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An experimental investigation of the sonic criterion for transition from regular to Mach reflection of weak shock waves

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Abstract. The signal speed, namely the local sound speed plus the flow velocity, behind the reflected shocks produced by the interaction of weak shock waves ($M_i < 1.4$) with rigid inclined surfaces has been measured for several shock strengths close to the point of transition from regular to Mach reflection. The signal speed was measured using piezo-electric transducers, and with a multiple schlieren system to photograph acoustic signals created by a spark discharge behind a small aperture in the reflecting surfaces. Both methods yielded results with equal values within experimental error. The theoretical signal speeds behind regularly reflected shocks were calculated using a non-stationary model, and these agreed with the measured results at large angles of incidence. As the angle of incidence was reduced, for the same incident shock Mach number, so as to approach the point of transition from regular to Mach reflection, the measured values of the signal speed deviated significantly from the theoretical predictions. It was found, within experimental uncertainty, that transition from regular to Mach reflection occurred at the experimentally observed sonic point, namely, when the signal speed was equal to the speed of the reflection point along the reflecting surface. This sonic condition did not coincide with the theoretical value.

1 Introduction

When a plane weak shock wave is incident on a rigid surface, two types of reflection are possible. For large angles of incidence, the reflected shock meets the incident shock at the reflection point on the reflecting surface, and this is known as regular reflection. If the angle of incidence is reduced, while keeping constant the Mach number M_i of the incident shock, a point will be reached beyond which the intersection of the reflected and incident shocks lies above the surface, and the intersection is joined to the reflecting surface by a third shock called the Mach stem. This condition is called Mach reflection in recognition of Ernst Mach who first observed and described the phenomenon.

Several criteria have been suggested in the literature to define the conditions which will result in a transition from regular to Mach reflection. Henderson and Lozzi (1975) proposed a mechanical equilibrium criterion, valid for strong shock waves only, based on the assumption that there will be no pressure discontinuity during the transition process. An alternative is the detachment criterion proposed by von Neumann (1963), which defines the set of conditions beyond which regular reflection is impossible, based on oblique shock theory, and applicable over all shock Mach numbers. Hornung et al. (1979) proposed the so called sonic criterion. The latter argued that, since there is a length scale associated with the Mach stem, but not with regular reflection, Mach reflection will not occur unless the corner signal, generated when the incident shock interacts with the leading edge of the reflecting surface, is able to overtake the reflection point. This requires that the signal speed namely the flow velocity plus the local speed of sound, behind the reflected shock must be equal to or greater than the speed of the reflection point along the reflecting surface.

In the case of weak shocks, for which the shock Mach number is less than 1.5, the mechanical equilibrium condition does not exist, and there is only a very small difference between the theoretical sonic and detachment conditions. Experimental differentiation between these conditions might therefore be expected to be difficult. Numerous workers (e.g., Henderson and Lozzi 1975; Henderson and Siegenthaler 1980; Ben-Dor et al. 1980) have shown that the observed transition from regular to Mach reflection of plane shocks reflecting from smooth rigid surfaces does not occur at the angle of incidence predicted by the detachment or the sonic condition. The results of Takayama et al. (1981) also illustrate that the point of transition is dependent on the smoothness of the reflecting surface, and it is generally accepted that the difference between theory and experiment is due to the fact that the classical theory assumes an inviscid flow. This assumption is clearly invalid for an event that takes place in the boundary layer over a reflecting surface.

The regular reflection of a plane shock wave from a rigid inclined surface is illustrated in Fig. 1. The incident shock is moving into region *0* which has uniform ambient conditions. Uniform conditions also exist in region *1* behind the incident shock, and those conditions can be calculated in terms of the incident shock Mach number and the ambient conditions in region *0*, using the Rankine-Hugoniot equations. The part of the reflected shock close to the reflection point is plane and

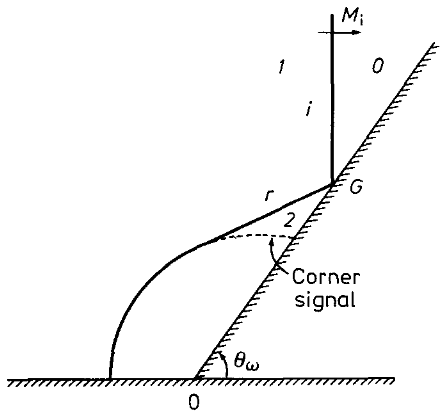

Fig. 1. Regular reflection of a plane shock from a rigid plane surface; incident shock, *i*, is moving with Mach number, M_i, into region *0* which has uniform ambient conditions; the reflected shock, *r*, is plane close to the reflection point, *G*, and curved where it has been overtaken by the rarefaction corner signal which was initiated at *O*, the leading edge of the wedge; regions *1* and *2* also have uniform flow conditions

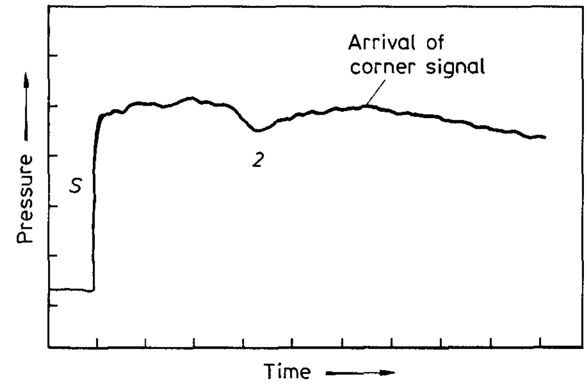

Fig. 2. Tracing of an oscilloscope record of the output from a piezo-electric transducer flush mounted in the reflecting surface during a regular shock reflection; there is a sharp rise, *S*, as the reflection point traverses the transducer, followed by a relatively constant pressure produced by region *2* in Fig. 1; the decrease of pressure is caused by the arrival of the corner signal; the pressure scale is uncalibrated and the time scale is 10 μs per division

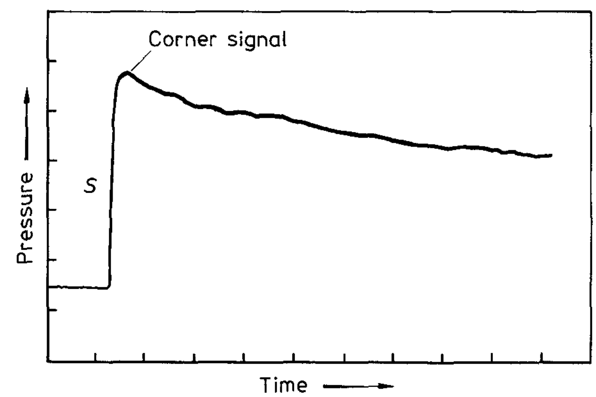

Fig. 3. Tracing of an oscilloscope record of the output from the piezo-electric transducer when the corner signal has just overtaken the reflection point; the pressure scale is uncalibrated and the time scale is 10 μs per division

bounds region *2*, which also has uniform flow conditions. Region *2* is further bounded by the corner signal from the leading edge of the wedge which moves at the signal speed within region *2* namely $u_2 + a_2$, where u and a are, respectively, the flow velocity and local sound speed of the gas.

In the configuration shown in Fig. 1, the flow field is self-similar so that the distance between the corner signal and the reflection point steadily increases, that is the speed of the corner signal is less than the speed of the reflection point along the reflecting surface. The sonic criterion for transition from regular to Mach reflection suggests that transition can only occur when the strength of the incident shock and the angle of the reflecting wedge are such that the speed of the corner signal is equal to or greater than that of the reflection point. The purpose of the experiments described here was to evaluate the sonic criterion for plane incident shocks in the range of shock Mach numbers from 1.1 – 1.4.

2 Experimental procedures

Two procedures were used to measure the signal speed behind reflected shocks. In the first, a piezo-electric transducer was flush mounted in the metal wedges which were used as the shock wave reflectors. Figure 2 shows a tracing of a pressure-time history from such a transducer. There is a sharp rise in pressure as the reflection point traverses the surface of the transducer. This is followed by a period of relatively uniform pressure, corresponding to region *2* in Fig. 1, which is terminated by the arrival of the rarefaction signal from the corner. Knowing the distance of the transducer from the leading edge of the wedge, the relative speeds of the reflection point and the corner signal were determined from the times of arrival of these events in the pressure-time record. Figure 3 shows a tracing of a pressure-transducer record in which the corner signal has just overtaken the reflection point so that there is no longer a region of uniform flow behind the reflected shock.

As can be seen in Figs. 2 and 3, the identification of the leading edge of the rarefaction wave is not exact and a photographic method was also used to determine the signal speed behind the reflected shocks. A weak spark was generated in a cavity behind a small aperture in the reflecting surface or in the floor of the shock tube at the leading edge of the reflecting wedge. The spark generator is illustrated in Fig. 4. The aperture permitted the passage of a small plasma jet from the spark and this generated a compression wave which could be observed by sensitive schlieren photography. A tracing of such a schlieren photograph is shown in Fig. 5.

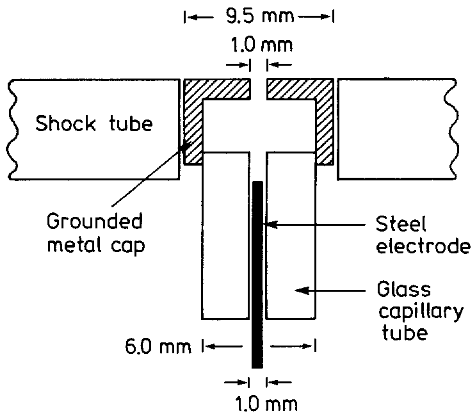

Fig. 4. Spark generator; a spark was produced in the metal cap by the discharge of a 10 μF capacitor at 900 V; a plasma jet was ejected through the 1 mm diameter aperture and produced a weak compression wave which could be visualized with a sensitive schlieren system

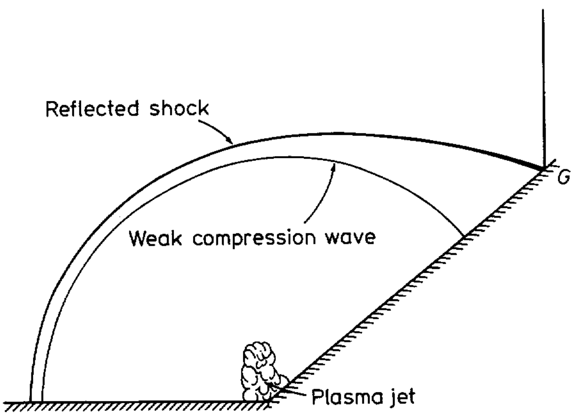

Fig. 5. Tracing of a schlieren photograph showing a regular shock reflection and a weak compression wave produced by a plasma jet at the leading edge of the wedge; when the jet was produced very close to the incident shock the compression wave combined with the reflected shock but could be accurately measured close behind the reflection point, *G* (A tracing is used here rather than a photograph because the weak compression wave can only be seen clearly in a photographically produced print)

The speeds of the incident shock and of the compression wave were measured from a series of such photographs taken using a multiply-pulsed schlieren system (Dewey and Walker 1975). To evaluate the validity of this technique a series of such photographs was also made of the compression wave generated in ambient air, and from these it was demonstrated that the observed compression waves were travelling at the speed of sound in the gas. By triggering the spark at the instant when the incident shock reached the leading edge of the wedge, a compression wave could be formed very close to the reflection point. This procedure called for very accurate timing, and it was necessary to do several experiments in order to obtain a compression wave in the desired position relative to the shock fronts.

There were advantages and disadvantages to both methods. The pressure signals were easy to record but it was difficult to identify precisely the leading edge of the rarefaction wave and thus to determine the time between the arrival of the shock front and the corner signal. In contrast the compression wave produced by the spark could be measured accurately from the schlieren photographs, but timing the spark so that it occurred just behind the shock front was difficult, and it was necessary to do several experiments before a correct configuration could be obtained. A further advantage of the schlieren technique was that it could be used and interpreted very close to, and past, the transition to Mach reflection.

Four series of experiments were performed using incident shocks with Mach numbers of 1.10, 1.17, 1.25, and 1.36. (These correspond to inverse pressure ratios across the shocks of 0.8, 0.7, 0.6 and 0.5, respectively.) The shocks were generated in a rectangular shock tube with internal dimensions of 25.4 × 7.62 cm. The observation section was 6 m from the diaphragm. Ambient air was used in the expansion section and the pressure of the air in the compression chamber was adjusted in each experiment in accordance with the measured ambient conditions so that sequences of experiments could be performed with the desired incident shock Mach numbers.

3 Experimental results

The results of the four series of experiments are shown in Fig. 6. The measured signal speed, $u_2 + a_2$, in the region immediately behind the reflection point has been expressed as a ratio of the speed of the reflection point along the reflecting surface, and this ratio, which we call the sonic number, has been plotted against the wedge angle. The ratios obtained from the pressure gauges and the schlieren photographs have been identified in the figure, but no significant difference could be detected between the two methods of measurement.

The curves plotted in Fig. 6 are the theoretical values of the sonic number calculated using the non-stationary two-shock model described by Dewey and McMillin (1985). These curves cannot be extended beyond the point at which there is no longer a two shock solution (the detachment point), because there is no adequate three shock theory describing a Mach reflection for incident shocks of the strengths considered in this project.

The experimental results plotted in Fig. 6 show that for large wedge angles the measured sonic numbers are similar to those calculated using two shock theory. As the transition condition is approached, the measured and theoretical values diverge, particularly at the lower Mach numbers. The wedge angles at which transition from regular to Mach reflection was observed to occur in these experiments have been identified by the triangles in Fig. 6. Interpolation of the

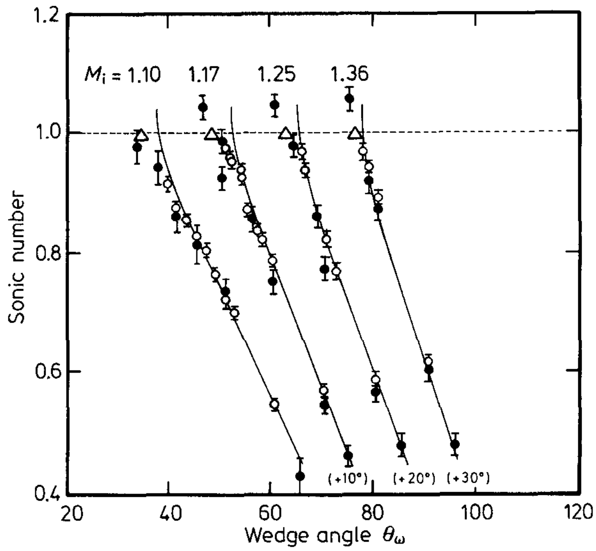

Fig. 6. Experimental and theoretical sonic numbers for the four shock strengths considered; the solid curves were calculated using non-stationary two-shock theory; open points were obtained from piezo-electric transducer measurements and the solid points from the schlieren photographs; the triangles mark the angles at which transition from regular to Mach reflection was observed (10°, 20° and 30° have been added to the $M_i = 1.17$, 1.25 and 1.36 results, respectively, so that the four sets of results can be distinguished)

Table 1. Summary of results

Shock Mach no.	Theoretical detachment angle	Theoretical sonic wedge angle	Experimental sonic wedge angle	Observed transition angle
1.10	37.07°	37.6°	33.5° ± 0.7	33° ± 0.5
1.17	42.06°	42.5°	39.4° ± 0.8	39° ± 0.5
1.25	45.12°	45.5°	43.4° ± 1.0	44° ± 0.5
1.36	47.42°	47.75°	46.2° ± 0.4	46° ± 0.5

experimental measurements indicates that transition does occur at the sonic point when the signal speed is equal to that of the reflection point, and the sonic point is clearly distinguishable from both the detachment condition and the theoretical sonic point. For each of the incident shock Mach numbers considered in these experiments the wedge angles identified by the theoretical sonic and detachment criteria, the wedge angles of the experimentally observed sonic condition and the observed angle at which transition from regular to Mach reflection occurred, have been listed in Table 1.

4 Conclusions

From the measurements described here and the results shown in Fig. 6, and Table 1, it seems clear that within experimental uncertainty, trasition from regular to Mach reflection of weak shock waves occurs at the experimentally observed sonic point, but this point does not coincide with the sonic point calculated using two shock theory.

The length scale argument of Hornung et al. (1979) explains why Mach reflection would not be expected to occur unless the sonic condition is satisfied, but not the physical reason why transition from regular to Mach reflection apparently occurs exactly at the sonic condition. The particle velocity in region *1* behind the incident shock has a component towards the reflecting surface. The effect of the reflected shock is to turn this flow so that it is parallel to the surface in region *2*. As the sonic point is approached the reflected shock has direction and strength which are close to the physical limit for an oblique shock to turn the flow parallel to the wedge surface. The theoretical limit is the detachment condition. The arrival of the corner signal, which is a rarefaction wave, both weakens and changes the direction of the reflected shock creating a two shock configuration which cannot be physically sustained, and transition to a three shock condition occurs.

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