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Experimental investigation of the asymptotically approached Mach reflection over the second surface in a double wedge reflection

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Abstract. The assumption that the Mach reflection which is formed over the second surface of a double wedge with angles θ_w^1 and θ_w^2 approaches asymptotically the Mach reflection which would have been obtained by an identical incident shock wave over a single wedge with an angle $\theta_w = \theta_w^2$ was verified experimentally. The verification of this assumption supports the shock polar analysis suggested by Ben-Dor et al. (1987) for the study of the reflection process of a planar shock wave over a double wedge. Measurements of the rate of approach to the asymptotic value are also provided.

1 Introduction

Ben-Dor et al. (1987) have studied, both analytically and experimentally, the reflection of a planar shock over a double wedge. In their analytical study they provided a detailed shock polar analysis of the reflection process, based on the assumption that the reflection over the second surface of the double wedge approaches asymptotically the reflection which would have been obtained over a single wedge with the same wedge angle.

A schematical illustration of a double wedge is shown in Fig. 1. θ_w^1 and θ_w^2 are the slopes of the first and second surfaces of the double wedge, respectively. The difference between these two slopes is $\Delta\theta_w = \theta_w^2 - \theta_w^1$. If $\Delta\theta_w > 0$ the double wedge is concave (Fig. 1 a), and if $\Delta\theta_w < 0$ the double wedge is convex (Fig. 1 b).

The domains of different types of reflection process over a double wedge in the (θ_w^1, θ_w^2)-plane are shown in Fig. 2. The line $\Delta\theta_w = 0$ divides the (θ_w^1, θ_w^2)-plane into the domains of concave and convex double wedges. The line $\theta_w^1 = \theta_w^{\text{det}}$ determines the type of reflection over the first surface of the double wedge. If $\theta_w^1 < \theta_w^{\text{det}}$ then the incident shock wave reflects over the first surface as a Mach reflection (MR), and if $\theta_w^1 > \theta_w^{\text{det}}$ then the initial reflection is regular (RR). The line $\theta_w^2 = \theta_w^{\text{det}}$ determines the type of reflection which is finally obtained over the second surface. If $\theta_w^2 < \theta_w^{\text{det}}$ then the final reflection over the second surface is a MR, and if $\theta_w^2 > \theta_w^{\text{det}}$ then the final reflection over the second surface is a RR. In the case of a concave double wedge ($\Delta\theta_w > 0$) and a Mach reflection over the first surface ($\theta_w^1 < \theta_w^{\text{det}}$) the Mach stem of the MR reflects eventually over the second surface. The type of its reflection depends upon whether $\Delta\theta_w$ is smaller or greater than θ_w^{det}. If $\Delta\theta_w < \theta_w^{\text{det}}$ then the Mach stem will reflect over the second surface as a MR, and if $\Delta\theta_w > \theta_w^{\text{det}}$ then it will reflect as a RR.

The above described transition boundaries give rise to seven domains of different types of reflection processes over a double wedge. They are numbered 1 to 7 in Fig. 2 and are summarized in more detail in Table 1.

The three reflection processes which are investigated in this study are those appropriate to domains 3, 4 and 6, for only in these three domains the final reflection over the second surface of the double wedge is a MR. In the following a detailed description of the reflection process in each of these three domains is given.

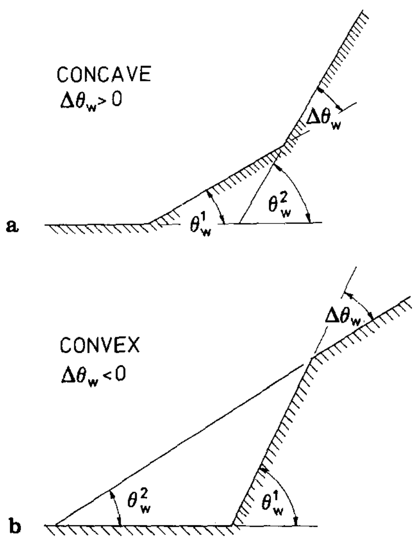

Fig. 1 a and b. A schematical illustration of a double wedge; **a** concave, **b** convex

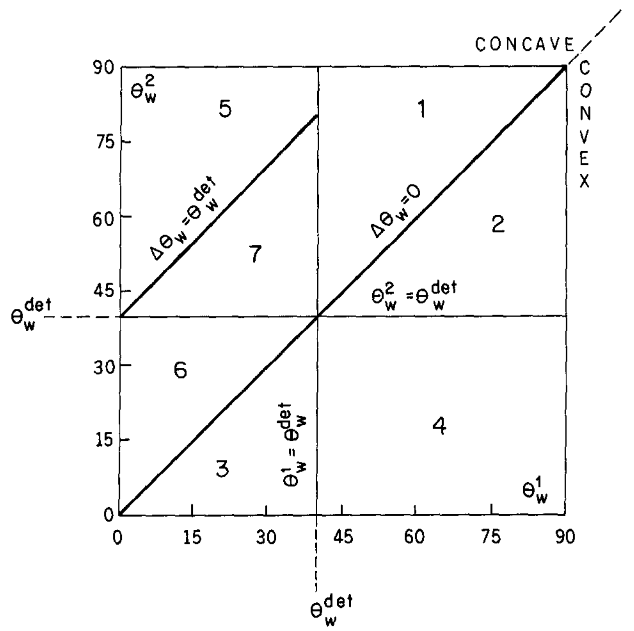

Fig. 2. Domains of different types of reflection processes over a double wedge

Table 1. A summary of the seven different reflection processes which can occur over convex and concave double wedges depending on the magnitude of the wedge angles θ_w^1, θ_w^2 and $\Delta\theta_w$ compared to the detachment wedge angle θ_w^{det} (referred to simply as "det" below); the numbers in the final column refer to the regions in the (θ_w^1, θ_w^2) plane of Fig. 2

	θ_w^1	θ_w^2	$\Delta\theta_w$	First surface	Second surface	Region
Convex	> det	> det	–	Regular	Regular	2
	< det	< det	–	Mach	Mach	3
	> det	< det	–	Regular	Mach	4
Concave	> det	> det	–	Regular	Regular	1
	< det	> det	> det	Mach	Regular → regular	5
	< det	< det	< det	Mach	Mach → Mach	6
	< det	> det	< det	Mach	Mach → regular	7

Domain 3

- Since $\Delta\theta_w < 0$, the double wedge is convex.
- Since $\theta_w^1 < \theta_w^{det}$, the incident shock wave reflects over the first surface as a MR.
- Since $\theta_w^2 < \theta_w^{det}$, the final reflection of the incident shock wave over the second surface is also a MR.

A schematical illustration of this reflection process is shown in Fig. 3.

Domain 4

- Since $\Delta\theta_w < 0$, the double wedge is convex.
- Since $\theta_w^1 > \theta_w^{det}$, the incident shock wave reflects over the first surface as a RR.

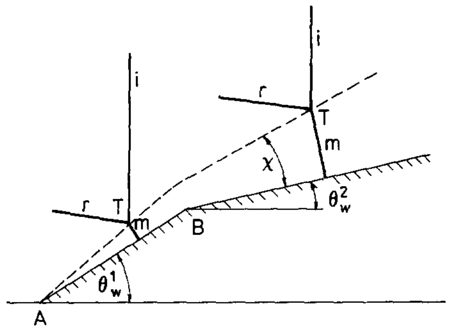

Fig. 3. A schematical illustration of the reflection process in domain 3 of Fig. 2

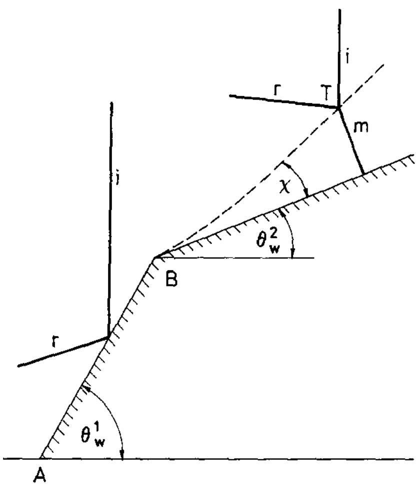

Fig. 4. A schematical illustration of the reflection process in domain 4 of Fig. 2

- Since $\theta_w^2 < \theta_w^{det}$, the final reflection of the incident shock wave over the second surface is a MR.

A schematical illustration of this reflection process is shown in Fig. 4.

Domain 6

Unlike the previous two cases, here $\Delta\theta_w > 0$, and therefore the double wedge is concave. Since $\theta_w^1 < \theta_w^{det}$, the incident shock wave reflects over the first surface as a MR. When the Mach stem of this MR collides with the leading edge of the second surface, for which $\Delta\theta_w < \theta_w^{det}$, it reflects over it as a MR. The two Mach reflections interact to create the final MR of the incident shock wave over the second surface, whose slope satisfies $\theta_w^2 < \theta_w^{det}$. A schematical illustration of this reflection process is shown in Fig. 5. Note that the interaction of the two triple points, T_1 and T_2, at point Q, results in two new triple points: T_3, associated with the MR of the incident shock wave over the second surface and an additional triple point, T_4, which "splits" the reflected shock wave.

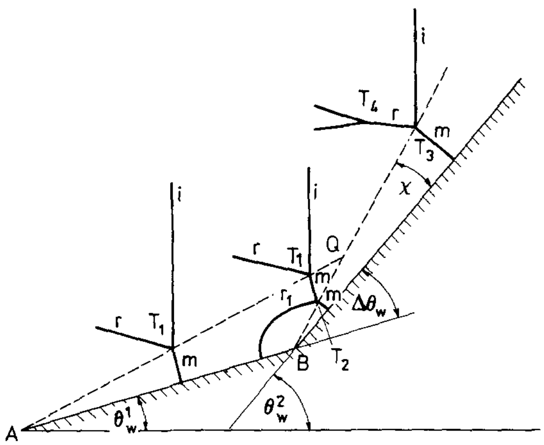

Fig. 5. A schematical illustration of the reflection process in domain 6 of Fig. 2

The previously mentioned assumption that the MR over the second surface approaches asymptotically the MR which would have been obtained by the same incident shock wave over a single wedge with an angle $\theta_w = \theta_w^2$ implies that the triple point trajectory angle χ of each of the Mach reflections shown in Figs. 3–5, over the second surface should approach asymptotically the value which would have been obtained with the same incident shock wave over a single wedge with an angle $\theta_w = \theta_w^2$. The verification of this assumption, which is basic to Ben-Dor et al. (1987) analysis, is the subject of the present experimental study.

2 Present study

In order to check the foregoing mentioned assumption, an experimental study was carried out in which the reflection process over a double wedge was recorded using high speed photography. Details of the high speed photography technique can be found in the papers by Dewey and Walker (1975) and Walker et al. (1982). The system consists of a giant ruby laser which can be pulsed in 50 μs intervals. The phenomenon was recorded with a rotating-mirror camera. Two experiments with very-nearly identical incident shock wave Mach numbers were conducted over each double wedge, with the first laser pulse of the second experiment delayed by 25 μs with respect to the first pulse of the first experiment so that is was possible to obtain multiple schlieren photographs of the reflection process over a double wedge in 25 μs intervals.

Once the two experiments were recorded, the trajectories of the triple points and the points where the feet of the Mach stems touch the reflecting surfaces were digitized. The digitized data were then evaluated to obtain specific details regarding the direction of propagation of the various triple points, their velocities and the velocities of the feet of the Mach stems along the reflecting surface.

In order to obtain the values of the triple point trajectory angle, χ, the velocity of the triple point, V_T, and the velocity of the foot of the Mach stem, V_G, to which the MR over the second surface was assumed to approach asymptotically, the reflection of an incident shock wave with an identical Mach number over a single wedge with an angle $\theta_w = \theta_w^2$ was recorded. The values of χ, V_T and V_G, which are constant over a single wedge, were measured directly from the photographs. The triple point trajectory angle, χ, was measured with a protractor to an accuracy of $\pm 0.5°$. The velocities of the triple point and the foot of the Mach stem were obtained by dividing their respective distances from the leading edge of the wedge by the time passed from the moment the incident shock wave collided with the leading edge of the wedge to the moment the photograph was taken. This time was calculated from

$$\Delta t = \frac{L_i}{V_i}$$

where L_i is the horizontal distance of the incident shock wave from the leading edge of the wedge and V_i is the velocity of the incident shock wave. The velocity of the incident shock wave was measured by two pressure transducers which were separated by 20 cm and were located just ahead of the test section of the shock tube.

3 Results and discussion

In the following, the experimental results of the foregoing described study are given for domains 3, 4 and 6 of Fig. 2. Note that only the asymptotic wave configuration is given over the second surface. The wave configurations which are obtained immediately after the reflection over the first surface interacts with the sudden change in the slope of the surface, are discussed in detail in Ben-Dor et al. (1987).

Domain 3

Experiments with two double wedges which are appropriate to domain 3 of Fig. 2 were performed. The geometry of the first double wedge was $\theta_w^1 = 40°$ and $\theta_w^2 = 25°$ ($\Delta\theta_w = -15°$). The incident shock wave Mach number was $M_i = 1.3$.

The triple point trajectory angle χ and its velocity in terms of Mach number, $M_T = V_T/a_0$, are shown in Fig. 6. The triple point trajectory angle over the first surface which has a slope of 40° is $\chi_1 = 1.2°$.

After the Mach stem of the MR over the first surface passes the leading edge of the second surface, the direction of propagation of the triple point, i.e. $\theta_w^1 + \chi_1$, decreases continuously and approaches the value appropriate to a MR with $M_i = 1.3$ over the single wedge with $\theta_w = 25°$, i.e. $\chi^s = 6.4°$. The velocity of the triple point, M_T, which was 1.74 over the first surface is also seen to be decreasing continuously and approaching asymptotically the value appropriate to a single wedge, i.e., $M_T^s = 1.523$. The velocity of the

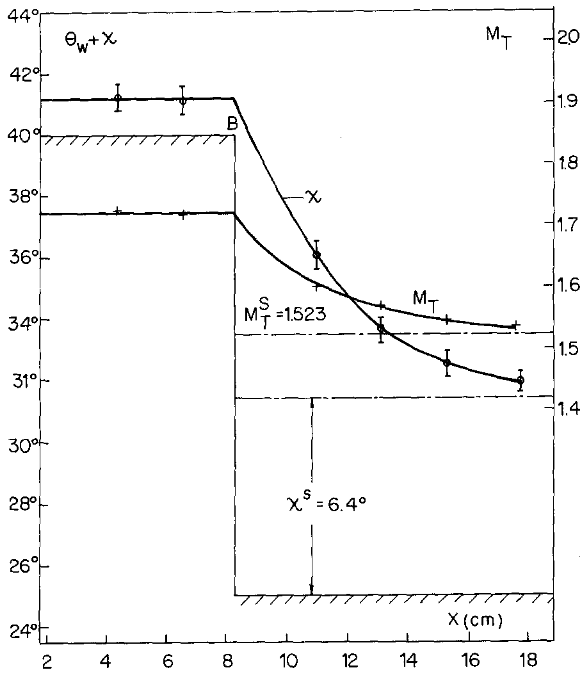

Fig. 6. The instantaneous triple point trajectory angle, χ, and triple point Mach number, M_T, of the Mach reflection of an incident shock wave with $M_i = 1.3$ over the second surface of a double wedge with $\theta_w^1 = 40°$ and $\theta_w^2 = 25°$ (domain 3 of Fig. 2)

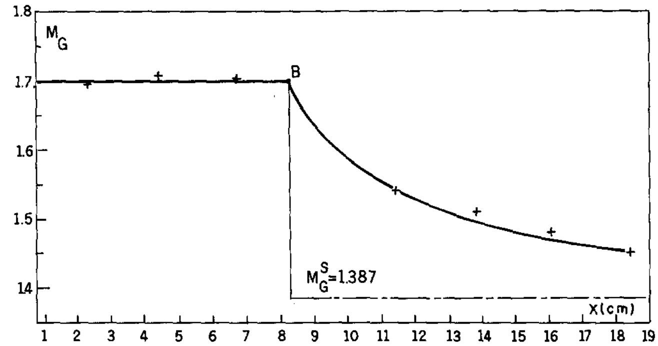

Fig. 7. The instantaneous Mach number, M_G, of the foot of the Mach stem of the Mach reflection of an incident shock wave with $M_i = 1.3$ over the second surface of a double wedge with $\theta_w^1 = 40°$ and $\theta_w^2 = 25°$ (domain 3 of Fig. 2)

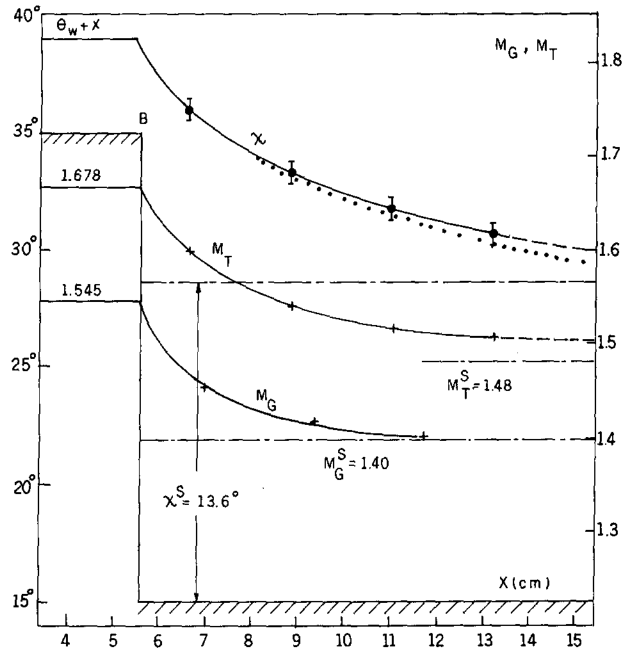

Fig. 8. The instantaneous triple point trajectory angle and Mach numbers of the triple point and the foot of the Mach stem of the Mach reflection of an incident shock wave with $M_i = 1.3$ over the second surface of a double wedge with $\theta_w^1 = 35°$ and $\theta_w^2 = 15°$ (domain 3 of Fig. 2)

foot of the Mach stem $M_G = V_G/a_0$, for this case is shown in Fig. 7. It is also seen to decrease continuously from its value of 1.7 over the first surface towards the value appropriate to a single wedge with $\theta_w = 25°$, i.e., $M_G^s = 1.387$.

At a distance of 18 cm from the leading edge of the double wedge the triple point trajectory angle χ and its velocity M_T have almost reached the values appropriate to a single wedge. However, M_G is still quite far from the asymptotic value it is assumed to reach at this distance (χ and M_T are about 1.5% and 1% larger than the asymptotic values, whereas M_G is still about 4.5% too large).

An additional experiment with a double wedge also appropriate to domain 3 of Fig. 2 is shown in Fig. 8. The geometry of the double wedge is $\theta_w^1 = 35°$, $\theta_w^2 = 15°$ ($\Delta\theta_w = -20°$) and the incident shock wave Mach number is again $M_i = 1.3$.

The experimental results again indicate that after the MR over the first surface passes the leading edge of the second surface the values of χ, M_T and M_G decreases towards the values appropriate to a single wedge with an angle $\theta_w = 15°$.

Unlike the previous case, here M_G is seen to reach its predicted asymptotic value at about $x = 12$ cm (x is measured from the leading edge of the double wedge) while M_T and χ are still about 1.5% and 4.9% larger than their assumed asymptotic values. Note that the $\pm$ 0.5° error bar in the measured value of χ could have resulted in a different curve which would still agree with all the measurements but would resemble a faster approach to the assumed asymptotic value. Such a curve is added to Fig. 8 in a dotted line. It results in a value which is only about 2.8% higher than the assumed limit.

Domain 4

The experimental results over a double wedge with $\theta_w^1 = 60°$ and $\theta_w^2 = 30°$ ($\Delta\theta_w = -30°$) and an incident shock wave with $M_i = 1.3$ are shown in Fig. 9. The reflection process over this double wedge starts with a RR over the first surface. When the reflection point of this RR reaches the leading edge of the second surface (point B), MR begins and a triple point forms.

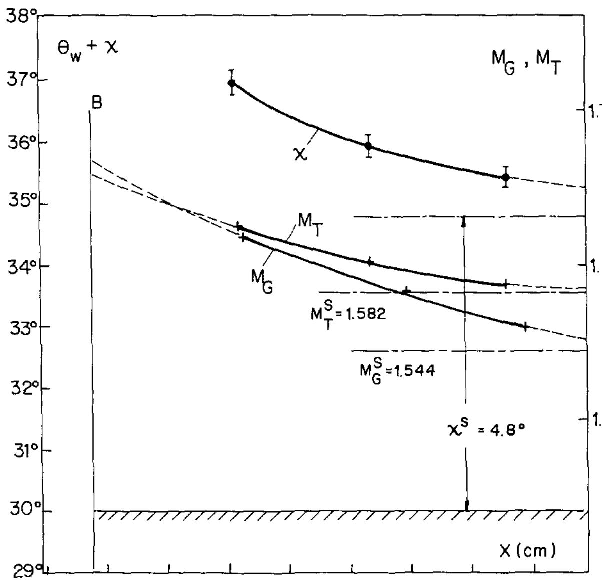

Fig. 9. The instantaneous triple point trajectory angle, and Mach numbers of the triple point and the foot of the Mach stem, of the Mach reflection of an incident shock wave with $M_i = 1.3$ over the second surface of a double wedge with $\theta_w^1 = 60°$ and $\theta_w^2 = 30°$ (domain 4 of Fig. 2)

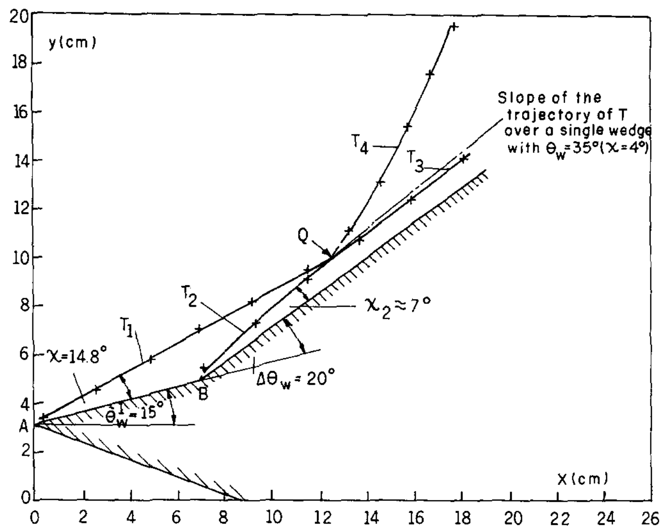

Fig. 10. The experimentally recorded trajectories of the four triple points T_1, T_2, T_3 and T_4 shown in Fig. 5, of the Mach reflection of an incident shock wave with $M_i = 1.3$ over the second surface of a double wedge with $\theta_w^2 = 15°$ and $\theta_w^2 = 35°$ (domain 6 of Fig. 2)

The experimental results in Fig. 9 again indicate that χ, M_T and M_G approach asymptotically their assumed limiting values. At about $x = 12$ cm (x is again measured from the leading edge of the double wedge), χ, M_G and M_T are about 1.3%, 0.4% and 0.25% larger than their respective asymptotic values. These small differences imply that the MR configuration has almost reached a configuration which would have been obtained by an incident shock wave with $M_i = 1.3$ over a single wedge with $\theta_w = 30°$.

Domain 6

Due to the complexity of the reflection process in this domain compared to those presented earlier for domains 3 and 4 the experimental results for this case are shown in a different way than those presented earlier.

Figure 10 shows the experimentally recorded trajectories of the four triple points T_1, T_2, T_3 and T_4 of Fig. 5. The first triple point T_1, is obtained when the incident shock wave with $M_i = 1.3$ reflects over the first surface which has an angle $\theta_w^1 = 15°$. The triple point trajectory angle of T_1 is $\chi_1 = 14.8°$. When the Mach stem of this MR collides with the leading edge, point B, of the second surface, which has an angle $\theta_w^2 = 35°$, it reflects over it as a secondary MR with triple point T_2. The experimental results indicate that unlike the trajectory of T_1 which is straight, the trajectory of T_2 is curved. This is probably due to the fact that the Mach stem (which serves as the incident shock wave in the secondary MR, see Fig. 5) is not a straight shock wave like the incident shock wave. Instead, it has a concave curvature. If, however, the trajectory of the T_2 is approximated by a straight line then it forms an angle, χ_2, of about 7° with the second surface. When the two triple points, T_1 and T_2, meet at point Q they interact to result in two new triple points, T_3 and T_4. T_3, the triple point of the MR of the incident shock wave over the second surface, is seen to approach a direction which is parallel to the trajectory which would have been obtained if an incident shock wave with $M_i = 1.3$ was reflected over a wedge with $\theta_w = 35°$, i.e., $\chi = 4°$. This direction is shown in Fig. 10 by a dash-dotted line. The trajectory of the triple point T_4, on the curved reflected shock wave, is also curved.

The evaluation of the velocity of T_3 resulted in $M_{T_3} = 1.659$. The velocity of an appropriate triple point over a single wedge would be $M_T = 1.673$. The difference, which is less than 1%, clearly suggests that the triple point T_3 has almost reached the asymptotic value it is assumed to reach. The velocities of T_1 and T_2, i.e., M_{T_1} and M_{T_2}, are 1.479 and 1.650, respectively. It should also be noted that the trajectory of T_4, which as mentioned earlier is curved, is seen to approach a straight line. The direction of this line with respect to the x-axis is 62.9°.

Finally, it is of interest to note that if one assumes that the Mach stem of the MR of the incident shock wave is straight and perpendicular to the wedge surface, then the location of point Q where the trajectories of the first two triple points, T_1 and T_2, intersect, can be calculated analytically using the following geometrical expression:

$$\overline{AQ} = L\frac{\sin(\Delta\theta_w + \chi_2)}{\sin(\Delta\theta_w + \chi_2 - \chi_1)} \tag{1}$$

where $\overline{AQ}$ is the distance from the leading edge of the double wedge, point A, to point Q and L is the length of the first surface, i.e., $L = \overline{AB}$.

Thus the location of Q in the (x, y)-plane is:

$$x_Q = \overline{AQ} \cos(\theta_w^1 + \chi_1) \quad (2\,a)$$

$$y_Q = \overline{AQ} \sin(\theta_w^1 + \chi_1) \quad (2\,b)$$

The analytical solution of the reflection at hand results in: $\chi_1 = 13.286°$ and $\chi_2 = 7.028$ (recall that the experimental results were 14.8° and 7°, respectively). Inserting these values into Eq. (1), together with $L = 72$ mm yields $\overline{AQ} = 140.2$ mm. Thus, from Eq. (2), one obtains $x_Q = 123.4$ mm and $y_Q = 66.4$ mm. The corresponding measured results as evaluated from the digitized data are 125 mm and 68 mm, respectively. The comparison between these results suggests that Eq. (1) and (2) could be used to predict quite accurately the location of point Q where T_1 and T_2 meet.

It should also be mentioned that experiments with $M_i = 1.3$ were repeated using the double wedge configuration at hand, i.e., $\theta_w^1 = 15°$, $\Delta\theta_w = 20°$ and $\theta_w^1 = 35°$, but with different values of L (L is the distance from the leading edge of the first surface, point A, to the leading edge of the second surface, point B). The experimentally obtained location of point Q agreed with that predicted by Eq. (1) and (2) to within 2.5% for all values of L in the range $15 < L < 72$ mm.

4 Conclusions

The assumption that the MR of the incident shock wave over the second surface of a double wedge approaches the MR which would have been obtained by the same incident shock wave reflecting over a single wedge with an angle θ_w equal to the slope of the second surface of a double wedge, i.e., θ_w^2, was investigated experimentally.

The experimental results clearly indicate that the assumption is correct. The triple point velocity (including its direction of propagation) and the velocity of the foot of the Mach stem of the MR are indeed found to approach the values appropriate to a reflection over a single wedge. This behavior can undoubtedly be attributed to the fact that shock wave stability is the governing mechanism of the phenomenon at hand.

It is also interesting to note that the experimental results presented in Figs. 6–10 indicate that the relaxation length (i.e., the distance travelled by the incident shock wave over the second surface until the asymptotic reflection is nearly obtained) is of the order of a few lengths of the first surface of the double wedge. If the relaxation length is defined as the distance at which the reflection has come to within 5% of the asymptotic values, then the relaxation lengths of the four cases presented in this study are about 1.25, 2, 3.5 and 1.5 times the length of the first surface of the appropriate wedge.

As mentioned earlier, an experimental study aimed at investigating the influence of the length of the first surface on the relaxation process revealed that such an influence, if it exists, is minimal.

The present study was limited to cases where the reflection of the incident shock wave over the first and second wedges were supposed to be either regular or single-Mach reflection. For stronger incident shock waves both complex and double-Mach reflections might be possible over the two surfaces. This would undoubtedly complicate the reflection processes and might also increase the relaxation lengths. However, it is hypothesized here, that the final reflection over the second surface of a double wedge will be that which would have been obtained over a single wedge with the same incident shock wave, no matter if it is a single, a complex or a double-Mach reflection.

It should also be mentioned that the fact that the shock configurations which are approached asymptotically in the cases of a reflection over a double wedge are similar to those which would have been obtained by the same incident shock wave over an appropriate single wedge, does not necessarily imply that the flow fields are also similar. This can most easily be justified if one recalls the paper by Ben-Dor and Glass (1978) where it was shown that different computer codes were capable of resulting in almost identical wave configurations which differed very much in their flow fields.

In summary, the present experimental study supports the shock polar analysis which was presented by Ben-Dor et al. (1987) for studying the reflection process over a double wedge.

Reference

Ben-Dor, G.; Glass, I. I. 1978: Nonstationary oblique shock wave reflections: actual isopycnics and numerical experiments. AIAA J. 16, 1146–1153

Ben-Dor, G.; Dewey, J. M.; Takayama, K. 1987: The reflection of a planar shock wave over a double wedge. J. Fluid Mech. 176, 483–520

Dewey, J. M.; Walker, D. K. 1975: A multiply pulsed double-pass laser schlieren system for recording the movement of shocks and particle tracers within a shock tube. J. Appl. Phys. 46, 3454–3458

Walker, D. K.; Scotten, L. N.; Dewey, J. M. 1982: Construction and evaluation of a simple and inexpensive rotating prism camera for recording movement of shocks and particle tracers within a shock tube. Proc. 15th int. cong. high speed photography and photonics (SPIE, vol. 348) pp. 125–130

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