WAVE PROPAGATION IN A MAGNETIC CYLINDER

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Abstract. The nature of oscillations in a magnetic cylinder embedded in a magnetic environment is investigated. It is shown that the standard slender flux tube analysis of a *kink* mode in a cylinder excludes the possibility of a second mode, which arises under photospheric conditions. Under coronal conditions, two widely separated classes of oscillation can be freely sustained, one on an *acoustic* time-scale and the other on an *Alfvénic* time-scale. The acoustic-type oscillations are always present, but the much shorter period, Alfvénic-type, oscillations arise only in *high* density (strictly, low Alfvén velocity) loops. An application to waves in fibrils is also given, and suggests (following Wentzel, 1979) that they are fast kink waves propagating in a density enhancement.

1. Introduction

In this paper we consider the propagation of magnetoacoustic waves in a magnetic cylinder embedded in a magnetic environment. Our analysis is a natural extension of the treatment in Edwin and Roberts (1982, Paper III; see Appendix for corrections) which considered the magnetic slab (see also Gordon and Hollweg, 1983). We compare and contrast the results for the two geometries. Our analysis for the cylinder complements and extends previous treatments (notably, Roberts and Webb, 1978, 1979; Wilson, 1979; Parker, 1979; Wentzel, 1979; Webb, 1980; Spruit, 1981a, b), which have provided only a partial view of the complex array of free modes supported by a magnetic flux tube.

The effects of gravity will be ignored, our emphasis being on an exploration of the role of magnetic structuring. Also, we will concentrate our attention on the two cases that most closely model the examples of solar interest, namely, the isolated flux tube of the photosphere, and the magnetic loop of the corona. In the case of the photosphere, gravitational effects are important, as stressed elsewhere (e.g., Roberts and Webb, 1978; Roberts, 1980, 1981c), and this should be borne in mind when considering our results for photospheric tubes. For coronal applications the neglect of stratification is less important.

In a slender flux tube (i.e., in a tube of radius much smaller than the wavelength of a disturbance), two characteristic speeds of propagation have been identified, namely the subsonic, sub-Alfvénic speed c_T of a symmetric pulsation (sausage mode), and the 'mean' Alfvénic speed c_k of a transversal (kink) mode. In terms of the sound speed c_0 and Alfvén speed v_A of a tube of gas density ρ_0 embedded in an environment of density ρ_e and Alfvén speed v_{Ae}, these two characteristic speeds may be written thus:

$$c_T = \frac{c_0 v_A}{(c_0^2 + v_A^2)^{1/2}}, \qquad c_k = \left(\frac{\rho_0 v_A^2 + \rho_e v_{Ae}^2}{\rho_0 + \rho_e}\right)^{1/2}. \tag{1}$$

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It may be noted that c_k is independent of the compressibility (i.e. sound speed) of both the cylinder and its environment and therefore is also the phase-speed of a long wavelength kink disturbance in an incompressible medium. Furthermore, c_k is the common speed of both the sausage and kink modes in the short wavelength, wide cylinder, limit for an incompressible medium.

Now it is well known that a uniform magnetic atmosphere can support two magnetoacoustic modes, the fast and slow waves. Similarly, in a structured atmosphere, such as an interface, two types of magnetoacoustic surface wave are possible. In a tube, the two classes of magnetoacoustic wave are manifest either as symmetric (sausage) oscillations or as asymmetric (kink) oscillations. These oscillations are characterised by the speeds c_T, c_k, and c_e (the external sound speed). In fact, the external sound speed c_e complements the speed c_T for the sausage mode. We show here that the kink mode is characterised by c_k and, curiously enough, by c_T (or its external equivalent $c_{Te} = c_e v_{Ae}/(c_e^2 + v_{Ae}^2)^{1/2}$, depending upon circumstances). Our treatment shows how to derive the behaviour of all of the free modes of a magnetic cylinder.

There are two other aspects of our investigation that require special mention. Firstly, as pointed out in Paper III for a magnetic slab, under coronal conditions a magnetic flux tube supports fast body oscillations. These modes are closely analogous to Love waves of seismology (see Love, 1911) and Pekeris waves of oceanography (see Pekeris, 1948); the dispersion relations for Love and Pekeris waves and coronal oscillations of a magnetic slab are exactly equivalent in the zero-β limit. Much the same kinds of oscillations occur in a magnetic cylinder, but now the governing dispersion relation in the zero-β limit is phase-shifted by $\pi/4$.

Magnetic Love and Pekeris waves, then, may occur in a coronal loop. They take the form of trapped modes of oscillation and occur only in those loops with locally reduced Alfvén speeds. For an essentially uniform coronal magnetic field fast oscillations of *high* density (*low* temperature) loops exist. The oscillations have periods on Alfvénic time-scales. Their possible relationships to observed coronal oscillations will be described elsewhere (Roberts *et al.*, 1983).

The theory of magnetic Love waves may also be applied to chromospheric fibrils. Such an application suggests that the waves observed propagating in fibrils (see Giovanelli, 1975) are magnetic Love waves freely propagating along density enhancements in the magnetic field. This is in accord with an earlier suggestion by Wentzel (1979).

A second aspect of cylindrical geometry that is of particular interest is the form that the dispersion relations take in the slender tube, long wavelength, limit. A knowledge of the approximate form of the dispersion relation provides a strong guide as to the possible form of a *soliton* that a flux tube might support. That a flux tube can support a soliton is known from the nonlinear analysis of the sausage mode in a magnetic slab (Roberts and Mangeney, 1982) or cylinder (Roberts, 1983), but the nonlinear behaviour of the kink mode is presently not known. A knowledge of the dispersion relation for a kink wave is a necessary preliminary to such an analysis.

2. The Dispersion Relations

We consider a uniform cylinder of magnetic field $B_0 \hat{\mathbf{z}}$ confined to a region of radius a, surrounded by a uniform magnetic field $B_e \hat{\mathbf{z}}$ (see Figure 1). The gas pressure and density within the cylinder are p_0 and ρ_0, outside p_e and ρ_e. Pressure balance implies

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}, \tag{2a}$$

where μ is the magnetic permeability, so the densities ρ_0 and ρ_e are related by

$$\rho_e/\rho_0 = \frac{2c_0^2 + \gamma v_A^2}{2c_e^2 + \gamma v_{Ae}^2}, \tag{2b}$$

where $c_0 = (\gamma p_0/\rho_0)^{1/2}$ and $v_A = B_0/(\mu\rho_0)^{1/2}$ are the sound and Alfvén speeds inside the cylinder, and $c_e = (\gamma p_e/\rho_e)^{1/2}$ and $v_{Ae} = B_e/(\mu\rho_e)^{1/2}$ are the corresponding speeds outside. (γ is the ratio of specific heats.)

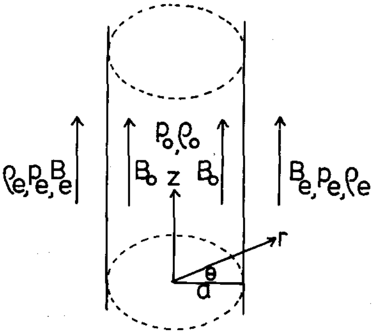

Fig. 1. The equilibrium configuration of a magnetic cylinder.

Linear perturbations about this equilibrium lead to the two equations (see, for example, Roberts, 1981a, b)

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial t^2} - (c_0^2 + v_A^2)\nabla^2\right)\Delta + c_0^2 v_A^2 \frac{\partial^2}{\partial z^2}\nabla^2 \Delta = 0, \tag{3a}$$

$$\left(\frac{\partial^2}{\partial t^2} - v_A^2 \frac{\partial^2}{\partial z^2}\right)\Gamma = 0, \tag{3b}$$

where ∇^2 is the Laplacian operator in cylindrical coordinates (r, θ, z), viz.,

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2},$$

and

$$\Delta \equiv \operatorname{div}\mathbf{v}, \qquad \Gamma = \hat{\mathbf{z}}\cdot\operatorname{curl}\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta},$$

for velocity $\mathbf{v} = (v_r, v_\theta, v_z)$. A corresponding pair of equations holds in the exterior of the cylinder.

If we now write

$$\Delta = R(r)\exp i(\omega t + n\theta + kz)\,, \tag{4}$$

then Equations (3a, b) imply that $R(r)$ satisfies the Bessel equation

$$\frac{\mathrm{d}^2R}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}R}{\mathrm{d}r} - \left(m_0^2 + \frac{n^2}{r^2}\right)R = 0\,, \tag{5}$$

where

$$m_0^2 = \frac{(k^2c_0^2 - \omega^2)(k^2v_\mathrm{A}^2 - \omega^2)}{(c_0^2 + v_\mathrm{A}^2)(k^2c_T^2 - \omega^2)}\,.$$

For a solution bounded on the axis ($r = 0$) of the cylinder we take

$$R(r) = A_0\begin{cases} I_n(m_0 r)\,, & m_0^2 > 0 \\ J_n(n_0 r)\,, & n_0^2 = -m_0^2 > 0 \end{cases} \quad (r < a)\,, \tag{6}$$

where A_0 is a constant and I_n, J_n are Bessel functions (see Abramowitz and Stegun, 1967) of order n.

In the external region, supposing that there is no propagation of energy away from, or towards, the cylinder $r = a$, we take

$$R(r) = A_1K_n(m_e r)\,, \qquad r > a\,, \tag{7}$$

where A_1 is a constant and m_e, given by

$$m_e^2 = \frac{(k^2c_e^2 - \omega^2)(k^2v_{\mathrm{A}e}^2 - \omega^2)}{(c_e^2 + v_{\mathrm{A}e}^2)(k^2c_{Te}^2 - \omega^2)}\,, \qquad c_{Te}^2 = \frac{c_e^2v_{\mathrm{A}e}^2}{c_e^2 + v_{\mathrm{A}e}^2}\,,$$

is taken to be positive.

Continuity of the radial velocity component v_r and the total (gas plus magnetic) pressure across the cylinder boundary $r = a$ then yields the required dispersion relations:

$$\rho_0(k^2v_\mathrm{A}^2 - \omega^2)m_e\frac{K_n'(m_e a)}{K_n(m_e a)} = \rho_e(k^2v_{\mathrm{A}e}^2 - \omega^2)m_0\frac{I_n'(m_0 a)}{I_n(m_0 a)} \tag{8a}$$

for *surface* waves ($m_0^2 > 0$), and

$$\rho_0(k^2v_\mathrm{A}^2 - \omega^2)m_e\frac{K_n'(m_e a)}{K_n(m_e a)} = \rho_e(k^2v_{\mathrm{A}e}^2 - \omega^2)n_0\frac{J_n'(n_0 a)}{J_n(n_0 a)} \tag{8b}$$

for *body* waves ($m_0^2 = -n_0^2 < 0$). (The dash denotes the derivative of a Bessel function: $K_n'(m_e a) \equiv (\mathrm{d}/\mathrm{d}x)K_n(x)$ evaluated at $x = m_e a$, etc.)

We will confine our attention to the cylindrically symmetric (sausage or pulsational) mode given by $n = 0$, and the asymmetric (kink or taut-wire) mode given by $n = 1$.

Dispersion relations (8a) and (8b) have been obtained previously by Wilson (1980) and Spruit (1982); see also Meerson *et al.* (1978). However, as mentioned in the introduction, their analyses do not provide a complete guide to the complex array of modes given by (8a, b).

3. The Structure of Waves in a Cylinder

To analyse the complex array of modes given by (8a) and (8b) we will confine our attention to four cases, chosen for their value in illustrating configurations of solar interest or for making clear a mathematical point. We begin by considering briefly the relatively simple case of an incompressible medium, turning thereafter to the photospheric tube (Section 3.2), the coronal loop (Section 3.3), and the chromospheric fibril (Section 3.4).

3.1. Incompressible modes

In the incompressible limit ($c_0^2 \to \infty$, $c_e^2 \to \infty$), m_0 and m_e become simply $|k|$, and so (8a) gives

$$\omega^2 = k^2\left(v_{Ae}^2 - \frac{\rho_0}{\rho_e}\, v_A^2 \phi_n\right)\Bigg/\left(1 - \frac{\rho_0}{\rho_e}\, \phi_n\right), \tag{9}$$

where $\phi_n = I_n(a|k|)K_n'(a|k|)/(I_n'(a|k|)K_n(a|k|))$. Thus, the sausage ($n = 0$) and kink ($n = 1$) modes are given explicitly. Equation (9) has been discussed by Uberoi and Somasundaram (1980).

The form of the incompressible dispersion relation is sketched in Figure 2. It is interesting to note that the phase-speed for the kink mode is *not* monotonic as a function of wavenumber k (here taken positive), but possesses a maximum (minimum) if $v_A > v_{Ae}$ ($v_A < v_{Ae}$); the sausage mode is monotonically decreasing (increasing) if $v_A > v_{Ae}$ ($v_A < v_{Ae}$). This feature of a maximum or minimum in the phase-speed of the kink wave is absent in the slab case (see Paper III, Figure 2), and so is evidently a reflection of the geometry of the magnetic field.

In the slender tube limit ($|k|\, a \ll 1$), Equation (9) yields a phase-speed $c = \omega/k$ given approximately by

$$c = \begin{cases} v_A\left[1 - \dfrac{\rho_e}{\rho_0}\left(1 - \dfrac{v_{Ae}^2}{v_A^2}\right)\dfrac{k^2a^2}{4}\, K_0(|k|\, a)\right], & \text{sausage mode} \\[2ex] c_k\left[1 - \dfrac{\rho_e\rho_0}{2(\rho_e + \rho_0)}\, \dfrac{(v_{Ae}^2 - v_A^2)k^2a^2}{(\rho_0 v_A^2 + \rho_e v_{Ae}^2)}\, K_0(|k|\, a)\right], & \text{kink mode.} \end{cases} \tag{10a, b}$$

The interest in these relations and their compressible counterparts will be discussed in the next section.

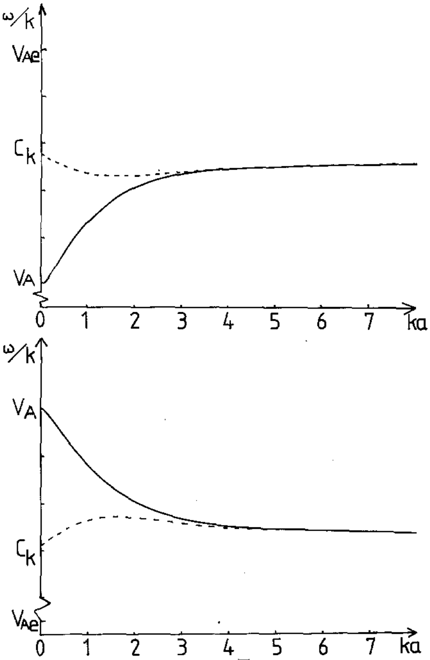

Fig. 2. The dispersion relation in the incompressible medium, giving the phase speed ω/k as a function of dimensionless wavenumber ka for the cases $v_{Ae} > v_A$ and $v_{Ae} < v_A$. (——: sausage mode; - - - -: kink mode.)

3.2. Photospheric Tubes

Figure 3 shows the behaviour of the sausage ($n = 0$) and kink ($n = 1$) modes of oscillation under conditions expected in photospheric flux tubes. (Recall, however, that the effects of stratification have been ignored.) There are several aspects of these curves which deserve comment.

A comparison of the curves for the sausage mode in a cylinder with those found previously for a magnetic slab (cf. Paper III, Figure 3) reveals that the sausage modes have much the same behaviour in the two geometries and, in particular, the speeds c_T and c_e are again significant. This is to be expected on physical grounds. On the other hand, a comparison of the behaviour of the kink modes in the two geometries (slab and cylinder) reveals two substantial changes, one in the behaviour of the fast mode and the other in the slow mode. That there are changes in not unexpected on physical grounds since, as Parker (1979) has pointed out, a cylinder oscillating transversely displaces less of its surroundings than its counterpart in the magnetic slab. The form of the changes requires detailed examination, to which we now turn.

Consider the kink mode with phase-speed close to c_k. We may develop (from (8a)) an expression for the dispersive correction to the slender tube result that $c = c_k$:

$$c = c_k\left\{1 - \frac{1}{2}\,\frac{\rho_0\rho_e(v_{Ae}^2 - v_A^2)}{(\rho_e + \rho_0)^2 c_k^2}\,\lambda^2(ka)^2 K_0(\lambda|k|\,a)\right\}, \qquad |k|\,a \ll 1\,, \qquad (11)$$

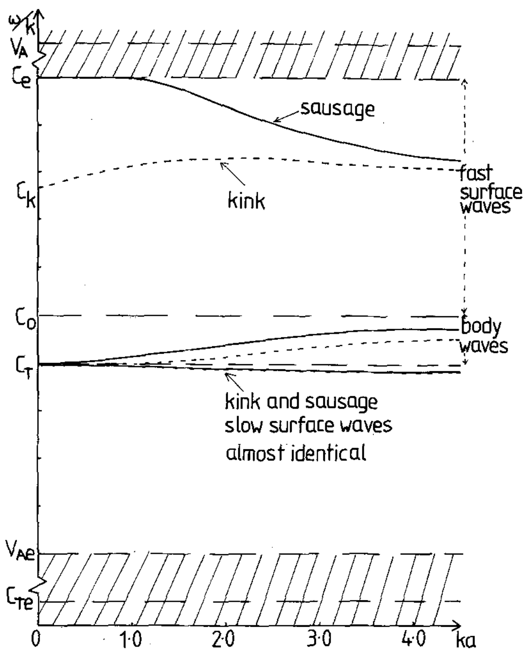

Fig. 3. The phase-speed of modes under photospheric conditions (i.e., $v_A > c_e > c_k > c_0 > v_{Ae}$). We have taken $v_A = 2.0c_0$, $c_e = 1.5c_0$, and $v_{Ae} = 0.5c_0$. The hatching denotes regions from which free modes (real ω and k) are excluded. (Only two of the infinitely many slow body waves are shown.)

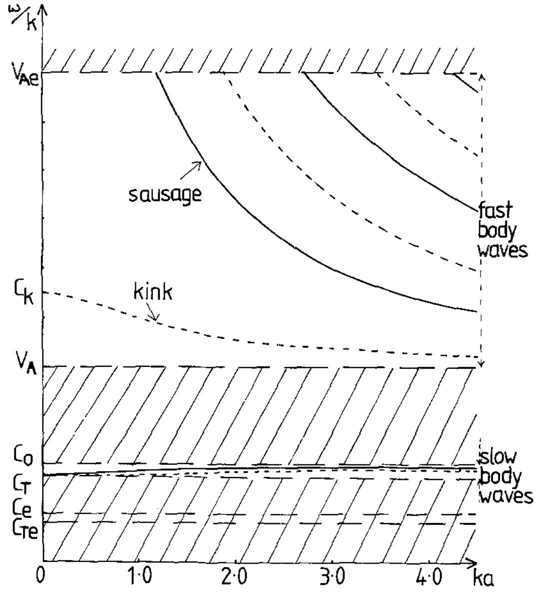

Fig. 4. The phase-speeds of modes under coronal conditions (i.e., v_{Ae}, $v_A > c_0$, c_e). We have taken $v_{Ae} = 5c_0$, $c_e = 0.5c_0$, and $v_A = 2c_0$. If $v_A > v_{Ae}$, the fast body waves are absent. (Only two of the infinitely many slow body waves are shown.) A similar diagram results for the case of a high density loop in a uniform magnetic field ($\rho_0 \gg \rho_e$, $B_0 = B_e$).

where

$$\lambda = \frac{(c_e^2 - c_k^2)^{1/2}(v_{Ae}^2 - c_k^2)^{1/2}}{(c_e^2 + v_{Ae}^2)^{1/2}(c_{Te}^2 - c_k^2)^{1/2}} > 0 \, .$$

This result is valid for $c_k < c_e$. (In the incompressible limit (11) reduces to (10b).)

It is interesting to note that this approximate dispersion relation is of the general form discussed recently by Roberts (1983). He considered the sausage mode in a cylinder and determined its evolution equation for weakly nonlinear disturbances. This evolution equation is akin to the Benjamin–Ono equation, which describes weakly nonlinear sausage waves in a magnetic slab (Roberts and Mangeney, 1982). The similarity between (11) and its counterpart for the sausage mode suggests that the kink wave may propagate nonlinearly as a solitary wave, at least when $v_{Ae} > v_A$. However, no investigation of the nonlinear kink wave is presently available.

We turn now to the slow kink wave indicated in Figure 3. The existence of *two* kink modes in a cylinder – aside from the body waves – is suggested by the order of Equations (3), and borne out by the analysis for a slab (Paper III, Figure 3) and a single interface. The slab geometry yielded a slow kink mode with phase-speed close to c_{T_e} in the slender tube limit and a fast kink mode with phase-speed close to v_{Ae}. The single interface dispersion relation yields two surface waves under appropriate circumstances (Roberts, 1981a). Since the analysis of the surface kink modes in a *wide* cylinder is equivalent to that for the single interface, this again indicates the existence of *two* kink modes.

We are thus lead to conclude that a magnetic cylinder has *two* kink (surface) modes, a fast and a slow mode. Under photospheric conditions the fast mode in a cylinder (see Equation (11)) has phase-speed close to c_k. The slow mode has phase-speed close to c_T (instead of v_{Ae} or c_{Te}, as found for a slab). Thus, in a cylinder c_T arises in both the sausage and the kink modes, thereby playing a dual role.

Now the available analyses for a slender flux tube in the kink mode of a cylinder (see Parker, 1979; Wilson, 1980; Spruit, 1982) all yield the fast kink c_k-mode, but *none* yields the slow kink (or, indeed, any speed other than c_k). To see how the slow kink may have a phase-speed close to c_T, consider the dispersion relation (8a) for $n = 1$. For a slender flux tube there are two possibilities to consider: (a) $m_0 a \to 0$ as $ka \to 0$, or (b) $m_0 a \to \nu$ as $ka \to 0$, for some finite ν. Case (a) yields the mode c_k. Case (b), under the supposition that there is a solution with $\omega \simeq kc_T$ for $|k| a \ll 1$, yields a solution of the form

$$\omega^2 = k^2 c_T^2 \left\{ 1 - \frac{c_T^2 (ka)^2}{\nu^2 (c_0^2 + v_A^2)} \right\}, \tag{12}$$

provided there exists $\nu > 0$ such that

$$\nu \frac{I_0(\nu)}{I_1(\nu)} - 1 = \frac{\rho_0 (v_A^2 - c_T^2)}{\rho_e (c_T^2 - v_{Ae}^2)} .$$

When $v_{Ae} = 0$, this latter equation requires that

$$c_0^2 + \tfrac{1}{2}\gamma v_A^2 < v_A^2 \left(\frac{c_e}{c_0} \right)^2 .$$

It is evident that this inequality is satisfied in Figure 3. (Notice, too, that it is not satisfied in the incompressible limit, $\gamma \to \infty$, which is in keeping with the discussion in Section 3.1 which showed that only one kink mode existed.)

3.3. Coronal loops

Figure 4 illustrates the behaviour of waves under coronal conditions, i.e. v_{Ae}, $v_A > c_e$, c_0. In such circumstances, just as for a magnetic slab (Paper III), there are no longer any surface modes but two classes of body waves can occur.* Of particular interest are the fast modes which arise only if $v_{Ae} > v_A$. Thus, for a coronal atmosphere with $B_0 = B_e$

* It should be noted that the distinction we draw between body and surface modes pertains only to their spatial structure *within* the loop. Both classes of wave are confined to the neighbourhood of the loop. Consequently, the term 'surface wave' is sometimes used to encompass both these types (see Wentzel, 1979).

and $p_0 = p_e$, fast body waves occur only if $\rho_0 > \rho_e$. This suggests that *dense loops* in the corona are able to sustain free vibrations with characteristic periods on an Alfvénic time-scale (since $v_A < (\omega/k) < v_{Ae}$).

The form of these fast modes is most conveniently exhibited for a cylinder of large radius (i.e., $ka \gg 1$). Setting $c_e = c_0 = 0$, we obtain (from (8b))

$$\tan\left(n_0 a - \frac{\pi}{4}\right) = \begin{cases} -\dfrac{n_0}{m_e}, & \text{sausage mode,} \\ \\ \dfrac{m_e}{n_0}, & \text{kink mode,} \end{cases} \tag{13}$$

which are analogous to Pekeris's and Love's equations, respectively, save for a phase-shift by $\pi/4$. The analogy with Love waves of seismology (see Love, 1911; Ewing *et al.*, 1957) has been made clear in Paper III. The analogy with the Pekeris (1948) mode of oceanography is described in Roberts *et al.* (1983).

It is also apparent from Figure 4 that, with the exception of the c_k-mode, the fast body waves have a low wavenumber cut-off. Since this cut-off occurs for $|k|\, a$ of order unity, only those wavelengths $(2\pi/k)$ that are *shorter* than the diameter of a coronal tube can propagate freely. For a typical loop radius of, say, one-tenth of a loop length, only high harmonics can arise – with the exception of the c_k-kink mode. The kink mode of oscillation of a high density coronal loop has a typical period* of

$$\tau = \frac{2L}{c_k}, \tag{14}$$

where $L = \pi/k$ and the fundamental harmonic in z has been selected. The sausage mode, by contrast, has a much shorter period of, say, one-tenth that of the kink mode, since it propagates only for $ka > 1.2$ and $a = \frac{1}{10}L$. The sausage and kink fast modes exist as free oscillations only in high density loops (if $B_e = B_0$). The slow modes, however, arise in both high and low density cylinders.

Finally, we note the dispersive correction to the fast kink mode for the coronal case (see (11) for the photospheric case). With $c_e = c_0 = 0$, we have

$$c = c_k \left\{ 1 - \frac{1}{2} \frac{\rho_0 \rho_e}{(\rho_0 + \rho_e)^2} \left(\frac{v_{Ae}^2 - v_A^2}{c_k^2} \right) \lambda^2 k^2 a^2 K_0(\lambda |k|\, a) \right\}, \tag{15}$$

where $\lambda = (v_{Ae}^2 - c_k^2)^{1/2}/v_{Ae} > 0$. Relation (15) is valid for $v_{Ae} > v_A$. If $v_{Ae} < v_A$, then we no longer have a free mode of oscillation.

* Formally, (14) may be compared with the characteristic period $2L/$(mean Alfvén speed) discussed by Ionson (1982); here the mean speed is c_k. However, Ionson is concerned with incompressible shear waves whereas we have discussed magnetoacoustic modes.

3.4. Waves in fibrils

Giovanelli (1975) has described in some observational detail the propagation of waves in chromospheric fibrils. The waves have periods of about 170 s, speeds of roughly 70 km s^{-1} and amplitudes of 5 km s^{-1}; they propagate along the fibrils. Giovanelli points out that the disturbances, though transverse to the fibrils, involve intensity fluctuations and so are *not* Alfvén waves. Furthermore, the wavefronts appear to cover only the width of a single fibril, rather than spreading outward in a coherent fashion (as seen, for example, in penumbral waves).

Wentzel (1979) has suggested that fibril waves are likely to be flux tube modes, the tube having an internal Alfvén speed that is lower than that in the surroundings (see also Spruit, 1983; Spruit and Roberts, 1983). We concur with Wentzel's suggestion entirely. In our terminology, for a low-β plasma, we would regard fibril waves as an example of the *fast kink* mode of Figure 4: a *propagating magnetic Love wave*. Fibril waves, then, arise only in regions with $v_{Ae} > v_A$ ($> c_e, c_0$), typically corresponding to regions of density enhancement. Giovanelli's remark that the wavefronts of fibril disturbances are confined to the fibril is then seen as entirely consistent with the requirement (imposed in the present paper) that the mode be laterally evanescent outside the cylinder.

4. Concluding Remarks

The discussion in Section 3 has emphasised the differences between cylinder and slab geometries as far as the dispersion diagrams for the kink modes are concerned. In particular, we have demonstrated the existence of a slow kink mode, under photospheric conditions, that is present in the long wavelength limit ($ka \ll 1$), but not given by available slender flux tube arguments.

The behaviour of several of the waves, under photospheric and coronal conditions, was examined in the small ka limit, showing the nature of the dispersive correction term to the limiting ($ka \to 0$) speed. The approximate dispersion relations so obtained indicate the possibility of solitary waves on flux tubes in which finite amplitude effects are allowed for. It is already known (Roberts, 1983) that the slow sausage mode can become a solitary wave when finite amplitude effects are included, and our present results for the fast kink mode suggests a similar possibility there (a verification of this, however, must await a detailed nonlinear analysis of the kink mode).

Under coronal conditions, we have shown that a tube of density/temperature inhomogeneity in an otherwise uniform magnetic field may oscillate with two distinct periodicities, a slow oscillation (with acoustic periods) and a fast oscillation (with Alfvénic periods). The fast kink oscillations are mathematically analogous to seismic Love waves, and so are conveniently referred to as *magnetic Love waves*. The fast sausage modes are analogous to Pekeris waves of oceanography.

For the *kink* mode in a coronal loop, we may expect a magnetic Love wave to have a characteristic period of $2L/c_k$, for a loop of length L. Such an oscillation can arise only if the Alfvén speed in the loop is lower than that in the environment. For example, for a coronal loop of length $L = 5 \times 10^4$ km and number density of 10^{15} m^{-3}, we obtain

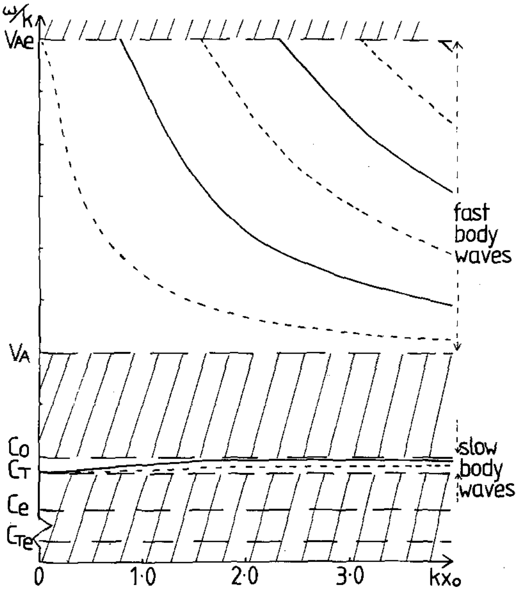

Fig. 5. Slab dispersion curves under conditions representative of the corona (illustrated for $v_{Ae} = 5c_0$, $c_e = 0.5c_0$, and $v_A = 2c_0$).

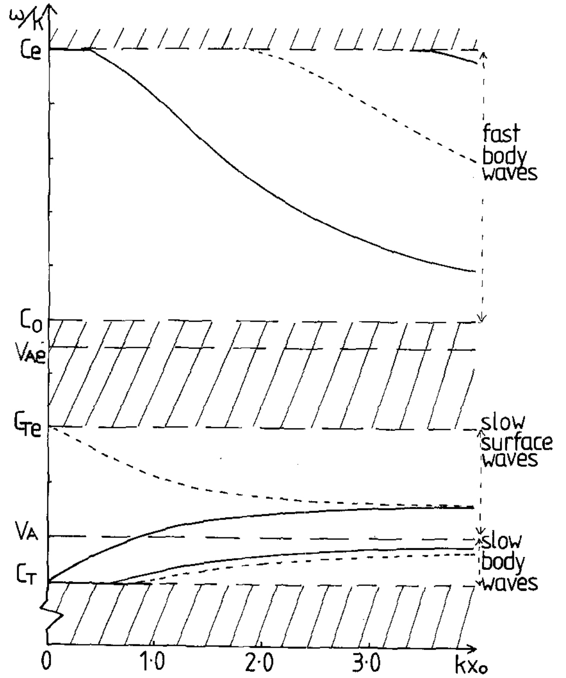

Fig. 6. Slab dispersion curves for $v_{Ae} = 0.95c_0$, $c_e = 1.5c_0$, and $v_A = 0.6c_0$.

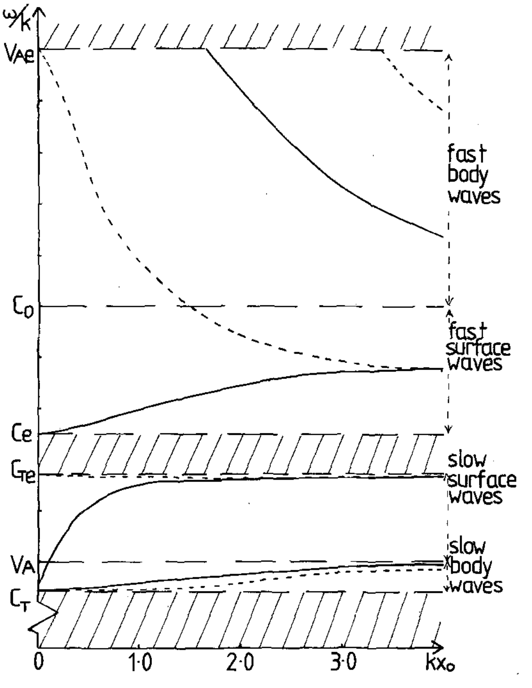

Fig. 7. Slab dispersion curves for $v_{Ae} = 1.5c_0$, $c_e = 0.75c_0$, and $v_A = 0.5c_0$.

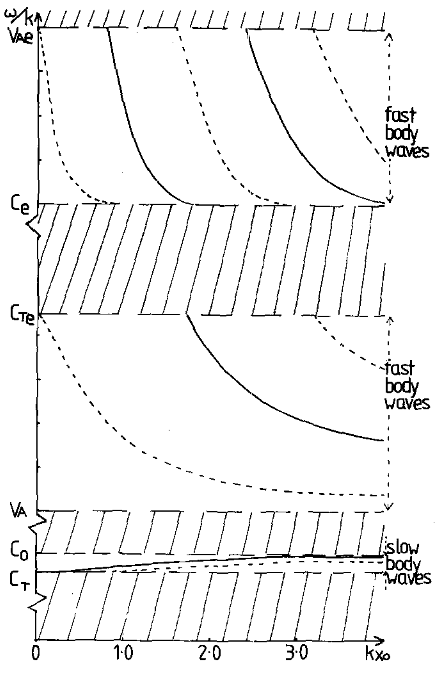

Fig. 8. Slab dispersion curves for $v_{Ae} = 5c_0$, $c_e = 4c_0$, and $v_A = 2c_0$.

a period of 10.2 s in a magnetic field $B_e = B_0 = 100$ G. The kink mode arises for all wavelengths.

By contrast, the sausage mode possesses a long wavelength cut-off, and can only propagate if ka is greater than about unity (see Figure 4). Thus, with the wavelength determined by the length of the loop, we see that the sausage mode of oscillation of a magnetic Pekeris wave can arise only in the form of multiple pulsations of the loop: the loop must be alternately compressed and expanded with many nodes along its length for this free oscillation to arise. For a loop of radius one-tenth of L, say, this indicates a pulsation with ten nodes along its length. The period of the mode is correspondingly reduced by the same factor. In the above illustration, the period of oscillation of the sausage mode would thus be of the order of a second. A more detailed discussion of coronal oscillations, and their potential use as a diagnostic tool for determining coronal magnetic field strengths, is given in Roberts *et al.* (1983).

Finally, in agreement with Wentzel (1979), we note that fibril waves (Giovanelli, 1975) are likely to be fast kink modes propagating along a density enhancement. Figure 4 is again appropriate. The fibril wave, then, is an example of a freely propagating magnetic Love wave, and in a magnetically controlled atmosphere can only occur where there are density enhancements (or, more specifically, regions of low Alfvén speed).

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Appendix: Waves in a Magnetic Slab

Our earlier investigation of waves in a magnetic slab (Edwin and Roberts, 1982, Paper III) unfortunately contained an error in the computational results for Figures 4 to 7. In brief, the pairing of sausage modes and the pairing of kink modes for fast body waves was spurious: single curves only is the correct behaviour. For the sake of clarity, we here present (in Figures 5 to 8) the corrected curves for Figures 4 to 7 of Paper III. No other changes in the text are necessary. The notation is as used in Paper III and repeated in the present paper.

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