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Why so much stability?

An approach to empirical studies of voting paradoxes: An update and extension

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1. Introduction

Since a previous note (Dobra and Tullock, 1981) an additional 30 cases of committee voting have been collected for which voters' ordinal rankings of alternatives are available. This update has been prepared to discuss selected cases from the sample in Section 2, and then some preliminary findings, analytical problems, and implications for the paradox of voting in Section 3.

By tradition, Public Choice theorists have pointed out many paradoxes of collective choice. The phenomenon of cyclical majorities and the more general issue of political equilibrium have proven to be fertile searching grounds. One class of paradoxes derives from the intransitivity of social orderings implied by the cycle. As Riker (1982) notes, this property of majority rule is most perplexing and paradoxical to those who hold 'Populist' beliefs that voting reflects a fair and meaningful amalgamation of individual preferences and must, therefore, reflect 'the will of the people.' If voting in general elections and committees is interpreted this way, then the existence of voting cycles and the implied intransitivity; or more generally, the impossibility of any rule for making social choices assuring transitive (sometimes equated with rational) outcomes, shakes the foundations of Populist principles of political philosophy.

Another paradox of cyclical majorities and simulations of voting games in general, concerns the stability of political equilibria. The body of theory regarding this issue is skillfully summarized by Shepsle and Weingast (1982) and can be reduced to a simple proposition: With sufficient assumptions regarding the states of participants' knowledge and the manipulability of

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decision rules, any motion can be defeated or avoided. Or, in short, any outcome is possible.

While universal intransitivity and instability are theoretical possibilities, numerous contributors to the debate on political equilibria (Tullock, 1967, 1981; Riker, 1980, e.g.) have maintained that these conclusions are out of contact with reality. In this current debate, contributors have sought to resolve the paradox by over-laying models of exchange (Tullock's log-rolling example, 1981) and institutions (Shepsle and Weingast 1982) to argue that these phenomena avert the endless voting and manipulation implied by the theory.

This research relates to these questions in several ways. First, using data from real voting situations we can confront the theory directly. Unfortunately, a rigorous statistical test will only be possible when the sample described in Section 2 is increased by a factor of ten at a minimum. Consequently, the first objective in preparing this note is to restate the request for examples of committee voting. With a larger sample specific hypotheses could be tested concerning, for example, the frequency of the non-existence of a Condorcet winner in the sample. This data could also be used to estimate how this frequency responds to changes in parameters like the numbers of voters and alternatives, and the characteristics of the institutional setting.

These probabilities and relationships have been derived theoretically using combinatorial techniques for limited sets of assumptions by Guilbaud (1952), Garmen and Kamien (1968), DeMeyer and Plott (1970), May (1971) and Fishburn and Gehrlein (1976, 1977). Hence, the data may eventually be used to test these kinds of results.

A second way that this research relates to these questions of political equilibria and social transitivity concerns the practical consequences observed in theoretically unstable situations. Specifically, using individual preference orderings of alternatives we can search for those instances where a Condorcet winner does not exist and which is believed to give rise to unstable and intransitive results. These conditions are 'believed' to give rise to instability and intransitivity because the major implication drawn from the cases available is that the permuted individual preference orderings that give us Condorcet's paradox are neither necessary nor sufficient conditions for instability and intransitivity.

2. The sample

Since the questions that an empirical study of voting may address are most significantly constrained by the nature of available data, let us first examine the cases of committee voting in the sample. These cases are listed

on Table 1 and it can be first noted that all cases involve relatively small numbers of voters. Also, in most cases where a Condorcet winner existed the number of voters was large relative to the number of alternatives as indicated by the column labeled 'n/m' (# voters/# alternatives). The table also shows the source of the example, the number of votes (# V) made by the committee, and an indication of the number and nature of voting cycles detected. 'Tie' indicates the existence of a tie-cycle such as in the first case listed on the table and reported on in the previous note. As can be seen from the Table, Condorcet winners appeared in 28 of the 32 cases (87.5%), and one complete cycle was found. Yet, as we shall see, this may understate the degree of stability and transitivity discovered in the sample.

The fourth case reported on Table 1 is similar to the first case on the Table in that it involved a faculty hiring decision and produced a tie-cycle. In this case a committee comprised of of 15 members of an economics department sought to choose between 3 alternatives: A – hire candidate A; B – hire candidate B; and C – hire both. The non-mutually exclusive nature of the choice set and the fact that Brown and Grofman reported some budgetary uncertainty associated with its feasibility, complicates the interpretation of this example.

Applying the Condorcet criterion for selecting a winner reveals that

Table 1.

Source	# V	Cycles	n/m
1. Political Science Faculty Search, (Dobra and Tullock, 1981)	2	1 tie	4/37 – 6/37
2. Public Choice Society, Duncan Black Award, 1981	1	0	6/3
3. Accounting Faculty Search, U.N.R.	1	0	5/10
4. Economics Faculty Search, (Brown and Grofman, 1978)	1	tie	14/3
5. College of Agriculture Dean Search, U.N.R.	1	0	10/5
6. Public Choice Class Grade Weighting, U.N.R.	6	0	16/3
7. Economics Department Personnel Committee Selection, U.N.R.	1	1	10/7
8. Demand Revealing Experiment, (Tideman, 1982)	18	1 tie	8/3 – 27/3
9. Mariner Spacecraft Trajectories (Dyer and Miles, 1976)	1	0	10/32
Total cases and cycles	32	3 tie-cycles 1 cycle	

alternative C would defeat candidate B (9–5), and that candidate B would defeat candidate A (8–6). Transitivity requires that alternative C defeat candidate A but, in fact, the motions tied. Brown and Grofman note, as may be seen from inspection of Table 2, that had the voter with the ordering 'B p C p A' reversed the order of his last two choices, i.e., changed his mind on which alternative was the worst, a full cycle would have been obtained.

Nonetheless, there was no clear-cut choice between hiring B only, and hiring both. According to Brown and Grofman (1978: 21):

> ... (c)onfronted with this situation, the non-voting chairman specified the department choice of B, his own most preferred alternative, and an offer was made to candidate B but not candidate A.

Hence, dis-equilibrium was avoided by the imposition of a dictatorial solution. But it should be noted that this solution agrees with the pair-wise choice of B over A.

It is interesting to compare these results with those which would have been obtained using Borda's method. The analysis of the first case on Table 1 showed that this method would have broken the tie-cycle observed in favor of the candidate who, except for being tied by other candidates that could be beaten by others, was a fairly obvious choice. Here, as shown on Table 2, candidates A and B tied behind C, the expensive compromise. This avoids the intransitivity implied by the Condorcet criterion but, of course, disagrees with the pair-wise choice of B over A which effectively treated C as an irrelevant alternative.

Additionally, had the voter with the ordering 'B p C p A' noted above changed the order of his last two choices, the Borda scores that would have resulted are shown in parentheses on the Table. The result would have been to break the tie between A and B in favor of A as in the above, disagreeing

Table 2. Individual preference orderings for the departmental vote

Preference orderings	# V	Borda scores		
		A	B	C
A p B p C	0	0	0	0
A p C p B	3	6	0	3
B p A p C	4(5)	4(5)	8(10)	0(0)
B p C p A	1(0)	0(0)	2(0)	1(0)
C p A p B	3	3	0	6
C p B p A	3	0	3	6
Totals	14	13(14)	13(13)	16(15)

more strongly with the pair-wise choice of B over A.

It is perhaps disconcerting that the chairman's dictatorial choice of B over C disagreed with both methods of aggregation. But closer examination of the preference orderings and the outcome of hiring only B in terms of the costs and benefits to individual department members helps rationalize the outcome. Note that six voters ranked C first and four ranked C second. Since the alternatives were not mutually exclusive and the expensive compromise could probably be obtained at no significant personal marginal cost to decision makers other than the chairman, the choice of C over A and B is consistent with unconstrained maximization. From the chairman's perspective, however, C would probably not have been a costless alternative. It would have undoubtedly involved some personal opportunity costs to negotiate and justify hiring both. In particular, it would appear difficult to justify hiring both when candidate B defeated A in a pairwise comparison. Had the two tied in that vote as they did using Borda's method, alternative C would clearly have been easier to justify.

Putting this kind of speculation aside, we can conclude that the permuted preferenced observed in this case were not sufficient conditions to generate either instability or, for that matter, significant intransitivity. While this may simply be due to the random chance that the chairman's first choice was B and not A, it should be noted that the stability of the outcome observed was enhanced by the potential manipulator's expected costs. Theoretically, these expected costs are implied in log-rolling and manipulation models, but from an empirical perspective we really have very little idea how these costs are influenced by alternative institutional settings that will render collective choices more or less stable and transitive.

A second case of interest is #7 on Table 1. In this case a committee of 10 department members were required to select a four member personnel committee from among themselves. Three members were disqualified for various reasons leaving seven candidates. The department chairman specified that the approval voting method be used. However, prior to the vote voters were asked to list candidates in a rank ordering indicating which of the four they specified was the most preferred committee member, second most preferred, etc.

Applying the Condorcet criterion to the rankings revealed a voting cycle involving C3, C4, and C5 as shown by Table 3. This result is but the first of a number of 'paradoxes' or inconsistencies provided by this example. Under the approval voting method actually used to select the committee C7 received the most votes (8). However, while C7 had the broadest base of support, he was generally not a top choice of voters and could have been defeated or tied by the candidates involved in the cycle. An additional inconsistency involving C7 is indicated by his Borda score which is also shown on Table 3. Using Borda's procedure, C7 drops from first to fourth

in the overall ranking. The Borda scores also reveal that this method could have resolved the cycle by choosing C3, but this, of course, would be inconsistent with the result of the pair-wise comparison between C3 and C4.

A final inconsistency in this example concerns the actual committee chosen using the approval method. As noted, C7 received the most votes. C3 and C5 tied with seven votes for second and third positions, and C4 and C6 tied with six votes for the final position. Faced with this situation the department chairman called for a run-off between the two which C6 won by two votes. Hence, the committee formed excluded C4, who was involved in the cycle and who received the second highest Borda score.

Because we do not have a complete ranking of all seven candidates for each voter, we do not know if these rankings remained constant between the two votes, and this final inconsistency is difficult to explain. Knowledge of the first three members of the committee could clearly have induced voters to change their pair-wise orderings of C4 and C6 to get a more 'balanced' committee. However, it is impossible to determine if this rationalization of the outcome has any credence. If no changes in the rankings occurred for the purpose of 'balancing' the committee, then the tie-breaking votes were cast by two voters who listed neither C4 nor C6 in their original ranking of their first four choices. Hence, instability was avoided by the coincidence of the ordering of C4 and C6 in two voters' least desirable alternatives.

Clearly, the lack of a Condorcet winner and the disagreement between alternative methods of aggregating departmental preferences suggest a paradox in this example. However, in reality, no such 'paradox', instability, or intransitivity was evident. The cycle involving the three candidates was discovered after the meeting had adjourned, and during the meeting there were no apparent attempts to manipulate the outcome. Hence, again, the non-existence of Condorcet winner does not necessarily imply either instability or intransitivity.

A final case of interest from Table 1 is #9 from Dyer and Miles' (1976) discussion of the selection of a pair of trajectories for the Mariner (now

Table 3. Economics department personnel committee selection

C_i	is defeated by C_j	ties	Borda score
C_1	C_3, C_4, C_5, C_7	C_6	13
C_2	$C_1, C_3, C_4, C_5, C_6, C_7$		1
C_3	C_4		22
C_4	C_5	C_6	18
C_5	C_3	C_7	18
C_6	C_3, C_5, C_7	C_1, C_4	12
C_7	C_3, C_4	C_5	15

called Voyager) space probes made in 1973. The selection process involved 10 teams of scientists organized to study specific aspects of the data derived from the probes. These teams were assigned to rank alternative pairs of trajectories according to their expected usefulness in their research. After initial identification of 105 feasible pairs of trajectories, 32 were selected by Jet Propulsion Laboratory engineers for the 10 teams to choose from. Four well known methods for aggregating preferences were then applied to these individual rankings. These methods were: Borda's method, Bentham's sum of cardinal utilities. Nash's multiplicative cardinal utilities, and the Condorcet criterion.

As discussed by Dyer and Miles, and later by Riker (1982), two of these methods agreed on one trajectory pair while the other two methods agreed on another pair. Riker (1982: 31) found this to be a 'deep ambiguity' and a demonstration that

> ... even if an omniscient observer .... knew the true tastes of every voter, it would still be impossible for him to predict the ... product of aggregating preferences unless he also knew the method of aggregation.

The impasse was resolved by modifying one of the winning pairs, presumably to make it more like the other, and then by persuading the teams to accept it. Hence, stability was achieved by manipulation of the alternatives, i.e., compromise. And, as for the issue of transitivity, these ambiguities are an illustration of the disagreement among voting methods explored by Ludwin (1976). From the perspective of the debate on the stability and transitivity of collective choice, however, more interesting than these ambiguities are the implications of this example for the proposition we offered at the outset. While the two examples described above indicate that the lack of a Condorcet winner is not a sufficient condition for observing instability or intransitivity, the last example suggests that it is not even a necessary condition.

3. Summary

The few cases that we have to analyze are, in effect, counter-examples to the celebrated Condorcet-Arrow counter-example. These counter-examples do not repudiate the work of the dis-equilibrium theorists, however, in fact, they may be viewed as giving their conclusions credence. For example, the cycles observed to date have, in two of three cases, occurred when the theory suggests they are most likely: when the number of alternatives is large relative to the number of voters. Also, because the permuted preferences are neither necessary not sufficient conditions for mani-

pulation, ambiguity, intransitivity, and/or instability, the implications of the universal instability theorists reflect possible states of nature. While these kinds of results are possible, however, the data suggests that they are improbable. Collective choice processes reach stopping points if only because of time constraints and the results are transitive and unambiguous as a general rule.

Clearly, these observations do not resolve the paradox of collective choice but expand it, pushing the avenue of inquiry into the realm of behavioral science, beyond the realm of the purely mathematical models that have dominated the literature. The current state of knowledge of the social dynamics of various choice settings, however, is extremely limited. Yet, the cases discussed above provides support for the kinds of rationalizations of the paradox of choice noted above, derived by overlaying models of exchange and institutions over the purely mechanical rules for aggregating preferences.

Behavioral constraints reflecting these social and political parameters reflect what Buchanan (1969: 44) has called 'choice-influencing costs.' Hence, potential manipulators not only recognize the value of getting their own way, but also the costs of moving the collective decision away from expected or alternative outcomes. The costs of achieving one's individual objectives in collective choice contexts are clearly similar to those found in private choices. Participants face a dead-weight opportunity cost of engaging in choice measured in a private numeraire good. In addition to this transaction cost, the process of exchange involves the transfer of some fungible asset, a price, to secure the cooperation of coalition members.

These behavioral constraints and their choice-influencing nature can be illustrated by Figure 1 – a straight-forward extension of economic theories of supply and demand behavior. The figure shows the ideal points of three committee members (X1, X2, and X3) in a single issue dimension X. In the absence of log-rolling, ambiguity in perceiving the nature of X, and if members vote their true preferences, we have an example of the 'median voter result.' When these assumptions are relaxed, however, the familiar dis-equilibrium results arise.

Under these relaxed conditions, X1 is no longer the obvious outcome but the median voter, V1 is still in a position to determine what the outcome will be. The curves drawn in the vertical marginal value dimension give an indication of what likely outcomes will be and the model clearly does not support the conclusion that any outcome is possible. Curve MC1 shows the marginal cost of moving away from X1 as perceived by the median voter. Similarly, MC2 and MC3 reflect the transactions and exchange costs (e.g., expected log-rolling costs and the costs of maneuvering around institutional rules) that voters V2 and V3 expect to incur to displace the equilibrium away from X1. MB2 and MB3, on the other hand, reflect the expect-

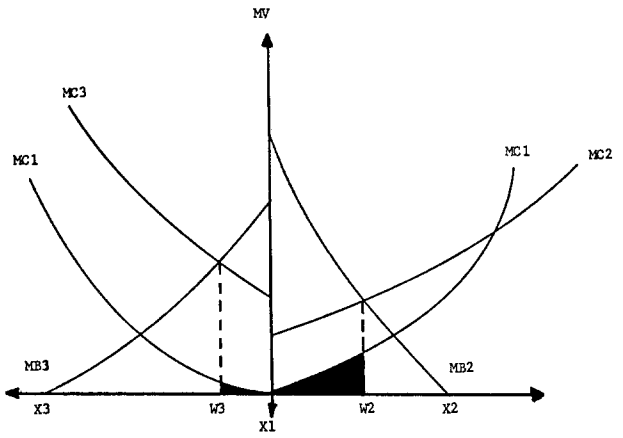

Figure 1.

ed benefits to V2 and V3 of displacing the equilibrium away from X1. Based on these perceptions of their expected costs and benefits of displacing the outcome, V2 and V3 would be willing to offer compromises to V1 of outcomes W2 and W3 for which compensation equivalent to the shaded areas below MC1, at a minimum, must be offered. A rent-seeking median voter, of course, could possibly extract the entire area under either V2 or V3's marginal benefit curve.

In any event, the model reduces to an analog of the theory of bi-lateral monopoly with the segment of the issue dimension from W2 to W3 representing the bargaining core. Note also that if dis-equilibrium reflected in greater than expected bargaining occurs, each committee members' marginal cost curve begins to shift upward to reflect the greater than expected opportunity costs of collective decision making. As these expected costs increase, the core collapses to X1, the median voter result.

This very simple model of individual choice behavior in collective choice settings uses nothing more than fundamental behavioral concepts of economic theory. Yet, it is capable of illustrating the fallacy of universal instability theorems. The actions of individuals considering these kinds of choice-influencing costs and *not* omniscient amalgamators of individual preferences determine the outcomes of collective choice processes. On this basis of this behavioral postulate this research has sought to develop a better understanding of institutions and exchange behavior as it relates to the conclusions of the theory. And, on the basis of this postulate we restate our request for additional examples of committee voting.

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