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Brittle Fracture of Rocks under Impulse Loads

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ABSTRACT
Brittle fracture of Tennessee Marble, Charcoal Granite and sandstone was studied using the Hopkinson split bar method. The study of the actual fracture behaviour of the rock was possible as no transducers were attached to the specimens themselves. The degree of brittleness of rock was determined by using the attenuation of the first wavefront in a fracturing specimen. There existed a maximum limit to the stress amplitude which could be transmitted through a rock specimen. Fracture is initiated in rock at the same stress under both static and dynamic loads.

Introduction

The study of the dynamic behaviour of rock has important applications in drilling and blasting. Little knowledge of the basic mechanisms of brittle fracture of rock under dynamic loads has been available. The most frequently used methods to test rock dynamically are the pulse and resonance methods. In both of these, the rock is subjected to low amplitude pulses, considerably below the failure values.

The Hopkinson split bar method which was originally designed for the testing of metals [1, 2] was chosen in the present study as a testing method. Kolsky [3] gives a discussion of the system with remarks on its limitations. The method was applied so that the fracture behaviour of the rock specimens could be observed. Measurements of the whole fracture process are possible since no strain transducers are attached to the specimen; all strainwave measurements are made on the metal bars.

In the previous work [4] the dynamic stress-strain curves were obtained. They, however, showed that the determination of the elastic modulus from the results even with short specimens was not reliable.

The consideration of the energies carried by a compressive pulse gave useful information. The energy entering the specimen and leaving it, and thus the energy absorbed in the specimen, could be determined. It was found that there existed a maximum energy limit which could be transmitted through a rock specimen by a given pulse. The limit was essentially a stress amplitude limit such that the stresses above this limit are not transmitted, but result in irreversible rock fracturing and consequent energy loss. The maximum transmitted energy depends thus on the duration of the pulse.

Model for Stress Attenuation in Rock

Consider a model where a cylindrical rock specimen is placed between two metal bars of the same diameter. The initial purpose is to determine how a stress wave is changed due to the reflections from the surface between the bars and the specimen.

The simple model of a plane wave travelling through the bars and the specimen, when force and particle velocity continuity at the interfaces is assumed, gives us the following relationships between the stress amplitudes.

$$\frac{\sigma_t}{\sigma_i} = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1} \tag{1a}$$

$$\frac{\sigma_r}{\sigma_i} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \tag{1b}$$

where: σ_i = stress wave amplitude incident at the interface;
σ_r = stress wave amplitude reflected from the interface;
σ_t = stress wave amplitude transmitted across the interface;
ρ = density;
c = longitudinal wave velocity.

Subscript 1 refers to the material of bar 1, i.e. through which the incident wave travels, and subscript 2 to the material of bar 2, i.e. into which the transmitted wave enters.

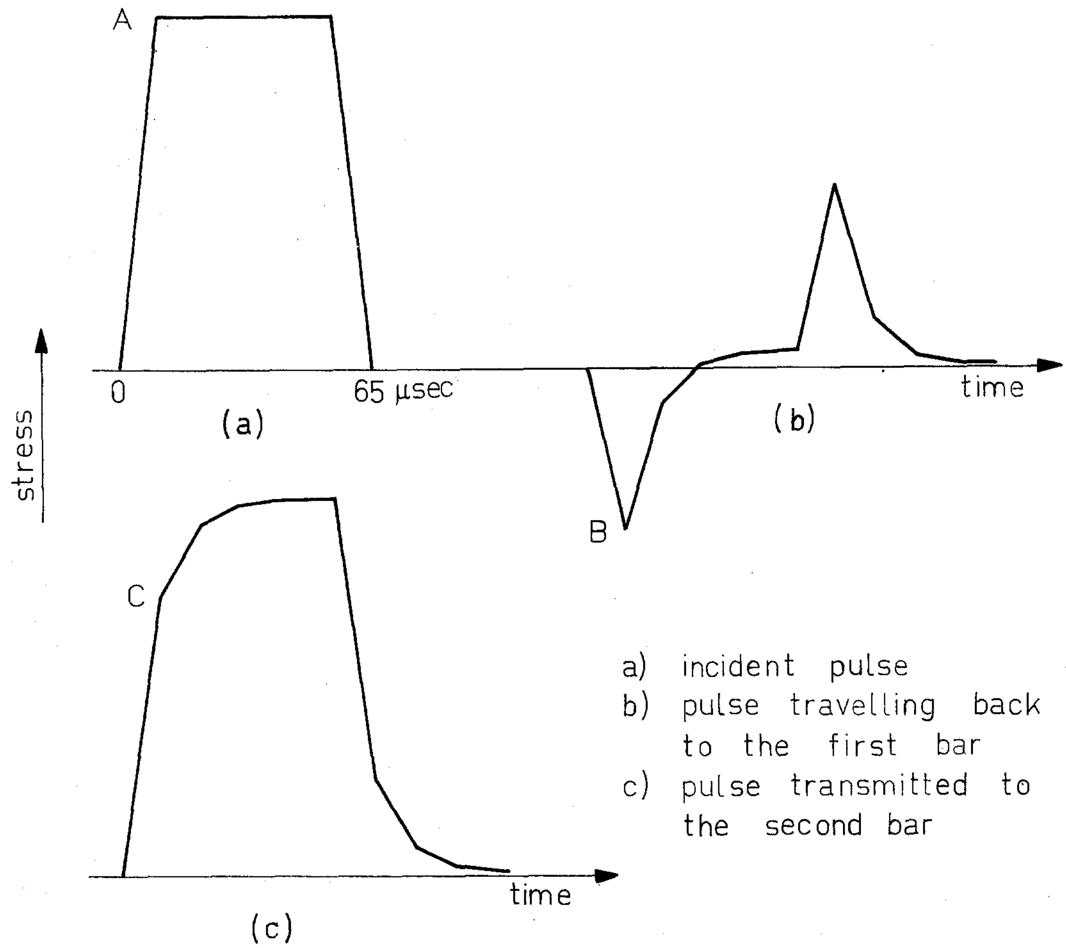

Figure 1. The behaviour of a pulse in the Hopkinson split bar system when a 25.4 mm long Tennessee Marble specimen is assumed to be placed between two steel bars of the same diameter as the specimen. Force and particle velocity continuity is assumed at the interfaces.

Fig. 1 illustrates the behaviour of the wave in a 25.4 mm long Tennessee Marble specimen placed between two steel bars.

The attenuation of the wave in the specimen can now be observed as the difference of the amplitudes entering the specimen and leaving it. From the difference between the incident and the reflected waves the stress wave amplitude which enters the specimen can be calculated, and from the transmitted stress wave amplitude, the amplitude at the final surface of the specimen. The difference between the amplitudes at the different surfaces of the specimen gives the stress attenuation in the specimen.

If it is assumed that the rock specimen behaves linearly up to the stress level of fracture initiation [5, 6] the presentation of the relationship of the stress amplitudes at the initial and final surfaces of the specimen can be shown as a step function. The first step is elastic and no amplitude attenuation occurs, the second step describes the behaviour of fracturing material with amplitude attenuation.

$$\sigma_{II} = h(\sigma_I)\sigma_I + g(\sigma_I) f(\sigma_I) \tag{2}$$

where:

$$\begin{aligned} h(\sigma_I) &= 1 \quad \text{when} \quad 0 \leqq \sigma_I < \sigma_c \\ h(\sigma_I) &= 0 \quad \text{when} \quad \sigma_I \geqq \sigma_c \text{ or } \sigma_I < 0 \\ g(\sigma_I) &= 1 \quad \text{when} \quad \sigma_I \geqq \sigma_c \\ g(\sigma_I) &= 0 \quad \text{when} \quad \sigma_I < \sigma_c \end{aligned} \tag{3}$$

σ_{II} = stress at the final surface of the specimen;
σ_I = stress at the initial surface of the specimen;
$f(\sigma_I)$ = function which gives the dependance of σ_{II} on σ_I when fracture propagates in the specimen;
σ_c = stress at which fracture is initiated.

In order to determine the function $f(\sigma_I)$ in (2) consider the parts of the stress amplitudes higher than σ_c. Then

$$\begin{aligned} \sigma_I - \sigma_c &= \sigma_a \\ \sigma_{II} - \sigma_c &= \sigma_b \,. \end{aligned} \tag{4}$$

The basic assumption made for the calculation of the attenuation in a fracturing specimen is that each differential increase in the stress amplitude σ_a at the initial surface of the specimen is attenuated in the specimen. The attenuation is assumed to depend linearly on the achieved stress at the final surface, measured by the fracture initiation stress, and exponentially on the length of the specimen.

$$d\sigma_b = d\sigma_a - \exp(\alpha l)[(\sigma_b/\sigma_c) d\sigma_a] \tag{5}$$

where: $d\sigma_a$ = increase in σ_a
$d\sigma_b$ = increase in σ_b
σ_a = stress above σ_c at the initial surface of the specimen
σ_b = stress above σ_c at the final surface of the specimen
α = material coefficient
l = length of the specimen

The solution of (5) with the initial condition $\sigma_b = 0$ when $\sigma_a = 0$ is

$$\sigma_b = \exp(-\alpha l)\sigma_c \left\{1 - \exp\left[-\exp(\alpha l)\frac{\sigma_a}{\sigma_c}\right]\right\} \tag{6}$$

which, taking account of (4), can be written as

$$\sigma_{II} = \sigma_c + \exp(-\alpha l)\,\sigma_c \left\{1 - \exp\left[-\exp(\alpha l)\frac{\sigma_I - \sigma_c}{\sigma_c}\right]\right\}. \tag{7}$$

From (2)

$$f(\sigma_I) = \sigma_{II}$$

when $h(\sigma_I) = 0$ and $g(\sigma_I) = 1$.

Equation (2) can finally be written in the form

$$\sigma_{II} = h(\sigma_I)\sigma_I + g(\sigma_I)\left\{\sigma_c + \exp(-\alpha l)\sigma_c\left[1 - \exp\left[-\exp(\alpha l)\frac{\sigma_I - \sigma_c}{\sigma_c}\right]\right]\right\} \tag{8}$$

where the definitions of $h(\sigma_I)$ and $g(\sigma_I)$ expressed by (3) are valid.

Experiments

The experiments were performed using the Hopkinson split bar method. The system consists of two 122 cm long 19.05 mm diameter stainless steel bars between which a specimen is sandwiched, and a pneumatic system by which a striker is caused to impact and initiate a pulse at the end of the incident bar.

The pneumatic system is composed of a 370 cm long brass tube through which the striker, a short piece of the same steel as the bars , is blown by compressed air. The length of the striker can be changed to control the length of the impact pulse, 65 μsec and 35 μsec pulses were used.

The waves produced by the impact were detected on each bar by two strain gauges placed on opposite sides of the bar and coupled so as to detect only the axial strain component. The distance of the gauges from the ends of the bar was determined so that incident, reflected and transmitted pulses could be recorded individually, i.e. without overlap, on the same oscilloscope image. A typical record of the waves is given in Fig. 2.

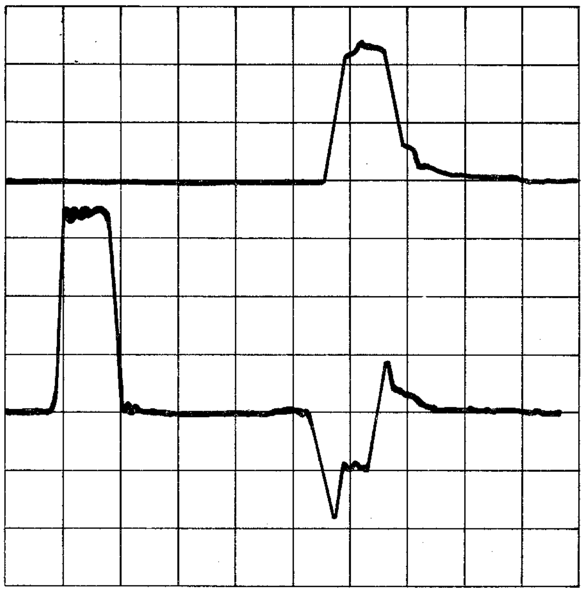

Figure 2. A typical recording of the incident, reflected (lower beam) and transmitted (upper beam) pulses, drawn from the oscilloscope image.

The rocks used in the present study were Tennessee Marble, Charcoal Granite and sandstone, the latter being fine-grained with dominant limestone layers. The specimens from the sandstone were taken perpendicular to the bedding. The diameter of all the specimens was the same as that of the bars.

Results

The amplitudes of the initial fronts of the incident, reflected and transmitted waves, indicated by letters A, B, and C in Fig. 1 respectively, were measured from the oscilloscope images. As a result of calibration, the readings could be transformed into stresses. The amplitudes at the ends of the specimen were calculated assuming force and particle velocity continuity at the interfaces between the specimen and the bars.

The results for the different rocks are presented in Figs. 3 and 4. The stress at the final surface of the specimen is given as a function of the stress at the initial surface of the specimen. The specimen length for each rock type is shown separately. The duration of the pulse did not cause any difference in the stress amplitude on the first front at the final surface of the specimen

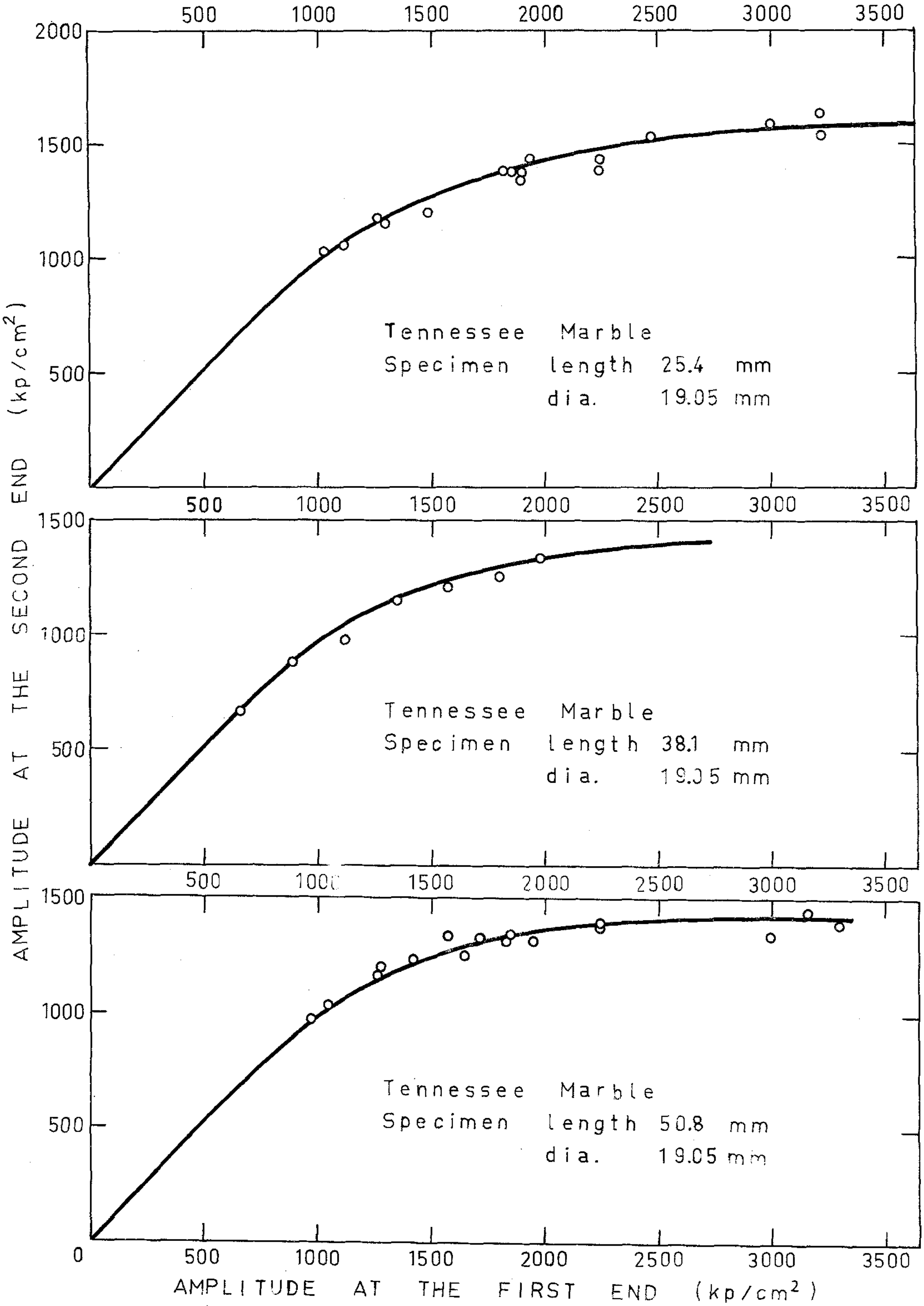

Figure 3. Stress amplitude of the first wavefront at the final surface of the specimen as a function of the stress amplitude at the initial surface of the specimen for Tennessee Marble.

and so the results with different pulse lengths are shown together.

The curves were calculated from (8) using values for the coefficient α which were obtained from the maxima of the transmitted amplitudes. From (8)

$$\sigma_{II}(\max = \sigma_c[1+\exp(-\alpha l)]$$

assuming σ_I increases without limit.

From (9), α can be calculated when $\sigma_{II}(\max)$ is known.

The values of α, together with the values of the fracture initiation stress (σ_c) obtained in static tests, are given in Table 1.

TABLE 1

The fracture iniation stress (σ_c) and the degree of brittleness (α) for Tennessee Marble, Charcoal Grauite and sandstone.

	σ_c [kp/cm^2]	α[1/cm]
Tennessee Marble	770	0.034
Charcoal Granite	1410	0.295
Sandstone	950	0.213

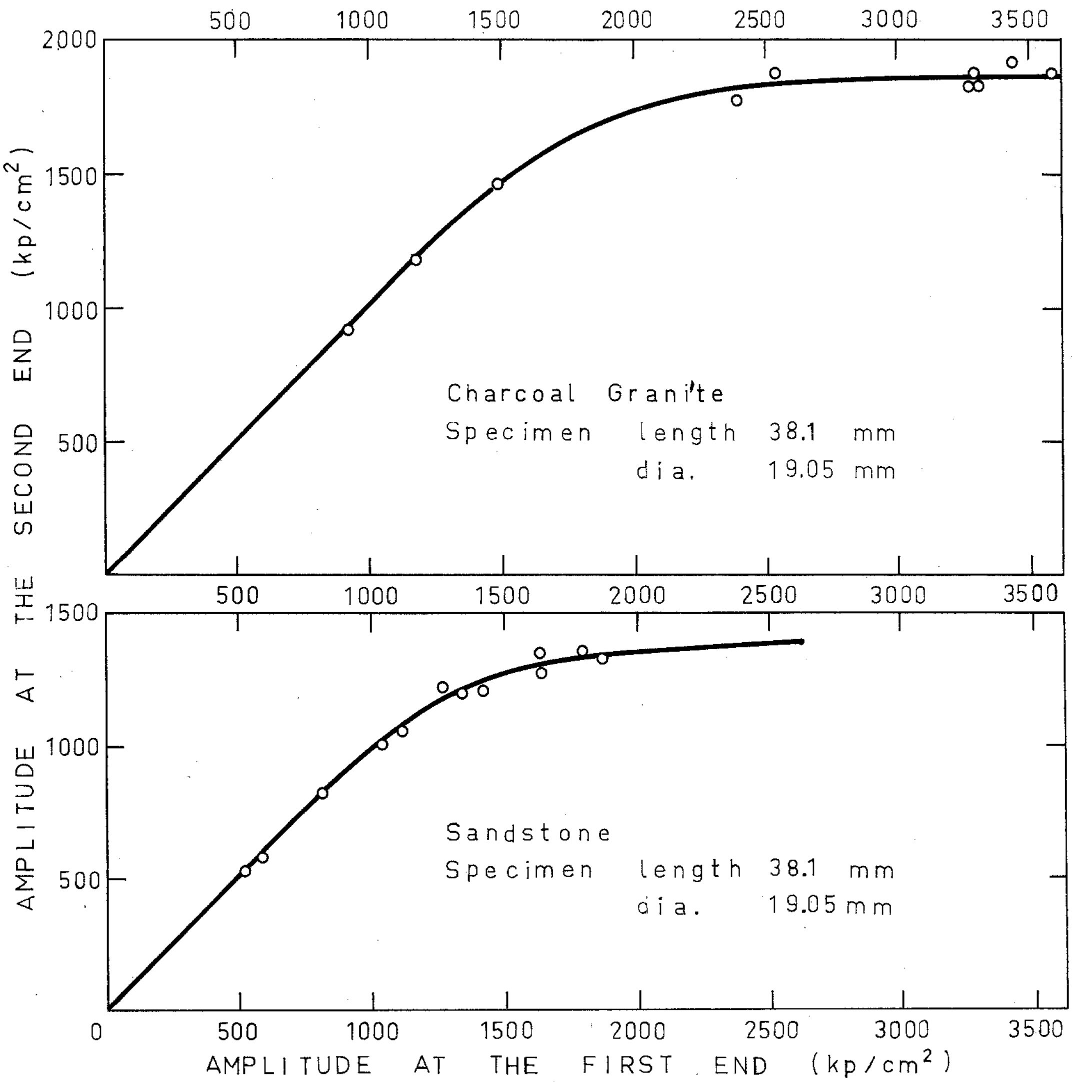

Figure 4. Stress amplitude of the initial wavefront at the final surface of the specimen as a function of the stress amplitude at the initial surface of the specimen for Charcoal Granite and sandstone.

Discussion

The calculated curves in Figs. 3 and 4 coincide well with the experimental results. This gives strong support to the assumption of the mode of attenuation of the stress amplitude of a compressive pulse in a fracturing rock specimen.

Fracture is characterized as brittle because practically no plastic deformation was observed before the fracture initiation which was recorded from the tested specimens. From the initiation of fracture to final failure rock may show also ductile type of behaviour. The ultimate dynamic strength of rock, which is often reported to be higher than the static strength, depends on the testing system and especially on the duration of the loading pulse, as the energy available for rock fracturing is the difference between the input energy and the energy which can be transmitted through the rock. Fracture initiation stress was recorded to be the same under static and dynamic loadings.

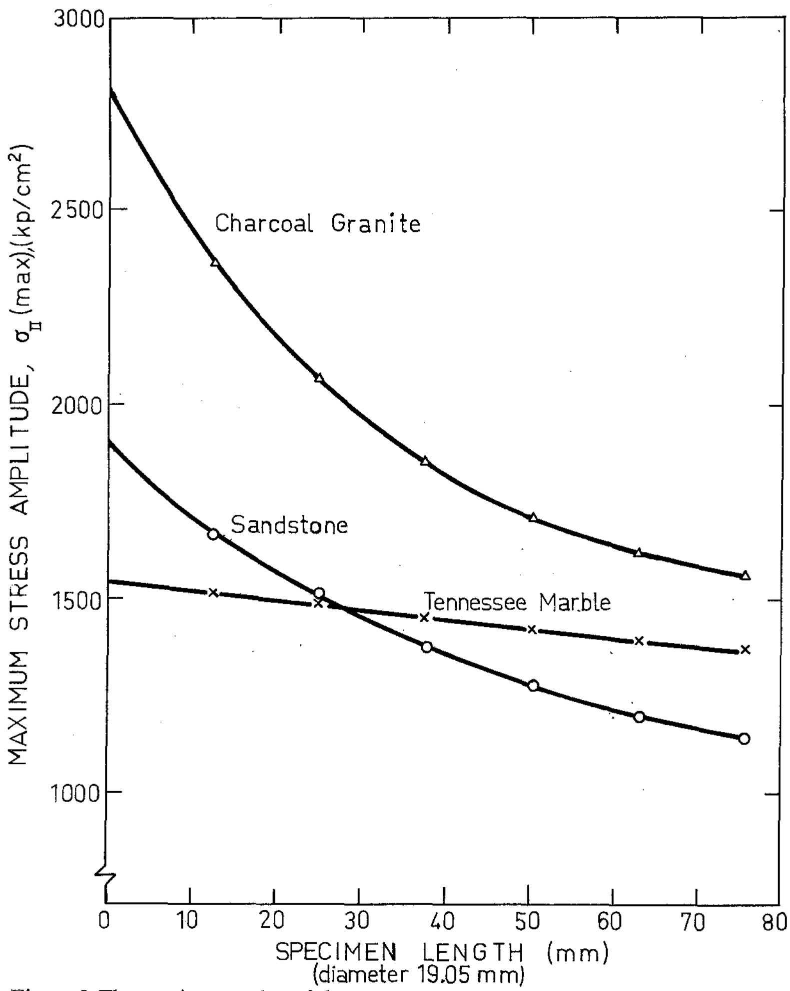

Figure 5. The maximum value of the stress amplitude of the initial wavefront at the final surface of a specimen as a function of the length of the specimen for different rocks calculated from (9).

The coefficient α is a characteristic of a material in fracture. This characteristic is connected to the rate at which the energy is absorbed in the fracture process. On the other hand, the coefficient α describes the degree of brittleness of the material. Fig. 5, where the maximum stress amplitude at the final surface of the specimen is given as a function of the length of the specimen, shows the difference in the rate of attenuation.

As was briefly pointed out earlier, there exists a maximum stress which can be achieved at the final surface of the specimen. This maximum stress depends on the degree of brittleness of the specimen and the specimen length. The maximum stress at the final surface of the specimen approaches the fracture initiation stress if the length of the specimen increases.

From (9) it is obvious that for positive values of α, the maximum absolute limit of the stress amplitude at the final surface of the specimen is twice the value of the fracture initiation stress. If α is restricted to positive values, brittle behaviour may be considered, however, it may be unrealistic to talk about brittle behaviour if α becomes negative.

This raises a question about the limit of the stress which it is reasonable to apply to rock in order to achieve a certain state of fracture. On the basis of the idea of brittle behaviour presented, twice the fracture initiation stress can be considered as a limit for the applied stress. A special study of whether there is a behaviour change of the rock after this limit is needed to elucidate its real meaning.

Rock is often said to behave differently under dynamic and static loads. This is especially a question of rock strength. This kind of behaviour has usually been explained to be dependent on the viscoelastic behaviour of rock, and different viscoelastic models have been developed. All these models, however, lack generality. The great gap has been that no realistic understanding of the mechanism of fracture under dynamic and static loads have been obtained.

The intense studies of rock fracture under static loads has increased our knowledge of brittle fracture, but the knowledge of the brittle behaviour under dynamic loads has been very limited. The present study, introducing a measure for the degree of brittleness, enables the study of the brittle behaviour of rock under short period impacts.

Conclusions

It has been shown previously by the author [4], and now in the present work, that there exists a limit for the energy which can be transferred through rock in the form of a compressive pulse. This limit is essentially a stress amplitude limit which is determined by the fracture initiation stress.

The amplitude at a point is, however, dependent on the distance from the point of impact and the degree of the brittleness of the material. The brittleness of the material can be determined experimentally using the Hopkinson split bar method in a way described in this paper.

There is no real evidence to support the belief that the behaviour of rock is completely different under dynamic loads from that under static loads. On the contrary, fracture in both cases begins at the same stress level as is shown in this study. After this, the main feature is the rate at which the fracturing material can absorb energy, described by the brittleness of the material. This behaviour can by no means be considered to be truly viscoelastic. Instead of obeying a continuous viscoelastic model, the behaviour may be divided into the behaviour before fracture initiation and the behaviour of the fracturing material.

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