies
Question 9:	
($11, 1.0)	11
($5, .70; $25, .3)	21
Question 10:	
($17, 1.0)	6
($10, .7; $30, .3)	26
Question 11:	
($16, 1.0)	6
($10, .7; $30, .3)	25
Question 12:	
($15, 1.0)	11
($5, .5; $25, .5)	18
Question 13:	
($20, 1.0)	13
($10, .5; $30, .5)	18

chosen, or the state of nature. In all three cases, a large majority of the subjects preferred the less certain alternative, with proportionately more subjects choosing the risky alternative than in any of the loss questions with real payoffs reported in table 2. These results are, of course, not consistent with the premise that the valuation function over gains, $u(\cdot)$, is concave, provided that overweighting of small probabilities is restricted to $p < .30$, well above the threshold of Kahneman and Tversky's suggested probability weighting function (recall figure 2).[10]

One can, of course, argue that responses to questions 9–11 merely refute the empirical specification of the probability weighting function suggested in Kahneman and Tversky (1979), but not the underlying theory, since it simply restricts overweighting to $p < .5$. Responses to questions 12 and 13, hypothetical payoff questions from gains series 1, suggest that this is not the case, however. Both show risk-loving over strictly positively valued alternatives in cases where the variable outcome alternative offered a 50–50 chance of a higher or lower payoff than the certain alternative.[11] This implies either overweighting of p .5, which would contradict the subcertainty assumption used to explain Allais-type common consequence violations of EU theory (Kahneman and Tversky, 1979, pp. 281-282), or convexity of the gains function, or some yet-to-be-specified framing effect.

The risk-loving reported in table 3 is in marked contrast to the risk aversion over high-probability gains, particularly those reported in table 2. Looking at these two tables, the least favorable outcome for the risky alternatives in table 3 all would have resulted in a reasonable sized gain, while the risky alternatives in table 2 all involved a relatively high probability of a $0 outcome. The implication is that subjects in our experiments are much

Table 4. Fair gambles involving gains and losses

	Choice frequencies
Question 14:[a]	
($0, 1.0)	14
(–$10, .5; $10, .5)	21
Question 15;[a]	
($0, 1.0)	20
(–$20, .5; $20, .5)	15

[a]One subject was indifferent between alternatives.

more willing to choose risky gains than prospect theory permits, provided the worst outcome leaves them in a reasonably favorable position relative to what they would have gotten from choosing the certain alternative.[12]

This risk-loving extends to fair gambles involving a 50–50 chance of winning or losing $x versus a zero change alternative. Questions 14 and 15 in table 4 show two gambles of this sort. A majority of the subjects preferred a 50–50 chance of ± $10 versus $0, with a sizable minority choosing ± $20 compared to a certain $0. In contrast, prospect theory predicts uniform preference for the zero outcome alternative, as a result of the relatively steeper slope of the loss function, compared to gains.[13] Framing questions 14 and 15 in terms of final net balance makes them quite similar to questions 12 and 13 in table 3 (recall that subjects were provided with $30 starting balances in both loss series). Thus, here, too, subjects were much more willing to take gambles than prospect theory predicts, provided the worst outcome was substantially better than zero.[14]

4. Testing between competing explanations of Allais-type common ratio violations of expected utility theory

A fundamental issue underlying our investigation concerned competing explanations of the common violations of EU theory reported in the literature. Our analysis focuses on Allais-type common ratio violations (common ratio violations include the *certainty effect* of Kahneman and Tversky and the Bergen Paradox of Ole Hagen as special cases). Allais-type common ratio violations involve rankings over pairs of prospects of the form:

Example 4:

A: x_2 with probability p	B: x_3 with probability q
x_1 with probability $1 - p$	x_1 with probability $1 - q$

Example 5:

C: x_1 with probability rp	D: x_1 with probability rq
x_2 with probability $1 - rp$	x_1 with probability $1 - rq$

where $p > q$, $|x_3| > |x_2| > |x_1|$, and $0 < r < 1$ (the term *common ratio* derives from the equality of prob(x_2)/prob(x_3) in A vs. B and C vs. D). An expected utility maximizer would prefer either A and C (if $p[u(x_2)-u(x_1)] > q[u(x_3)-u(x_1)]$), or else B and D (if the opposite were true). However, using primarily hypothetical choice alternatives, researchers find a systematic tendency for subjects to prefer A and D when the outcomes involve gains, and B and C when they involve losses (Kahneman and Tversky, 1979; MacCrimmon and Larson, 1979; references cited in Machina, 1983b).[15]

A key element of all nonexpected utility theories is to explain these, and other, systematic violations of expected utility theory. In explaining these violations, the theories make predictions that have yet to be subject to intense investigation. Our question is whether these new behavioral relations are satisfied in the data. We consider predictions based on rank-dependent expected utility theory (Yaari, 1987; Chew, Karni and Safra, 1987; Segal 1987), regret theory (Loomes and Sugden, 1982), prospect theory (Kahneman and Tversky, 1979), and Machina's (1982) generalized expected utility theory.

4.1. Rank-dependent expected utility theory (RDEU)

RDEU, like prospect theory, replaces probabilities with decision weights. In RDEU, the decision weight of a probability p_i depends on the other p_i and on the rank of outcome $x_i$compared to the other outcomes. Unlike prospect theory, the probability weighting function in RDEU is defined over the entire interval [0, 1], and does not induce first-degree stochastic dominance violations.

Segal (1987) explores the characteristics of RDEU theory required to unravel the empirical paradoxes of EU theory, in particular the common consequence and common ratio violations of EU theory. Segal shows that a necessary condition for RDEU to explain the common consequence effect is for the probability weighting function to be concave, so that the worst (lowest-ranked) outcome gets overweighted compared to its untransformed probability. Further, the function $g(p) = 1 - f(1-p)$, where f is the probability weighting function, must be increasingly elastic in p to organize Allais-type common ratio violations.[16]Segal goes on to show that if a decision maker behaves in accordance with RDEU and the common consequence or the common ratio effect, so that $g(p)$ is convex, then he cannot satisfy a generalized common ratio effect: a common ratio effect violation of EU theory in cases where prospects A and B in example 4, and prospects C and D in example 5, are mean-preserving spreads (Segal, 1987, theorem 1). In contrast, regret theory, prospect theory, and generalized expected utility theory all permit characteristic Allais-type violations of EU theory under these conditions, or "fanning out" of the local expected utility function, to use Machina's (1982) characterization of these violations (see section 4.4 below).

Table 5 recycles the answers to the gains questions in table 2, testing for Allais-type common ratio violations of EU theory. In sets 1 and 2, the alternatives in each pair involved mean-preserving spreads with equal expected value. Set 3 contains a more standard set of prospects, where the B alternative in each pair is riskier and yields a higher expected value than the corresponding A alternative.[17] We use it to compare the extent of Allais-type

Table 5. Common ratio effects with positive outcomes

Questions
Set 1:
A_1 ($14, .20) or B_1 ($20, .14)
A_2 ($14, 1.0) or B_2 ($20, .70)
Set 2:
A_1 ($12, .20) or B_1 ($20, .12)
A_2 ($12, 1.0) or B_2($20, .60)
Set 3:
A_1 ($18, .20) or B_1 ($27, .16)
A_2 ($18, 1.0) or B_2 ($27, .80)

		Choice frequencies		
Possible choice patterns[a]	Patterns consistent with	Set 1[b] Real	Set 2	Set 3
A A	EU	21	16	2
B B	EU	9	1	14
B A	Allais-type violations	23	6	13
A B	Not Allais-type violations	7	7	3

[a]Stochastically dominated alternatives listed first.
[b]Combined results from gains series 1 and 2.

violations of expected utility theory in the first two sets of questions with a more standard representation.

Looking at set 1, expected utility theory fails to explain more than a random number of responses: 50% of the data in set 1 versus 50% on the basis of chance factors alone. Allais-type violations (choice pattern BA) explain substantially more of the data than expected on the basis of chance (38.3% versus 25%), with this difference being significant at the 5% level. In set 2, expected utility theory does moderately better than chance alone, while Allais-type violations organize somewhat less of the data (20% of the observations) than one would expect on the basis of chance factors alone. Set 3 in table 5, which constitutes a more standard common ratio effect manipulation, looks much like set 1, since EU theory does no better than chance factors alone would predict, and Allais-type violations account for 40.6% of the data and 81.3% of the deviations from EU theory. The frequency of Allais-type violations in set 3 is only slightly greater than in set 1, where the corresponding numbers are 38.3% and 77.6%, respectively. Since, in terms of RDEU with a concave probability weighting function, we observe violations of EU theory when predicted (set 3), and when not predicted (set 1), the data suggest that there is an alternative explanation for these violations that the RDEU formulation fails to capture.

4.2. Regret theory

Loomes and Sugden (1982) and Loomes (1988a; 1988b) argue that Allais-type violations of expected utility theory can be explained on the basis of disappointment theory and/or regret theory. Regret theory is also capable of explaining a host of paradoxical results in choice under uncertainty, most notably the preference reversal phenomena (Grether and Plott, 1979). Regret theory entails a distinctive prediction. Because regret theory *is* concerned with the juxtaposition of consequences (that is, because regret–rejoicing is based on comparisons between what is received as a result of choosing one action compared with what might have been received under the same state of the world had the individual chosen differently), regret theory predicts the possibility of an individual's preferences varying with the way questions are formulated. For example, consider the following alternative formulations of identical prospects:

Example 6:

A: Winning $0	if 1-28	(28%)	B: Winning $20	if 1-28	(28%)
Winning $14	if 29-68	(40%)	Winning $0	if 29-68	(40%)
Winning $0	if 69-100	(32%)	Winning $0	if 69-100	(32%)

Example 7:

A: Winning $14	if 1-40	(40%)	B: Winning $20	if 1-28	(28%)
Winning $0	if 41-100	(60%)	Winning $0	if 29-100	(72%)

Regret theory predicts that preferences are likely to vary systematically with these different formulations, since question 6 calls into play these regret–rejoicing considerations, whereas the standard formulation in question 7 is likely to mask these considerations (Loomes, 1988a, 1988b). In particular, the convexity condition on the outcome evaluation function in regret theory predicts greater preference for the B alternative under the regret–rejoicing formulation (example 6) than in the standard formulation (example 7) (Loomes, 1988b). In contrast, if the gambles in examples 6 and 7 all involved nonpositive outcomes (losses), this same convexity condition requires greater preference for the A alternative in the regret–rejoicing formulation (example 6) than in the standard formulation (question 7).

Table 6 reports results from four sets of questions designed to test these predictions. For regret effects to have an impact on our data, we should observe, for questions involving gains, a shift in favor of the less certain alternative under the regret–rejoicing formulation as compared to the standard formulation. The first set of questions, involving choices over prospect ($14, .40) versus prospect ($20, .28), shows a modest shift between formulations in the predicted direction. In contrast, in the second set of questions, prospect ($14, .20) versus prospect ($20, .16), there is a very modest difference in the *opposite* direction to that predicted under regret–rejoicing.

For losses, the regret–rejoicing formulation predicts greater preference for the more certain alternative than the corresponding standard formulation. The first set of questions, involving choices over prospect (–$14, .40) versus prospect (–$14, .28), shows a very modest difference in the predicted direction. In contrast, the second set of questions,

Table 6. Tests of regret–rejoicing theory[a]

Gains:			
	More certain prospect ($14, .40)	Less certain prospect ($20, .28)	
Regret–rejoicing formulation	28		7
Standard formulation	30		3
		$\chi^2 = 1.61$ (.20)	
	More certain prospect ($14, .20)		Less certain prospect ($20, .16)
Regret–rejoicing formulation	17		13
Standard formulation	16		16
		$\chi^2 = .28$	
Losses:			
	More certain prospect (–$14, .40)		Less certain prospect (–$20, .28)
Regret–rejoicing formulation	11		25
Standard formutlation	9		25
		$\chi^2 = .14$	
	More certain prospect (–$18, .20)		Less certain prospect (–$27, .16)
Regret–rejoicing formulation	22		12
Standard formulation	28		7
		$\chi^2 = 2.02$ (.16)	

[a]Differences in total number of responses under different formulations result from changes in number of subjects expressing indifference.

prospect (–$18, .20) versus prospect (–$27, .16), shows a rather large difference in the *opposite* direction to that predicted under regret–rejoicing.

Work reported by Loomes and Sugden generally shows statistically significant regret–rejoicing effects over substantially larger sample populations than employed here. But there are exceptions, even cases involving significant differences opposite to those predicted by regret theory (Loomes and Sugden, 1987, p. 128). The questions posed here provide an independent examination of regret–rejoicing effects, involving the first analysis (to our knowledge) over losses, and using different experimental procedures then Loomes and Sugden, since they typically employ graphical representations of their prospects (see

	1 — 30	31 — 50	51 — 100
A	11.50	1.50	1.50
B	7.50	7.50	1.50
	30	20	50

Fig. 3. Choice representative employed in Loomes (1988a).

figure 3) in contrast to the numerical formulations employed here (see examples 6 and 7 above).[18] Further experiments are needed to explore the impact of both these factors, for if regret effects fail to be found regularly over losses, or under the numerical formulations employed here, this would suggest important limitations on the applicability of the theory.

4.3. Prospect theory

Prospect theory, like RDEU, uses the properties of the probability weighting function to explain common violations of expected utility theory. For a fixed ratio of probabilities, the ratio of the corresponding decision weights is closer to unity when the probabilities are low than when they are high, $w(rq)/w(rp) > w(q)/w(p)$, where $0 < r < 1$. This property of the probability weighting function, called subproportionality, is used to explain Allais-type common ratio violations. This can be seen as follows: in the case of gains, under prospect theory, preference for A over B in example 4 implies $u(x_2)/u(x_3) > w(q)/w(p)$ (since $u(x_1) = 0$ for $x_1 = 0$), and preference for D over C in example 5 implies that $u(x_2)/u(x_3) < w(rq)/w(rp)$, or $w(rq)/w(rp) > w(q)/w(p)$.

Allais-type common ratio violations over gains invariably start with subjects preferring the certain alternative, as in example 4. The risk-loving reported in table 3 over strictly positive outcomes provides an unusual opportunity to test the subproportionality explanation of these violations, since it permits us to look for common ratio violations starting from a position of risk-loving. In this case, the common ratio manipulation should have no effect on the majority of subjects who initially preferred the risky (B) alternative, and should induce some of the subjects who initially preferred the certain (A) alternative to switch to the risky alternative following the probability changes.

Table 7 reports the results of these tests using questions 9 and 10 from table 3. In set 1, 11 subjects violate EU theory, with 7 of the 11 moving opposite to the direction that subproportionality predicts. The results in set 2 are similar. In this case, a bare majority of subjects satisfied EU theory (51.1%), while 10 of the 16 violations of EU theory violated subproportionality as well. The frequency with which subjects violated subproportionality, relative to the total violations of EU theory, is not significant at conventional levels under a maintained hypothesis of subproportionality. However, in both cases, the majority of subjects violating EU also violated subproportionality, and the frequency of these violations was more than one would anticipate on the basis of the repeat reliability likely to be

Table 7. Common ratio tests over gains with strictly positive outcomes

Set 1:			
	A_1 ($17, .20; $10, .80)	or	B_1 ($30, .06; $10, .94)
	A_2 ($17, 1.0)	or	B_2 ($30, .30; $10, .70)
Set 2:			
	A_1 ($11, .20; $5, .80)	or	B_1 ($25, .06; $5, .94)
	A_2 ($11, 1.0)	or	B_2 ($25, .30; $5, .70)

Possible choice patterns[a]	Patterns consequent with	Choice frequencies	
		Set 1	Set 2
A A	EU	3	8
B B	EU	18	9
B A	Prospect theory	4	6
A B	Not EU and not prospect theory	7	10

[a]Stochastically dominated alternatives are listed first.

encountered in data of this sort.[19] As such, these results support those of section 3.3, suggesting breakdowns in the probability weighting function characteristics specified in prospect theory, or failure of some other property of the theory.

Violations of subproportionality here also imply fanning in of subjects' indifference curves, contrary to hypothesis II—fanning out of indifference curves in the unit probability triangle—in Machina's (1982, 1983a) generalized expected utility theory. The next section reports additional, and substantially more dramatic, violations of hypothesis II.

4.4. Generalized expected utility theory: Tests of hypothesis II

Machina's (1982; 1983a) generalized expected utility analysis identifies four classes of systematic violations of the independence axiom that have been reported in the literature (of which the common ratio effect is one), and shows that all four follow from a single assumption, termed hypothesis II, on the shape of individual preference functionals.[20]Hypothesis II states that in moving from one probability distribution to another that (first-order) stochastically dominates it, the local (linear in the probabilities) utility function retains the same degree of concavity, or becomes more concave at each point. In other words, preferences tend to vary systematically as a consequence of an expected income effect, so that distributions that ex ante involve more income yield similar or more risk-averse choices.

In terms of the unit probability triangle, this yields indifference curves that are parallel (as expected utility theory predicts) or that "fan out" relative to the origin. Figure 4a illustrates these common patterns of fanning out over gains (preference for alternatives A and D), with figure 4b showing common patterns for losses (preference for alternatives B and C). The fanning out pattern explains these Allais-type common ratio violations in both cases.

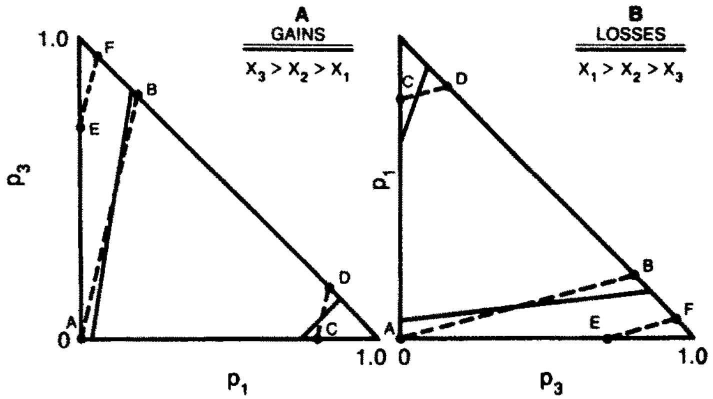

Fig. 4. a) Indifference curves (solid lines) fanning out to explain Allais-type common ratio violations for gains (preference for A over B and D over C). Fanning out also requires preference for prospect E over F. Dashed lines represent linear combinations of the prospects employed. b) Indifference curves (solid lines) fanning out to explain Allais-type common ratio violations for losses (preference for B over A and C over D). Fanning out also requires preference for prospect F over E. Dashed lines represent linear combinations of the prospects employed.

Fanning out of indifference curves in the unit probability triangle corresponds to the *light hypothesis* of weighted utility theory (Chew and MacCrimmon, 1979; Chew and Waller, 1986), and is a characteristic of several other generalizations of expected utility theory as well (see Machina, 1983b). In the case of gains, the obvious question is whether choices over pairs of prospects that stochastically dominate A and B—prospects that lie in the northwest corner of the unit probability triangle, such as E and F in figure 4a—continue to satisfy the requirements of fanning out, namely preference for the E alternative on the part of those choosing A. The corresponding question for losses is whether choices over prospects that are dominated by A and B—prospects that lie in the southeast corner of the unit probability triangle, such as E and F in figure 4b—continue to satisfy the requirements of fanning out, namely preference for F on the part of those choosing B.

We conducted several tests of fanning out over gains in the northwest corner of the unit probability triangle. In all cases, fanning out explained less than 50% of the deviations from expected utility theory. Choice set 1.1 in table 8 (figure 5a) shows the most dramatic collapse of fanning out observed. Expected utility theory organizes less than half the data (44.4%), with fanning out organizing only 20% of the deviations from expected utility theory. In contrast, fanning in organizes as much of the data as does expected utility theory here (44.4%), which is well beyond what one would expect on the basis of chance factors alone ($p < .01$).

Choice set 1.2 replaces alternative A_2 in set 1.1 with A_3, the difference being that A_3promises a payoff of either $27 or $18, while A_2 leaves a 6% chance of getting $0. Further, A_2 involves a probability mix of A_3 and B_2 so that $A_3 - B_3$ lies equally to the northwest of

Table 8. Tests of hypothesis II

Set 1:

1.1. A_1 (\$18, .90) or B_1 (\$27, .72)
A_2 (\$27, .74; \$18, .20) or B_2 (\$27, .90)

1.2. A_2 (\$18, .90) or B_1 (\$27, .72)
A_3 (\$27, .50; \$18, .50) or B_3 (\$27, .90)

Possible choice patterns[a]	Patterns consistent with	Choice frequencies	
		Set 1.1	Set 1.2
A A	EU	6	14
B B	EU	10	11
B A	Fanning out	4	3
A B	Fanning in	16	8

Set 2:

2.1. A_1 (–\$27, .74; –\$8, .20) or B_1 (–\$27, .90)
A_2 (–\$18, .90) or B_2 (–\$27, .72)

2.2. A_3 (–\$27, .50); –\$18, .50) or B_3 (–\$27, .90)
A_2 (–\$18, .90) or B_2 (–\$27, .72)

Possible choice patterns[a]	Patterns consistent with	Choice frequencies	
		Set 2.1	Set 2.2
A A	EU	21	18
B B	EU	3	3
B A	Fanning out	2	5
A B	Fanning in	8	7

Set 3:

3.1. A_1 (–\$20, .74; –\$14, .20) or B_1 (–\$20, .88)
A_2 (–\$14, .90) or B_2 (–\$20, .63)

3.2. A_3 (–\$20, .60; –\$14, .40) or B_3 (–\$20, .88)
A_2 (–\$14, .90) or B_2 (–\$20, .63)

Possible choice patterns[a]	Patterns consistent with	Choice frequencies	
		Set 3.1	Set 3.2
A A	EU	7	3
B B	EU	11	17
B A	Fanning out	1	5
A B	Fanning in	15	7

[a]Stochastically dominated alternatives are listed first.

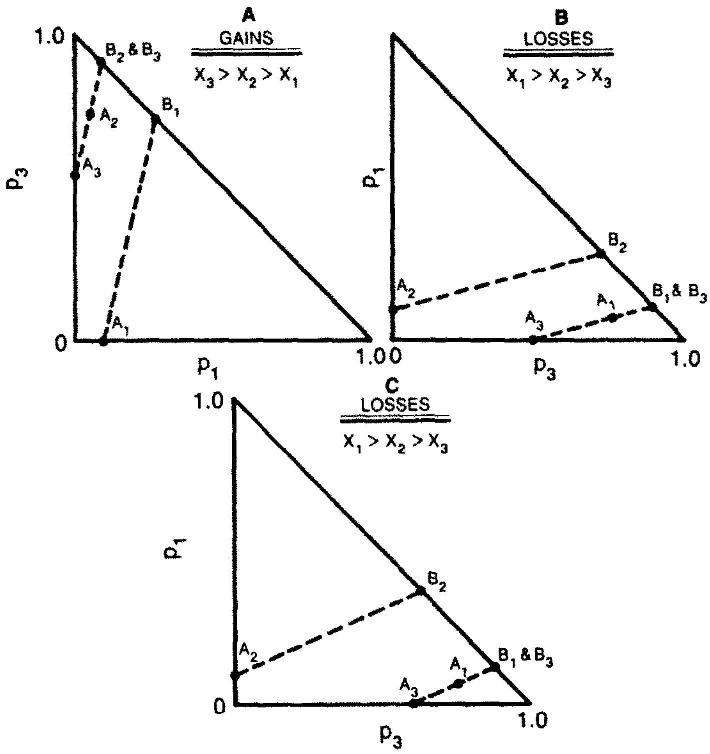

Fig. 5. Prospects employed in tests of hypothesis II. Dashed lines represent linear combinations of the prospects employed.

$A_1 - B_1$, promising similar ex ante increases in expected income in both cases (see figure 5a). Now, expected utility theory organizes 69.4% of the data, while clear fanning in no longer explains more observations than expected on the basis of chance (22.2% versus 25%) and organizes somewhat fewer, but still a majority, of the deviations from expected utility theory.[21] In moving to the northwest corner of the unit probability triangle, eliminating the chance for losses on the more certain alternative makes it considerably more attractive, promoting greater conformity with EU theory, and less frequent fanning in.

Results similar to these were observed for losses in the southeast corner of the unit probability triangle.[22] In set 2.1 (figure 5b), EU theory explains a majority of the data (74.6%), while fanning in organizes most of the deviations (80%) from expected utility theory. Replacing alternative A_1 in set 2.1, with its 6% chance to escape without loss, with A_3 in set 2.2, which guarantees a loss if it is chosen, reduces the attractiveness of the A alternative and improves the performance of the fanning out hypothesis, although it still fails to explain a majority of the deviations from expected utility theory. In set 3.1 (figure 5c), EU theory organizes 52.9% of the data compared to 50% on the basis of chance factors alone. Fanning out breaks down completely, since fanning in organizes 44.1% of the data, well beyond expectations based on chance factors alone ($p > .01$). Choice set 3.2 replaces A_1 in set 3.1, with its 6% chance to escape without losses, with A_3, which guarantees losses should it be chosen.[23] The certainty of losses reduces the attractiveness of the A alternative, thereby producing greater conformity with EU theory and increasing the frequency of clear fanning out, although fanning out still fails to explain a majority of the deviations from EU theory.

The results of these exercises suggest certain behavioral regularities. Sure gains are much more attractive than uncertain gains with equal expected value and some small chance of a zero payoff. Introducing a chance for a zero payoff into both alternatives promotes choice on the basis of the highest valued payoff alternative. Under appropriate conditions, this produces fanning out, as in the standard Allais-type common ratio violations of EU theory (see figure 4a). At times, it produces fanning in, as in set 1.1 in table 8. Note, however, that fanning in is reduced substantially once we reintroduce a certain positive payoff and pair it against an uncertain alternative with some possibility of losses, as in set 1.2 in table 8.

Conversely, certain losses are much less attractive than uncertain losses with equal expected value and some chance of escaping losses completely (a zero payoff). Introducing a chance to escape losses on both alternatives promotes choice on the basis of the lowest loss alternative. This produces fanning out and standard Allais-type common ratio violations (see figure 4b), but it can also produce fanning in, as in sets 2.1 and 3.1 in table 8. Here, too, fanning in is mitigated by reintroducing certain losses, as in sets 2.2 and 3.2 in table 8.

The certainty effects identified here for both gains and losses operate in much the way that prospect theory suggests. However, the results in section 3.3 indicate that the probability weighting mechanism that prospect theory employes to explain these Allais-type common ratio violations is suspect. Rather, some sort of risk-dimension approach (Slovic and Lichtenstein, 1968; Schoemaker, 1982), where payoffs and probabilities are viewed as separate stimuli, and decisions are made in a decomposed fashion comparing alternatives on a piecemeal basis, may well be guiding choices.

5. Summary and conclusions

Responses to four series of questions involving risky alternatives are reported. Our answers to the primary research questions posed are as follows: First, there are systematic and significant quantitative differences over real and hypothetical choice questions, but the qualitative conclusions reached regarding the substantive behavioral issues in question do not differ between hypothetical and real choices. Second, choices over gambles fail to reflect complete asset integration, since choices with the same net balance position tend to produce risk-loving (or risk neutrality) when they involve high-probability losses, and risk aversion when they involve high-probability gains. Third, the reflection effect proposed in prospect theory is present on average (but is far from universal) when one of the alternatives involves a positive probability of a zero payoff. However, we also find risk-loving over a number of prospects with all positive payoffs, in cases where prospect theory predicts risk aversion. This calls into question prospect theory's arguments regarding the extent to which the gains function is concave and/or the probability weighting function exhibits subcertainty. Finally, we do not observe regret effects in our data, we identify conditions generating large systematic violations of hypothesis II in previously unexplored areas of the unit probability triangle, and we observe Allais-type common ratio violations of expected utility theory under conditions where RDEU argues that they should not be observed.

Our overall conclusion is that none of the alternatives to expected utility theory considered here consistently organize the data, so we have a long way to go before having a complete descriptive model of choice under uncertainty. Our results regarding the inability of any single alternative to EU theory to consistently organize choices are similar to those of other independent investigators (Camerer, 1989; Harless, 1987; Starmer and Sugden, 1987a, 1987b). Each of these studies investigates somewhat different test procedures, and interested readers are encouraged to look at these articles on their own, since we cannot do justice to them here. One solution to the failure of the formulations investigated here is to develop hybrid models with more free parameters (see Chew and Epstein, 1987, and Luce and Narens, 1985, for developments along these lines). The problem with this approach is that it leaves the practitioner with no consistent empirical regularities, in terms of deviations from EU theory, to help in organizing new data sets.

A few things are clear, however. First, any successful descriptive model will have to give up asset integration. This has important implications for choice, in particular that framing questions in terms of gains or losses, even though net outcomes are the same, will have systematic effects on choices. Second, comparing certain gains with uncertain gains with some chance of a $0 payoff tends to produce risk aversion, while comparing certain losses with uncertain losses with some chance of a $0 payoff tends to produce risk-loving or risk neutrality. These are two of the key differences of prospect theory compared to standard expected utility theory.

Notes

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1. The delay period and voucher system was used in efforts to maximize the chances that subjects would treat their starting balances over real losses as genuine losses from their endowments.

2. There was a 15-minute rest period between answering the two blocks of questions. Subjects were not allowed to talk to each other during the rest period.

3. Recent articles by Holt (1986) and Karni and Safra (1987) show conditions under which answers to questions in a series of questions, only one of which will be paid off on, will not be independent of the set of questions posed. This criticism applies to both the Becker, DeGroot, and Marschak (1964) procedure for determining certainty equivalent values of gambles, and to simple elicitation of preferences over a series of paired alternatives, such as

that employed here. Responses to each question are independent of the set of questions posed if the independence axiom holds or if subjects choose as if each gamble will actually be selected (as in the isolation effect of prospect theory). To test for independence, Camerer, (1989) permitted subjects to change their choices once the question to be played out had been selected. Few subjects changed their choices. Further, the strong repeat reliability obtained here for six identical questions posed in series 1 and 2 (see the next to last paragraph in this section) is also consistent with independence, since there were substantial differences in the other questions included in the two series. Since the data below show frequent violations of the independence axiom, we must conclude that subjects treated each gamble separately, either in response to our urging them to do so, or out of a natural kind of isolation effect (consistent with the absence of asset integration found below).

4. Each subject made their own draw in both stages. This is more time-consuming than a single draw for all subjects, but permits the use of the law of large numbers in planning the cost and design of the experiment. It also serves to enhance the credibility of the experimenters, since it minimizes the probability that we had rigged the procedures to achieve the worst outcome from the subjects' point of view.

5. Given the size of the starting cash balances, complete asset integration would result in positive net balances in all cases. Framing outcomes in terms of these positive net balances would, according to the reflection effect in prospect theory, result in changing choices from risk-loving to risk aversion, and vice versa, in choosing over these losses. As noted below, we do not observe this effect.

6. The exception here is that losses that will result in ruin, or bankruptcy, are likely to produce risk-loving (see Machina, 1982a, for an example).

7. In both cases, we simply omit responses of indifference. There was a total of 33 subjects in this phase of the experiment, so the reader can readily compute the frequency of this class of responses.

8. The payoff and probability ratios in questions 7′ and 8′ match those of problems 9 and 10 in Tversky and Kahneman (1986).

9. The reflection effect holds for individual respondents, while Kahneman and Tversky's work typically compares responses across groups of subjects. Hershey and Shoemaker (1980) calculate the potential for overstatement and understatement of the reflection effect for individual respondents. Their analysis is based on within-subject comparisons. Also see Chew and Waller (1986, p. 66) who, using hypothetical payoffs, report limited tests for a reflection effect, finding that on a within-subject basis, less than half of all respondents exhibited a reflection effect of the kind prospect theory predicts.

10. For example, in question 11, preference for the second alternative implies, according to prospect theory, that $u(16) < u(10) + w(.3)[u(30)-u(10)]$, or $[u(16)-u(10)]/[u(30)-u(10)] < w(.3) \leq (.3)$, if there is no overweighting of [=.3. Assuming that the gains function $u(\cdot)$ is strictly concave and continuous with $u(0) \geq 0$, then it is well known that $u(t)/t$ decreases with t for all $t > 0$. Thus $u(6)/6 > u(20)/20$ or $u(6)/u(20) > .3$. Consider now $v(t) \equiv [u(t+10) \equiv u(10)]$ so that $v(0)=0$. Since $v'(t) = u'(t+10)>0$ and $v''(t) = u''(t+10)<0$, v is concave, too. It follows then that for all $t \geq 0$, $v''(t)/t$ decreases with t, or $.3 < v(6)/v(20)=[u(16)-u(10)]/[u(30)-u(10)]$, directly counter to the preferences implied by responses to question 11.

11. Statistical tests of the null hypothesis of risk aversion (preference for the certain alternative) in questions 12 and 13 show that we cannot reject the null hypothesis at conventional significance levels (p = .13 and .24 in 12 and 13, respectively). However, compared to the reversal rate subjects have in answering questions of this sort (Camerer, 1989, reports an average reversal rate of 31.6%), we would reject the more neutral hypothesis that reference for the risky alternative is simply due to chance at better than the .01 level in both cases. Further, comparing responses to hypothetical and real payoffs on questions 9–11, we find more subjects choosing the variable outcome alternative with real as compared to hypothetical payoffs. These are the primary exceptions to the rule of more conservative choice over real as compared to hypothetical gains questions mentioned in section 2 above.

12. The risky alternative in gains question 2′ in table 1 has a lower expected value ($11) than the certain alternative, while question 1′, which comes closest in structure to the questions in table 3, exhibits the lowest frequency of risk aversion.

13. Relative to the average reversal rate of 31.6% reported in Camerer (1988), we can reject a null hypothesis of preference for the certain payment of $0 at $p < .01$ and $p = .07$ in questions 12 and 13, respectively. Note that the one repeat question asked in our experiments, in the hypothetical loss series, showed a reversal rate of less than 5%.

14. Thaler and Johnson (in press) attribute these violations to a "house money" effect, whereby people are more

willing to take gambles, given unexpected winnings. There is clearly some element of this in their data, and in ours—for example, 20 out of 32 subjects (62.5%) preferred the gamble (–\$6, .7; \$14, .3) versus (–\$3, .7; \$7, .3) in the real-loss series, compared to 27 out of 59 (45.8%) in hypothetical gambles. However, with 45.8% of the subjects choosing the riskier alternative for hypothetical choices, where there can be no house money effect, this hardly seems to provide the full explanation for the absence of strong risk aversion that prospect theory suggests over gambles of this sort.

15. Rats also violate the independence axiom in the Allais-type direction over choice problems with real payoffs (Battalio, Kagel, and MacDonald, 1985; Kagel, MacDonald, and Battalio, 1989).

16. Yaari (1987) developed a special case of RDEU theory (called *dual* EU) in which the utility function for wealth is linear. Yaari shows that the probability weighting function must be concave to organize common consequence violations of expected utility theory, and that $g(p)$ must satisfy subproportionality, so that $g(q)/g(p) \leq q(rq)/g(rp)$, where $0 < q < p < 1$ and $0 < r < 1$, to explain common ratio effect violations. The subproportionality requirement here is similar to the requirement in prospect theory, although the probability weighting functions are quite different between the two formulations.

17. The payoff and probability ratios match those of problems 9 and 10 in Tversky and Kahneman (1986).

18. In fact, we were motivated to examine regret theory here by Loomes's suggestion that it would be interesting to compare choices under our representation with his.

19. Using the repeat reliability frequency of 31.6% (see note 11 above), the frequency with which subjects deviated from subproportionality, given that they deviated from EU theory, is significant at better than the 5% level in both cases.

20. These involve 1) the common consequence effect, the most famous specific example of which is the so-called *Allais Paradox,* 2) the common ratio effect, 3) oversensitivity to changes in small probability-outlying events, and 4) the utility evaluation effect (see Machina, 1982, 1983a).

21. Looking at choices between alternatives A_2-B_2 and A_3-B_3 permits us to test the betweenness axiom of EU theory that is shared by a number of rival formulations as well (for example, weighted utility theory). If gamble $X > Y$ (where > indicates preference), then betweenness requires that $X > rX + (1-r)Y > Y$, $0 < r < 1$: any gamble that is a probability mix of X and Y must be between them in preference. Betweeness implies linear indifference curves in the unit probability triangle. It requires that subjects choose the less risky or more risky gamble both times from pairs of gambles that lie on a straight line within the triangle. In set 1, betweenness was violated by 30.3% of all subjects. Ten of thirteen subjects who violated betweenness chose A_3 over B_3 and B_2 over A_2, consistent with an inverted ordering (concave indifference curves in the unit probability triangle), and inconsistent with portfolio theory (Coombs and Huang, 1976).

22. Experiments with rats also show strong violations of fanning out over previously unexplored areas of the unit probability triangle (Kagel, MacDonald, and Battalio, 1989; MacDonald, Kagel, and Battalio, 1989).

23. In sets 2 and 3, 31.4% and 54.5% of the subjects, respectively, violate betweenness. Eight of 11 and 15 of 18 of the violations in sets 2 and 3 chose A_1 over B_1 and B_3 over A_3, which is consistent with a folded ordering (convex indifference curves in the unit probability triangle), as portfolio theory (Coombs and Huang, 1976) suggests.

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